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# STRING QUANTUM GRAVITY

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## 1. INTRODUCTION

Perhaps the main challenge in theoretical physics today is the unification of all interactions including gravity. At present, string theories appear as the best candidates to achieve such an unification. However, several technical and conceptual problems remain and a quantum theory of gravity is still non-existent. Continuous effort over the last quarter of a century has demonstrated the many difficulties encountered in repeated attempts to construct such a theory and has also indicated some of the particular properties which an eventual complete theory will have to posses. The amount of work in that direction can be by now presented in two different sets which have most evolved (and remain) separated: (i) conceptual unification (introduction of the uncertainty principle in general relativity, the interpretation problem and the concept of "observables", Q.F.T. in curved space time and by accelerated observers, Hawking radiation and its consequences , the Wheeler-De Witt equation and the "wave function of the universe"...) (ii) grand unification (the unification of all interactions including gravity from the particle physics point of view, in which, gravity is considered as a massless spin two particle (the graviton), such as in supergravities, Kaluza-Klein theories and the more succefull: superstrings).

Most of the work in the part (i) ("conceptual unification") treats gravity in the context of point particle field theory, that is what we call conventional quantum gravity. Few are the works in such part, which have incorporated the novelty of strings. On the other hand, most of the work done on strings do not treat the connection with the main problems of quantum gravity. (The main motivation and the impact of modern string theory is to give a consistent quantum theory of gravity, but, unfortunately, most of the work done on strings do not address to this problem).

Whatever the final theory of the world will be, if it is to be a theory of everything, we would like to know what new understanding it will give us about the singularities of classical general relativity. If string theory would provide a theory of quantum gravity, it should give us a proper theory (not yet existent) for describing the ultimate state of quantum black holes and the initial (very early) state of the universe. That is, a theory describing the physics (and the geometry) at Planck energies and lengths.

## 2. A QUANTUM THEORY OF GRAVITY MUST BE FINITE

Many attempts have been done to quantize gravity. The problem most often discussed in this connection is the one of renormalizability of Einstein theory (or its various generalizations) when quantized as a local quantum field theory. Actually, even deeper conceptual problems arise when ones tries to combine quantum concepts with General Relativity. Let us begin by an argument showing conceptually that a consistent quantum theory of gravity must be *finite*. In other words, it is not possible to conceive a renormalizable Q.F.T. when the gravitational interaction is included.

What is a renormalizable Q.F.T.? This is a theory with some domain of validity characterized by energies E such that $E < \Lambda$. Here, the scale $\Lambda$ is characteristic of the model under consideration: (e.g., $\Lambda = 1$ Gev for QED, 100 Gev for the standard model or $10^{16}$ Gev for GUT, etc. One always applies the QFT in question till infinite energy (or zero distance) for virtual processes and finds usually ultra-violet infinities. These divergences reflect the fact that the model is unphysical for energies $\Lambda << E << \infty$. In a renormalizable QFT these infinities can be absorbed in a finite number (usually few) parameters like coupling constants and mass ratios, which are not predicted by the model in question. One would need a more general theory valid at energies beyond $\Lambda$ in order to compute these renormalized parameters (presumably from others more fundamental). For example, $M_W/M_Z$ is calculable in a Grand Unified Theory, whereas it must be fitted to its experimental value in the standard electro-weak model.

Now, what about quantization of gravity ? The relevant energy scale is the Planck mass ($M_{Planck} \sim 10^{19}$ Gev). At this mass, the Schwarzschild radius ($r_s$) equals the Compton wavelength ($\lambda_c$) of a particle. Then, if we imagine particles heavier than $M_{Planck}$, their size $\lambda_c$ will be smaller than $r_s$. In other words, to localize them in a region of size $\lambda_c$ will be in conflict with what we know about the Schwarzschild radius from general relativity. Such heavy objects ($M_{Planck} \sim 10^{-5}$ g) can not behave (if they really exist) as usual point particles do in relativistic QFT. This means that $M_{Planck}$ gives the order of magnitude for the heaviest point particles. There cannot be point particles beyond $M_{Planck}$ in a relativistic QFT as soon as gravity is included. This shows that we can not conceive a renormalizable QFT including gravity since there can not exist a theory at energies higher than $M_{Planck}$ whose ignorance is responsible for the infinities of quantum gravity. (If ultraviolet divergences appear in a quantum theory of gravity, there is no way to interpret them as coming from a higher energy scale as it is usually done in QFT). Hence, a consistent theory including gravity must be *finite*. All dimensionless physical quantities must be computable in it. These conceptual arguments are consistent with all failed attempts to construct renormalizable field theories of quantum gravity.

Another consequence of these arguments is the following: Since a quantum theory of gravitation would describe the highest possible mass scale, such model must also include all other interactions in order to be consistent and true. That is, one may ignore higher energy phenomena in a low energy theory, but the opposite is not true. To give an example, a theoretical prediction for graviton-graviton scattering at energies of the order of $M_{Planck}$ must include *all* particles produced in a real experiment. That is, in practice, all existing particles in nature, since gravity couples to *all matter*.

These simple arguments, based on the renormalization group [1] lead us to an important conclusion: a consistent quantum theory of gravitation must be a *theory of everything* (TOE). So rich a theory should be very complicated to find and to solve. In particular, it needs the understanding of the present desert beween 1 and $10^{16}$ Gev. There is an additional dimensional argument about the inference that a Quantum theory of gravity implies a TOE. There exist only three fundamental physical magnitudes: length, time and energy and hence three fundamental dimensional constants: c, h and G. All other parameters being dimensionless, they must be calculable in a unified quantum theory including gravity, and therefore, a theory like this must be a TOE. From the purely theoretical side the only serious candidate at present for a TOE is string theory. Ŭnfortunately, most of the research work done on strings consider the strings in Minkowski space-time. All string models exhibit particle spectra formed by tower of massive particles going up to infinite mass and hence passing by $M_{Planck}$. If these states are to be considered as point particles, one arrives, for masses larger than $M_{Planck}$, to the clash between general relativity and quantum mechanics described above. A solution of this paradox could be that the particle spectrum of string models is at energies $E >> M_{Planck}$ very different from what we know today on the basis of perturbation theory in flat (10 or 26 dimensional) space-time. The results we will present here about strings in strong gravitational fields support this suggestion.

Since the most relevant new physics provided by strings concerns quantization of gravity, we must, at least, understand string quantization in a curved space-time. Actually, one would like to extract the space-time and the particle spectrum from the solution of string theory, but we are still far from doing that explicitly.

Practically, all what we know about strings comes from their study in flat critical (10 or 26) dimensional space-time. It must be noticed that expanding in perturbation around the Minkowski metric is not better since the non-trivial features appear in the strong curvature regimes, in the presence of horizons and of the intrinsic singularities. Curved space-times, besides their evident relevance in classical gravitation are also important at energies of the order of the Planck scale. At such energy scales, the picture of particles propagating in flat space-times is no longer valid and one must take into account the curved geometry created by the particles themselves. In other words, gravitational interactions are at least as important as the rest and can not anymore being neglected as it is usually the case in particle physics.

As a first step in the understanding of quantum gravitational phenomena in the framework of string theory, we started in 1987 a programme of string quantization in curved space-times. A summary of the developpements and results till now in this programme is given in what follows.

## 3. STRINGS IN PHYSICALLY RELEVANT CURVED SPACE-TIMES

Until now, gravity has not completely been incorporated in string theory: strings are more frequently formulated in *flat* space-time. Gravity appears through massless spin two-particles (graviton). One disposes only of partial results for strings in curved backgrounds, and these mainly concern the problem of consistency (validity of quantum conformal invariance) through the vanishing of the beta-functions. The non linear quantum string dynamics in curved space-time has only been studied in the slowly varying approximation for the geometry (background field method) where the field propagator is essentially taken as the flat-space Feynman propagator.

Clearly, such approximations are useless for the computation of physical quantities (*finite parts*) such as the mass operator, scattering amplitudes and critical dimension in strong-curvature regimes. Our aim is to properly understand strings in the context of gravity (classical and quantum). As a first step in this program we proposed to study Q.S.T. (Quantum String Theory) in curved space-times. There are different kind of effects to be considered here :

*1) ground state and thermal effects :* these are associated to the fact that in general relativity there are no preferred reference frames, and one has the possibility of having different choices of time. This arises the possibility for a given quantized field or string theory to have different alternative well defined Focks spaces (different "sectors" of the theory), (which may be or may be not related by Hawking radiation). Associated with this: The presence of "intrinsic" statistical features (temperature, entropy) arising from the non-trivial structure (geometry, topology) of the space-time and not from a superimposed statistical description of the quantum matter fields themselves. For a detailed account of them see refs [2-4] and sections below.

*2) Curvature effects :* these will modify the mass spectrum, the critical dimension and the scattering amplitudes of the strings. We will discuss them below.

*3) Conceptual aspects :* In addition to those discussed above there are conceptual aspects to consider here, which appear when strings are restricted to live in causally disconnected regions of the space-time, that is in the presence of event horizons, and which imply quantum fluctuations of the event horizon and of the light-cone itself.

*4)* Finally, a word of warning on *the question of conformal invariance* in the metrics we consider. It is well known [5] that for a single string moving in a background, the conditions of quantum conformal invariance coincide with the *vacuum* Einstein equations ( modulo string corrections). As such they require, at tree level, $R_{\mu\nu} =0$. Certainly, de Sitter space time , cosmological backgrounds, gravitational shock-waves,... do not satisfy such a condition. Our interpretation of this point [6] is that such backgrounds are simply *not* candidate string ground states (vacua). Yet these physically relevant metrics can play the role of effective backgrounds felt by a single string as it moves in the presence of many others. An example of a situation of this kind is the effective Aichelburg-Sexl (AS) metric felt by one string as it collides with another at very high (Planck) energies [7-9]. Here too one would not identify the AS metric as a possible string vacuum; nonetheless, this metric is physically relevant to the description of the planckian energy collision process.

The price to pay for simplifying the true many-body problem into that of a single string moving in a effective, external metric will be indeed some (hopefully small) violation of unitarity. The main physics conclusions should, however, retain their validity at some semi-quantitative level, especially in the region of validity of a semi-classical approximation (since the conformal anomaly is an $O(\hbar^2)$ effect ).

In flat space-time, the string equations of motion are linear and one can solve them explicitely, as well as the quadratic constraints. It should be recalled that the constraints in string theory contain as much physical information as the equations of motion. In curved space-time, the string equations of motion are highly non-linear (these equations are of the type of non-linear sigma models) and they are coupled to the constraints. Thus, right and left movers interact with each other and also with themselves. In flat Minkowski space-time, it is always possible to choose a gauge in which the physical time $X^0(\sigma,\tau)$ (or a light cone combination of it) and the world -sheet time $\tau$ are identified. In curved space-time, the relation between the world sheet time $\tau$ and the physical time $X^0(\sigma,\tau)$ is involved, and in general not exactly known. However, it is possible to find the proportionality between $\tau$ and $X^0$ in some well defined asymptotic regimes or space-time regions. For all the physically relevant space-times we have studied, we have found well defined regimes in which $X^0(\sigma,\tau) \approx \tau$. In ref [10] , we have proposed a general scheme to solve the string equations of motion and constraints, both classically and quantum mechanically. The principle is the following: we start from an exact particular solution and develop in perturbations around it. We set

$$X^A(\sigma,\tau) = q^A(\sigma,\tau) + \eta^A(\sigma,\tau) + \zeta^A(\sigma,\tau) + \ldots..$$

Here $q^A(\sigma, \tau)$ is an exact solution of the string equations and $\eta^A(\sigma,\tau)$ obeys the linearized perturbation around $q^A(\sigma,\tau)$. $\zeta(\sigma,\tau)$ is a solution of the second order perturbation around $q^A(\sigma,\tau)$. Higher order perturbations can be considered systematically. The choice of the starting solution is upon physical insight. Usually, we start from the solution describing the center of mass motion of the string $q^A(\tau)$, that is the point particle (geodesic) motion. The world-sheet time variable is here identified with the proper time of the center of mass trajectory. Even at the level of the zero order solution, gravitational effects including those of the singularities of the geometry are fully taken into account.It must be noticed that in our method, we are treating the space-time geometry *exactly* and taking the string oscillations around $q^A$ as perturbation. So, our expansion corresponds to low energy excitations of the string as compared with the energy associated to the geometry. In a cosmological or in a black hole metric, our method corresponds to an expansion in $\omega/M$, where $\omega$ is the string frequency mode and M is the universe mass or the black hole mass respectively. This can be equivalently considered as an expansion in powers of $(\alpha')^{1/2}$. Actually, since $\alpha' = (l_{Planck})^2$, the expansion parameter turns out to be the dimensionless constant

$$g = l_{Planck} / R_c = 1 / ( l_{Planck} M) = \omega / M$$

where $R_c$ characterizes the curvature of the space-time under consideration and M its associated mass (the black-hole mass or the mass of the Universe in the cases before mentioned). So, our expansion is well suited to describe strings (test strings) in strong gravitational fields. In most of the interesting situations, one clearly has $g \ll 1$.

The constraint equations must also be expanded in perturbations. The classical $(\text{mass})^2$ of the string is defined through the center of mass motion (or Hamilton-Jacobi) equation. The conformal generators (or world-sheet two dimensional energy-momentum tensor) are bilinear in the fields

$\eta^A$ $(\sigma,\tau)$. In order to obtain these constraints to the lowest non-trivial order it is necessary to keep first-and second-order fluctuations. Notice the difference with field theory where the first order fluctuations are enough to get the leading order approximation around a classical solution.

If we would like to apply our method to the case of flat spacetime, the zero order solution $q^A(\tau)$ plus the first order fluctuations $\eta^A$ $(\sigma,\tau)$ would provide the exact solution of the string equations.

