FPRINT-93-35

Fermilab Library
0 1160 0023927 1

10

PHASE TRANSITION IN CHERN-SIMONS SUPERCONDUCTIVITY

S.S.Mandal, S.Ramaswamy and V.Ravishankar *

Department of Physics

Indian Institute of Technology

Kanpur, U.P., 208016, INDIA

Indian Inst. Tech., Kanpur

ABSTRACT

We present an analysis of the finite temparature Chern-Simons superconductivity model. Based on our analytical and numerical results for the magnetic susceptibility, conductivity and the dielectric constant, we present evidence for a phase transition in the system to the normal state. This transition is also accompanied by the restoration of Parity and Time reversal symmetries.

T

PACS numbers: 74.20.Kk, 74.25.-q, 11.90.+t

* *e-mail: vravi@iitk.ernet.in*

Chern-Simons (CS) gauge theories have had a profound impact on our understanding of condensed matter systems in two spatial dimensions. This has been especially so in the case of the Fractional Quantum Hall effect[1](FQHE) and in the possibility of a novel kind of Parity and Time reversal (P, T) violating mechanism for superconductivity, originally proposed in connection with high-T_c superconductivity[2].

The purpose of this letter is to present the results of a comprehensive study of the finite temperature (FT) Chern-Simons superconductivity (CSS). We use the mean field approach for the CS magnetic field and compute the effective action for the photon field and the electromagnetic response of the system. On the basis of our analytical and numerical results, we have found evidence for a phase transition from the superconducting to a normal (non-magnetic, insulating and a weakly dielectric) state, signalled by an abrupt change in the electromagnetic properties of the system. This transition also coincides with the restoration of the P, T symmetries, whose violation is characteristic of CSS.

The FT analysis of CSS has been considered in several earlier works[3-7]. To our knowledge none of these considers genuinely high temperatures. One conclusion[3] - which has perhaps led to a decline of interest in this topic- that the Meissner effect (ME) persists for all finite temperatures- is not corroborated by our studies. Ref. 5 does point to a transition based on a partial numerical computation and a low temperature analysis which is, however, somewhat dependent on assumptions on the numerical values of the parameters of the theory. Ref. 6 has also argued for a Kosterlitz-Thouless (KT) transition; however their estimate of T_{cr} does not coincide with our finding. Recently, Ref. 8 has analysed the normal state behaviour, along with a computation of the change in the magnetic permeability, which qualitatively agrees with our result. We believe that the merit of our work lies in its complete characterization of the electromagnetic properties of the system, both in the superconducting and normal phases which we feel is significant, given the present interest in this model.

The MF approach is standard and we shall not repeat the details. The relevant lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\nu}{2}\epsilon_{\mu\nu\alpha}a^\mu\partial^\nu a^\alpha + \psi^\dagger iD_0\psi - \frac{1}{2m}|D_k\psi|^2 + eA_0\rho \tag{1}$$

displays the nonrelativistic (spinless) fermions coupled minimally to the Maxwell (A_μ) and the CS (a_μ) gauge fields along with the kinetic energy terms for the respective gauge fields. The last term neutralises the system by providing for a uniform positive background charge density. We need the knowledge of the partition function

$$Z = Tr\,exp[-\beta(\mathcal{H} - \mu\mathcal{N})] \tag{2}$$

(where $\mathcal{H}$ is the hamiltonian, $\mathcal{N}$ is the conserved particle number and $\beta = 1/T$ is the inverse of the temperature) which will allow us to compute the thermodynamic properties of the system.

