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ON THE U(1)-PROBLEM, THE STRONG CP-PROBLEM AND THE η AND η' MASSES

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Abstract

A disscussion of the *U(1)-problem* and its solution is presented in the context of Anisotropic ChromoDynamics (ACD), a theory which describes finite energy quantum fluctuations around a non perturbative vacuum. This solution has not the difficulty of the strong CP-problem which is cause of trouble for other proposed solutions. In addition a calculation of the masses and mixing angle of η and η' mesons is carried out in good agreement with experimental values.

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The values of the masses of the lightest mesons have something very peculiar when looked at from the point of view of the quark model. In fact, the $I = 0$ vector meson $\omega(783)$ and the $I = 1$ $\rho(770)$ have slightly different masses, while, as concerns the pseudoscalars, the mass of the $I = 0$ meson $\eta(547)$ is very different from that of the pion ($I = 1$). In addition the other $I = 0$ vector meson, the $\phi(1020)$, is known to be a quasi-pure $|s\bar{s}\rangle$ state, implying an almost ideal mixing angle between the $SU(3)_f$ octet (the ω) and singlet components (the ϕ); while for the pseudoscalar η and η', the mixing angle is far from ideal ($\theta_p \simeq -20°$). It is widely accepted that the pseudoscalar mesons play a crucial rôle in the mechanism of spontaneous breaking of the chiral symmetry. In particular the observed small values of the current masses of the u and d quarks (suggested by the validity of PCAC hypothesis) implies the existence of three $I = 1$ Goldstone bosons (massless in the limit of $m_q \to 0$) that are readily identified with the π mesons, whose masses on a hadronic scale are very small. In the chiral limit ($m_{u,d} \to 0$) the structure of QCD Lagrangian is very simple and shows the presence of a $SU(2) \times U(1)$ symmetry which is spontaneously broken (otherwise the hadron spectrum would posses parity doublets). The application of the Goldstone theorem to that spontaneous symmetry breaking implies the existence of four Goldstone bosons, three of which can be identified with the pions but the fourth cannot be identified with the η, due to its high mass. At first glance one is naturally lead to predict a structure of the masses and mixing angle very similar to that of the vector mesons, the η being the pseudoscalar counterpart of the ω, and very close in mass to its $I = 1$ partners, the π-mesons; but as we have seen there is a clear deviation from these simple and natural expectations. This discrepancy has been called the *U(1)-problem*. Thus, the question is: Where does the explicit violations of *U(1)* $I = 0$ come from? The importance of the *U(1)-problem* is appreciable from the huge literature on this subject, which analyzes the basic structure of QCD and its non perturbative vacuum fluctuations (instanton physics). G. 't Hooft * [3], proposed the first solution of the

* For extensive reviews see the, though discordant, Physics Reports [1] and [2].

U(1)-problem on the classical QCD vacuum ($\langle F^a_{\mu\nu}\rangle = 0$) in the late 1976. He used the fact that this vacuum consists of pure-gauge configurations of the vector potential A^a_μ which may be classified by topological argumentation and therefore characterized by an integer topological "winding number" $\nu = \frac{1}{16\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$. From a quantum mechanical point of view these different configurations are separated by finite energy barriers which can be crossed by vacuum fluctuation called "instantons", so that the real QCD vacuum is a linear superposition, with well defined phases of gauge-field sector with different winding number. Such superposition are uniquely labelled by an angle θ, and represent the new QCD-vacuum, the so called θ-vacuum on which the theory is built. 't Hooft theory is also based on the existence of the well known Adler, Bell, Jakiw (A.B.J.) anomalous term in the divergence of the *U(1)*-axial current [4]:

$$\partial^\mu J^5_\mu = 2m_q J^5 + \frac{N_f g^2}{16\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a, \tag{1}$$