### 3.1 Strings in Black-Hole Space-times

We have applied our method to describe; the quantum string dynamics in the Schwarzschild geometry and computed the effects of the scattering and interaction between the string and the black hole [11]. We have analyzed the string equations of motion and constraints both in the Schwarzschild and Kruskal manifolds and their asymptotic behaviours. The center of mass motion is explicitely solved by quadratures. We found the first and second order quantum fluctuations, $\eta$ and $\zeta$ around the center of mass solution. We give an "in" and "out" formulation of this problem. We define an in-basis of solutions in which we expand first and second-order quantum fluctuations and define left and right oscillation modes of the string in the asymptotically flat regions of the space-time. The ingoing solutions are defined by selecting the behaviour at $\tau \to -\infty$ equal to a purely positive frequency factor $e^{-in\tau}$ (in-particle states). The ingoing ( $\tau \to -\infty$ ) and outgoing ( $\tau \to +\infty$ ) coefficient modes are related by a linear-Bogoliubov transformation describing transitions between the internal oscillatory modes of the string as a consequence of the scattering by the black hole. We find two main effects: (i) a change of *polarization* of the modes without changing their rigth or left character and (ii) a *mixing* of the particle and antiparticle modes changing at the same time their right or left character. That is, (i) if in the ingoing state, the string is in an excited mode with a given polarization, then in the outgoing state, there will be non-zero amplitudes for modes polarized in any direction and (ii) an amplitude for an *antiparticle* mode polarized in the same direction but with the right (or left) character reversed.

We have studied the conformal generators ($L_n$) and the constraints. An easy way to deal with the gauge invariance associated with the conformal invariance on the world sheet is to take the light-cone gauge in the ingoing region. Time evolution from $\tau \to -\infty$ to $\tau \to +\infty$ conserves the physical or gauge character of the modes. The independent physical excitations are those associated with the transverse modes. We solve for the second-order fluctuations in a mode representation, and then we get the conformal generators. These generators can be computed in terms of the ingoing basis ( $\tau \to -\infty$ ) or alternatively, with the outgoing basis ( $\tau \to +\infty$ ). The conservation of the two-dimensional energy-momentum tensor yields $L_n{}^{in} = L_n{}^{out} + \Delta L_n$ , where $\Delta L_n$ describes excitations between the internal (particle) states of the string due to the scattering by the black hole. We find the mass spectrum from the $L_0 = 0$ constraint, which is formally the *same* as in flat space-time. This is a consequence of the *asymptotically flat* character of the space-time and of the absence of bound states for D>4. (If bound states would exist, they would appear in the $(\text{mass})^2$ operator besides the usual flat space spectrum). The critical dimension at which massless spin-two states appear is D=26, the *same* as in flat space-time.

We have studied the elastic and inelastic scattering of strings by a black hole. The Bogoliubov coefficient $A_n$ describes elastic processes and $B_n$ inelastic ones. By elastic amplitudes we mean that the initial and final states of the string corresponding to the nth mode are the same. We find that pair creation out of the in-vacuum takes place for $\tau \to +\infty$ as a consequence of the scattering by the black hole. (Each pair here is formed by a right and left mode). The explicit computation of the coefficients $A_n$ and $B_n$ has been performed in an expansion at first order in $(R_s/b)^{D-3}$ , $R_s$ being the Schwarzschild radius and, b the impact parameter of the center of mass of the string. The elastic scattering cross section of the stringby the black hole is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} = \left(\frac{d\sigma}{d\Omega}\right)_{c.m} \mid < n^{out} \mid n^{in} > \mid^2 = \left(\frac{d\sigma}{d\Omega}\right)_{c.m} \left[1 - \sum_{n' \neq n} \mid B_{n'} \mid^2\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} = \left(\frac{d\sigma}{d\Omega}\right)_{c.m} \left[1 + \left(\frac{\pi p \alpha'}{b}\right)^2 \left(\frac{R_s}{b}\right)^{2(D-3)}\right] \sum_{n' \neq n} \left| F\left(\frac{m}{p}, \frac{n'b}{\alpha' p}\right) \right|^2$$

where $\left(\frac{d\sigma}{d\Omega}\right)_{c.m} = C_D \; R_s^{\,D-2} \; \Theta^{-(D-1+1/(D-3))} \left[1 + \left(\frac{D-3}{D-2}\right)\frac{m^2}{p^2}\right]^{(D-2)/(D-3)}$

$$C_D = (D-3) \; [\; (\pi)^{1/2} \Gamma(D/2)/\Gamma((D-1)/2) \;]^{(D-2)/(D-3)}$$

Here, the function F is a dimensionless number [11] and $\alpha'$ is the string tension. The factor $C_D$ is typical of D-dimensions. $(d\sigma / d\Omega)_{c.m}$ is the center of mass scattering cross section for large impact parameter b and small scattering angle $\Theta$. For D=4, we recover the analogue of Rutherford's formula:

$$\Theta_{D=4} = \frac{R_s}{b}\left(2 + \frac{m^2}{p^2}\right) \quad , \quad \frac{d\sigma}{d\Omega}_{D=4} \underset{\Theta \ll 1}{=} \frac{4 R_s^2}{\Theta^4} \left(1 + \frac{m^2}{2p^2}\right)^2$$

We see that quantum string corrections to the Rurherford's scattering are of order $\alpha'^2$. Quantum corrections to the scattering of particles by a black hole have been obtained previously in the framework of point-like particles [12].

It should be noticed that in the point particle theory, the interaction of particles with the Schwarzschild geometry is static. Pair creation only takes place via Hawking radiation. In contrast, the interaction of strings with the Schwarzschild geometry exhibits *new* features due to the *composite* character of the strings. The infinite set of oscillator modes constituting the string becomes excited during the scattering by the influence of the black hole field. Actually, any localized external field would lead to qualitatively similar effects. As a result, particle transitions between the ingoing and the outgoing final states take place, giving rise to the phenomenom of *particle transmutation* [11,13]. A more detailed description of this phenomenon is given below. This effect is proportional to $B_n{}^2$ and describes inelastic scattering since the final state is different from the initial one. It must be noticed that $B_n$ is of order of $\alpha'$ and therefore this is a very small effect at least energies of order of Planck energy are reached. This particle transmutation effect is a genuine *stringy* effect which *does not* exist in the context of point-particle field theory. On the other hand, this effect is *not* related to Hawking radiation and the presence of an event horizon is not essential here. The Hawking radiation and related phenomena appearing in usual quantum field theory have an analogue in string theory [2-4]. They are linked to the possibility for a given field or string theory to have different non-equivalent descriptions (different choices of the physical time and thus different possible definitions of particle and vacuum states).

### 3.2 Particle Transmutation from the Scattering of Strings and Superstrings in Curved Space-time

More recently, we have given a general formulation of the scattering of strings by a curved space-time, for both open and closed bosonic strings and for supersymmetric ones [13]. We consider space-times which admit flat regions in order to define ingoing and outgoing scattering states, as it is the case for the shock-wave space-times, for the space-time of very thin and straight cosmic strings, and for asymptotically flat geometries. Due to the interaction with the geometry, the string excitation state changes from the ingoing to the outgoing situation. Therefore, if the string ingoing state described a given particle with mass m and spin s, the outgoing state may describe a different particle with mass m' and spin s'. When (m,s) = (m',s') this is an elastic process, and we have the same particle state in the initial and final states, although the momentum and spin polarization may change. Otherwise, the process is *inelastic;* the initial particle (m,s) *transmutes* into a different final one (m',s'). At first order in $(\alpha')^{1/2}$, ($\alpha'$ being the string tension), for string oscillations small compared with the energy scales of the metric, outgoing and ingoing oscillator operators are related by a linear, or Bogoliubov transformation. For open strings, transitions from the ground state to a state with an *even* number of creation operators are non-zero. (These final states describe very heavy particles of mass greater than the Planck mass). Transitions from the ground state to states with an *odd* number of creation operators *vanish*. For closed strings, the more relevant transitions are those from the ground state to a dilaton, to a graviton and to massive states, and the transmutation of a *dilaton into a graviton*. For supersymmetric strings, massless particles cannot transmute among themselves (this is true to all orders in $(\alpha')^{1/2}$). Several properties of the particle transmutation processes can be derived directly from the symmetry properties of the geometry. Particle transmutation amplitutes for strings in black hole spacetimes and gravitational shock-waves have been explicited computed in ref.[13].

### 3.3 String Propagation through Gravitational Shock Waves

Recently, gravitational shock wave backgrounds have raised interest in the context of both field theory and strings [7-9,14-21]. These metrics are relevant to the particle scattering at the Planck energy scale. They correspond to boosted geometries in the limit in which the velocity of the source tends to the speed of light and the mass of the source tends to zero in an appropriate way. The ultrarelativistic limit of the Schwarschild solution is the Aichelburg-Sexl geometry (point particle source). The gravitational shock waves corresponding to the ultrarelativistic limit of the

Kerr-Newmann geometry, to ultrarelativistic cosmic strings and other ultrarelativistic extended sources, have been recently found [17,18].
Remarkable enough, the string equations of motion [8,9] and the Klein-Gordon equation [14, 8] have been *exactly* solved in this geometry. The mass spectrum and the critical dimension are the same as in flat space time but there is non-trivial elastic and inelastic scattering of the string by the shock wave [8]. We have found the *exact non-linear* transformation relating the ingoing ( $\tau < 0$ ) and outgoing ( $\tau > 0$ ) string mode operators (and zero modes) before and after the collision with the gravitational the shock wave. This transformation contains all the information about the scattering and interaction of the string with the shock wave geometry. The linearized transformation at first order in $(\alpha')^{1/2}$ is a Bogoliubov transformation. As in the black-hole case, transitions take place between the internal modes of the string. Here too, corrections to the point particle scattering cross section are of order $\alpha'^2$. For large impact parameters, the scattering angle and cross section in the black hole and in the shock wave geometry are very similar.

More recently, we have performed this treatment for a general shock wave space time of *any localized source*. We have computed the *exact* expectation values of the total number and mass square operators of the string and show that they are *finite*, which generalize ours previous results in the Aichelburg-Sexl geometry [20,21]. We have studied the energy-momentum tensor of the string, computed the expectation values of all its components and show they are *finite*. The ingoing-outgoing ground state transition amplitude $< O_< | O_> >$ expresses as a sum of terms, which can be interpreted [20,21] as a n-leg scalar amplitude with vertex operators inserted at $\tau=0$ (a line of pinchs at the intersection of the world sheet with the shock wave), and for which we have found integral representations. The integrands posses equally spaced real pole singularities *typical* of string models in flat space-time. The presence and structure of these poles *is not* at all related to the structure of the space time geometry (which may or may be not singular). We give a sense to these integrals by taking the principal value prescription, yielding a well defined finite result. For the expectation values of the mass and number operators, we find similar integral representations. The integrands factorizes into two pieces: a part (given by the Fourier transform of the density matter of the source) which characterizes the shock wave geometry and the function

$$tg ( \alpha' \pi p^2 ) \Gamma (\alpha' p^2) / \Gamma ( \alpha' p^2 + 1/2 ) \ ,$$

which depends only of the string. This integrand posses real singularities (poles) like the tree level *string* spectrum. A *physical* interpretation of such poles is that they correspond to all higher string states which become excited after the collision through the shock wave. Moreover, the quantum expectation values of the string energy-momentum tensor $T^{AB}(X)$ admit similar integral representations (with the same structure of integrands and poles). Instead of studying the local dependence (as it would be appropriated for classical or cosmic strings), we integrate $T^{AB}(X, X^0)$ over an spatial volume completely closing the string at time $X^0$, since the fundamental string here describes particle states. That is, we define

$$\tau^{AB} ( X^0 ) = (-G)^{1/2} T^{AB} ( X, X^0 ) d^{D-1} X$$

For asymptotic times $X^0 \to \infty$, we find

$$\tau^{AB} = p^A p^B /p^0 \ ,$$

which is precisely the energy-momentum tensor of a point particle integrated over a spatial volume. Even for $X^0 \to \infty$, the $\tau^{AB}$ of the string is not trivial because of the constraints $P^{0\,2} = X'^{\,2} + X^{\,2}$. We find vacuum polarization effects induced by the shock wave on the string oscillators. There is a stress in the longitudinal and transverse directions . We have also computed the *fluctuations*

$$( \Delta\tau^{AB} )^2 = < \tau^{AB\ 2} > - < \tau^{AB} >^2$$

These fluctuations are *finite* and *non-zero* even for the AB components where $< \tau^{AB} > = 0$. In particular, for the energy density, the expectation value is trivial (it coincides with the flat space time value equal to the mass) but exhibits *non-trivial* quantum fluctuations.

We want here to compare these string results with those known for point particle QFT in shock-wave backgrounds [22]. For point particle QFT, no vacuum polarization effects arise in these backgrounds since the ingoing (< ) and outgoing(>) creation and annhilation operators do not get mixed (there is no Bogoliubov transformation in such context). Therefore, no particle creation effects takes place for point particle field theories in these geometries. On the contrary, for strings particle transmutations as well as polarization effects on the energy-momentum tensor appear in shock-wave space times. These effects can be traced back to the mixing of creation and annhilation in (<) and out (>) string oscillators.

### 3.4 Strings in Cosmological Backgrounds

We have quantized strings in de Sitter space-time first [10] . We have found the mass spectrum and vertex operator. The lower mass states are the same as in flat space-time up to corrections of order $g^2$ but heavy states *deviate* significantly from the linear Regge trajectories. We found that there exists a *maximum* (very large) value of order $1/g^2$ for the quantum number N and spin J of particles. There exists real mass solutions only for

$$N < N_{max} = \pi / 2g^2 + O( g^{-2/3}) \quad , \quad g = 10^{-61} .$$

Moreover, for states in the leading Regge trajectory, the mass monotonously increases with J up to the value

$$J_{max} = 1/ g^2 + O(1)$$

corresponding to the maximal mass $m^2_{max} = 0.76 + O(g^2)$ . Beyond $J_{max}$ the mass becomes complex. These complex solutions correspond to unstable states already present here at the tree ( zero handle) level.