We consider the mean field properties first. The mean field ansatz smears the CS magnetic field ($b = e\rho/\nu$) to a uniform value which yields Landau levels with $\epsilon_n = (n+1/2)\omega_c$ being the energy eigenvalues and $\omega_c = eb/m$ being the cyclotron frequency. We fine tune as usual the CS parameter to $\nu = Ne^2/2\pi$, to achieve N completely filled levels. The chemical potential $\mu(\beta)$ is to be determined by the constraint of fixed density ρ. The low temperature results are exponentially small corrections to the value at $T = 0$, in agreement with Ref 3. Thus, $\mu = \epsilon_F - exp(-\beta\epsilon_F)/2\beta$ (ϵ_F is the Fermi energy at $T = 0$), the equation of state is given by $P = \pi\rho^2/m + 2\rho\, exp(-\beta\epsilon_F/2N)/N\beta$, and the specific heat $C_v = \rho(\beta\omega_c)^2 exp(-\beta\epsilon_F/2N)/2N$. Clearly, the temperature effects are negligible. The thermal properties at high temperatures are more interesting and relevant. Contrary to Ref 3, μ is not a perturbation over ϵ_F any longer; indeed, the small $\beta\omega_c$ limit is equivalent to a small thermal volume for the system, as given by $u = 2\pi\beta\rho/m \equiv \lambda_T^2\rho << 1$. In this region, an Euler-Mclaurin expansion yields the fugacity to be $exp(\beta\mu) = u + u^2/2 + (1 + 1/4N^2)u^3/6 + ..$, from which we obtain the virial expansion thus: $P\beta = \rho[1 + \lambda_T^2\rho/4 + (1 - 9/N)(\lambda_T^2\rho)^2/36 + ..]$. The second and the third virial coefficients therefore read[9], $B_2 = \lambda_T^2/4$ and $B_3 = (1 - 9/N)\lambda_T^4/36$. A physical quantity of immediate relevance is the response of the free energy to a weak external (Maxwell) magnetic field. It is given by

$$\frac{\delta F}{\delta(\delta B^E)}\Big|_{\delta B^E=0} = \frac{\pi\beta e\rho^2}{6m^2}(3 + \frac{1}{N}). \tag{3}$$

The above result is significant[10]: since F is not a minima at $\delta B^E = 0$, the system has no reason to forbid external magnetic fields, which suggests an absence of ME, atleast upto order β. To vindicate this anticipation, consider the fluctuations (coming from the fermion loop) in the gauge fields about the mean field configuration. The effective one loop action that ensues involves the fermion vacuum polarization tensor $\Pi^{\mu\nu}$ acting on the fluctuating gauge fields $\tilde{A}_\mu$ and $\tilde{a}_\mu$. The effective lagrangian is then (dropping the $\sim$ for simplicity),

$$\mathcal{L}_{eff} = \mathcal{L}_0 - \frac{1}{2}\mathcal{A}_\mu\Pi^{\mu\nu}\mathcal{A}_\nu$$

where $\mathcal{A}_\mu = (A_\mu + a_\mu) + A_\mu^E$ also has a piece coming from external (probe) fields and $\mathcal{L}_0$ is the tree level term. Further, resolve

$$\Pi^{\mu\nu} = \Pi_0(q^2g^{\mu\nu} - q^\mu q^\nu) - (\Pi_0 - \Pi_2)\delta^{\mu i}\delta^{\nu j}(\vec{q}^2\delta^{ij} - q^iq^j) + i\Pi_1\epsilon^{\mu\nu\lambda}q_\lambda,$$

where the form factors (in the limit $\omega, \vec{q}^2 \to 0$) have the structure $\Pi_0 = \Pi_0^{(1)} + \Gamma^2/\vec{q}^2$, $\Pi_1 = \Pi_0^{(1)}\omega_c$. At small $\beta\omega_c$, they have the simple form (upto $O(\beta)$)

$$\Pi_1 = 0;\ \Pi_2 = \frac{e^2\beta\rho}{12m^2};\ \Gamma^2 = e^2\beta\rho. \tag{4}$$

Note that at $T = 0$, Π_1 exactly cancels the CS term in $\mathcal{L}_0$, and this was argued to be essential for superconductivity[2]. The static response (linear) is now readily evaluated. The magnetic susceptibility χ,

the dielectric constant ϵ and the conductivity σ_{ij} are respectively given by

$$\chi = -\Pi_2;\ \epsilon^{-1} = 1 + \frac{\Gamma^2}{\vec{q}^2 + \Gamma^2};\ \sigma_{ij} \equiv \epsilon^{ij}\sigma = 0 \tag{5}$$

