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CHIRAL LAGRANGIAN AND LOW-LYING MESON RESONANCES

T. N. Pham
Centre de Physique Théorique,
Centre National de la Recherche Scientifique, UPR A0014,
Ecole Polytechnique, 91128 Palaiseau Cedex, France.
CPT

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ABSTRACT

The quartic derivative terms of the chiral Lagrangian for $\pi\pi$ scattering are reanalysed using forward dispersion relation. It is shown that the $\pi^+\pi^0$ and $\pi^0\pi^0$ total cross sections above 1 GeV make contributions to the Skyrme and non-Skyrme terms, 10 and 30% respectively, of the low-energy contributions. For both the quartic terms, The high-energy contribution tends to cancel the finite width corrections, leaving a strength given by vector and scalar meson dominance , consistent with low-energy $\pi\pi$ data. In particular our analysis shows clearly that vector meson dominance, as often used in pion physics, can now be justified in a reliable manner.

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In a previous paper [1] on the determination of the Skyrme and non-Skyrme terms in the $SU(2) \times SU(2)$ chiral Lagrangian for low-energy pion physics, we have evaluated the low-energy contributions to these two chiral-invariant quartic terms ($O(p^4)$ terms) and found that the low-lying vector and scalar meson resonances produce a large contribution to these terms comparable with the experimental values obtained from the pion-pion scattering data [2]. This indicates that, like the pion charge-radius, low-energy pion-pion interactions are dominated by low-lying resonances, as found in a recent analysis [3]. Although it is quite plausible that vector meson dominance hypothesis as used in the pre-QCD days is a good description of low-energy hadron physics, e.g the proton charge-radius and the nucleon isovector anomalous magnetic moment, to this days, there is no really convincing reason for its validity. With the chiral Lagrangian approach to a phenomenological description of pion physics and the dispersion relation for the scattering amplitude, we now seem to understand, at least, on a qualitative level, that vector meson is the dominant contribution to the low-energy pion physics. This is because of the positivity argument for the Skyrme term enables us to reliably estimate the high-energy contributions in terms of the total $\pi\pi$ cross sections. The purpose of this paper is to evaluate the high-energy contributions to these $O(p^4)$ terms of the $SU(2) \times SU(2)$ chiral Lagrangian. Including high mass meson resonances and using the high-energy $\pi\pi$ cross sections data which are consistent with the standard Regge-pole model or the additive quark model predictions, we find that the contributions to the Skyrme term from the high-energy region above 1 GeV about 10% of the ρ meson resonance contribution. The contribution to the non-Skyrme term is somewhat larger, about 30% of the low-energy contribution.

Our starting point is the standard quadratic minimal $SU(2) \times SU(2)$ chiral Lagrangian for low-energy pion interactions up to the $O(p^2)$ terms where p is a typical pion momentum.

$$\mathcal{L}_0 = \frac{f_\pi^2}{8} \mathrm{Tr}\, \partial_\mu M \partial_\mu M^\dagger + \frac{m_\pi^2 f_\pi^2}{8} \mathrm{Tr}\, [\, m(M + M^\dagger)] \tag{1}$$

where m is the quark mass matrix in the $SU(2)$ space. In the standard non-linear realisation of chiral symmetry and for $SU(2) \times SU(2)$, we have

$$M = exp(2if\phi), \qquad \phi = \frac{1}{\sqrt{2}} \sum_i \tau_i \phi_i, \qquad f^{-1} = f_\pi \simeq m_\pi \tag{2}$$

where ϕ is the pion field operators and f_π is the charged pion decay constant measured in $\pi^+ \to l^+ \nu$ decay.

This Lagrangian can also be generalised to the chiral $SU(3) \times SU(3)$ as first given in the above exponential form for the meson coupling matrix M by Cronin [4] and has become the standard lore in low-energy physics [5].