From the analysis of the mass spectrum, we find that the critical dimension for bosonic strings in de Sitter space-time is D=25 (instead of the value 26 in Minkowski space-time). This result is confirmed by an independent calculation of the critical dimension from the path intergral Polyakov's formulation, using heat-kernel techniques: we find that the dilaton $\beta$- function in D-dimensional de Sitter space-time must be

$$\beta^{\phi} = ( D + 1 - 26 ) / ( \alpha' 48\pi^2 ) + O(1)$$

It is a general feature of de Sitter space-time to lower the critical dimensions in one unit. For fermionic strings we find D=9 instead of the flat value D= 10.

We have found that for the first order amplitude $\eta^i(\sigma, \tau)$, ( i = 1,....D-1 refers to the spatial components), the oscillation frequency is

$$\omega_n = [ n^2 - (\alpha' m H)^2 ]^{1/2} ,$$

instead of n, where H is the Hubble constant. For high modes $n >> \alpha' mH$, the frequencies $\omega_n \sim n$ are real. The string shrinks as the universe expands. This shrinking of the string cancels precisely the expanding exponential factor of the metric and the invariant spatial distance does not blow up. Quantum mechanically, these are states with real masses ($m^2H^2<1$). This corresponds to an expansion time $H^{-1}$ very much bigger than the string period $2\pi/n$, that is, many string oscillations take place in an expansion period $H^{-1}$ (in only one oscillation the string does not see the expansion).

For low modes $n < \alpha' mH$, the frequencies become imaginary. This corresponds to an expansion time very short with respect to the oscillationtime $2\pi/n$ ("sudden" expansion, that is the string "does not have time" to oscillate in one time $H^{-1}$). These *unstable* modes are analyzed as follows. The n=0 mode describes just small deformations of the center of mass motion and it is therefore a physically irrelevant solution. When $\alpha' mH > 1$ relevant unstable modes appear. Then, the n=1 mode dominates $\eta^i(\sigma,\tau)$ for large $\tau$. Hence, if $\alpha' mH > (2)^{1/2}$, $\eta^i$ diverges for large $\tau$, that is fluctuations become larger than the zero order and the expansion breaks down. However, the presence of the above unstability is a true feature as it has been confirmed later by further analysis [6].

The physical meaning of this instability is that the string grows driven by the inflationary expansion of the universe. That is, the string modes couples with the universe expansion in such a way that the string inflates together with the universe itself. This happens for inflationary (ie accelerated expanding) backgrounds. In ref. [6] we have studied the string propagation in Friedman-Robertson-Walker (FRW) backgrounds (in radiation as well as matter dominated regimes) and interpreted the instability above discussed as Jeans-like *instabilities*. We have also determined under which conditions the universe expands,when distances are measured by stringy rods. It is convenient to introduce the *proper amplitude* $\chi^i = C\eta^i$, where C is the expansion factor of the metric. Then, $\chi^i$ satisfies the equation

$$\ddot{\chi}^i + [\, n^2 - \ddot{C}/C \,]\, \chi^i = 0$$

Here dot means $\tau$-derivative. Obviously, any particular (non-zero) mode oscillates in time as long as $\ddot{C}/C$ remains $< 1$ and, in particular, when $\ddot{C}/C < 0$. A time- independent amplitude for $\chi$ is obviously equivalent to a fixed proper ( invariant) size of the string. In this case, the behaviour of strings is stable and the amplitudes $\eta$ shrink (like $1/C$).

It must be noticed that the time component, $\chi^0$ or $\eta^0$, is always well behaved and no possibility of instability arises for it. That is the string time is well defined in these backgrounds.

i) For non-accelerated expansions (e.g. for radiation or matter dominated FRW cosmologies) or for the high modes $n >> \alpha' Hm$ in de Sitter cosmology, string instabilities do not develop (the frequencies $\omega_n \sim n$ are real). Strings behave very much like point particles: the centre of mass of the string follows a geodesic path, the harmonic-oscillator amplitudes $\eta$ shrink as the univers expands in such a way to keep the string's proper size constant. As expected, the distance between two strings increase with time, relative to its own size, just like the metric scale factor C.

ii) For inflationary metrics (e.g. de Sitter with large enough Hubble constant), the proper size of the strings grows (like the scale factor C) while the co-moving amplitude $\eta$ remains fixed ("frozen"), i.e. $\eta \sim \eta(\sigma)$.

Although the methods of references [10] and [6] allow to detect the onset of instabilities, they are not adequate for a quantitative description of the high instable (and non-linear) regime. In ref. [23] we have developped a new quantitave and systematic description of the high instable regime. We have been able to construct a solution to both the non-linear equations of motion and the constraints in the form of a systematic asymptotic expansion in the large C limit, and to classify the (spatially flat) Friedman-Robertson Walker (FRW) geometries according to their compatibility with stable and/or unstable string behaviour. An interesting feature of our solution is that it implies an asymptotic proportionality between the world sheet time $\tau$ and the *conformal time* T of the background manifold. This is to be contrasted with the stable (point-like) regime which is characterized by a proportionality between $\tau$ and the *cosmic time*. Indeed, the conformal time (or $\tau$) will be the small expansion parameter of the solution: the asymptotic regime (small $\tau$ limit) thus corresponds to the large C limit only if the background geometry is of the inflationary type. The non linear, high unstable regime is characterized by string configurations such that

$$|X'^{o}| << |\dot{X}^{o}| \quad , \quad |\dot{X}^{i}| << |X'^{i}|$$

with $$X^{o}(\sigma, \tau) = C\,L(\sigma) \quad , \quad L(\sigma) = (\delta_{ij}\, X'^{i} X'^{j})^{1/2}$$

$$X^{i}(\sigma,\tau) = A^{i}(\sigma) + \tau^{2} D^{i}(\sigma)/2 + \tau^{1+2\alpha} F^{i}(\sigma)$$

where $A^{i}$, $D^{i}$ and $F^{i}$ are functions determined completely by the constraints, and $\alpha$ is the time exponent of the scale factor of the metric : $C = \tau L^{-\alpha}$.

For power-law inflation : $1 < \alpha < \infty$, $X^{o} = \tau L^{1-\alpha}$.

For de Sitter inflation : $\alpha = 1$, $X^{o} = \ln(-\tau HL)$.

For Super-inflation: $0 < \alpha < 1$, $X^{o} = \tau L^{1-\alpha} + \text{const.}$

Asymptotically, for large radius $C \to \infty$, this solution describes string configurations with expanding proper amplitude.

These highly unstable strings contribute with a term of negative pressure to the energy-momentum tensor of the strings. The energy momentum tensor of these highly unstable strings (in a perfect fluid approximation) yields to the state equation

$$\rho = -P\,(D-1),$$

$\rho$ being the energy density and P the pressure ( $P < 0$). This description corresponds to *large radius* $C \to \infty$ of the universe.

See Table 1 below for a summary of the stability and and instability regimes

**Table I**

| | $R(\eta)$ | $R(t)$ | Allowed Regimes |
|---|---|---|---|
| flat | const | const | stable |
| standard | $\eta^{\alpha}$ , $\alpha > 0$ | $t^{\beta}$ , $0 < \beta < 1$ | stable |
| linear | $\exp(K\eta)$ | $Kt$ , $KL < 1$ | stable |
| power – law | $\eta^{-\alpha}$ , $\alpha > 1$ | $t^{\beta}$ , $\beta > 1$ | stable & unstable |
| de Sitter | $-(H\eta)^{-1}$ | $e^{Ht}$ , $\alpha' MH < 1$ | stable & unstable |
| de Sitter | $-(H\eta)^{-1}$ | $e^{Ht}$ , $\alpha' MH > 1$ | unstable |
| super | $\eta^{-\alpha}$ , $\alpha < 1$ | $t^{-\beta}$ , $\beta > 0$ | unstable |

**Table caption**

*Table I*

A classification of spatially flat FRW backgrounds, according to their asymptotic compatibility with stable and unstable string configurations. Here "unstable" denotes the regime, discussed in this paper, in which the string proper size grows in time like the scale factor, while "stable" refers to a regime in which the solution to the string equations can be consistently expanded around a geodesic, characterized by the **mass parameter** $M$.

(In this Table $\eta$ denotes the conformal time and $t$ the cosmic time).

For *small radius of the universe*, highly unstable string configurations are characterized by the properties

$$|\dot{X}^{0}| \gg |X'^{0}| \quad , \quad |X'^{i}| \ll |\dot{X}^{i}|$$

The solution for $X^i$ admits an expansion in $\tau$ similar to that of the large radius regime. The solution for $X^0$ is given by L/C , which corresponds to *small radius* $C \to 0$, and thus to small $\tau$. This solution describes, in this limit, string configurations with shrinking proper amplitude, for which $CX'^i$ behaves asymptotically like C, while $CX^i$ behaves like $C^{-1}$. Moreover, for an ideal gas of these string configurations, we found:

$$\rho = P\,(\,D-1\,)\,,$$

with *positive pressure* which is just the equation of state for a gas of massless particles.

More recently [24], these solutions have been applied to the problem in which strings became a dominant source of gravity. In other words, we have searched for solutions of the Einstein plus string equations. We have shown, that an ideal gas of fundamental strings is not able to sustain, alone, a phase of isotropic inflation. Fundamental strings can sustain, instead, a phase of anisotropic inflation, in which four dimensions inflate and, simultaneously, the remaining extra (internal) dimensions contract. Thus, fundamental strings can sustain, simultaneously, inflation and dimensional reduction. In ref. [24] we derived the conditions to be met for the existence of such a solution to the Einstein and string equations, and discussed the possibility of a successful resolution of the standard cosmological problems in the context of this model.

### 3.5 Strings Falling into Space-time Singularities and Gravitational Plane-wave Backgrounds

Recently [25] , we have studied strings propagating in gravitational-plane wave space-times described by the metric

$$dS^2 = F(\,U, X, Y\,)\, dU^2 - dU\, dV + dX^i\, dX^j\,,$$

where $F(U, X, Y) = W(U)(X^2 - Y^2)$ and $W(U \to 0) = \alpha / |U|^{\beta}$.

U and V are ligth cone variables; $\alpha$ and $\beta$ are positive constants. These are vacuum space-times. The space time is singular on the null plane U=0.

The string equations in this class of backgrounds are linear and *exactly solvable*. In the light cone gauge $U = \alpha' p \tau$ and after Fourier expansion in the world sheet coordinate $\sigma$, the Fourier components $X_n(\tau)$ and $Y_n(\tau)$ satisfy a one-dimensional Schrodinger-type equation but with $\tau$ playing the role of the spatial coordinate and $p^2 W(\alpha' p \tau)$ as the potential [26]. (Here p stands for the U-component of the string momentum). We studied the propagation of the string when it approaches the singularity at U=0 from U< 0. We find different behaviours depending on whether $\beta < 2$ or $\beta \geq 2$. For $\beta<2$, the string coordinates X and Y are regular everywhere, that is, the string propagates smoothly through the gravitational singularity U=0. For strong enough singularity ( $\beta \geq 2$), the string goes off to $X=\infty$ grazing the singularity plane U=0. This means that the string *does not* go accross the gravitational wave, that is the string *can not reach* the U>0 region. For particular initial configurations, the string remains *trapped* at the point X=Y=0 in the gravitational wave singularity U=0. The case in which $\beta=2$ and then $W(U) = \alpha / U^2$ is explicitely solved in terms of Bessel functions.

The string propagation in these singular space-times has common features with the fall of a point particle into a singular attractive potential $-\alpha / x^{\beta}$. In both cases, the falling takes place when $\beta \geq 2$. The behaviour in $\tau$ of the string coordinates $X^A(\sigma,\tau)$ is analogous to the behaviour of the Schrödinger equation wave function $\Psi(x)$ of a point particle. However, the physical content is different. The string coordinates $X(\sigma,\tau)$ are dynamical variables and not wave functions. Moreover, our analysis also holds for the quantum propagation of the string: the behaviour in $\tau$ is the same as in the classical evolution with the coefficients being quantum operators. At the *classical*, as well as at the *quantum* level, the string propagates or does not propagate through the gravitational wave depending on whether $\beta < 2$ or $\beta \geqslant 2$, respectively. In other words, *tunnel effect does not takes* place in this string problem.

It must be noticed that for $\tau \to 0-$, i.e. $U \to 0-$, the behaviour of the string solutions is *non-oscillatory in* $\tau$ whereas for $\tau \to \infty$, the string oscillates. This *new type* of behaviour in $\tau$ is analogous to that found recently for strings in cosmological inflationary backgrounds [23, 24]. For $\beta = 2$, it is possible to express the coefficients characterizing the solution for $\tau \to 0$ in terms of the oscillator operators for $\tau \to \infty$ .

It must be noticed that the spatial (i.e. fixed $U = \tau$) proper length of the string grows indefinitely for $\tau \to 0$ when the string approaches the singularity plane. Here too, this phenomenon is analogous to that found for strings in cosmological inflationary backgrounds. Moreover, this analogy can be stressed by introducing for the plane-wave background the light-cone coordinate U by

$$d\tilde{U} = W(U)\, dU \qquad , \qquad \text{i.e.} \qquad U = (\alpha)^{1/2}\ |U|^{\,1-\beta/2} \,/\, (1-\beta/2)$$

Then, $\tilde{U}$ is like the conformal time in cosmological backgrounds. For instance, $W(U) = \alpha/U^2$ mimics de Sitter space for $\tau \to 0$, and in this case we have, as in de Sitter space, $U=(\alpha)^{1/2} \ln(\alpha p \tau)$.