Note that χ is independent of $\vec{q}^2$, is very weakly negative and is strictly zero at $\beta = 0$. This implies the absence of ME (atleast upto $O(\beta)$). There is a weak Faraday edge current in the system, causing an infinitesimal uniform attenuation of the magnetic field in the medium. Appropriately, the conductivity tensor σ_{ij} (which is characteristically Hall like in the model) also vanishes which signifies the absence of the supercurrent. Finally, the system behaves like an ordinary dielectric medium with a screening length given by Γ^{-1}. Thus at high temperatures, the theory is essentially free and strictly so at $\beta = 0$.

A further understanding of the changes occurring in the behaviour of the system is obtained by an analysis of the excitations of the system. It is simpler (and instructive) to consider the neutral system (i.e., excluding the Maxwell field) first. At zero temperature, the cancellation of the CS term is crucial for the existence of the superfluid mode, which then has the dispersion relation $\omega = c|\vec{q}|$ ($c = \sqrt{\pi\rho/m^2}$ being the velocity of phonon). An exponentially weak mass[2] does develop at low temperatures but this does not destroy superfluidity. However at high temperatures the form factor Γ^2 dominates as the mass term. In fact the dispersion relation now becomes that of a non-relativistic particle, $\omega \approx \vec{q}^2/2M$, $M = \Gamma^2/2\nu$, signifying the demise of superfluidity. This is most strikingly revealed by the changes in the energy and specific heat contributed by the mode. At low temperatures $U^{ph} \approx \zeta(3)/\pi\beta^3c^2 \Rightarrow C_v^{ph} = 3\zeta(3)T^2/\pi c^2$. At high temperature $(1/A)lnZ^{ph} = \pi MT/12$ comes out independent of temperature! Consequently $U^{ph}, C_v^{ph} \approx 0$. Thus, heating the system only serves to dissolve the mode ($\Gamma^2 \to 0$ as $\beta \to 0$) and does not energize it.

A similar change in the specific heat is also reflected in the charged system. At low temperature there are two modes with the dispersion (at low momentum and ignoring the exponentially small low temperature correction) $\omega_+^2 \approx \vec{q}^2 + C_1^2$; $\omega_-^2 \approx \vec{q}^2/C_2 + C_1^2/C_2^2$, where $C_1 = e^2N/2\pi$ and $C_2 = e^2mN^2/4\pi^2\rho$. Correspondingly one has $C_v \approx (C_1^3\beta/2\pi)\,exp(-\beta C_1) + (C_1^3\beta/2\pi C_2^2)\,exp(-\beta C_1/C_2)$ as the contribution of the two modes. At high temperatures the Coulomb interaction gets mediated by the non-relativistic mode, $\omega \approx \vec{q}^2/2M$, which does not contribute to the thermodynamic properties. The magnetic interaction tends to a long range mode, $\omega^2 = (1 + \Pi_2)\vec{q}^2 + \Gamma^2$. Its contribution to the specific heat $C_v \approx 3\zeta(3)T^2/\pi + e^2\rho\zeta(3)T/12\pi m^2$ approaches the 2+1 photonic value as $T \to \infty$.

The above features strongly suggest a phase transition at an intermediate temperature. To explore the behaviour at such temperatures we have carried out a numerical computation which bears out our expectation. Figures 1-4 summarize our findings. We have studied the system for two extreme values of $N(= 10, 1000)$, and $e^2/m = 10^{-5}, \rho/me^2 = 10^{-1}$ and $e^2 = 10^5 cm^{-1}$. Many interesting features are apparent: χ, σ and ϵ^{-1} (figs.1, 2) undergo a sharp variation at $T_{cr} \sim 0.18\omega_c$ and $.04\omega_c$ for N=1000 and 10

respectively. Clearly, the transition is to a non-magnetic, insulating and weakly dielectric state and is driven by a corresponding sharp fall in the values of Π_1 and Π_2 (fig.3), and the dominance of Γ^2 as the mass term (fig.4). Indeed, we find that T_{cr} is determined by the conditions $\Pi_1/\Gamma \to 0$; $\Pi_2 \to 0$, which is evident in figs.3,4. Finally, the vanishing of the off-diagonal Hall conductivity demonstrates the restoration of P,T symmetries.

