8 FPRINT-93-34

Strongly Interacting W's and Z's and the Existence of a Heavy Fourth Generation of Fermions

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ABSTRACT

By employing the dictum that axiomatic principles are devoid of predictive power, we find that the elastic unitarity constraint, applied to strong $W_L W_L$ scattering, does not alter the assumed spectrum of intermediate states. We consider intermediate states involving a heavy Higgs and heavy fermions of a hypothetical fourth generation doublet. In contrast to recent studies, we find no p-wave resonance, and therefore no violation of the S parameter upper bound. We conclude that the elastic unitarity constraint sheds no light on the existence of a heavy fourth generation.

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APR 22 1993

Long ago, it was of interest to determine whether a strongly interacting linear sigma model (LSM), in all its apparent simplicity, could prove capable of generating the complex hadronic spectrum [1]. In recent times, the familiar isomorphism between the pions and the longitudinal modes of the standard model gauge bosons, made precise through the equivalence theorem, has led to a resurgence of interest in the strongly interacting LSM [2,3,4]. Motivation rests on the understanding that $W_L W_L$ scattering provides a unique probe of the mechanism responsible for electroweak symmetry breaking [5]. Evidently, triviality bounds [6] provide at best a narrow window within which a strongly interacting Higgs sector could exist. However, it is important to know what resonance structure to expect should such a window exist. The specific question that we address is: Do intermediate states involving heavy fermions of a hypothetical fourth generation doublet provide enough binding to produce a p-wave resonance? As emphasized by Truong in Ref. 4, this question is of special interest since precision weak-interaction measurements constrain the spin-1 content of a strongly interacting Higgs sector via the S parameter upper bound [7].

Study of the singularity structure of the scattering amplitude requires trading crossing symmetry for elastic unitarity, in a non-unique way. In Ref. 3 and Ref. 4, the method of Padé approximants is used to show that, for a large fermion mass, it is possible to dynamically generate a p-wave resonance. If this result is correct, then the S parameter bound can serve to exclude a heavy fourth generation of fermions [4]. We will argue that the use of the Padé method in Refs. [1-4] is based on the notion that elastic unitarity should be imposed for the purpose of making predictions. Our approach is conceptually novel in that, in sync with current lore, we ensure that unitarity per se yields no predictive power, a point of view clearly orthogonal to S-matrix theory (in the bootstrap sense.) That axiomatic constraints like unitarity and causality do not uniquely determine S-matrix elements was an important lesson learned with the advent of QCD. A priori, there are an infinite number of S-matrices consistent with the most general physical principles [9]. For example, in the context of a non-abelian gauge field theory, changing gauge group and fermion content certainly does not affect the unitarity of the theory.

We find that the I=1 singularity structure is insensitive to the heavy fermion mass. Furthermore, the only nearby pole of the full amplitude is seen to be the physical Higgs pole. Therefore, we find that there is no violation of the S parameter upper bound for any value of the heavy fermion mass. We conclude that elastic unitarity, imposed as a constraint on strong $W_L W_L$ scattering, yields no information concerning the existence of a heavy fourth generation of fermions.

Exploitation of the model-independent low-energy structure of the theory is essential to our approach. Assuming a custodial SU(2) symmetry, the most general effective Lagrangian including terms with four derivatives is given by[1]

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + \frac{C_1}{16\pi^2}\,\mathrm{Tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right)\mathrm{Tr}\left(\partial_\nu \Sigma \partial^\nu \Sigma^\dagger\right)$$

1 The coefficients, normalized in this way, are of $O(1)$ in the sense of naive dimensional analysis [8].

$$+\frac{C_2}{16\pi^2}\,\mathrm{Tr}\left(\partial_\mu\Sigma\partial_\nu\Sigma^\dagger\right)\mathrm{Tr}\left(\partial^\mu\Sigma\partial^\nu\Sigma^\dagger\right). \tag{1}$$