We label with the indices < and > the operators in the regions $U < T$ and $U > T$ respectively (i.e. before and after the collision with the singularity plane $U=0$). We compute the total mass squared and the total number of modes $< N_> >$ after the string propagates through the singularity plane $U=0$ and reaches the flat spacetime region $U > T$. This has a meaning only for $\beta < 2$. For $\beta \geqslant 2$, the string does not reach the $U > T$ region and hence there are no operators >. In particular, there are no mass squared $M_>{}^2$and total number $N_>$ operators for $\beta \geqslant 2$. For $\beta < 2$ , $< M^2{}_> >$ and $< N_> >$ are given by [25]

$$< M_>{}^2 > = m_0{}^2 + 2\,\alpha'^{-1} \sum_{n=1}^{\infty} n\,[\ |B_n{}^x|^2 + |B_n{}^y|^2\ ]$$

$$< N_> > = 2 \sum_{n=1}^{\infty} [\ |B_n{}^x|^2 + |B_n{}^y|^2\ ]$$

where $B_n{}^x = -B_n{}^y \underset{n\to\infty}{\ } \approx (2n/\alpha' p)^{\beta-2} \quad , \quad o < \beta < 1$

Here the expectation values refer to the ingoing (e.g., < ) ground state .

$< M_>{}^2>$ is finite for $\beta <1$ but diverges for $1\leq \beta <2$. $< N_> >$ is finite for $\beta <3/2$ but diverges for $3/2 \leq \beta <2$. The physical meaning of these infinities is the following: These divergences are due to the infinite transverse extent of the wave-front and not to the short distance singularity of W(U) at U=0. The gravitational forces in the transverse directions (X,Y) transfer to the string a finite amount of energy when the transverse size of the shock wave front is finite. When the size of the wave front is infinite, the energy transfered by the shock-wave to the string impart large elongation amplitudes in the transverse directions ( X,Y ) which are responsible of the divergence of $< M_>{}^2>$. This question has been analyzed in detail in ref.[24].

For a sourceless shock-wave with metric function

$$F(U, X, Y) = \alpha\ \delta(U)\ (X^2 - Y^2),$$

we find that $B_n{}^x = B_n{}^y = (\alpha p \alpha' / 2in)$ and these are the same coefficients as those corresponding to $W(U) = \alpha / U^{\beta}$ with $\beta = 1$. This is related to the fact that both functions W(U) have the same scaling dimension.The string propagation is formally like that of a Schrödinger equation with a Dirac delta potential: the string *passes across* the singularity at U=0 and *tunnel effect* is present. The string scattering in this sourceless shock wave is very similar to the string scattering by a shock-wave with a non-zero source density [20,21].

We have also computed $< M_>{}^2>$ and $< N_> >$ for a metric function

$$F( U, X, Y ) = \alpha \ \delta(U) ( X^2 - Y^2 ) \ \theta ( \rho_0 - X^2 + Y^2 )$$

where $\theta$ is the step function and $\rho_0$ gives the transverse size of the wave-front. This F belongs to the shock-wave class with a density source we have treated in refs. [20,21]. Here too $< M_>{}^2>$ is finite as long as $\rho_0$ is finite. This shows explicitly that the divergence of $< M_>{}^2>$ is due to the infinite transverse extent of the wave-front and not to the short distance singularity of W(U) at U=0. More generally, for a string propagating in a schock wave spacetime with generic profile

$$F ( U , X, Y ) = \delta( U ) f ( X, Y ) ,$$

we have found the exact expressions of $< M_>{}^2>$ and $< N_> >$ in ref. [20,21] . When f(X,Y) has infinite range, the gravitational forces in the X,Y directions have the possibility to transfer an infinite amount of energy to the string modes. We have computed all the components of the string energy-momentum tensor near the U=0 singularity.

The propagation of classical and quantum strings through these singular space-times is physically *meaningful* and provides new insigths about the physics of strings on curved space-times.

Strings in gravitational plane-wave backgrounds have been studied in ref.[26]. However, this problem has subtle points which were overlooked there. The analysis done in ref.[26] by analogy with the Schrödinger equation is not enough careful. The mass and number operators are expressed in terms of the transmission coefficient $B_n$. In ref. [26] the cases in which $B_n = \infty$ , mean that there is no transmission to the region U>0, and then , there is no mass operator, neither number operator (since there is no string) in that region. This is the situation of falling to U=0 for $\beta \geqslant 2$ which we mentioned above. Therefore, $M_>{}^2$ and $N_>$ make sense only for $\beta < 2$ and any statement about $M_>{}^2$ and $N_>$ for $\beta \geqslant 2$ is meaningless.

## 3.6 Strings in Topologically Non-trivial Backgrounds: Scattering of a Quantum (Fundamental) String by a Cosmic String

In ref [27], we have studied a (quantum) fundamental string in a conical space-time in D dimensions. This geometry describes a straight cosmic string of zero thickness and it is a good approximation for very thin cosmic strings with large curvature radius. The space-time is locally flat but globally it has a non-trivial (multiply connected) topology. There exists a conelike singularity with azimuthal deficit angle

$$\delta\Phi = 2\pi\ ( 1-\alpha ) = 8\pi G\mu .$$

$G\mu$ is the dimensionless cosmic-string parameter, G is the Newton constant and $\mu$ the cosmic-string tension (mass per unit of length). $G\mu$ $10^{-6}$ for standard cosmic strings of grand unified theories.

The string equations of motion and constraints are exactly solvable in this background. The string equations are free equations in the Cartesian-type coordinates $X^0$, X , Y, $Z^i$, $(3 < i < D-1)$, but with the requirement that

$$0 < \arctan (Y/X) < 2\pi\alpha .$$

The exact solution in the light-cone gauge is given in ref.[27]. The string as a whole is deflected by an angle

$$\Delta = \delta\Phi / 2.$$

A string passing to the right ( left) of the topological defect is deflected by + (-)$\delta\Phi$. This deflection does not depend on the impact parameter, nor on the particle energy due to the fact that the interaction with the space-time is of purely topological nature. In the description of this interaction we find essentially two different situations:

(i) The string does not touch the scatterer body. A deflection $\pm \Delta$ at the origin and a rotation in the polarization of modes takes place. In this case there is no creation or excitation of modes (creation and annhilation operators are not mixed) and we refer to this situation as elastic scattering.

(ii) The string collides against the scattering center; then in addition to being deflected, the internal modes of the string become excited. We refer to this situation as inelastic scattering. In the evolution of the system, continuity of the string coordinates and its $\tau$-derivatives at the collision time $\tau = \tau_0$ is required. In this case, the relation between the ingoing and outgoing oscillators is given by an exact Bogoliubov transformation. In addition to a change in the polarization, there are mode excitations which yield final particle states *different* from the initial one. This provides another example of the particle transmutation process described above. Notice that we are dealing with a single (test) string. That is, the initial and final states are *one particle* but *different* states. Also notice that the particle states *transmute* at the classical (tree) level as a consequence of the interaction with the space-time geometry. In the present case, this is a topological defect.

We explicitly proved [27], that the conformal $L_n$ generators $Ln_<$ built from the ingoing modes are identical to the $L_{n>}$ built from the outgoing modes. The mass spectrum is the same as in the standard Minkowski space-time and the critical dimension is the same (D=26 for bosonic strings).

Let us notice that strings in conical space-times were considered in ref. [28] but only for deficit angles $2\pi(1\text{-}1/N)$, N being an integer, where the scattering is *trivial*. (In that case, the space becomes an orbifold). In our work, we have solved the scattering problem for general deficit angles where it is nontrivial. Let us also notice that the condition of conformal invariance (vanishing of the $\beta$ function) is identically satisfied everywhere in the conical space-time, except eventually at the origin. If such difficulty arises, this space-time will simply not be a candidate for a string ground state (vacuum). Anyway, this geometry effectively describes the space-time around a cosmic string.

*Can the string split ?* An interesting question here is whether the string may split into two pieces as a consequence of the collision with the conical singularity. When the string collides against the conical singularity, since the deflection angles to the right and to the left of the scattering center are different, one could think that the splitting of the string into two pieces will be favored by the motion. Such splitting solution exists and is consistent. However, its classical action is larger than the one without splitting. Therefore, this splitting may take place *only quantum mechanically*. In fact, such a possibility of string splitting always exists and already in the simplest case for strings *freely* propagating in *flat* space-time. The free equations of motion of strings in flat space-time admit consistent solutions which describe splitting but once more their action is larger than the solution without splitting.

When the string propagates in curved space-time the interaction with the geometry modifies the action. In particular, the possibility arises that the action for the splitting solution becomes smaller than the one without splitting. Therefore, string splitting will occur *classically*.

Finally, we have computed exactly and in closed form the scalar particle (lowest string mode) quantum scattering amplitude in the conical space of the cosmic string. For that, it is neccessary to know first, the solution of the Klein-Gordon equation in conical space-time in D dimensions. We have found the ingoing solution which satisfies the massive free equation with the non-trivial requirement to be periodic in the azimuthal angle $\Phi$ with period $2\pi\alpha$. This prevents the usual asymptotic behaviour for large radial coordinate $R \to \infty$. The full wave function is the sum of two terms. For D=3, this solution has been found in refs. [29] and [30]. The incident wave turns out to be a finite superposition of plane waves without distortion. They propagate following wave vectors rotated from the original one by a deflection + $\Delta$ and periodically extended with period $2\pi\Delta$. This incident wave although undistorted suffers multiple periodic rotations as a consequence of the multiply connected topology. In addition, the second term describes the scattered wave with scattering amplitude

$$f(\Theta) = \frac{1}{2\pi} \frac{\sin \pi/\alpha}{(\cos \pi/\alpha + \cos \Theta/\alpha)}$$

In the scalar (ground state) string amplitude, we have ingoing ( $\tau < \tau_0$ ) and outgoing ( $\tau > \tau_0$ ) zero modes and oscillator modes averaged in the ingoing ground state $|0_< \rangle$. Inserting the in and out wave function solutions in this matrix element yields four terms, each of these terms splitting into four other ones corresponding to the natural four integration regions of the double $\tau$-domain. The detailed computation is given in ref.[27]. (For this computation it is convenient to work in the covariant formalism where all string components are quantized on equal footing). The effect of the topological defect in space-time on the string scattering amplitudes manifests through the nontrivial vertex operator (which is different from the trivial one $e^{ik.x}$, and through the fact that ingoing and outgoing mode operators are related by a Bogoliubov transformation which makes the expectation value on the ingoing ground state $| 0_< \rangle$ non-trivial. In the $\alpha = 1$ limit (that is, for the cosmic string mass $\mu = 0$), we recover the flat space Minkowski amplitude. If the oscillator modes n=0 are ignored, we recover the point-particle field-theory Klein-Gordon amplitude.

### 3.7 Strings in Rindler space and horizon regularization

Although Rindler space-time is flat, it possesses a space-time structure including an event horizon, similar to a black-hole manifold. The Rindler manifold is defined by

$$x^1 + x^0 = e^{\alpha(X^1 + X^0)}$$
$$x^i = X^i \qquad i = 2,......,D.$$

where $(x^1, x^0)$ and $(X^1, X^0)$ are respectively inertial or Minkowskian

(Kruskal-like) and accelerated or Rindler (Schwarzschild-like) coordinates [31]. (See ref. [32] for a new approach to Q.F.T in these coordinates, which is appropriate for the description of the Hawking-Unruh effect for strings).The constant $\alpha$ defines the proper acceleration of the Rindler observers ($\alpha$ is equal to the surface gravity in the black-hole case). The event horizon is at $x^1 \pm x^0 = 0$. This corresponds to $X^0 = \pm\infty$ and $X^1 = -\infty$.

Let us discuss some features of Rindler space which are important for our study of strings in this space. The Rindler transformation maps the right-hand wedge $x^1 > |x^0|$ of Minkowski space onto the whole Rindler space $-\infty \leqslant X^1, X^0 \leqslant +\infty$ (the whole Minkowski space can be covered using four different Rindler patches). As is known, a point particle quantum field in Rindler space is in a thermal state with temperature $T=\alpha/2\pi$. In addition, ultraviolet divergences arise in the free energy and entropy of quantum fields from the existence of a horizon in the space-time. The same problem appears in the case of a four (or D-dimensional) black-hole [33]. This phenomenon is illustrated by considering (for simplicity) a free massive scalar field,with modes of positive frequency $\lambda$. The total number of wave modes with frequency less than $\lambda$ is given by [3]

$$\pi \mathcal{N}_\lambda = \int_{-H}^{a(\lambda)} dX^1 \int \prod_{i=1}^{D-2} \frac{dk^i}{2\pi} \sqrt{\lambda^2 - (m^2 + k^{i\,2})\alpha^2 e^{2\alpha X^1}} ,$$

$$a(\lambda) = (1/\alpha) \ln(\lambda/m\alpha) .$$

This has been evaluated in the WKB semiclassical approximation ($\lambda > \alpha$) , which is enough to study the ultraviolet behaviour of the quantities interesting us. Here H is a large cut-off ($H < 1/\alpha$ ) on the negative Rindler coordinate $X^1$. The free energy (F) and entropy (S) at temperature T are given by

$$F = \frac{\Gamma(D)}{(4\pi)^{D-1/2}\,\pi^D} \; \frac{\alpha}{(D-2)} \; e^{\alpha H (D-2)}$$

$$S = \frac{\Gamma(D)}{(4\pi)^{D-1/2}\ \pi^{D}} \ \frac{2\pi D}{(D-2)}\ e^{\alpha H(D-2)}$$

We explicitely see that F and S need the ultraviolet cut-off to be finite. This cut-off H shifts the horizon by replacing the light-cone $x^1 = x^0$ as a boundary of Rindler space-time by the hyperbola

$$(x^1)^2 - (x^0)^2 = e^{-2\alpha H}$$

This is equivalent to considering the following mapping defining the accelerated coordinates

$$x^1 - x^0 + \varepsilon = e^{\alpha(X^1 - X^0)}$$
$$x^1 + x^0 + \varepsilon = e^{\alpha(X^1 + X^0)}$$

where

$$\varepsilon = e^{-\alpha H} \qquad , \qquad -H \leqslant X^1 \leqslant +\infty$$

The horizon is now at a finite distance $|X^1 \pm X^0| \sim (1/\alpha)\ \ln(1/\varepsilon)$. The longitudinal Rindler coordinate $X^1$ must be bounded bellow (it can not be $-\infty$ but $-\alpha \ln \alpha\varepsilon$). This regularization reflects the fact that a classical description of the geometry is not longer valid at distances of the order of the Planck length, here, $\varepsilon \sim l_{\mathrm{Planck}}$.
In string theory, in order to quantize the string properly in Rindler's manifold, this horizon regularization is needed [3]. The need for such a cut-off appears already at the level of the definition of the positive frequency modes of the string with respect to Rindler's time.