On that basis he argued that in the limit $m_q \to 0$, there is no conserved charge associated with J^5_μ. *U(1)* is thus explicitly violated and no $I = 0$ extra-Goldstone boson can exist, as experimentally observed. However 't Hooft solution cannot yield, by a calculation, the values of the masses of $\eta - \eta'$ mesons, in other word is only a " in principle" solution. Moreover the notion of θ-vacuum, while solving the *U(1)-problem*, it introduces a new, much more severe problem: the strong CP-violation, that characterize the structure of θ-vacuum, unless $\theta = 0$. The experimental evidence that strong interaction does not violate CP-symmetry implies that : $\theta < 10^{-9}$ [5], and, as a consequence, one have to justified how QCD realize such a remarkable fine tuning. The gravity of the strong CP-problem is evident in the huge literature on this subject.

A second solution of the *U(1)-problem* is due to Witten and Veneziano [6] who showed, by means of the $\frac{1}{N_c}$ expansion technique, that the leading diagrams associated to $q\bar{q}$-annihilation into gluons cause an explicit breaking of the axial *U(1)*-symmetry, thus the fourth extra-Goldstone boson cannot exist. No quantitative determination of the masses and mixing angle could however be carried out and the strong CP-problem is still present here.

In this contribution I present the result of a calculation of m_η, $m_{\eta'}$ and θ_P that has been carried out in the framework of the dynamical realization of QCD around what I believe to be the most probable ground state of QCD, the Chromo Magnetic Liquid (CML). The basic principles of this theory, which has been called Anisotropic Chromo Dynamics (ACD), are described in [7]; the reader should consult necessarily this literature. Here I only remark that the CML consists of an extremely dense set of needle-shaped "Weiss" domains, inside which a very large chromo-magnetic field is condensed, trapping the chromo-electric lines of force in the direction of the needle. As a consequence of this space-time topology the dynamics of quark and gluon fields inside a single domain becomes 1+1 dimensional, yielding a natural and simple explanation of color confinement and freedom at short distances, which are both well known properties of QCD_2 [8] . It is also well known that the smoothness at short distances, which characterizes low dimensional theories –and QCD, as I observe it– has a counterpart in the singular behaviour in the massless (infrared) limit. A few years ago [9] it was showed how such singularity of ACD can explain that, in the limit of small quark masses, the ensuing chiral symmetry is realized *à la* Goldstone rather than *à la* Weyl-Wigner. This means that should exist the Goldstone boson associated to the *U(1)*-symmetry of the chiral QCD-Lagrangian; but experimentally, as I already said, this particle is not observed. This is the *U(1)-problem* which I'm now in a position to solve quantitatively.

In this framework I studied the diagram in figure 1 which represents the second order contribution of the ACD Hamiltonian to $q\bar{q}$-annihilation. Note that it corresponds to the contributions considered in [6], but here the physics is totally different, in particular in this diagrams the gluons are not dynamical (in ACD they are called "longitudinal instantaneous gluons") and they can only "live" into the "Weiss" domain. This diagram can be calculated in a straightforward way in ACD, and the analytic expression of its matrix elements with pseudoscalar states can be written:

$$A = \frac{\int_0^{+\infty} dq_1 dq_2 u_{1f_1}(q_1) K^{f_1 f_2}(q_1,q_2) u_{2f_2}(q_2)}{\left\{\int_0^{+\infty} dq_1 |u_{1f_1}(q_1)|^2 \int_0^{+\infty} dq_2 |u_{2f_2}(q_2)|^2\right\}^{1/2}} \tag{2}$$

