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KURTOSIS OF THE DENSITY FIELD

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Abstract

We discuss the non–linear growth of the excess kurtosis parameter S_4 of the smoothed density fluctuation field δ in an Einstein–de Sitter universe. We assume Gaussian primordial density fluctuations with scale–free power spectrum and analyze the dependence of S_4 on primordial spectral index n, after smoothing with a Gaussian filter. As already known for the skewness ratio S_3, the kurtosis parameter is a *decreasing* function of n, both in exact perturbative theory and in the Zel'dovich approximation. The parameter S_4 provides a powerful statistics to test different cosmological scenarios.

We assume that present–day structures in the universe formed by gravitational instability from Gaussian (G) fluctuations δ in a pressureless fluid with matter density $\rho = \rho_b[1+\delta]$, where ρ_b is the background mean density. We analyse the dependence of the induced–by–gravity excess kurtosis of the density field, smoothed with a G filter, namely the parameter $S_4 \equiv [\langle\delta^4\rangle - 3\langle\delta^2\rangle^2]/\langle\delta^2\rangle^3$, on an initial (scale–free) power spectrum $P(k) \propto k^n$. To do this, we take advantage of the exact perturbative technique ([2], [3]) and the Zel'dovich approximation (ZA) [4]. The kurtosis describes important features such as sharpness or stretchiness of the mass distribution and the extent of its rare–event tail. Moreover, it is possibly related to the *initial* sign of the skewness – as predicted in some non–Gaussian models – which is important for the final galaxy clustering pattern (see e.g. [6]).

According to the perturbative theory ([2], [3]), the lowest order non–zero connected contribution to the induced–by–gravity kurtosis parameter S_4 is

$$\langle\delta^{(1)2}\rangle^3\, S_4 \;=\; 6\,\langle\,\delta^{(1)2}\,\delta^{(2)2}\,\rangle_c \;+\; 4\,\langle\,\delta^{(1)3}\,\delta^{(3)}\,\rangle_c\,, \tag{1}$$

where $\delta^{(n)}$ is the n–th order approximation to the density fluctuation. We compute the parameter S_4 of an initial G density field in a flat universe with no cosmological constant. The integral expression of the kurtosis ratio of the smoothed density field δ_R in the exact perturbative case results

$$S_4 \;=\; \frac{24}{\sigma_R^6}\int \frac{d\mathbf{k}_1\,d\mathbf{k}_2\,d\mathbf{k}_3}{(2\pi)^9}\;\widetilde{W}_R(k_1)\,\widetilde{W}_R(k_2)\,\widetilde{W}_R(k_3)\,\widetilde{W}_R(|\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3|)\,\times$$

$$P(k_1)\,P(k_2)\,\Big[P(k_3)\,J^{(3)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \;+\; 2\,J^{(2)}(-\mathbf{k}_2,\mathbf{k}_2+\mathbf{k}_3)\,J^{(2)}(\mathbf{k}_1,\mathbf{k}_2+\mathbf{k}_3)\,P(|\mathbf{k}_2+\mathbf{k}_3|)\Big]\,, \tag{2}$$

where $\widetilde{W}_R(k)$ is the Fourier transfom of the G window function and σ_R^2 is the mass variance on (filtering) scale R. The kernels $J^{(n)}$ are symmetric homogeneous (with degree 0) functions of the wavevectors $\mathbf{k}_1, \ldots, \mathbf{k}_n$, describing the effects of non-linear collapse (tidal and shear forces). (A discussion of the properties of the kernels $J^{(n)}$ and their Feynman diagrammatic representation may be found in [3].) The expression of S_4 in the ZA is obtained substituting the kernels $J^{(n)}$ with the corresponding $J_{ZA}^{(n)}$ in Eq.(2), which are given in [4]. The angular integrations in Eq.(2) may be analytically performed in the unsmoothed-case. One obtains $S_4 = 60,712/1,323 \approx 45.9$ in the perturbative case [2] and $S_4 = 88/3 \approx 29.3$ in the ZA. These unsmoothed-case results are independent of the primordial spectral index, as is the skewness ratio S_3 [5]. We calculate S_4 by an Adaptive Multidimensional Monte Carlo Integration subroutine, for scale-free power spectra $P(k) \propto k^n$ with n in the range $-3 \leq n \leq 1$. Due to the assumed primordial scale-invariance, S_4 only depends on the primordial spectral index n, and not on the scale R. In Fig.1, we plot the kurtosis ratio S_4 of the G-smoothed density field versus the spectral index n, for both the perturbative approximation and ZA. It clearly appears that S_4 *strongly* depends on the primordial spectral index n. In particular the kurtosis parameter is a *decreasing* function of n. Our results can be used to constrain both the probability distribution and the power spectrum of primordial density fluctuations and thus help to select the most reliable mechanism for the generation of large-scale structures. More details can be found in [1].

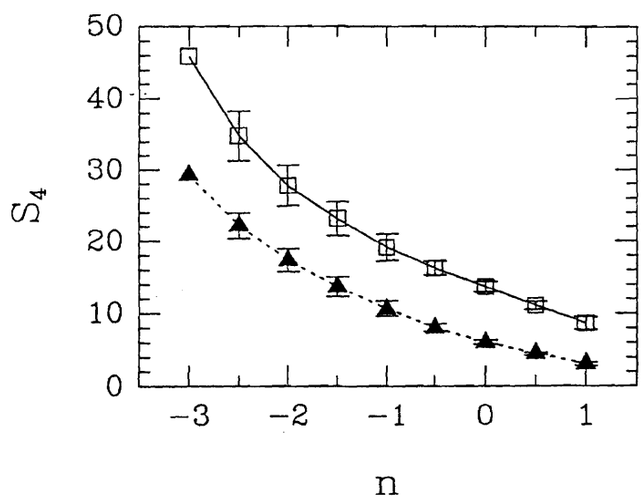

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Figure 1. The kurtosis ratio S_4 of the density field for power-law spectra $P(k) \propto k^n$ and Gaussian filter versus the primordial spectral index n, for both the perturbative (squares and solid line) and Zel'dovich approximations (triangles and dotted line). Note that the values at $n = -3$ correspond to the unsmoothed cases. Error bars refer to the uncertainty estimated from the Monte Carlo Integration.

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