The Goldstone boson fields (w^+,w^-, and z) are contained within the field variable $\Sigma = exp(\frac{i\vec{\tau}\cdot\vec{w}}{v})$. C_1 and C_2 are undetermined constants which characterize the underlying theory at low energies. In general, there are contributions to C_1 and C_2 from all heavy degrees of freedom, as well as continuum contributions arising from goldstone boson loops. The contributions to these low-energy constants arising from intermediate states involving the Higgs boson and degenerate[2] heavy fermions of a fourth generation doublet have been calculated perturbatively in Ref. 10 using an on-shell subtraction scheme. They are given by

$$\begin{aligned} C_1^H(\mu) &= \frac{1}{4}\left[-\left(\frac{9\pi}{4\sqrt{3}}-\frac{37}{9}\right)-\frac{1}{6}\log\left(\frac{\mu^2}{M_H^2}\right)\right]+2\pi^2\left(\frac{v^2}{M_H^2}\right)\\ C_2^H(\mu) &= \frac{1}{4}\left[-\left(\frac{2}{9}\right)-\frac{1}{3}\log\left(\frac{\mu^2}{M_H^2}\right)\right], \end{aligned} \tag{2}$$

and

$$\begin{aligned} C_1^f &= -\frac{N_c}{12}\left[\frac{1}{2}+6\left(\frac{2}{a}+\frac{(4-a)}{a^2}\int_0^1\log\left(1-ax\left(1-x\right)\right)dx\right)\right]\\ C_2^f &= \frac{N_c}{12}, \end{aligned} \tag{3}$$

where $a \equiv \frac{M_H^2}{M_f^2}$. Note that for definiteness we use values of the low-energy constants extracted from perturbation theory. However, we stress that we could equally well consider the most general couplings of fields with *any* quantum numbers to the goldstone bosons, and estimate the values of these couplings using naive dimensional analysis. The uncertainty associated with a change of the C_i of $O(1)$ should certainly not exceed the inherent uncertainty that accompanies any unitarization scheme. In fact, we find that our basic conclusions are insensitive to natural changes in scale. For example, we can replace the C_i^H by the values that obtain from coupling a scalar to the goldstone bosons in the most general way [11]. In this case there is an undetermined parameter that can be related to the scalar width. If, instead of choosing the perturbative standard model value for the width, we choose one-half of that value, as is the case when the existence of a narrow p-wave resonance is assumed [12], our results are unaffected.

To order s^2, the relevant partial wave amplitudes of definite custodial isospin are given by

$$\begin{aligned} a_0\left(s\right) \equiv a_{00}\left(s\right) =& \alpha_0 s\Big\{1-\frac{\alpha_0 s}{\pi}\left[\log\left(\frac{-s}{\mu^2}\right)-6\left(2C_1+C_2\right)\right]\\ &-\frac{\alpha_0 s}{\pi}\left[\frac{7}{18}\log\left(\frac{s}{\mu^2}\right)-\frac{11}{108}-\frac{2}{3}\left(4C_1+5C_2\right)\right]\Big\},\\ a_1\left(s\right) \equiv a_{11}\left(s\right) =& \alpha_1 s\Big\{1-\frac{\alpha_1 s}{\pi}\left[\log\left(\frac{-s}{\mu^2}\right)-12C_2\right] \end{aligned} \tag{4a}$$

2 The heavy fermions are taken to be degenerate in order to avoid introducing isospin breaking terms.

$$+\frac{\alpha_1 s}{\pi}\left[\log\left(\frac{s}{\mu^2}\right)-\frac{1}{3}+12\left(C_2-4C_1\right)\right]\Bigg\}, \tag{4b}$$

$$a_2(s)\equiv a_{20}(s)=\alpha_2 s\Bigg\{1-\frac{\alpha_2 s}{\pi}\left[\log\left(\frac{-s}{\mu^2}\right)-12C_2\right]$$
$$-\frac{\alpha_2 s}{\pi}\left[\frac{11}{9}\log\left(\frac{s}{\mu^2}\right)-\frac{25}{54}-\frac{4}{3}\left(8C_1+7C_2\right)\right]\Bigg\}, \tag{4c}$$