where $u(k)$ is the reduced radial wave function in the momentum space and

$$K^{f_1 f_2}(q_1,q_2) = \frac{3}{4}\left(\frac{\mu^2}{2\pi}\right)^2 \frac{m_{f_1} m_{f_2}}{E_{f_1 q_1} E_{f_2 q_2}} \int dq \frac{q^2}{(q^2+\lambda^2)^2} \left\{ \frac{E_{f_1(q_1-q)} + E_{f_2(q_2-q)}}{E_{f_1(q-q_1)} E_{f_2(q-q_2)} \left[(E_{f_1(q-q_1)} + E_{f_2(q-q_2)})^2 - (E_{f_1 q_1} - E_{f_2 q_2})^2 \right]} + \right. \\ \left. - \frac{E_{f_1(q_1+q)} + E_{f_2(q_2-q)}}{E_{f_1(q+q_1)} E_{f_2(q-q_2)} \left[(E_{f_1(q+q_1)} + E_{f_2(q-q_2)})^2 - (E_{f_1 q_1} - E_{f_2 q_2})^2 \right]} \right\} \tag{3}$$

with

$$E_{fk} = \sqrt{k^2 + m_f^2}. \tag{4}$$

By making use of the amplitude in figure 1 I can compute the matrix element $\langle 0 \mid \partial_\mu J_5^\mu \mid \eta', \beta \rangle$ (figure 2), with

$$\mid \eta', \beta \rangle = \int d^3\vec{k}\ \phi_\beta(\vec{k}) \frac{\delta_{f,f'}}{\sqrt{3}} \frac{\delta_{s,s'}}{\sqrt{3}} \frac{\delta_{c,c'}}{\sqrt{3}} b^\dagger_{scf}(\vec{k}) d^\dagger_{s'c'f'}(\vec{-k}) \mid 0 \rangle \tag{5}$$

$$\phi_\beta(\vec{k}) = \frac{2}{\pi^{\frac{1}{4}}} \left(\frac{\sqrt{2}}{\beta m}\right)^{\frac{3}{2}} \frac{1}{\sqrt{4\pi}} e^{-(\frac{k}{\beta m})^2} \tag{6}$$

If this matrix element (or anything else belonging to the Fock space) is different from zero then the axial current is not conserved and the *U(1)-problem* is solved because one of the hypotheses of the Goldstone theorem is not verified. The result of my calculation is very simple: I get (c being an uninteresting constant)

$$\langle 0 \mid \partial_\mu J_5^\mu \mid \eta'(0) \rangle = \frac{c}{\sqrt{m_q}}, \tag{7}$$

that shows how this matrix element does not vanish when $m_q \to 0$, but it actually diverges. This result shows that the severe singularities of ACD in the massless quarks' limit not only insure the Goldstone realization of the associated chiral symmetry, but it solves the *U(1)-problem* as well.

In order to get a quantitative prediction of the masses and mixing angle I sandwich the diagram in figure 1 between the pseudoscalar wave functions of the $\frac{1}{\sqrt{2}}(\mid u\bar{u}\rangle + \mid d\bar{d}\rangle)$

and $\mid s\bar{s}\rangle$ quark states which have been computed by solving the relativistic Schrödinger equation for the $q-\bar{q}$ states, as described in reference [10]. I obtain the following mass matrix

$$M = \begin{pmatrix} M_\pi + \alpha & \beta \\ \beta & M_{s\bar{s}} + \gamma \end{pmatrix}, \tag{8}$$

where the mass $M_{s\bar{s}}$ has been determined as a function of the constituent quark masses m_u and m_s, from the solutions of the relativistic Schrödinger equation. Setting the constituent quark masses to be

$$\begin{aligned} m_u \simeq m_d &= 108\ MeV \\ m_s &= 180\ MeV \end{aligned} \tag{9}$$

I compute

$$\begin{aligned} \alpha &= 217\ MeV \\ \beta &= 124\ MeV \\ \gamma &= 62\ MeV. \end{aligned} \tag{10}$$

Taking for $M_{s\bar{s}}$ the calculated value of $774\ MeV$, I diagonalize the mass matrix (8) and obtain

	Computed	Experimental
M_η	$484\ MeV$	$547\ MeV$
$M_{\eta'}$	$926\ MeV$	$958\ MeV$
θ_P	-29°	$-17.3^o \pm 1.8^o$