For the chiral $SU(2) \times SU(2)$, the above chiral Lagrangian seems to describe rather well the low-energy $\pi\pi$ scattering data, such as the scattering lengths and phase shift in the region near the threshold. However, to describe the interactions at higher energies, it is necessary to add to the minimal Lagrangian terms quartic in the momenta which produces $O(p^4)$ terms in the scattering amplitudes. There could be a derivative coupling symmetry breaking mass term, such as the one needed in the $SU(3) \times SU(3)$ chiral Lagrangian to account for the Callan-Treiman relation and the symmetry-breaking effects in the decay constants of the pseudoscalar meson octet [6]. For $\pi\pi$ scattering, these terms, like the non-derivative coupling mass terms, give rise to $O(m_\pi^2)$ effects which are small in the low-energy region considered here , and will therefore be ignored. Then the $SU(2) \times SU(2)$ chiral Lagrangian, up to $O(p^4)$ terms is given by

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_Q \tag{3}$$

with the quartic term $\mathcal{L}_Q$ given by

$$\mathcal{L}_Q = \frac{1}{32e^2}\mathrm{Tr}([\partial_\mu M M^\dagger, \partial_\mu M M^\dagger]^2) + \frac{\gamma}{8e^2}[\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger)]^2 \tag{4}$$

The first term in Eq. (4) is denoted as the Skyrme term and the other term is called the non-Skyrme term [2]. The decomposition of the quartic terms in this way is more convenient than other alternative forms [5] for our present purpose, since, as shown previously [1] their coefficients are all positive-defined, a consequence of the positivity of the total cross sections in the dispersion relation. The $\pi\pi$ scattering amplitudes up to $O(p^4)$ terms now can be obtained from the total Lagrangian Eq. (3) and are given previously [1] . Then using the dispersion for the $\pi^+\pi^0$ and $\pi^0\pi^0$ forward scattering amplitudes we arrive at the following sum rules for the strengths of the quartic terms:

$$\begin{aligned} \frac{1}{e^2} &= \frac{f_\pi^4}{\pi}\int_{4m_\pi^4}^{\infty} \frac{\sqrt{s(s-4m_\pi^2)}\,\sigma_{tot}^{\pi^+\pi^0}\,ds}{(s-2m_\pi^2)^3} \\ \frac{\gamma}{e^2} &= \frac{f_\pi^4}{2\pi}\int_{4m_\pi^4}^{\infty} \frac{\sqrt{s(s-4m_\pi^2)}\,\sigma_{tot}^{\pi^0\pi^0}\,ds}{(s-2m_\pi^2)^3} \end{aligned} \tag{5}$$

The $\pi\pi$ scattering amplitudes are then completely determined up to $O(p^4)$ terms in terms of only two structure-dependent constants e^2 and γ. This is the advantage of the chiral Lagrangian approach. We also note again here that since the $\pi\pi$ total cross sections are measurable in the available energy range, e^2 and γ, in principle, could be evaluated with any desired accuracy. This allows us to estimate corrections to the resonance contributions. This is another advantage of working with forward

dispersion relation. Infact, though a dispersion relation for the pion form factor can be written down to obtain the vector meson dominance result for the pion charge-radius, the corrections to the ρ dominance are not easy to obtained since the absorptive part of the pion form factor is not positive-defined and directly measurable.

Consider now the low-energy contributions to e^2 and γ. In terms of the isospin channel total cross sections, the total cross sections $\sigma_{tot}^{\pi^+\pi^0}$ and $\sigma_{tot}^{\pi^0\pi^0}$ are given by

$$\sigma_{tot}^{\pi^+\pi^0} = \frac{1}{2}\sigma_{tot}^{I=1} + \frac{1}{2}\sigma_{tot}^{I=2}, \qquad \sigma_{tot}^{\pi^0\pi^0} = \frac{1}{3}\sigma_{tot}^{I=0} + \frac{2}{3}\sigma_{tot}^{I=2} \tag{6}$$

Since the $\pi\pi$ scattering are dominated by a few partial waves in the low energy region, the total measured $\pi\pi$ cross sections for energies up to 1 GeV can be parametrized in terms of partial wave cross sections. Thus, below 1 GeV , apart from a negligible contribution from the $I = 2$ cross section $\sigma_{tot}^{I=2}$ and a small infrared-divergence logarithmic term of the form $\ln(m_\pi^2/M^2)$, where M is an energy cut-off parameter above which the total cross section falls off rapidly ; the Skyrme term essentially comes from the ρ meson resonance and the non-Skyrme term get the main contribution from the large $I = 0$ S-wave cross section. Measurements for the $\pi\pi$ cross sections in the low energy region have been made long ago [7]. For a description of the data, it is convenient to parametrize the partial-wave amplitudes in a form which satisfies current-algebra at low energy and elacstic unitarity. We have [8, 9]