where $\alpha_0\equiv\frac{1}{16\pi v^2}$, $\alpha_1\equiv\frac{1}{96\pi v^2}$, and $\alpha_2\equiv\frac{-1}{32\pi v^2}$. Each curly bracket consists of three terms, corresponding to the low-energy theorem, and the $O(s^2)$ contributions in the direct- and the crossed-channel respectively. *Note that we have been careful to preserve the crossing properties of the undetermined coefficients* [14].

Our unitarization scheme corresponds to a simple bubble-sum with amplitude given by

$$t_i(s)=\frac{\alpha_i s}{1+\frac{\alpha_i s}{\pi}\left[\log(\frac{-s}{\mu^2})+R_i(\mu^2)\right]}. \tag{5}$$

The R_i's are obtained by matching against the *direct-channel* piece of the chiral expansion. By inspection of Eq. (4) we find $R_0=-6(2C_1+C_2)$ and $R_2=R_1=-12C_2$. The "complementarity" between the I=1 and I=2 channels that follows from $R_2=R_1$ is investigated elsewhere in detail [13,14].

Inspection of Eq. (3) reveals that the I=1 and I=2 amplitudes are independent of the heavy fermion mass, in sharp contrast with the Padé result of Ref. 3 and Ref. 4. Only the I=0 amplitude has non-logarithmic contributions that depend on the Higgs and fermion masses. This is not surprising; the values of C_1 and C_2 given in Eq. (3) are the low-energy manifestation of a scalar-dominated theory. Unitarization simply restores the basic properties of the assumed underlying theory.

In Fig. 1 we schematically depict the complex s-plane. With a rather conservative choice of cutoff, given by $\Lambda=4\pi v\simeq$ (3 TeV), and with $M_H=M_f=$ 1 TeV, we see that the only pole in the theory is the "physical" Higgs boson. In Fig. 2 we display the partial wave amplitudes of definite custodial isospin for values of the tree Higgs mass of 0.75 TeV and 1 TeV. For values of M_f above 250 GeV, the fermionic contributions to the I=0 amplitude amount to a negligible renormalization of the physical Higgs mass, and so we neglect them in the graph. The complementary character of the non-resonant I=1 and I=2 amplitudes is clearly evident. We also display the Padé prediction for the I=1 amplitude, with M_H=0.75 TeV and M_f=1 TeV.

The approximation of neglecting crossed-channel contributions clearly works best near an s-channel pole. Since our primary goal is to investigate the possibility of a p-wave resonance for definite values of C_1 and C_2, this sort of approximation is ideally suited to the task. More importantly, we argue that if one wants to play the unitarization game, then one is *required* to make this approximation. We have argued that no S-matrix element should be uniquely determined by unitarity alone. Yet, we see in Eq. (5) that if t_1 is resonant, the width of the resonance is automatically fixed to the weak scale analogue of the KSRF relation [12]. However, we need not worry. This prediction is not a consequence of imposing elastic unitarity, but rather of neglecting the left-hand cut. This is easily seen by including left-hand cut contributions in a way that respects the low-energy structure of Eq. (4), and yet avoids double-counting of graphs [15]. Eq. (5) then becomes

$$t_i(s) = \frac{\alpha_i s + \beta_i s^2 \left[\log\left(\frac{s}{\mu^2}\right) + B_i(\mu^2)\right]}{1 + \frac{\alpha_i s}{\pi}\left[\log(\frac{-s}{\mu^2}) + R_i(\mu^2)\right] + \frac{\beta_i}{2\pi}s^2\left[\left(\log(\frac{-s}{\mu^2})\right)^2 + 2B_i(\mu^2)\log(\frac{-s}{\mu^2}) + M_i(\mu^2)\right]}, \tag{6}$$

