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MORE ABOUT THE TIME ANALYSIS OF TUNNELLING PROCESSES: ANSWER TO THE COMMENTS BY LEAVENS ON OLKHOVSKY—RECAMI'S APPROACH TO THE "TUNNELLING TIME PROBLEM".(*)

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ABSTRACT – In our recent review article [*Phys.Rep.***214**(1992)339] we put forth an analysis of the main theoretical definitions of the sub-barrier tunnelling and reflection times, and proposed a new definition for such durations which seems to be self-consistent within conventional quantum mechanics. In this note we criticize —showing them to be unjustified— the reasons that led Leavens in a very recent paper to regard our definitions as "seriously flawed" and to object to our analysis of the dwell–time approaches. At last, the results of Leaven's calculations of the average transmission times "by our expressions" are shown to be incorrect, in the sense that they are not derived really from our equations.

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We are, first of all, thankful to Leavens[1] for his interest in our recent review–paper[2], and his rapid attempt to calculate transmission times by the expression forwarded by us. We are going to show, however, that his critical comments[1] on our analysis[2] of the dwell–time approaches[3–7] are not well grounded, while his calculations[1] claiming to use some expressions of ours[2] do not appear to be correct, in the sense that they are not based actually on our equations.

To begin with, let us recall that, in our eqs.(30)-(31) for the average transmission and reflection times, $< \tau_{\rm T} >$ and $< \tau_{\rm R} >$, the related, temporal integrations[2] had to run from $-\infty$ to $+\infty$. On the contrary, in ref.[1] they were replaced by time integrals running only from zero to ∞ [see eqs.(4)-(5) and (12) therein]. This is not admissible in our approach —which does of course deal with a collision process—, for the following reasons. First of all, to assume that the incident wave-packet is prepared strictly at a finite time $t = 0$, *i.e.* starts passing through the point $x_{\rm i}$ at an instant $t = 0$, means to introduce a sharp forward front, getting a certain contribution from large *above-barrier* speeds (which ought to be cut off, according to condition (7) in ref.[2]). Even more important, such a "preparation" is incompatible with choice (7), adopted in ref.[1] for the initial wave-packet, since in any self-consistent formalism a wave-packet with tails infinitely extended in space (namely, for $\mid x - x_0 \mid \longrightarrow \infty$) —as commonly required in order to avoid infinite velocities— does inevitably possess tails infinitely extended also in time (*i.e.*, for $\mid t - t_0 \mid \longrightarrow \infty$). With that choice, therefore, the substitution of integrals of the type $\int_0^\infty {\rm d}t$ for our integrals $\int_{-\infty}^\infty {\rm d}t$ does mean cutting off the temporal extension for $t < 0$ of the wave-packet moving along the x-axis; and this artificial, and physically unjustified, cut-off does even depend on x. No wonder, therefore, that in ref.[1] unphysical results were obtained when calculating the average transmission times inside the barrier by their (but not our!) equation (12). We can here stress, at variance with the author of ref.[1], that the quantitative evaluations performed by Zakhariev (briefly presented by us in ref.[8], and qualitatively depicted in ref.[2]) show clearly enough that a noticeable contribution to all time averages comes just from the integration from $-\infty$ to zero.

For clarity's sake, let us underline again that in our approach the initial wave-packet $\Psi_{\rm in}(x,t)$ is not regarded as prepared at a certain instant of time, but it is expected to flow through any (initial) point $x_{\rm i}$ during the infinite time interval $(-\infty, \ +\infty)$, even if with a *finite* time–centroid x_0. The value of such centroid x_0 is essentially defined by the phase of the weight amplitude $g(E - \bar{E})$, and is equal to 0 when $g(E - \bar{E})$ is real. Actually, it is an usual procedure in the collision theory (differently from the case of the decay theory!) to assume the initial packet to be prepared during a rather long (ideally infinite) time interval, in a remote past [adiabatic hypothesis]; see, *e.g.*, sect.1 of Chapter 5 in Golberger and Watson's book, ref.[5].

Let us now pass to the dwell–time question, interestingly raised by Leavens (we are grateful to him, incidentally, for the offered opportunity of expanding our considerations

about the significance of the dwell–time $\bar{\tau}^{\rm Dw}$). In our recent review[2], we did not reject the concept of dwell–time as a whole, but only showed some shortages of the dwell–time approaches developed in refs.[3--7]. In fact, let us consider our equation (16) of ref.[2] (all notations being defined there):

$$\bar{\tau}^{\rm Dw}(x_{\rm i}, x_{\rm f}; k) \;=\; \left(\int_{-\infty}^{\infty} {\rm d}t \int_{x_{\rm i}}^{x_{\rm f}} {\rm d}x\; \rho(x,t)\right) \left(\int_{-\infty}^{\infty} {\rm d}t\; J_{\rm in}(x_{\rm i},t)\right)^{-1} \quad , \tag{1}$$

and its equivalence —discovered in ref.[9], and stressed in ref.[1]— with the expression

$$\bar{\tau}^{\rm Dw}(x_{\rm i}, x_{\rm f}; k) \;=\; \left(\int_{-\infty}^{\infty} {\rm d}t\, t\, J(x_{\rm f},t) \;-\; \int_{-\infty}^{\infty} {\rm d}t\, t\, J(x_{\rm i},t)\right) \left(\int_{-\infty}^{\infty} {\rm d}t\; J_{\rm in}(x_{\rm i},t)\right)^{-1} \quad , \tag{2}$$

whose notations are still to be found in ref.[2]. This equivalence *reduces* the difference between our definition $< \tau_{\rm T} >$ of the average transmission time and quantity $\bar{\tau}^{\rm Dw}$ *to* the difference between the average made with use of the positive–definite probability density ${\rm d}t J_+(x,t)/\int_{-\infty}^{\infty} {\rm d}t J_+(x,t)$ and the average made with use of the ordinary "probability density" ${\rm d}t J(x,t)/\int_{-\infty}^{\infty} {\rm d}t J(x,t)$. Generally speaking, the last expression is *not* always positive definite, as it was explained at page 350 of ref.[2], and hence does not possess any direct physical meaning. A clear physical meaning can be attributed to the dwell–time expressions (1)-(2) only when[2] $x_{\rm i} \longrightarrow -\infty$ and $x_{\rm f} \geq a$.

In ref.[1] it has been critically commented also on our view about performing actual averages over the physical time. We cannot agree with those comments. Let us re-emphasize that, within conventional quantum mechanics, the time $t(x)$ at which a particle (wave-packet) passes through the position x is "statistically distributed" with the probability densities ${\rm d}t J_\pm(x,t)/\int_{-\infty}^{\infty} {\rm d}t J_\pm(x,t)$, as we explained at page 350 of ref.[2]. This distribution meets the requirements of the time–energy uncertainty relation.

The last object of the criticism in ref.[1] refers to the impossibility, in our approach, of distinguishing between "to be transmitted" and "to be reflected" particles at the leading edge of the barrier. One may answer just by recalling that the presence of the related interference terms in $\rho(x,t)$, in $J(x,t)$ and even in $J_\pm(x,t)$ seems to be an inevitable consequence of the superposition principle, valid for wave functions in conventional quantum mechanics.

Let us end by observing that, although Leavens' criticism on our paper[2] seems to be due to incorrect interpretation of our formulas and reasoning, nevertheless his aim of comparing the definitions proposed by us for the tunnelling times not only with conventional, but also with non–standard quantum mechanics may be regarded as stimulating

and worth of further investigation.

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