$$\begin{aligned} f_{11}(s) &= \frac{(L/12)(s-4m_\pi^2)}{D_{11}(s)} \\ f_{00}(s) &= \frac{(L/12)(s-m_\pi^2/2)}{D_{00}(s)} \end{aligned} \tag{7}$$

with $L = (4\pi f_\pi^2)^{-1}$ and the partial wave amplitudes defined as

$$f_{Il}(s) = \frac{exp(i\delta_{Il})\sin\delta_{Il}}{\rho(s)}, \qquad \rho(s) = \sqrt{\frac{(s-4m_\pi^2)}{s}} \tag{8}$$

The D functions for P-wave and S-wave, respectively, can be represented approximately by [8, 9]

$$\begin{aligned} D_{11}(s) &= \Big(1 - a(s-m_\pi^2/2) + (L/12)(s-4m_\pi^2)[h(s) - h(m_\pi^2/2) - i\rho(s)]\Big) \\ D_{00}(s) &= \Big(1 - b(s-m_\pi^2/2) + (L/12)(s-m_\pi^2/2)[h(s) - h(m_\pi^2/2) - i\rho(s)]\Big) \end{aligned} \tag{9}$$

The h function is given by

$$h(s) = \frac{2}{\pi}\frac{(s-4m_\pi^2)^{1/2}}{s}\, ln\left(\frac{\sqrt{s}+\sqrt{s-4m_\pi^2}}{2}\right) \tag{10}$$

For $a = 0.0357\, m_\pi^2$ [1], the $P-$ wave phase shift passes through 90° at $m_\rho = 775\,\mathrm{MeV}$ and yields value of 155 MeV for the width of the ρ meson, consistent with the current experimental data of (151.5 ± 1.2) MeV [10]. Note that the expression for the $\rho \to \pi\pi$ decay rate is precisely the prediction given by the KSFR relation for the $g_{\rho\pi\pi}$ coupling constant [11]. The $I = 0$ $S-$ wave phase shift can also be fitted with similar form for $D_{00}(s)$, as in Eq. (9) with $b = -m_\sigma^{-2} \simeq -0.04\, m_\pi^{-2}$ at least for data up to 700 MeV. This looks like a broad scalar meson resonance with mass $m_\sigma \simeq 675\,\mathrm{MeV}$. Since our main interest is the resonance contributions and the coefficients of the infrared divergence $\ln(m_\pi^2/M^2)$ terms, in the followings, for simplicity, we shall neglect the unitarity corrections represented by $h(s)$. The partial waves then take the following simplified form:

$$\begin{aligned} f_{11}(s) &= \frac{\gamma_1(s-4m_\pi^2)}{(m_\rho^2 - s - i\gamma_1(s-4m_\pi^2)\rho(s))} \\ f_{00}(s) &= \frac{\gamma_0(s-m_\pi^2/2)}{(m_\sigma^2 - s - i\gamma_0(s-m_\pi^2/2)\rho(s))} \end{aligned} \tag{11}$$

where

$$\gamma_1 = \frac{m_\rho^2}{48\pi f_\pi^2}, \qquad \gamma_0 = \frac{m_\sigma^2}{8\pi f_\pi^2} \tag{12}$$

m_ρ and m_σ are respectively the masses of the ρ meson and the hypothetical broad scalar meson resonance. Putting the cross sections obtained by this parametrisation into Eq. (5), we find

$$\begin{aligned} \frac{1}{e^2} &= -\left(\frac{1}{96\pi^2}\right)\ln\frac{m_\pi^2}{m_\rho^2} + (\text{finite terms as } m_\pi \to 0) + \left(\frac{1}{2} + \arctan\frac{m_\rho}{\Gamma_\rho}\right)\frac{f_\pi^2}{m_\rho^2} \\ \frac{\gamma}{e^2} &= -\left(\frac{1}{12\pi^2}\right)\ln\frac{m_\pi^2}{m_\sigma^2} + (\text{finite terms as } m_\pi \to 0) + \left(\frac{1}{2} + \arctan\frac{8\pi f_\pi^2}{m_\sigma^2}\right)\frac{1}{3}\frac{f_\pi^2}{m_\sigma^2} \end{aligned} \tag{13}$$

Note that the infrared divergence terms $\ln(m_\pi^2/\mu^2)$ which can be obtained in chiral perturbation theory with standard technique as $m_\pi \to 0$ in the chiral symmetry limit appear in our result with the energy scale μ replaced by the m_ρ and m_σ. Similar expression for the pion charge-radius has also been obtained previously with dispersion relation [13] .