In further analyzing the numerical results, we recall that the system has been studied in the low $\vec{q}^2$ limit, ie, $\vec{q}^2l^2 << 1$, ($l = \sqrt{1/eb}$ being the magnetic length). Consequently any mass scale $> \sqrt{2\pi\rho/N}$ is to be taken as decoupled from the theory. Thus soon after T_{cr} the larger mass M_+ decouples, restoring thereby the P,T symmetry. The subsequent singularity in M_+^2 and its change of sign (which is caused by a change of sign of Π_1) could be significant but its interpretation lies beyond the domain of our perturbative analysis. It is apparent that the HT limit (characterized by the Γ^2 dominance), is valid much earlier than where our analytic results (4) are, i.e., soon after T_{cr}. Further, it can also be asserted that the transition would be accompanied by a sharp change in the specific heat. And significantly, our numerical results are not crucially dependent on our specific choices for the parameters of the theory. In fact, the temperature dependence always arises in the form $\beta\omega_c$. Further, we observe that the variation of T_{cr}/ω_c with N is small and is presumably an $1/N$ effect.

Our perturbative results are inadequate to shed light on the physical cause of the transition. It is possible that the underlying mechanism[6] is a KT transition due to the melting of vortex-antivortex pairs. A more refined analysis would perhaps establish this with certainty. Further a general fixed point analysis of the renormalization group flow in CS theories has been carried out by Avdeev et. al.[11]. It remains to be investigated if the rich phase structure they find can be mapped to the transition found here.

Finally, we conclude with the following remarks: numerically, even at $T = 0$, M_+ is substantially larger than M_- (fig.4) which is very near the BCS mass. Thus any attempt to see experimentally the P,T-violation has to contend with this fact. Next, the cancellation of the CS term is not exact at $T \neq 0$, (or for that matter, even at $T = 0$, in models where the fermion may have a spin[12]). This apparent drawback has motivated[13] alternative models for high T_c superconductors. However, this study has shown that such an inexact cancellation does not destroy superconductivity. Coupled with our finite temperature results, we thus see that, as a model for 2-D superconductivity, CSS is neither unrealistic nor pathological. CS superconductivity may thus possess an intrinsic significance for quasi planar systems - in as much as the remarkably successful Laughlin ansatz does for FQHE.

Acknowledgements: We thank J.K.Bhattacharjee, A.Singh and R.Shankar for discussions. S.R. thanks the Department of Science and Technology, Government of India for support during the course of this work.

REFERENCES

[1] S.C.Zhang, T.H.Hanson and S.Kivelson, Phys. Rev. Lett. 62, 821 (1989); See also the review by S.C.Zhang, Int. Journ. Mod. Phys. B6, 25 (1992).

[2] A.L.Fetter, C.B.Hanna and R.B.Laughlin, Phys. Rev. B39, 9679 (1989); Y-H.Chen, F.Wilczek, E.Witten and B.I.Halperin, Int. Journ. Mod. Phys. B3, 1001 (1989); J.Lykken, J.Sonnenschein and N.Weiss, Phys. Rev. D42, 2161 (1990); X.G.Wen and A.Zee, Phys. Rev. B41, 240 (1990).

[3] S.Ranjbar-Daemi, A.Salam and J.Strathdee, Nucl. Phys. B340, 403 (1990).

[4] J.E.Hetric, Y.Hosotani and B.H.Lee, Ann. Phys. 209, 151 (1991).

[5] J.E.Hetric and Y.Hosotani, Phys. Rev. B45, 2981 (1992).

[6] Y.Kitazawa and H.Murayama, Phys. Rev. B41, 11101 (1990).

[7] P.K.Panigrahi, R.Ray and B.Sakita, Phys. Rev. B42, 4036 (1990).