### 3.8 The Hawking-Unruh effect in string theory

As it is known, reparametrization invariance of the world sheet coordinates allows to choose the world sheet metric in the conformal form and this still allows the reparametrizations

$$x_+ + \varepsilon = f\,(x'_+ + \ell)$$
$$x_- + \varepsilon = g\,(x'_- + \ell)$$

where

$$x_{\pm} = \sigma \pm \tau \quad , \quad x'_{\pm} = \sigma' \pm \tau'$$

and we have introduced the constants $\varepsilon$ and $\ell$ whose meaning will be clear in the sequel. In this way, we can choose the light-cone gauge:

$$V = X^0 + X^1 = p_+\, x_+$$

where the proportionality constant $p_+ > 0$ is the momentum of the center of mass of the string. An important step in field as well as string quantization is the definition of positive frequency states and its associated ground state. Canonical states for different coordinate systems are physically different (each timelike vector field leads to a separate indication of what constitutes a positive frequency). There are different ways (and there are thus an ambiguity) in choosing such a basis.This makes it possible for a given field or string theory to have different alternative well -defined Fock spaces (different "sectors" of the theory). Usually, the string is described in the Minkowski (inertial) frame where

$$\partial_{x+}\partial_{x-}\, X^A = 0 \ , \qquad A= 0, 1, \ldots\ldots, D\text{-}1$$

and then,positive frequency modes are defined with respect to the inertial time $X^0$. In the light-cone gauge, $X^0$ is proportional to $\tau$ and therefore the modes $\phi_n \sim e^{\mp i n x_{\pm}}$ define the (inertial) particle states

of the string.
Obviously, if we consider the ( $\sigma'$, $\tau'$ ) parametrization of the string world sheet defined above, i.e.,

$$\sigma' + \ell = [\, G(\sigma + \tau + \varepsilon) + F(\sigma - \tau + \varepsilon)\,](1/2) \,, \qquad F = f^{-1}$$
$$\tau' = [\, G(\sigma + \tau + \varepsilon) - F(\sigma - \tau + \varepsilon)\,](1/2) \,, \qquad G = g^{-1}$$

for which we have

$$\partial_{x'_+}\partial_{x'_-} X^A = 0 \,,$$

positive frequency modes with respect to $\tau'$, namely $\Psi_n \sim e^{\mp i\lambda_n x'_\pm}$ do not define positive frequency modes with respect to $X^0$, i.e. are not inertial particle states of the string. However, they are positive frequency modes with respect to another inequivalent (accelerated) time $X'^0$ defined by [2]

$$X^1 - X^0 = p_- \, f(X'^1 - X'^0)$$
$$X^1 + X^0 = p_+ \, f(X'^1 + X'^0)$$

here, the mappings f and g are the same which act on the world sheet variables. This corresponds to the description of the string in an accelerated reference frame $\{X'^0, X'^1, \ldots\ldots\ldots, X'^{D-1}\}$. In this description , we can always choose an accelerated, (in particular Rindler) light -cone gauge. Thus, the modes $\Psi_n \sim e^{\mp i\lambda_n x'_\pm}$ are accelerated particle modes of the string.
The manifold ( $\sigma'$, $\tau'$ ) is the convenient world sheet parametrization for a string in an accelerated space-time. This is true in either flat or curved space-time [ although in the last case the free equation of motion, as well as the choice of the light-cone gauge, may only be valid asymptotically for $x'_\pm \to \pm\infty$ ].

An important point here is that of the boundary conditions which the transformations f in this context must satisfy: *Not all* conformal mappings are suitable in this context, but only those satisfying appropriate boundary conditions. As it is known, $\sigma$ lies in a *finite* interval, and $\tau$ ranges from $-\infty$ to $+\infty$. In order to have $\tau'$ ranging from $-\infty$ to $+\infty$ and $\sigma'$ lying in a *finite* interval , suitable boundary conditions on the mappings must be imposed. The condition on $\sigma'$ is satisfied only is $\varepsilon \neq 0$ . This yields

$$0 < \sigma' < L_\varepsilon \quad \text{where} \quad L_\varepsilon = F(\varepsilon + 2\pi) - F(\varepsilon)$$

$$\ell = F(\varepsilon) \qquad , \quad F = f^{-1}$$

The condition on $\tau'$ (and hence on $X'^0$) is not particular to strings. It is required to get a consistent quantization (of fields or strings) in accelerated manifolds and to have a complete ingoing (outgoing) basis [32]. On the other hand, it may occurs that the accelerated manifold cover only a bounded region (namely $X^1 > |X^0|$) of the original (inertial) one. Similarly, the manifold $(\sigma',\tau')$ may cover only a domain ( $\sigma > |\tau|$) of the inertial or global world sheet. All these considerations are satisfied by taking mappings such that

$$u_{\pm} = f(\pm\infty)$$

where $u_+$, $u_-$ are constants which can take independently, finite or infinite values, and $u_+ > u_-$. This means that the inverse mapping $F=f^{-1}$ has singularities at $X^1 - X^0 = u_{+}$ ., i.e.,

$$F(u_{\pm}) = \pm\infty$$

Singularities of these mappings describe the asymptotic regions of the space-time. Critical points of f, i.e.,

$$f'(\pm\infty) = 0$$

describe event horizons at $X^1 - X^0 = u_-$ and $X^1 - X^0 = u_+$. In this case, $u_-$ and $u_+$ are finite and the manifold covers the region $u_- < X^1 + X^0 < u_+$ of the global space-time. If $u_\pm = \pm\infty$, there are no horizons. In this approach, the well-known Rindler's space corresponds to the mapping

$$f = e^{\alpha U} \qquad\qquad (U = X^1 - X^0)$$

In this case, $u_- = 0$ and $u_+ = +\infty$ ; $X^1 - X^0 = u_- = 0$ is an event horizon and the manifold covers the right-hand wedge $|X^1 - X^0| > 0$ of Minkowski space-time.
(in order to cover the whole plane, four Rindler's patches are needed).
For the string world sheet we have

$$x_\pm + \varepsilon = e^{\alpha(x'_\pm + \ell)}$$

and we see how the presence of the parameter $\varepsilon$ is necessary in order to have a finite string period in the manifold $(\sigma', \tau')$. For the Rindler's mapping we have

$$\ell = 1/\alpha \log \varepsilon$$
$$L_\varepsilon = 1/\alpha \log ( 2\pi/\varepsilon + 1 )$$

For $\varepsilon \rightarrow 0$ , there is a stretching effect of the string due to the presence of an event horizon in the world sheet.
These mappings are not without consequences but change the ground state and, in general, the quantum states, except for mappings belonging to the O(2,1) group, that is except for the Möbius or bilinear transformations. Under the mappings f, the states transform as

$$| \; > \quad \rightarrow \quad e^{i\widehat{G}} \; |' \; >$$

$$\hat{G} = \Sigma\, \theta_n\, (C_n C_n - C_n^+ C_n^+)$$

here $a_n | \rangle = 0$ defines the standard (inertial) Minkowskian ground state and $C_n | ' \rangle = 0$ defines the accelerated one. The operators $a_n$ and $C_n$ are related by a Bogoliubov transformation. [$\hat{G}$ is an operatorial representation for the Bogoliubov transformation, with Bogoliubov coefficients $\cosh\theta_n$ and $\sinh\theta_n$].

The vacuum expectation value of the two dimensional (world sheet) energy-momentum tensor transforms as

$$< T_{\mu\nu} > = < T'_{\mu\nu} > + \Theta_{\mu\nu} + P_{\mu\nu}$$

$P_{\mu\nu}$ is any conserved traceless tensor taking into account the dependence of $T_{\mu\nu}$ on the quantum state. It represents the non-local part of $T_{\mu\nu}$. $\Theta_{\mu\nu}$ depends on the mapping and represents the local part. In the conformal gauge, the constraints

$$< T_{\mu\nu} > = 0 \qquad , \qquad < T'_{\mu\nu} > = 0$$

yield the equations

$$d^2/dx'^2_{\pm}\, \Psi_{\pm} - 12\pi\, \mathcal{U}_{\pm}(x'_{\pm})\, \Psi_{\pm} = 0.$$

for the mappings f and g [2]. This is a zero-energy Schrödinger equation.

Here $\mathcal{U}_+$ and $\mathcal{U}_-$ are arbitrary functions of the indicated variables. By giving the "potentials" $\mathcal{U}_{\pm}$, this equation allows to determine the "wave functions"

$$\Psi_- = 1/\sqrt{f'} \quad , \quad \Psi_+ = 1/\sqrt{g'} \qquad ,$$

that is the mappings f and g. Notice that because $\mathcal{U}_{\pm}$ are arbitrary functions (compatible with the boundary conditions), these equations do not yield additional constraints on the mappings f, g but a way of connecting the mapping to a potencial.
The first term of the equation is the Schwarzian derivative of f:

$$D[f] = (f'''/f') - (3/2)(f''/f')^2 \ ,$$

which is invariant under the Möbius or bilinear transformations. Under these O(2,1) transformations, f becomes a new function but D[f] is invariant determining the same ground state of the string. In particular, $\mathcal{U}_+ = \mathcal{U}_- = 0$ determines f, and g as

$$f = (\alpha\ x'_- + \beta\ )\ /\ (\ \gamma\ x'_- + \delta) \qquad , \qquad (\alpha\delta - \beta\gamma) = 1.$$
$$g = (\alpha\ x'_+ + \beta\ )\ /\ (\ \gamma\ x'_+ + \delta)$$

Interestingly, the ground state defined by this mapping can be considered as a reference or "minimal" state at zero temperature with respect to which other ground states corresponding to non-zero potencials appear as excited or thermal ones. For constant potential, i.e. $\mathcal{U}_+ = \mathcal{U}_- =$ constant > 0, we have

$$\Psi_{\pm} = Ae^{-Kx'_{\pm}} \qquad , \qquad f = (1/A^2\alpha)\ (e^{\alpha x'_{\pm}} - \varepsilon).$$

This is the Rindler mapping. Here A is a normalizing constant (we choose $A^2 = \alpha^{-1} e^{-\alpha\rho}$) and K is the zero-energy transmission coefficient

$$K = \alpha/2 = \sqrt{12\pi\mathcal{U}_0} \qquad , \qquad (\ \mathcal{U}_{\pm} = \text{constant} = \mathcal{U}_0\ )$$

The surface gravity is the zero-energy transmission coefficient and the parameter $\varepsilon$ arises naturally as an integration constant. For $\varepsilon \to 0$, this mapping defines an event horizon at $x_{\pm} = 0$ ( $x_{\pm}' = -\infty$ ) and carries an intrinsic temperature $T_s = \alpha/2\pi$ , as it can be seen by taking $\tau = i\tau$ ( $x_{\pm} = x \pm i\tau$ ) and then $0 < \tau < 2\pi/\alpha$. The temperature is

$$T_s = \sqrt{12\, u_0 / \pi}\ .$$

This temperature $T_s$ characterizes the ground state spectrum of the string in Rindler space [2,3]. The expectation values of the Rindler number mode operator, as well as the square mass operator, in the ground state of the string follow a thermal distribution with temperature $T_s$. This is the same Hawking-Unruh value which appears in the field theoretical context. Here the frequencies of the Rindler modes are given by $\lambda_n = n\lambda_0$ , that is, they differ in a large factor $\lambda_0 = 2\pi\alpha /( \ln 2\pi/\varepsilon + 1 )$ from the inertial ones (n). Physically, this factor reflects the indefinite increasing of the string length when it approaches the event horizon. However, if one measures the frequency in dimensionless units (1,2,....) instead of multiples of $\lambda_0$, then , the temperature of the ground state is a (very large) pure number

$T_0 = ( T_s / \lambda_0 ) = 1/ 4\pi^2 \ln ( 2\pi/\varepsilon + 1 )$.

We see that with appropiate boundary conditions, the holomorphic mappings of reparametrization invariance of string theory can be interpreted as a change of coordinate frame in the space-time in which strings are embedded. These mappings change the ground state in the quantized theory except for transformations belonging to the O(2,1) group. This allows to discuss in a natural way the Hawking-Unruh effect in string theory. We also see that in order to discuss these effects, a change of global coordinate frame in the space-time, must be accompagnied by a similar change on the world sheet. The boundary conditions satisfied by these mappings are the same in string theory as well as in Q.F.T., but for strings, the mappings carry an additional parameter $\varepsilon$ (notice that $\ell = \ell(\varepsilon)$ ). This additional parameter plays a fundamental role in the string context. $\varepsilon$ acts as a regulator or cut-off to avoid the presence of an event horizon in the world sheet and to get a finite string period. Its magnitude is of the order of the Planck length.