Figure 4 show the mixing angle θ_p as a function of quark masses, I remind you that this angle is defined by the following two equations:

$$\begin{aligned} \mid \eta \rangle &= \cos\theta_p \mid \eta_8 \rangle - \sin\theta_p \mid \eta_0 \rangle \\ \mid \eta' \rangle &= \sin\theta_p \mid \eta_8 \rangle + \cos\theta_p \mid \eta_0 \rangle \end{aligned} \tag{11}$$

The sensitivity of these results on the input constituent quark masses can be appreciated from figure 3–5, where I have reported contour plots in the $m_{u,d}$ - m_s plane for the three above quantities.

In conclusion I can summarize the results of this contribution saying that I have solved quantitatively the *U(1)-problem* within the framework of ACD. Differently from the usual "in principle" resolutions of Refs. [3] and [6], my calculation does not suffer from the further, disquieting strong CP-problem (whose many proposed solutions have so far received no confirmation). In fact CML has zero winding number because the vacuum fluctuations (the instantons) cannot cross the infinite (in the limit of the ultraviolet cut-off $\Lambda \to +\infty$) energy barriers that characterize the structure of the CML.

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Figure Captions

Fig. 1 Box diagram responsible for quark mixing.

Fig. 2 The matrix element which is singular in the chiral limit.