[8] I.E.Aronov, E.N.Bogachek, I.V.Krive and S.A.Naftulin, JETP Lett., 56, 283(1992).

[9] B_2 does not contain the statistical interpolation parameter $e^2/2\pi\nu$, and as such differs from the value obtained by D.Arovas, J.R.Schreiffer, F.Wilczek and A.Zee, Nucl. Phys. B251, 117 (1985). This fact may be an indicator of the limitation of the MF approach.

[10] Recall that at $T = 0$, the free energy has a cusp at $B^E = 0$, which motivated Chen *et al*[8], to argue that there would be a Meissner effect.

[11] L.V.Avdeev, G.V.Grigoryev and D.I.Kazakov, Nucl. Phys. B382, 561(1992).

[12] B.Chakraborty, S.Ramaswamy and V.Ravishankar, Int. Jour. Mod. Phys. B6, 3513 (1992).

[13] See, e.g., N.Dorey and N.E.Mavromotos, Nuc. Phys. B386, 614 (1992) and the references therein.

FIGURE CAPTIONS

Fig. 1: The dependence of χ (— , – – –), $\sigma(T)/\sigma(0)$ (– · –, · · ·) on T/ω_c for N =1000, 10. σ is shown for a typical value $2\pi\bar{q}^2 l^2 = 10^{-4}$.

Fig. 2: The variation of ϵ^{-1} (— , – – –) as a function of T/ω_c for N = 1000,10.

Fig. 3: The dependence of $\Pi_1(T)/\Pi_1(0)$, $\Pi_2(T)/\Pi_2(0)$ on T/ω_c for N=1000. Note that they are highly coincident.

Fig. 4: The masses $M_{\pm}^2$ (– · –, —) and Γ^2 (– – –) as functions of T/ω_c are shown in the units of $M_-^2(0)$ for N=1000.

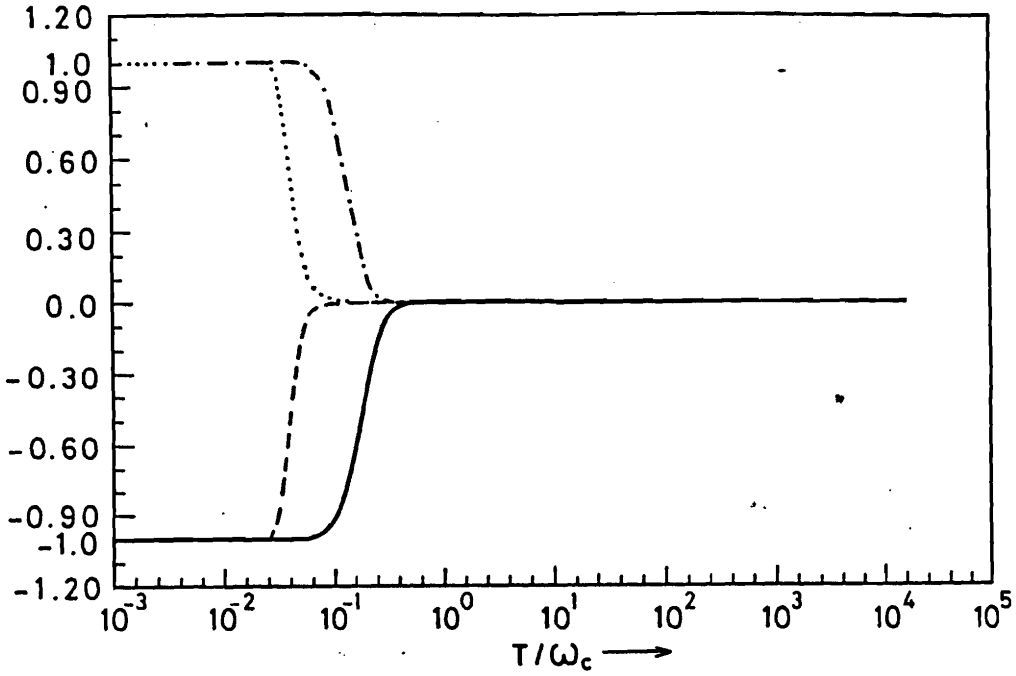
1.20
1.0
0.90
0.60
0.30
0.0
-0.30
-0.60
-0.90
-1.0
-1.20
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
T/ω_c