*Application to the Black Hole:*

The results described above about the Hawking-Unruh effect in string theory apply also to curved space-time. For the most important metrics in General Relativity, the presence of isometry groups allows the maximal analytic extension of the (D-dimensional) manifold to be performed through the extension of a relevant two-dimensional manifold containing the time axis and a suitable spatial coordinate. This maximal analytic extension is performed by the class of mappings we considered here, where $X^1$, $X^0$ are Kruskal (maximal) type coordinates and $X'^1$, $X'^0$ are of Schwarzschild type[2,3]. The major features of the string quantization in Rindler space presented here also hold for a string in a Schwarzschild space-time. An appropriate light-cone gauge can be introduced for left or right movers treated separately in the null Kruskal and Schwarzschild coordinates. The shifting of the horizon is equivalent to considering a shifting $\varepsilon$ in the mapping

$$r_k \pm t_k + \varepsilon = e^{K\,(R^* \pm T)}$$

$r_k$, $t_k$ are Kruskal coordinates, $R^*$,T are Schwarzschild's ones.

Here $\varepsilon \sim (M_{Planck}/\ M)^{D-2/D-3}$, M is the black -hole mass and K is the surface gravity $K \sim (M_{Planck}/\ M)^{1/(D-3)}$.

The expectation value of the string number operator of Schwarschild string modes taken in the Kruskal ground state of the string gives a Planckian distribution at the temperature $T_S = K/2\pi$ or at the dimensionless temperature

$$T_0 = 1/4\pi^2 \ln (\ 2\pi\ /\varepsilon\ ) \quad >> 1$$

One can consider different higher dimensional black hole space-times, namely a 26-dimensional or a four dimensional black hole with extra 22 dimensions compactified in a torus. Intermediary situations can also be envisaged but it must be noticed that the qualitative properties of the string quantization will be the same since they depend upon the horizon structure in the two variables R, T (or $X^0$, $X^1$, for Rindler space).
It can be noticed that the Hagedorn temperature ($T_m$) in this context is

$$T_m = 1/(\alpha')^{1/2} \simeq M_{Planck} \text{ . Then ,}$$

$$T_s/T_m \simeq (M_{Planck}/M)^{1/D-3}$$

and we have

$$T_s \lesssim T_m \quad ,$$

since the basic requirement of the present semiclassical treatment is

$$M \gtrsim M_{Planck} \, .$$

### 3.9 Accelerated strings

In ref.[34] , we have considered a simple model for *accelerated strings*. In the general case such a string must be affected by the action of some external force causing the acceleration. In order to make our study more concrete we consider that the string is open and that there are "heavy" particles at its ends (e.g. monopoles [35]); the force causing the acceleration being applied to these "heavy" particles. In such a simplified model all the information concerning the interaction of the string with the external forces is reduced to the specific choice of the boundary conditions. Then , one can consider a string whose ends are moving with uniform acceleration, that is they follow accelerated world lines (hyperbolae). Here the equations of motion and constraints are the usual ones but the boundary conditions are non-linear .

The main result of our study is the proof of the existence of a *critical value of the acceleration.* For accelerations smaller than the critical one, there exist equilibrium configurations of the accelerated string which represent its"rigid-body-like" motion. For accelerations larger than the critical one such a rigid equilibrium configuration is impossible; the *length* of the string begings to *grow* infinitely.

First, we have considered a string moving as a rigid body without any excitation, and we have found that there exists a critical value of the acceleration $a_c$ beyond which such a solution does not exist. Alternatively, this configuration can be interpreted as the static equilibrium configuration for a string in a homogeneous gravitational field, the inverse of the critical parameter ( $a_c^{-1}$ ) giving the minimal distance from the string to the horizon. Then, we study linear small perturbations (string excitations) around this equilibrium or non-excited accelerated string solution. After linearization of the constraints and boundary conditions around this zero-order solution, a careful analysis of their solutions and the stability problem shows that: (i) For $a < a_c$ , the perturbations describe small oscillations, whereas that, for $a > a_c$ the solution contains an exponentially growing factor with time, that is, for $a > a_c$ the *equilibrium is unstable*. (ii) For the value $a = a_c$ , at which the instability develops, the growing factor in time is not exponential but a power of time. The notion of *limiting acceleration* and its *associated instability* naturally arises in the interpretation of our problem when the string is considered at rest in a homogeneous gravitational field : in this case we show that there exists a critical field strength, for which negative energy modes of the string perturbations arise. (This instability happens for the same value of the acceleration as we have found for the case of a uniformly accelerated string, which is exactly what corresponds to the equivalence principle).

If for a given distance L between the (uniformly) accelerated string's ends, one tries to increase adiabatically the value of the acceleration, one finds out that the form of the accelerated equilibrium string will also be adiabatically changing until the moment when the string's acceleration reaches the critical value $a_c$. The string with this acceleration appears to be unstable. Hence, for higher values of the acceleration, the equilibrium configurations are already impossible.

The effect of the existence of a critical acceleration and of an accelerated string instability is of quite general nature. No extended system of size $l$, say, can move as a rigid body with an acceleration $a$ greater than $l^{-1}$. The string is an elastic body and for a given distance L between the ends of the string, the string size in the direction of the acceleration depends not only on L but also on its tension and on the acceleration itself. That is why for the critical value of the acceleration, the size of the string in the direction of the acceleration, i.e:

$$a^{-1} ( 1 - \cosh^{-1} \zeta_0 ) \quad , \quad ( \zeta_0 = 1.2 ) \quad ,$$

is less than the distance to the horizon $a^{-1}$. It is interesting to notice that the same situation occurs when the string is placed into the static gravitational field of a black hole, namely the equilibrium configurations of strings are possible until the distance between the string and the event horizon becomes less than some critical value [36] .

<u>*What happens to the accelerated string after it loses its stability?*</u>

At the beginning of this process, the length of the string in the direction of the acceleration becomes larger and larger until a part of the string crosses the horizon; after that, the string completely loses its rigidity. The causal signal propagating along the part of the string which is moving behind the horizon can reach the string ends. That is why this part of the string will be almost freely propagating and one may expect that the length of the string will be permanently growing.

This model allowed us to simplify the discussion because an external force inducing the aceleration was implied to the string ends while the string itself was not affected by any additional force. Let us make now some general remarks based on the results obtained from this simplified model.

Let us consider a closed string moving as a whole with some acceleration $a$. Of course this string must be affected by an external force. We may assume, for example, that the string possesses electric charge and that it is placed into a homogeneous electric field. An important point is that a uniformly accelerated string as well as any other system with internal degrees of freedom becomes a source of radiation, accompanied by the string's excitation process. The excitation of the accelerated system continues until the radiation of the modes (which decreases the energy of the system) becomes comparable to the radiation accompagnied by the string's excitation. This effect is also valid for classical systems provided the system with internal degrees of freedom (a string in our case) at the beginning already is not at its lowest energy state. The quantum description of this process corresponds to the Unruh effect. The nature of the radiation depends of the nature of fields which interact with the system. In the case of the string, one of such fields must be the gravitational one. If the closed string is moving with acceleration $a$, or if it is at rest in a static gravitational field near the event horizon, then the equilibrium excitation of the string is characterized by the Unruh temperature $T_s = a/2\pi$. But the string heated up to this temperature must have a mass

$$M = 4\pi T_s \qquad \text{and a size} \qquad L = C\, T_s^{1/2}\, \alpha'^{3/4}.$$

On the other hand, the size L of the equilibrium string cannot exceed the distance $a^{-1}$ to the horizon. This means that there must exist some *limiting acceleration*

$$a_c < \left(\frac{2\pi}{C^2}\right)^{1/3} \frac{1}{\alpha'^{1/2}} \qquad , \quad C = 2\pi\, [(\,D\text{-}2\,)/6]^{1/2}\,.$$

Any attempt to accelerate a string beyond this value will result in an infinitely *growing mass* and *size* of the string. It is interesting to notice that the characteristic Unruh temperature connected with the limiting acceleration $a_c$ is $T_c = (2\pi C)^{-2/3}\ (\alpha')^{-1/2}$ and it is comparable with the Hagedorn temperature which arises as a *limiting temperature* in string thermodynamics. Related issues have been also discussed in refs. [37].

## 4. RECENT PROGRESSES

### 4.1 Exact Integrability of Strings in D-dimensional De Sitter Spacetime

In order to go further in the understanding of the string dynamics in de Sitter space-time, we have given recently[38], a different formulation of this problem, and searched for its ***exact*** solvability. We have shown the complete integrability of the string propagation in D-dimensional de Sitter space-time[38].

We found that the string equations of motion -which correspond to a non compact O(D,1) symmetric sigma model- plus the string constraints, are equivalent to a generalized Sinh-Gordon equation. In D = 2, this is the Liouville equation, in D = 3, this is the standard Sinh-Gordon equation and in D = 4, this equation is related to the $B_2$ Toda model. We have shown that the presence of instability is a general exact feature of strings in de Sitter space, as a direct consequence of the strong instability of the generalized Sinh-Gordon Hamiltonian (which is unbounded from below), irrespective of any approximative scheme. Bäcklund transfomations for this generalized sinh-Gordon equation, which relate expanding and shrinking string solutions were exhibited[38]. We found all classical solutions in D = 2 and physically analize them. In D = 3 and D = 4, we found the asymptotic behaviours of the solutions in the instability regime. The exact solutions exhibit asymptotically all the characteristic features of string instability, namely: the logarithmic dependence of the cosmic time u on the world sheet time $\tau$ for $u \to \pm\infty$, the stretching (or the shrinking) of the proper string size and the proportionality between t and the conformal time.

The string equations of motion in curved space-time are generalized non-linear sigma models. De Sitter space-time is maximally symmetric, the string equations in D-dimensional de Sitter space-time correspond to a non-compact O(D,1) symmetric sigma model in two dimensions. This model is integrable.

In addition, the two-dimensional (world-sheet) energy-momentum tensor is required to vanish by the constraints. For our purposes here, it is convenient to consider de Sitter space-time as a D-dimensional hyperboloid embedded in D+1 dimensional flat Minkowski space-time of coordinates $(q^0, q^1, ...., q^D)$. The complete de Sitter is manifold is the hyperboloid

$$-(q^0)^2 + \sum_{i=1}^{D} (q^i)^2 = 1.$$

The string system of equations can be then simplified by choosing an appropriate basis for the string coordinates in the D+1 dimensional flat embedding space. The construction of this basis is analogous to the reduction of the O(N) symmetric non linear sigma model [39]. We find that the string system of equations is equivalent to a generalized Sinh-Gordon equation

$$\partial_\eta \partial_\xi \alpha(\xi,\eta) - e^{\alpha(\xi,\eta)} + e^{-\alpha(\xi,\eta)} \sum_{i=4}^{D+1} u_i v_i = 0$$

where $\quad \xi = (\sigma + \tau)/2 \quad , \quad \eta = (\sigma - \tau)/2 \quad ,$

and the function $\alpha(\xi,\eta)$ is defined by the scalar product

$$e^{\alpha(\xi,\eta)} = -\partial_\xi q \cdot \partial_\eta q$$

The vector fields $u_i$ and $v_i$ take into account the embedding dimensions beyond three and relate to $\partial_\xi{}^2 q$ and $\partial_\eta{}^2 q$ respectively. In D = 2, we have $u = v = 0$, and the string equation reduces to the Liouville equation

$$\partial_\eta \partial_\xi \alpha - e^{\alpha(\xi,\eta)} = 0$$

In D = 3, we find $u = u(\xi)$, $v = v(\eta)$, and then, using the reparametrization invariance on the world sheet, we change variables

$$d\xi'/d\xi = \sqrt{u(\xi)} \qquad , \qquad d\eta'/d\eta = \sqrt{v(\eta)}$$

such that

$$\alpha(\xi,\eta) = \hat{\alpha}(\xi',\eta') + \frac{1}{2}\log\left[u(\xi) v(\eta)\right] .$$

Then, the string equation reduces to the Sinh-Gordon equation

$$\partial_{\eta'} \partial_{\xi'} \hat{\alpha} - e^{\hat{\alpha}(\xi',\eta')} + e^{-\hat{\alpha}(\xi',\eta')} = 0$$

In D = 4, we find that the scalar product $u_i . v_i = \cos\beta$ is determined by the following equation

$$\partial_{\eta'} \partial_{\xi'} \beta - e^{-\hat{\alpha}} \sin\beta = 0 \quad ,$$

and the string equation reduces to

$$\partial_{\eta'}\partial_{\xi'}\hat{\alpha} - e^{\hat{\alpha}} + e^{-\hat{\alpha}}\cos\beta = 0,$$

as it was found in ref.[40].

The result that the string propagation in D-dimensional de Sitter space-time follows a generalized sinh-Gordon equation (in particular, in D = 3, this is just the standard sinh-Gordon model), shows that the presence of instability is a ***general exact*** feature of string propagation in de Sitter space-time, irrespective of any approximation scheme or of any particular solution. The strong attractive potential corresponding to the generalized sinh-Gordon equation is unbounded from below and this indicates that the string time evolution tends to the absolute minima at $\alpha = -\infty$ and $\alpha = +\infty$. (This unstable behaviour is explicitely exhibited by the form of the solutions). The function $\exp[\alpha(\sigma,\tau)]$ is a measure of the proper length of the string. The invariant length of the string is given by

$$ds^2 = \frac{1}{2H^2}\, e^{\alpha(\sigma,\tau)}\,(d\sigma^2 - d\tau^2)$$

and it grows infinitely when $\alpha \to +\infty$. Similarly, when $\alpha \to -\infty$, the string collapses to a point. This infinite stretching and this infinite shrinking is a typical feature of string instabilities, here appearing as a direct consequence of the generalized Sinh-Gordon equation, without the need of searching for explicit solutions.