If we ignore a small contribution from these chiral-log terms, we find

$$\frac{1}{e^2} = 0.93 \left(\frac{f_\pi^2}{m_\rho^2}\right), \qquad \frac{\gamma}{e^2} = 0.74 \left(\frac{1}{3}\frac{f_\pi^2}{m_\sigma^2}\right) \tag{14}$$

to be compared with the zero-width approximation value

$$\frac{1}{e^2} = \frac{f_\pi^2}{m_\rho^2}, \qquad \frac{\gamma}{e^2} = \frac{1}{3}\frac{f_\pi^2}{m_\sigma^2} \tag{15}$$

given previously [1, 12]

Note that the effects of the finite width corrections on the non-Skyrme term are quite important and reflect the $I = 0$, S-wave $\pi\pi$ scattering data as discussed above and therefore is a more accurate determination of the strength of the non-Skyrme term than the zero-width approximation value.

In the region 1–2 GeV, the total $\pi\pi$ cross sections receive contributions from higher mass resonances and a background of inelastic cross sections. Each resonance contributes to the quartic a term proportional to $f_\pi^4\Gamma/m_r^5$, which are suppressed by a factor $(m_0/m_r)^5$ (m_0 being the low-lying meson resonance mass) compared to the low-lying meson resonance term. We have

$$\frac{1}{e^2} = \sum\nolimits_i 16\pi(2J_i+1)\,\frac{\Gamma_i}{m_i}\frac{f_\pi^4}{m_i^4}, \qquad \frac{\gamma}{e^2} = \sum\nolimits_i \frac{8\pi(2J_i+1)}{3}\,\frac{\Gamma_i}{m_i}\frac{f_\pi^4}{m_i^4} \tag{16}$$

where J_i, m_i, Γ_i are respectively the spin, mass and width of the resonance. Many high mass resonances are now well established in the region 1–3 GeV. Using current data on the high mass meson resonances [10] , we find

$$\begin{aligned} \frac{1}{e^2} &= 0.013\,(\rho_3(1690))\frac{f_\pi^2}{m_\rho^2} \\ \frac{\gamma}{e^2} &= [\ 0.011\,(f_0(975)) + \ 0.13\,(f_2(1275)) \\ &\quad + (0.005 - 0.015)\,(f_0(1400))\ ]\frac{1}{3}\frac{f_\pi^2}{m_\sigma^2} \end{aligned} \tag{17}$$

Note that because of the spin factor $2J+1$, the $f_2(1275)$ contribution is the most important resonance contribution in this region.

The non-resonance cross sections from 1.5–3 GeV constitute the bulk of the total cross sections. From the data by Walker [14] and assuming the $\pi\pi$ cross sections are isospin independent, if we follow the quark model prediction [16] , we obtain

$$\sigma_{\rm NR}^{\pi^+\pi^0} = \sigma_{\rm NR}^{\pi^0\pi^0} = 20\,{\rm mb} \tag{18}$$

This gives

$$\frac{1}{e^2} = 0.06 \left(\frac{f_\pi^2}{m_\rho^2}\right) , \qquad \frac{\gamma}{e^2} = 0.075 \left(\frac{1}{3}\frac{f_\pi^2}{m_\sigma^2}\right) \tag{19}$$

where $m_\sigma = 675\,\mathrm{MeV}$.

For the $\pi\pi$ total cross sections at high energy, above 2 GeV, there are experimental measurements in the region 2–4 GeV. At these energies the elastic cross sections are negligible compared to the total cross sections. The $\pi^+\pi^+$ and $\pi^+\pi^-$ cross sections seem to reach a constant value of 15 mb [14]. It appears that, like the pp total cross sections, the $\pi^+\pi^+$ total cross sections are also dominated by Pomeron exchange [7, 15] and stay constant in the $3 - 30\,\mathrm{GeV}$ region. Assuming that the total cross sections are independent of isospin, as predicted by the quark model [16], we have then, for energy above 3 GeV [7]

$$\sigma_{tot}^{\pi^+\pi^0} = \sigma_{tot}^{\pi^0\pi^0} = 17\,\mathrm{mb} \tag{20}$$