Fig. 3 η mass as a function of $m_{u,d}$ and m_s

Fig. 4 η' mass as a function of $m_{u,d}$ and m_s

Fig. 5 Mixing angle as a function of $m_{u,d}$ and m_s

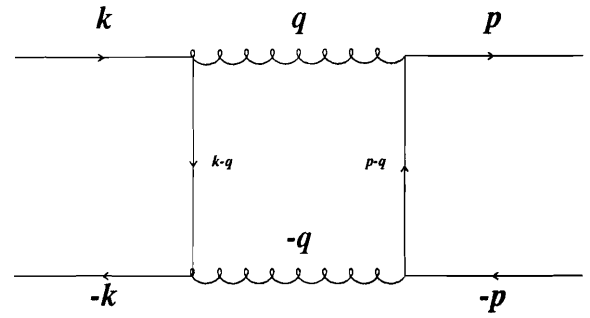

Fig. 1

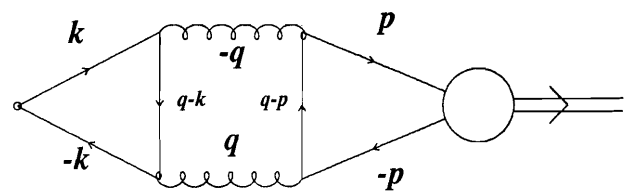

Fig. 2

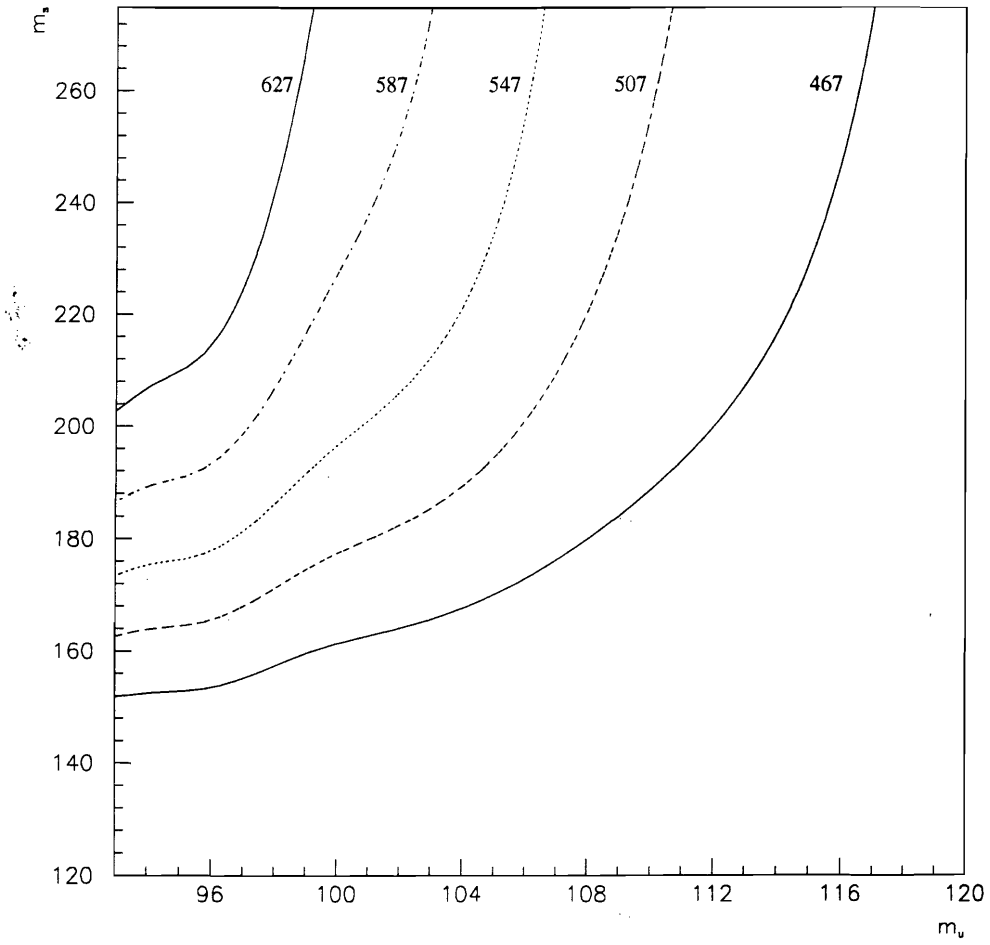
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Fig. 3

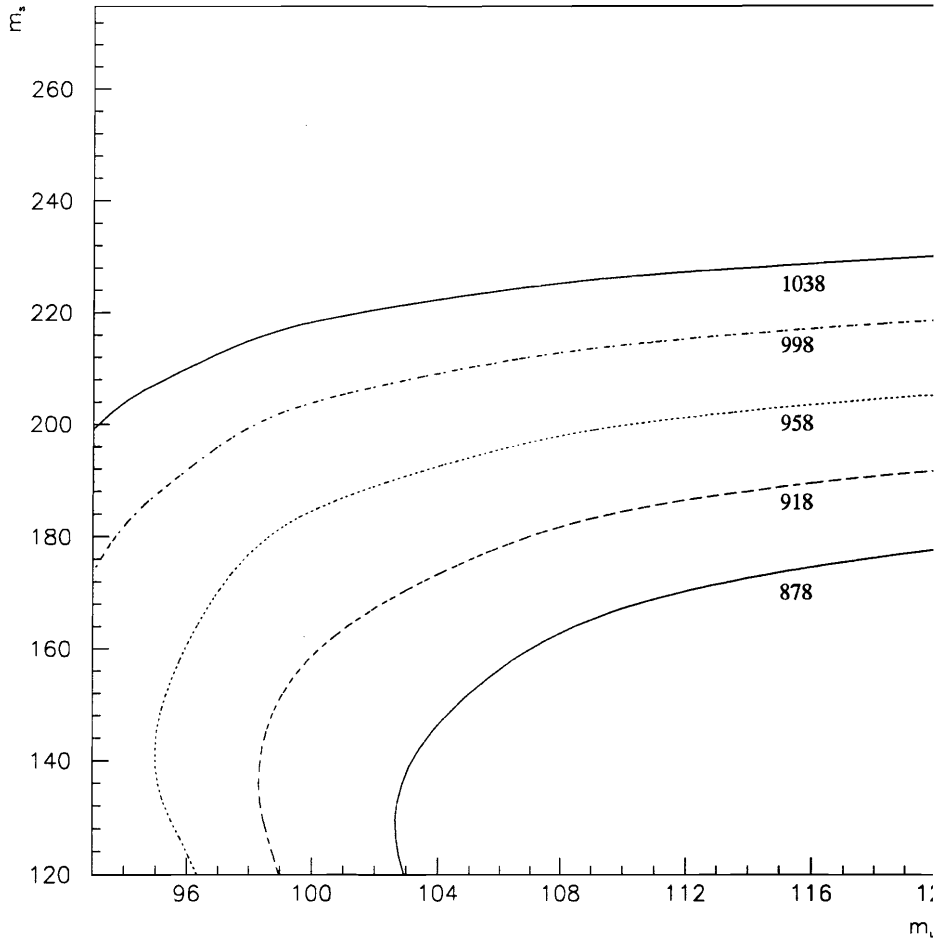
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Fig. 4