The string equations of motion and constraints enjoy an exact symmetry transformation which is defined by a first order differential equation in $(\sigma, \tau)$ and whose compatibility condition yields the equations of motion. This is a Bäcklund transformation which relates solutions of a expanding string metric of radius R into solutions of the contracting or 'dual' string metric of radius $\tilde{R} = R^{-1}$.

We relate solutions $\alpha$ and $\tilde{\alpha}$ of the generalized sinh-Gordon equations through this Bäcklund transformation.

We also analyze solutions of this problem. First, we solve the D = 2 case. In this case, we find ***all*** the solutions. The solution corresponding to the center of mass of the string (geodesic motion) appears in a separate sector from the solutions describing the truly string properties. In two dimensions, they are strings winded around de Sitter universe and evolving with it. These solutions depend on two arbitrary functions which just reflect the conformal invariance on the world sheet. (There are no further arbitrary functions since there are no transverse degrees of freedom in D = 2). Here, $\tau \in [0,\pi]$ and a half of the string evolution $(0 < \tau < \pi/2)$ corresponds to the expansion time $(0 < u < \infty)$ of the de Sitter universe, u being the cosmic time. [Similarly, $(\pi/2 < \tau < \pi)$ corresponds to the contraction phase $(-\infty < u < 0)$]. The string can wind n times around de Sitter space (here a circle), n being an integer. In such case, the string evolution period is reduced to $\Delta\tau = \pi/2n$ ( instead of being $\pi/2$), i.e; in $\tau$ the string expansion (and contraction), is n times faster.

Asymptotically, for $u \to \pm\infty$, the cosmic time u depends logarithmically on $\tau$. In the context of cosmological backgrounds, this logarithmic behaviour is typical of strings in de Sitter universe. In these asymptotic regions, the proper length of the string stretches infinitely, i.e. the conformal factor $\exp[\alpha(\sigma,\tau)]$ blows up.

This is the same unstable behaviour found in inflationary backgrounds [6, 23, 24,], as well as for strings falling into space-time singularities [25]. In such asymptotic regimes, $\tau$ is proportional to the conformal time, another relation typical of strings in the presence of strong gravitational fields or near space-time singularities: in the asymptotic regimes where the string stretches or shrinks indefinitely, (and is thus unstable), the string evolution is governed by the conformal time.

The solution corresponding to the center of mass, is the trajectory of a massless particle. Transverse dimensions are absent in D = 2 and then, only massless states appear.

We also analyze solutions for $D = 3$ and $D = 4$. We find that for $\tau \to 0$,

$$\alpha(\sigma,\tau) = -2 \log \tau \quad \to -\infty$$

This behaviour reflects the string instability in de Sitter space, as a consequence of the unboundness of the Sinh-Gordon potential.

The study of *exact string solutions in 2+1 dimensional de Sitter spacetime* is performed in ref [41]. Strings generically tend to inflate or either to collapse. The world sheet time $\tau$ interpolates between the cosmic time ($\tau \to \pm\infty$) and conformal time ($\tau \to 0$). For $\tau \to 0$ the typical string instability is found, while for $\tau \to \pm\infty$, a new string behaviour appears. In that regime, the string expands (or contracts) but not with the same rate as the universe does.

## 4.2 String Instabilities in D-dimensional Black Holes

The string dynamics in black hole spacetimes is very complicated to solve (even asymptotically and approximately). In ref [11] the study of string dynamics in a Schwarzschild black hole was started and the scattering problem was studied for large impact parameters. Stable oscillatory behaviour of the string was found for the transversal (angular) components, accompanied by the particle transmutation process already described. In ref [42] the study of the *unstable sector* of strings in black hole backgrounds was started, and the emergence of string instabilities found. By unstable behaviour, we mean here the following characteristic features: non oscillatory behaviour in time, or the emergence of imaginary frequencies for some modes, accompanied of an infinite stretching of the proper string length. In addition, the spatial coordinates (some of its components) can become unbounded. Stable string behaviour means the usual oscillatory propagation with real frequencies, (and the usual mode- particle interpretation), the fact that the proper string size does not blow up, and that the string modes remain well-behaved.

We express the first order string fluctuations $\eta^\mu$, ($\mu$= 0, ....,D-1) in D-dimensional Reissner-Nordstrom-DeSitter spacetime as a Schrödinger type equation for the amplitudes $\chi^\mu = q^R \eta^\mu$, $q^R$ being the radial center of mass coordinate. We find the asymptotic behavior of the longitudinal and transverse string coordinates ($\chi^+$, $\chi^-$, $\chi^i$) with i= 2,.....,D-1, at the spatial infinity, near the horizon and near the spacetime singularity. Here + and - stand for the longitudinal (temporal and radial) components respectively, and i for the transverse (angular) ones. We analyse first a head-on collision(angular momentum L=0), that is, a radial infall of the string towards the black hole. Then, we analyse the full L=0 situation. We consider Schwarzschild, Reissner- Nordstrom and De Sitter spacetimes (described in static coordinates which allow a better comparison among the three cases). In all the situations (with and without angular momentum) and for the three backgrounds we find the following results:

The time component $\chi^+$ is *always stable* in the three regions (near the infinity, the horizon and the singularity), and in the three backgrounds (black holes and De Sitter spacetimes).

The radial component $\chi^-$ is *always unstable* in the three regions and in the three backgrounds. In the Schwarzschild case, the instability condition for the radial modes -which develop imaginary frequencies near the horizon- can be expressed as

$$n < \frac{\alpha' m \sqrt{D-3}}{R_s} \left[ D - 2 - \left( \frac{D-3}{2} \right) \frac{m^2}{E^2} \right]^2 ,$$

where $\alpha'$, m and E are the string tension, string mass and energy respectively. The quantity within the square brackets is always positive, thus the lower modes develop imaginary frequencies when the typical string size $\sim(\alpha' m\sqrt{D-3})$ is larger than the horizon radius. Notice the similarity with the instability condition in De Sitter space,

$n < \alpha' m r_H$, $r_H$ being the De Sitter horizon radius.

In the Schwarzschild black hole, the transverse modes $\chi^i$ are stable (well behaved) everywhere including the spacetime singularity at $q^R = 0$. In the Reissner-Nordstrom (RN) black hole, the transverse modes $\chi^i$ are stable at infinity and outside the horizon. Imaginary frequencies appear, however, inside a region from $r_- < q^R < r_+$ to $q^R \to 0$, where $r_\pm = M \pm \sqrt{M^2 - Q^2}$, M and Q being the mass and charge of the black hole respectively. For the extreme black hole (Q=M), instabilities do not appear. There is a critical value of the electric charge of a Reissner-Nordstrom black hole, above which the string passing through the horizon passes from the stable to unstable regime. In De Sitter spacetime, the only stable mode is the temporal one ($\chi^+$). All the spatial components exhibit instability, in agreement with the previous results in the cosmological context [6, 23, 24]. A sumary of this analysis is given in Table 2:

| Region | Mode | Schwarzschild | Reissner-Nordstrøm | De Sitter |
|---|---|---|---|---|
| $q^R \to 0$ | $\Xi^+$ | stable | stable | stable |
| | $\Xi^-$ | unstable | unstable | unstable |
| | $\Xi^i$ | stable | unstable | unstable |
| $q^R \to q^R_H$ | $\Xi^+$ | stable | stable | stable |
| | $\Xi^-$ | unstable | unstable / stable | unstable |
| | $\Xi^i$ | stable | stable | unstable |
| $q^R \to \infty$ | $\Xi^+$ | stable | stable | stable |
| | $\Xi^-$ | unstable | unstable | unstable |
| | $\Xi^i$ | stable | stable | unstable |

**Table 2** Regimes of string stability in black hole and De Sitter spacetimes: Here stable means well behaved string fluctuations and the usual oscillatory behavior with real frequencies. Unstable behavior corresponds to unbounded amplitudes $(\Sigma^{\pm}, \Sigma^{i})$ with the emergence of non oscillatory behavior or imaginary frequencies, accompanied by the infinite string stretching of the proper string length. $\Sigma^{+}, \Sigma^{-}$ and $\Sigma^{i}$, $(i = 2, \ldots, D-1)$, are the temporal, radial and angular (or transverse) string components respectively.

Imaginary frequencies in the transverse string coordinates $(\chi^{i})$ appear in the case in which the local gravity, i.e. $(\partial_r a/2,)$ is negative (that is repulsive effects). Here,

$$a(r) = 1 - (R_s/r)^{D-3} + (\tilde{Q}^2/r^2)^{D-3} + \frac{\Lambda}{3} r^2 ,$$

where $\tilde{Q}^{2(D-3)} = 8\pi G Q^2/[(D-2)(D-3)]$, and $\Lambda$ is the cosmological constant. That is why the transverse modes $(\chi^{i})$ are well behaved in the Schrwarzschild case, and outside the Reissner-Nordstrom event horizon. But close to $q^R \to 0$, $a'_{RN}<0$ (Reissner-Nordstrom has a repulsive inner horizon), and the gravitational effect of the charge overwhelms the attractive effect of the mass; in this case instabilities develop. In the Reissner-Nordstrom-De Sitter spacetime, unstable string behavior appears far away from the black hole where De Sitter solution dominates, and inside the black hole where Reissner-Nordstrom solution dominates. for M=0 and Q=0, we recover the instability criterium [10,6] $\alpha' m \Lambda/6 > 1$ for large enough Hubble constant (this is in agreement with the criterium given in reference [43]. See also ref. [34].

We find that in the black hole spacetimes, the transversal first order fluctuations $(\chi^{i})$ obey a Schrödinger type equation (with $\tau$ playing the role of a spatial coordinate):

$$\ddot{\chi}^i_n + \{ n^2 + [\alpha' L/(q^R)^2]^2 - \ddot{q}/qR \} \chi^i_n = 0.$$

By using the geodesic solutions for $q^R$, we obtain the full t-dependence of the potential. We find that near the spacetime singularity $q^R=0$, the potential is $\gamma(\tau-\tau_0)^{-2}$, (where $\tau_0$ is the proper time of arrival to the singularity at $q^R=0$). The dependence on D and L is concentrated in the coefficient $\gamma$. Thus the approach to the black hole singularity is like the motion of a particle in a potential $\gamma(\tau-\tau_0)^{-\beta}$, with $\beta = 2$. And, then like the case $\beta=2$ of strings in singular gravitational waves [27] (in which case the spacetime is simpler and the exact full string equations become linear). It should be stressed that for black holes, what determines the possibility of instabilities is not the type of singularity (Reissner Nordstrom or Schwarzschild), nor how it is approached (L=0 or L=0), but the sign of the coefficient $\gamma$, i.e. the attractive character of the potential $(\tau-\tau_0)^{-2}$. The coefficient $\gamma$ is given by

$$\gamma_{Sch} = \begin{cases} 2(D-3)/(D-1)^2, & L=O \\ 2(D-1)/(D+1)^2, & L=0 \end{cases}$$

for the Schwarschild black hole, and

$$\gamma_{RN} = \begin{cases} -2(D-3)/(D-2)^2, & L=O \\ -2(D-2)/(D-1)^2, & L=O \end{cases}$$

for the Reissner-Nordstrom black hole.

Here $\gamma>0$ for strings in the Schwarzschild spacetime, for which we have regular solutions $\chi^i$; while $\gamma<0$ for Reissner-Nordstrom, that is, in this case we have a singular potential and an unbounded behavior (negative powers in $(\tau-\tau_0)$) for $\chi^i{}_{RN}$. The fact that the angular coordinates $\chi^i{}_{RN}$ become unbounded means that the string makes infinite turns around the spacetime singularity and remains trapped by it.

For $(\tau-\tau_0) \to 0$, the string is trapped by the black hole singularity. In Kruskal coordinates $(u_K(\sigma,\tau), v_K(\sigma,\tau))$, for the Schwarzschild black hole, we find

$$\lim_{(\tau-\tau 0) \to 0} u_K v_K = \exp\ [2KC(\sigma)(\tau-\tau_0)^P]$$

where $K = (D\text{-}3)/(2R_s)$ is the surface gravity, $P>0$ is a determined coefficient that depends on the dimensions, and $C(\sigma)$ is determined by the initial state of the string. Thus, $u_K v_K \to 1$ for $(\tau-\tau_0) \to 0$, that is the string approaches the spacetime singularity $u_K v_K = 1$. The proper spatial length element of the string at fixed $(\tau-\tau_0) \to 0$, between $(\sigma,\tau)$ and $(\sigma+d\sigma,\tau)$, stretches infinitely as

$$dS^2{}_{(\tau-\tau 0) \to 0} \to \quad C(\sigma)'^2\, (\tau-\tau_o)^{-(D-1)P}\ d\sigma^2$$

The same conclusions can be drawn for the quantum propagation of strings. The $\tau$ dependence is the same because this is formally described by the same Schrödinger equation with a potential $\gamma(\tau-\tau_0)^{-2}$, the coefficients of the solutions being quantum operators instead of C-numbers. The $\tau$ evolution of the string near the black hole singularity is fully determined by the spacetime geometry, while the $\sigma$-dependence (contained in the overall coefficients) is fixed by the state of the string.