Putting this into Eq. (5), and evaluating the integrals from $s = N^2$ to $s = \infty$, we find for the high-energy contribution:

$$\frac{1}{e^2} = 0.017 \left(\frac{f_\pi^2}{m_\rho^2}\right) , \qquad \frac{\gamma}{e^2} = 0.024 \left(\frac{1}{3}\frac{f_\pi^2}{m_\sigma^2}\right) \tag{21}$$

We have thus seen that the total non-resonance, high-energy contributions to the quartic terms are rather small, being suppressed by the factor m_ρ^2/N^2 relative to the low-energy contributions. The total contribution of the $\pi\pi$ cross sections above 1 GeV is then

$$\frac{1}{e^2} = 0.10 \left(\frac{f_\pi^2}{m_\rho^2}\right) , \qquad \frac{\gamma}{e^2} = 0.24 \left(\frac{1}{3}\frac{f_\pi^2}{m_\sigma^2}\right) \tag{22}$$

From Eq. (14) and Eq. (22) , we obtain for the total strength of the quartic terms

$$\frac{1}{e^2} = 1.03 \left(\frac{f_\pi^2}{m_\rho^2}\right) , \qquad \frac{\gamma}{e^2} = 0.98 \left(\frac{1}{3}\frac{f_\pi^2}{m_\sigma^2}\right) \tag{23}$$

To summarise, we have estimated the contributions to the two quartic terms in the chiral Lagrangian for $\pi\pi$ scattering and found that the Skyrme term is dominated

by the ρ meson and that the total $\pi\pi$ cross sections above 1 GeV make only a small contribution, of the order 10% of the ρ meson contribution. This clearly justifies the use of the vector meson dominance approximation in the analysis of $\pi\pi$ scattering data and confirms a recent analysis [3] which conclude that vector meson dominance model agrees rather well with the data on the Skyrme term. For the non-Skyrme term, we found that the total cross sections above 1 GeV produces 30% of the low-energy contribution, However the finite width correction and the high-energy contribution to these two $O(p^4)$ terms tends to cancel each other, leaving the Skyrme term given simply by the tree graph ρ meson term and the non-Skyrme term by the hypothetical scalar meson σ pole contribution. This seems to be consistent with the fact that the values of the parameter a and b obtained from the $\pi\pi$ data at low energy [9] (defined in Eq. (9)) agree well with the prediction given in Eq. (23), as if all the $O(p^4)$ terms could be given simply as the ρ and σ contribution in the zero-width pproximation.

We have seen that vector meson dominance for for the Skyrme term can now be justified using dispersion relation. Then it is more convenient for $\pi\pi$ scattering and other low-energy processes involving pion and ρ meson to use a chiral Lagrangian with vector meson from which a tree-graph amplitude can be obtained by integrating out the vector meson field. The resulting amplitudes will then obey crossing symmetry and their low-energy expansion can be obtained in a straightforward manner. For convenience we write down here the Lagrangian with the ρ meson. We note that there is a simple and economical way of constructing a vector meson chiral Lagrangian as shown by Weinberg [17] . In this non-linear realisation of chiral symmetry, the ρ meson field transforms as gauge field under chiral transformation [17] . Let ρ_μ be this ρ meson field, the minimal Lagrangian for ρ meson is [12, 17]

$$\mathcal{L}_\rho = -\frac{1}{8}\mathrm{Tr}\, F_{\mu\nu}^2 + \frac{1}{4}m_\rho^2 \mathrm{Tr}\left(\rho_\mu - \frac{2i}{g}v_\mu\right)^2 \tag{24}$$

where $F_{\mu\nu}$ is the covariant field strength tensor for ρ_μ defined in ref.[12] and v_μ transform also as gauge field under chiral $SU(2) \times SU(2)$ (this Lagrangian can be generalised to an $SU(3)$ vector meson octet for $SU(3) \times SU(3)$). In terms of the non-linear field ξ , we have, in standard notation

$$v_\mu = \frac{1}{2}[\xi^\dagger, \partial_\mu \xi], \qquad p_\mu = \frac{1}{2}\{\xi^\dagger, \partial_\mu \xi\} \tag{25}$$

where $M = \xi^2$. Note that p_μ transforms as covariant derivative and v_μ transforms as gauge field under the non-linear chiral transformation U.