where $\beta_0 \equiv (\frac{-7}{18\pi})(\alpha_0)^2$, $\beta_1 \equiv (\frac{1}{\pi})(\alpha_1)^2$, and $\beta_2 \equiv (\frac{-11}{9\pi})(\alpha_2)^2$ (see Eq. (4).) The B_i are the low-energy constants associated with heavy particle exchange in the crossed-channel. The M_i are undetermined constants that appear at two-loop order in the chiral expansion. We see that it is by neglecting the contribution to the imaginary part of the inverse amplitude involving B_1 that we are able to predict the KSRF relation. Therefore, the predictive power of Eq. (5) is not a result of imposing unitarity, but rather a result of neglecting a class of graphs associated with heavy particle exchanges in the crossed-channel, which are manifest at $O(s^2)$ in the chiral expansion. It is important to note that the above does not constitute a new derivation of the KSRF relation. In fact, all justifications of the KSRF relation, including the original current algebra derivation [16] , *require* the tacit assumption that the left-hand cut of the I=1 scattering amplitude is effectively absent [17]. We find it powerful evidence in favor of our scheme that, by ensuring that predictive power come from a source other than elastic unitarity, we arrive at a consistent derivation of the KSRF relation.

The method of Padé approximants, as applied in Refs. [1-4], also predicts the KSRF relation in the I=1 channel, and yet the $O(s^2)$ crossed-channel contributions are *included.* Therein lies its downfall; the neglect of crossed-channel contributions can no longer serve as the source of predictive power, and so the crossed-channel contributions necessarily appear in the wrong place. Yet if this is the case, then why do both unitarization schemes of the bubble-sum type and the Padé method provide a good parametrization of the π-π phase shift data? The reason is straightforward. One can say that the bubble-sum method works well because the crossed-channel contributions which are neglected are small, whereas the Padé method works well because the crossed-channel contributions *which are included in the wrong place* are small. Since these misplaced contributions appear in the real part of the inverse amplitude, in the current context it is quite understandable that unphysical poles are present. Of course, there are well defined instances in field theory where crossed-channel contributions decouple. For example, the O(N) model is exactly solvable to leading order in $\frac{1}{N}$ precisely because left-hand cut contributions first appear at $O(\frac{1}{N^2})$ [18]; not surprisingly, to leading order in $\frac{1}{N}$, the [1,1] Padé approximant yields the exact result [19]. In this spirit, it is interesting to note that if we assume that the crossed-channel contributions that appear at $O(s^2)$ in Eq. (4) are much smaller than the direct-channel contributions at the same order, then our unitary amplitude, Eq. (5), *is* the [1,1] Padé approximant of Eq. (4). However, our unitary amplitude with crossed-channel contributions included, Eq. (6), is clearly unrelated to any Padé approximant. The moral of this story is that the Padé method, which is ideally suited to problems in potential theory, should be applied only with great care to problems where crossing symmetry is important.

Our conclusions are not surprising. The effective field theory viewpoint implies that one gets out essentially what one puts in. Once we saturate the low-energy constants of chiral perturbation theory with

contributions from a scalar-dominated underlying theory, information regarding the intermediate-energy spectrum is, in a sense, exhausted. The elastic unitarity constraint does not, and should not, change the character of the assumed underlying theory, albeit a strongly interacting one, e.g., by inducing a prominent vector contribution.

Acknowledgements

We are grateful to D. Chao, C.B. Chiu, D. Dicus, E.C.G. Sudarshan, S. Thomas, and U. van Kolck for useful discussions and suggestions. This work was supported in part by the Department of Energy Grant No. DEFG05-85ER40200.