Fig. 1

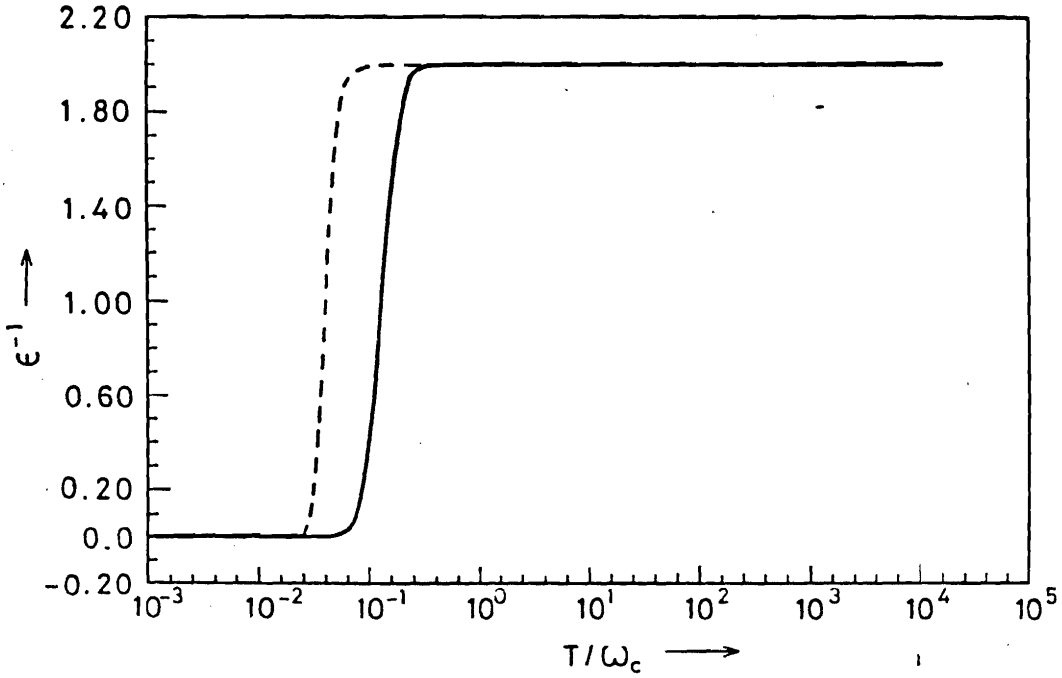
2.20
1.80
1.40
1.00
0.60
0.20
0.0
-0.20
ϵ^{-1}
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
T/ω_c