It must be noticed that in cosmological inflationary backgrounds, the unstable behavior manifests itself as non-oscillatory in $(\tau-\tau_0)$ (exponential for $(\tau-\tau_0) \to \infty$ , power-like for $(\tau-\tau_0) \to 0$); the string coordinates $\eta^i$ are constant (i.e. functions of $\sigma$ only), while the proper amplitudes $\chi^i$ grow like the expansion. In the black hole cases, and more generally, in the presence of spacetime singularities, all the characteristic features of string instability appear, but in addition the spatial coordinateshi (some of its components) become unbounded. That is, not only the amplitudes $\chi^i$ diverge, but also the string coordinates $\eta^i$, what appears as typical feature of strings near black hole singularities. A full description of the string behavior near the black hole singularity will be reported elsewhere [44].

The perturbative analysis of the equations of motion we have done, is valid in the stable regimes and allows to discover the presence of instabilities. In the unstable cases, one has to describe the unstable non linear regime non-perturbatively. Asymptotic solutions describing the highly unstable string regime will be reported elsewhere [44].

### 4.3 The Two-dimensional Stringy Black Hole New Approach and a Pathology

*Contextual Background*

The space-time metric associated to the SL(2,R)/U(1) coset model (and its different versions), and interpreted as a two dimensional black hole [45-47] arised enormous attention recently (see references [48-50] for instance).

Very recently, in ref. [51], we looked at this problem from our string gravity point of view, that we started and developped (see our references here for instance), independently and before the interest on two dimensional black holes flourished.

The two dimensional black hole model is interesting in the sense that it is an exact solution of the renormalization group equations of string theory. As it is known, these equations define the backgrounds in which strings propagate consistently.

Although two dimensional models have many attractive tractable aspects and can be used to test and get insights on particular features, D=2 is not for string theory neither for gravity the most physically appealing dimension.

Strings in two dimensional black hole backgrounds have been extensively treated in the literature in the context of the conformal Field Theory (CFT) techniques. (See ref. [48] for a complete description). In ref. [51] and here, we present a different view and a different approach to this problem which yields in a simple and physical way the full exact quantum result; generalizes and yields new features with respect the already known C F T descriptions.

We study string propagation in the two-dimensional black hole background of refs [45-47]

$$dS^2 = k\ [\ dr^2 - \tanh^2 r\ dt^2\ ]$$
$$\Phi = \log(\cosh^2 r) \quad , \quad 0 < r < \infty\ ,\ -\infty < t < +\infty$$

where k is a positive integer and $\Phi$ is the dilaton field. (Classically, the string propagates in this metric, and $\Phi$ is decoupled from them). In Kruskal null type coordinates (u ,v) this metric reads

$$dS^2 = k\ du\ dv\ /\ (1 - uv) \quad , \quad 0 < u,v < \infty .$$

We completely solve the string equations of motion and constraints in the Lorentzian and on the Euclidean (i.e. $t = i\Theta$) regimes. The solutions fall into four types. In the Lorentzian regime in which there are no compact dimensions in the space-time, the $\sigma$-dependence can be completely gauged away, and all the solutions describe just the geodesics of a massless point particle. The only physical degree of freedom is the one associated to the center of mass of the string; we are left more with a point particle field theory rather than a string theory.

In the Euclidean regime, the Schwarzchild imaginary time is periodic, $(0 \leq \Theta \leq 2\pi)$, and the $\sigma$-dependence of the string solutions remains. The solution must be also euclidean in the world sheet ($\sigma$

becames purely imaginary and identified to the imaginary time). There are two types of euclidean solutions : (i) a string winding $n$ times along the $\Theta$-direction in the "cigar" manifold from r=0 to r= $\infty$, and (ii) a string winding $n$ times along $\Theta$ in the "trumpet" manifold $dS^2 = ds^2 + \coth^2 s \, d\Theta^2$, $0 < s < \infty$. It must be noticed that these are *rigid* strings staying around the manifolds without any oscillation. In D=2, $e^{in\sigma}$ does not describes the string fluctuations, thus the name modes ("winding modes" or "momentum modes", as commonly refered (see for example ref. [48]), are misleading.

For these solutions, the metric on the world-sheet is

$$dS^2_{cigar} = k \frac{n^2}{1+e^{2n\tau}} \left(d\tau^2 + d\sigma^2\right)$$
$$dS^2_{trumpet} = k \frac{n^2}{1-e^{-2n\tau}} \left(d\tau^2 + d\sigma^2\right)$$

The proper string length (between two points $\sigma$ and $\sigma+d\sigma$ at fixed $\tau$) is

$$\Delta S_{cigar} = \sqrt{k} \frac{n}{\sqrt{1+e^{2n\tau}}} \, d\sigma \ , \ \Delta S_{trumpet} = \sqrt{k} \frac{n}{\sqrt{1-e^{2n\tau}}} \, d\sigma$$

In D=2 the string world sheet and the physical space-time can be identified; the world sheet covers completely (generically, $n$ times) the physical space. The trumpet manifold has a curvature singularity at s=0. Near s=0, i.e. $\tau \to 0$, $\Delta S_{trumpet}$ blows up, the string stretches infinitely. This typical feature of string instability [6, 23-25, 34, 38, 41, 42], just corresponds here to the location of the string in the open extremity of the trumpet. The euclidean black hole manifold is non singular, and the string length ($\Delta S_{cigar}$,) is always finite in it, since the cigar radius is finite. We also discuss the cosmological version of the black hole string solutions (in the euclidean regime); in this case the string solutions are the same as the trumpet solution.

We recall that in D =2 de Sitter space-time with Lorentzian signature, the general solution is a string wound $n$ times around de Sitter space (the circle $S^1$) and evolving with it. For $\tau \to 0$, $\Delta S_{de\ Sitter} \to \infty$; the string expands (or contracts, $\Delta S_{de\ Sitter} \to 0$) with the universe itself.

It is interesting to point out the relation between $\tau$ and the physical time: $\tau$ interpolates between the Kruskal time ($\tau \to 0$) and the Schwarzschild time ($\tau \to \infty$). This is like strings in cosmological backgrounds in which t interpolates between the conformal time ($\tau \to 0$) and the cosmic time ($\tau \to \infty$). (And like strings in gravitational wave backgrounds). The logarithmic relation between the Schwarzchild time and $\tau$ for $\tau \to 0$ is exactly like the cosmic time and $\tau$ in de Sitter space.

In this two dimensional black hole, the string takes an infinite time $\tau$ to approach the curvature singularity u v = 1, and then, never reaches it. This is to be contrasted with the higher (D-) dimensional black hole [42] and discussed in the section above. For all D>2, the string falls down into the physical singularity u v = 1 in a finite proper time $\tau$, and classically as well as quantum mechanically, the string is trapped by the singularity [42].

We study the string quantization in this background (in the lorentzian and in the euclidean regimes) and follow the spirit of ref. [10], in which one starts from the c.m. motion, and then takes the $\sigma$-dependence as fluctuations around. Here, in the lorentzian case, there are no $\sigma$-fluctuations, and the quantization of the center of mass Hamiltonian

$$\mathcal{H} = \frac{1}{2k}\left[(p_r - i\tanh r)^2 - p_t^2 \coth^2 r\right]$$

$$\mathcal{H} \to \hat{\mathcal{H}} = \frac{1}{2k} h \quad , \quad \partial_t = -iE \quad , \quad h = -\left[\frac{d^2}{dr^2} + 2\coth(2r)\frac{d}{dr} + E^2\coth^2 r\right] \quad ,$$

yields the full and exact description of the system.

The operator h is the same as the $L_0$ operator of ref. [48] (here after refered as DVV). They only differ by additive and multiplicative constants, (but as operators are identically the same). We solve the eigenfunction problem

$$h\,\Psi = \lambda\Psi$$

describing the scattering of the (masless) tachyon field by the black hole, which yields a Schrödinger type equation with an energy-dependent potential

$$V_{eff} = -\frac{E^2+1/4}{\sinh^2 r} + \frac{1}{4\cosh^2 r} - E(E-1) - \lambda + 1 \overset{r\to 0}{=} -\frac{\gamma}{r^2} + O(1)\ , \quad \gamma \equiv E^2 + 1/4$$

Since $\gamma > 1/4$, there exists absortion (fall into the event horizon r=0) for all energy E. This is like the D>2 black holes.

The constant $\lambda$ is determined from the $r \to \infty$ behaviour, by requiring the tachyon to be massless

$$\lambda - 1 = (1 - c^2)\, p^2$$

p being the momentum, p = E/c.

The eigenfunctions $\Psi$ are given in terms of hypergeometric functions

$$\Psi^{\pm}_{E,c}(r,t) = e^{-iEt}(\sinh r)^{\mp iE} F\left(\frac{1}{2} \mp \frac{iE}{2}(1+\frac{1}{c}), \frac{1}{2} \mp \frac{iE}{2}(1-\frac{1}{c}); 1 \mp iE; -\sinh^2 r\right)$$

They exhibit the typical behaviour of the wave functions near the black hole horizon

$$\Psi^{\pm}_{E,c}(r,t) \overset{r\to 0}{=} e^{-iE(t \pm \log r)}\ ,$$

$\Psi^+$ describing purely incoming particles at the future event horizon. ($\Psi^-$ describes outcoming particles from the past horizon). The tachyon field incident from spatial infinity is partially absorbed and reflected by the black hole

$$\Psi^{+}_{E,c}(r,t) \overset{r\to\infty}{=} e^{-ipct}\left[e^{-ipr} + S(p,c)\, e^{ipr}\right]$$

We find

$$S(p,c) = S_{coul}(p)\ \tilde{S}(p,c)$$

where

$$S_{coul}(p) = 2^{-2ip}\frac{\Gamma(ip)}{\Gamma(-ip)} = 2^{-2ip}e^{2i\arg\Gamma(ip)}$$

$$\tilde{S}(p,c) = \left[\frac{\Gamma\left(\frac{1}{2} - \frac{ip}{2}(c+1)\right)}{\Gamma\left(\frac{1}{2} - \frac{ip}{2}(c-1)\right)}\right]^2$$

$S_{coul}$ takes into account the large r interaction and the purely elastic scattering; $\tilde{S}$ describes the genuine black hole features:

$$|S_{coul}(p)| = 1$$

$$|\tilde{S}(p,c)| = \left[\frac{\cosh\frac{\pi p}{2}(c-1)}{\cosh\frac{\pi p}{2}(c+1)}\right]^2 < 1\,,$$

which describes the absorption by the black hole.

S(p,c) exhibits an infinity sequence of imaginary poles at the values ip=n, (n=0,1,...), which are like the Coulombian-type poles, and also an infinite sequence of imaginary poles at ip(c+1) = 2n+1, n=0,1... (in the $\tilde{S}$ part).

It must be stressed that S(E,c) depends on two physical parameters: E and c. For each energy E, we have a monoparametric family of solutions depending on c, each c yields a different S-matrix. c is a *purely quantum mechanical parameter*, which is not fixed by any special requirement, and can take any value; c accounts for a renormalization of the speed of light. Classically, we choose our units such that c=1. But, quantum mechanically, c is no more unit in this problem. We find that c is related to the parameter k of the WZW model:

$$c = \sqrt{\frac{k}{k-2}}$$

and we reproduce the results of ref [48]. In ref [48], the effect of the renormalization of the speed of light is also present (although it has not been noticed). Classically, before quantization, DVV have c=1, but after quantization, since they choose k = 9/4, they have c=3. This can be seen from the asymptotic behaviour of the metric for $r\to\infty$.

Classically, $dS^2(r\to\infty) = dr^2 + d\Theta^2$, and after CFT quantization of the SL(2,R)/U(1) model :

$$dS^2_{DVV} \overset{r\to\infty}{=} 2(k-2)\left[dr^2 + \frac{k}{k-2}d\theta^2\right] = \frac{4}{c^2-1}[dr^2 + c^2 d\theta^2]$$

Notice that in the classical limit $k\to\infty$, c takes its classical value. The S-matrix of ref. [48] is a particular case of our results for c=3:

$$S(p, c=3) = S_{DVV}(p) = 2^{-2ip}\frac{\Gamma(ip)}{\Gamma(-ip)}\left[\frac{\Gamma\left(\frac{1}{2}-2ip\right)}{\Gamma\left(\frac{1}{2}-ip\right)}\right]^2$$

[The factor $2^{-2ip}$ is missing in ref.[48]].

Notice that nothing special happens, however, at the conformal invariant k=9/4 point. The physical relevance of the k=9/4 point in the two dimensional black hole scattering matrix is not clear. Also notice that the presence or absence of conformal anomaly (it vanishes for k=9/4) is totally irrelevant for the black hole singularity.

The computation of the Hawking radiation in this problem follows directly from the by now well known treatment of QFT in curved space-time and does not present any particular feature here. The vacuum spectrum is a Planckian distribution at the temperature $1/2\pi$.

We also quantize the system in the euclidean regime and exactly compute the partition function of the two dimensional stringy black hole. It is reported in ref. [51].

In conclusion, the quantization of the center of mass of the string, i.e. of the classical solution, with the requirement that $m^2=0$, yields all the physics of the problem. We have obtained the full exact quantum result without introducing any correction 1/k to the space time metric, neither to the dilaton.

The two dimensional CFT constructions can be avoided in a problem like the two dimensional stringy black hole. They introduce a lot of technicality in a problem in which all the physics can be described by the straightforward quantization of a two-dimensional massless scalar particle. Perhaps the CFT tools would be really necessary for the problem of interacting (higher genus) strings in the two dimensional black hole background, problem, which unfortunately, has not been treated until now.

Finally, let us comment about the pathological feature of the renormalization of the speed of light for strings in these two dimensional curved backgrounds. Normally, c is never affected by quantum corrections. Once one chooses units such that c=1 classically, this value remains true in the quantum theory (relativistic quantum mechanics, QFT, string theory, etc). In the D=2 stringy black hole, it turns out that $c = \sqrt{k/(k-2)}$ for the quantum massless particle described by the string. We think that this pathology is specific of the two dimensional string dynamics in such curved background.

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