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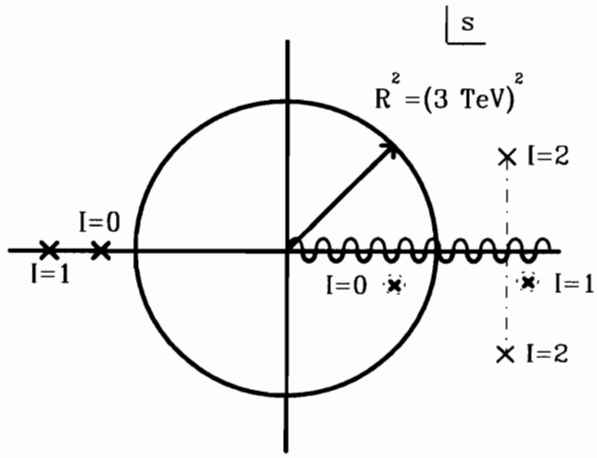

Figure 1: Schematic depiction of the complex s-plane for characteristic values of the input parameters, $M_H = M_f = 1$ TeV. The only pole below the cutoff is the physical Higgs pole.

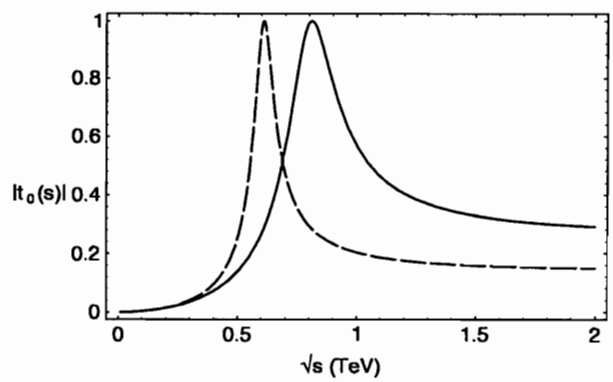

Figure 2a: I=0 s-wave amplitude. The dashed line corresponds to M_H=0.75 TeV and the solid line to M_H=1 TeV.

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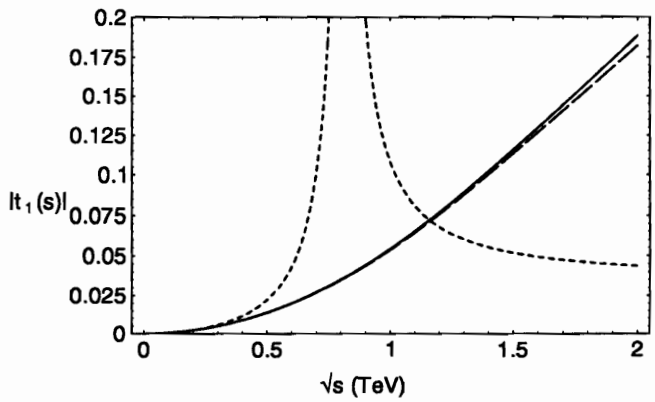

Figure 2b: I=1 p-wave amplitude. The dashed line corresponds to M_H=0.75 TeV and the solid line to M_H=1 TeV. The dotted line corresponds to the Padé method prediction for M_H=0.75 TeV and M_f=1 TeV

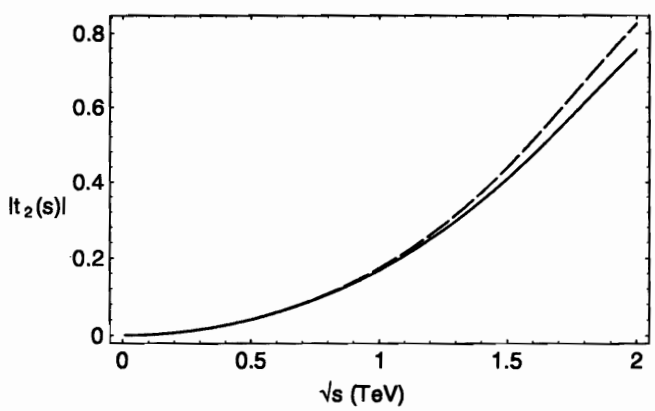

Figure 2c: I=2 s-wave amplitude. The dashed line corresponds to M_H=0.75 TeV and the solid line to M_H=1 TeV.