Fig. 2

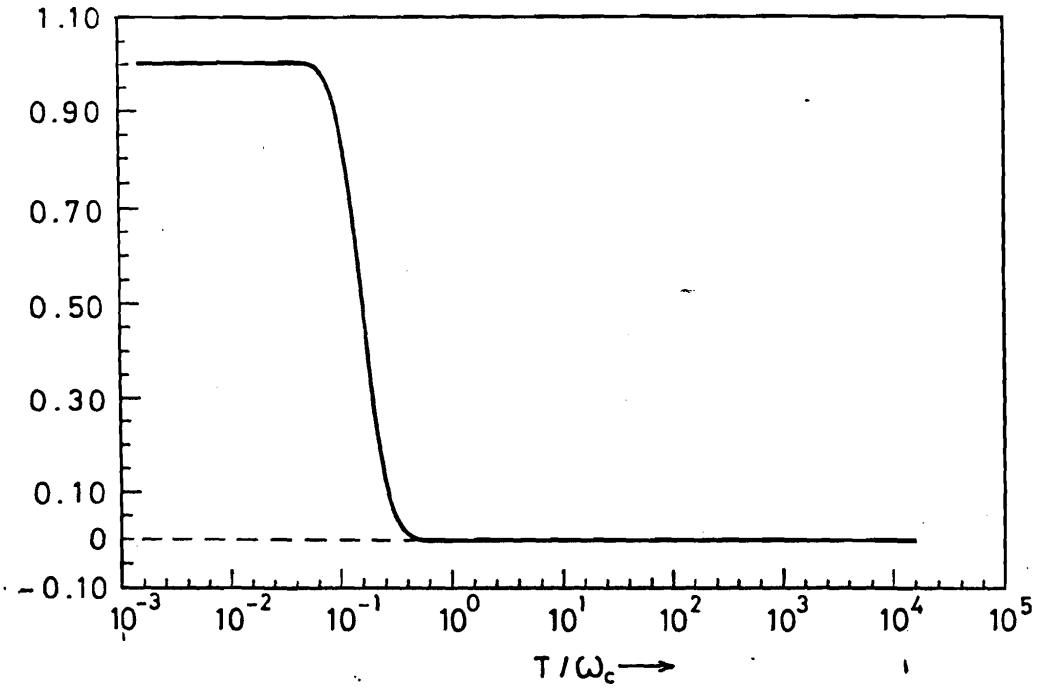
1.10
0.90
0.70
0.50
0.30
0.10
0
-0.10
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
$T/\omega_c \longrightarrow$

Fig. 3

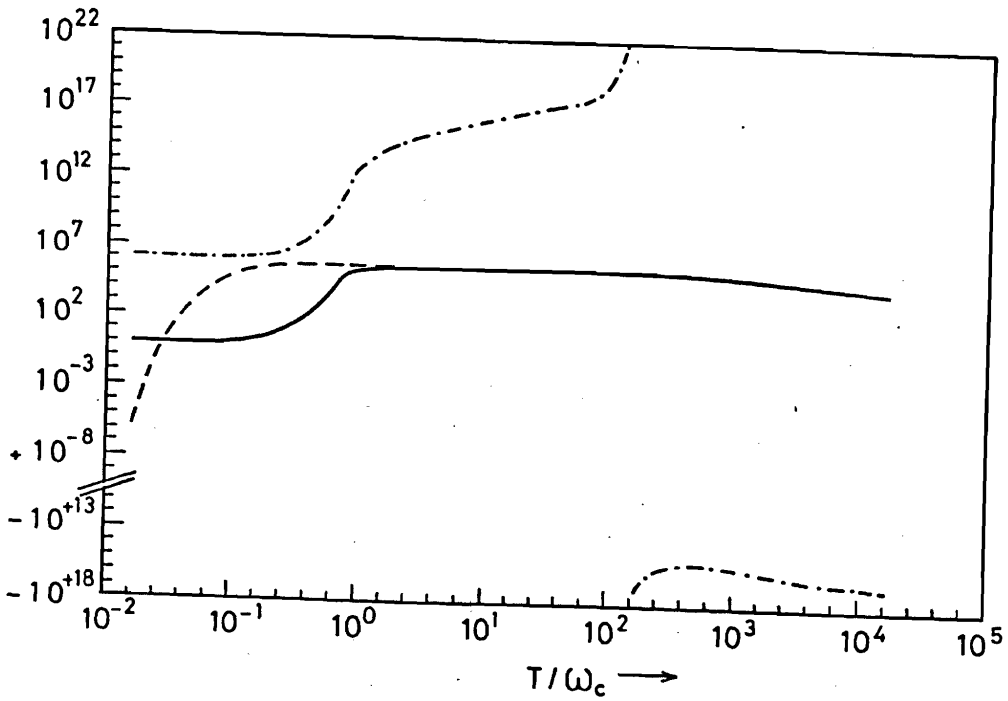
10^{22}
10^{17}
10^{12}
10^{7}
10^{2}
10^{-3}
$+10^{-8}$
-10^{+13}
-10^{+18}
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
$T/\omega_c \longrightarrow$

Fig. 4