The Lagrangian Eq. (24) contains an interaction of pion with ρ meson and a contact term of the form

$$\mathcal{L}_{\rm I} = -i\frac{m_\rho^2}{g}\,{\rm Tr}\,\rho_\mu v_\mu - \frac{m_\rho^2}{g^2}\,{\rm Tr}\,v_\mu v_\mu \tag{26}$$

As shown previously [17] , The quadratic terms in pion momenta ($O(p^2)$ terms) are cancelled by the ρ-exchange terms, leaving only quartic $O(p^4)$ terms for the ρ-induced $\pi\pi$ interactions. For this reason, it is more convenient to work with a new ρ field which transforms as

$$\rho_\mu \to U\rho_\mu U^{-1} \tag{27}$$

under chiral $SU(2)\times SU(2)$. Then all the interaction terms can be transferred to the ρ kinetic terms and the $\rho\pi\pi$ vertex is quadratic in pion momenta and the induced $\pi\pi$ interactions can be obtained immediately as the quartic contact terms. For this purpose, this new ρ field must have the form

$$\rho_\mu({\rm new}) = \rho_\mu - \frac{2i}{g}v_\mu \tag{28}$$

so that the mass term in Eq. (24) is invariant under U. Calling this new field ρ_μ from now on, we have

$$\mathcal{L}_\rho = -\frac{1}{8}{\rm Tr}\,\rho_{\mu\nu}^2 + \frac{1}{4}m_\rho^2{\rm Tr}\,\rho_\mu{}^2 \tag{29}$$

with $\rho_{\mu\nu}$ transforms as covariant derivative and takes the form

$$\rho_{\mu\nu} = D_\nu\rho_\mu - D_\mu\rho_\nu + \frac{ig}{2}[\rho_\mu,\rho_\nu] + \frac{2i}{g}[p_\mu,p_\nu] \tag{30}$$

with $D_\nu\rho_\mu$

$$D_\nu\rho_\mu = \partial_\mu\rho_\mu + [v_\nu,\rho_\mu] \tag{31}$$

We can now read off the Skyrme term from Eq. (29). It is simply the term

$$\mathcal{L}_{\rm SK} = \frac{1}{2g^2}{\rm Tr}\,[p_\mu,p_\nu]^2 \tag{32}$$

which can be written in the usual form as in Eq. (4) by noting that

$$\begin{aligned} L_\mu &= M\partial_\mu M^\dagger = -2\xi p_\mu\xi^\dagger \\ R_\mu &= M^\dagger\partial_\mu M = 2\xi^\dagger p_\mu\xi \end{aligned} \tag{33}$$

and identifying

$$\frac{1}{e^2} = \frac{1}{g^2}, \qquad g = \frac{m_\rho}{f_\pi} \tag{34}$$

We have shown that by a redefinition of the vector meson field, a simple derivation of the Skyrme term can be obtained from the Weinberg formulation. This reminds us of the equivalence between the linear and the non-linear σ model. In the linear form, the cancellation between the non-derivative σ-exchange terms and the 4π contact terms leaves only an effective $\pi\pi$ interactions quadratic in pion momenta. By redefining the pion field, we can generate the derivative $\pi\pi$ interactions from the linear σ model [18] directly from derivative contact term. The quartic term is then due to the σ- exchange graphs. The amplitude $A(s,t,u)$ (in standard notation) is then

$$A(s,t,u) = \frac{2}{f_\pi^2}(s - m_\pi^2)\left(\frac{m_\sigma^2}{m_\sigma^2 - s}\right) \tag{35}$$

Similarly, the s^2-term for the ρ-induced $\pi\pi$ scattering amplitude obtained from Eq. (29) is of the form

$$\begin{aligned} A(\text{rho}) &\sim \frac{s^2}{g^2} + \frac{1}{g^2}\frac{s^3}{m_\rho^2 - s} \\ &= \frac{1}{g^2}\frac{s^2\, m_\rho^2}{m_\rho^2 - s} \end{aligned} \tag{36}$$

which is just the vector meson dominance form. A similar result is found for the pion charge-radius. This again shows that vector meson dominance can be described by the Lagrangian given in either Eq. (24) or Eq. (29) .

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