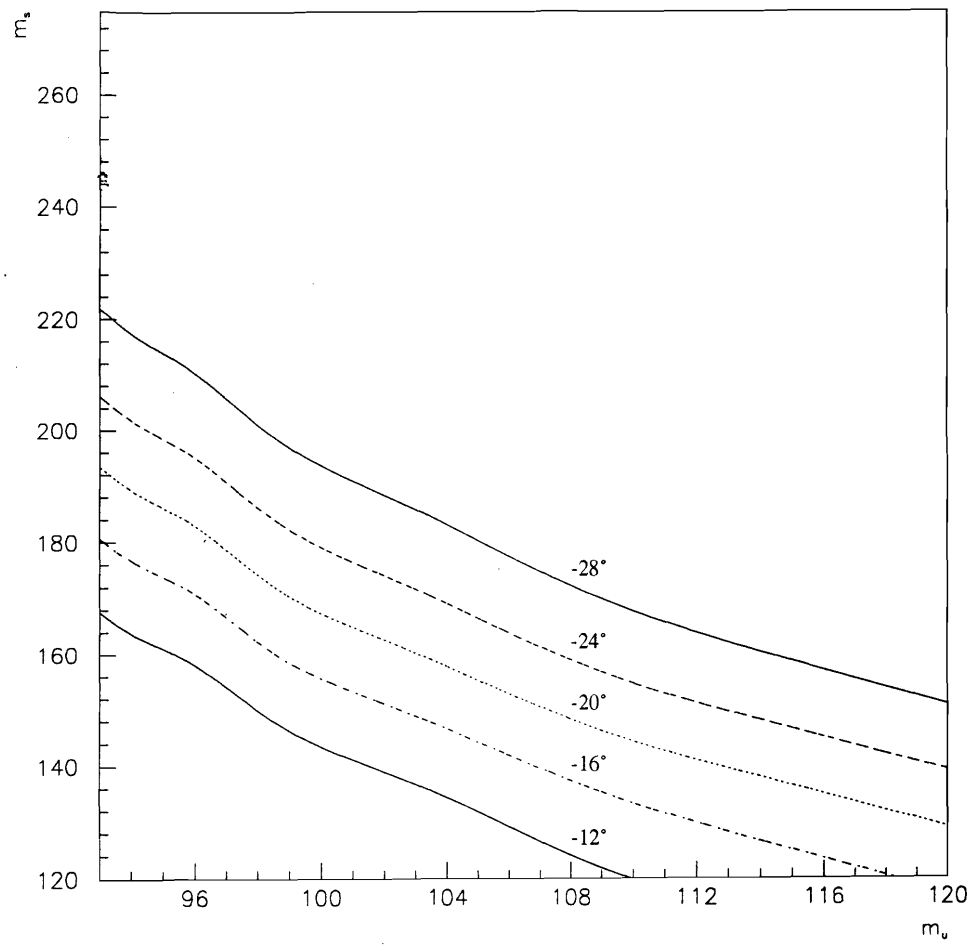
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Fig. 5