8/10 10

Inflation, Gravitational Waves and the Cosmic Microwave Background: Reconciling CDM with COBE?

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Abstract. We examine the production of tensor (gravitational wave) perturbations in a universe where inflation is driven by a scalar field interacting via an exponential potential. In such a scenario, the Universe undergoes a power-law rather than exponential expansion and density perturbations have a power-law but non scale-invariant spectrum. We show that models which lead to only slight departures from scale-invariant density perturbations also produce significant gravitational wave perturbations that lead to significant anisotropies in the Cosmic Microwave Background. For a spectral index $n < 0.95$, more than half the quadrupole anisotropy detected by COBE could be due to tensor, rather than scalar, perturbations. This result has profound implications for the Cold Dark Matter (CDM) model of galaxy and large-scale structure formation. If the standard version of CDM, in which the scalar perturbations are scale invariant and tensor perturbation production is negligible, is normalised to produce the correct level of CMB anisotropy it fails to account for the small scale velocities and clustering properties of galaxies. On the other hand, if non-scale-invariant fluctuations are generated by the inflationary mechanism we suggest, a CDM model can simultaneously account for these properties and the COBE quadrupole anisotropy. The model may still have problems with large-scale clustering and bulk streaming motions but observations on such scales are less well established and interpretation is made difficult by any scale-dependent bias.

Key Words: radiation mechanisms: gravitational - galaxies: clustering, formation - cosmic microwave background - cosmology: theory - large-scale structure of the Universe - early Universe.

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1 Introduction

One of the attractive features of the inflationary model for the evolution of the very early Universe is that it leads to a prediction of the amplitude, spectrum and statistics of the density fluctuations in the Universe on very large scales. These fluctuations manifest themselves as temperature fluctuations in the cosmic microwave background radiation (CMBR) and create the distribution of linear density irregularities from which galaxies and clusters subsequently form. Besides offering a self-consistent picture for the origin of galaxies and large-scale clustering, these predictions permit inflationary universe scenarios to be tested against observations.

Unfortunately, models of large-scale structure formation based upon the inflation picture have fared very badly when confronted with observed data. The most successful such model has been the Cold Dark Matter (CDM) model, but this has suffered a series of severe setbacks at the hands of the observers. Longstanding difficulties for CDM have been posed by the high amplitude of the cluster-cluster correlation function (Bahcall & Soneira 1983) and the apparently large amplitude and coherence length of the peculiar velocity field (Lynden-Bell *et al.* 1988; Bertschinger *et al.* 1990). More recently, the APM angular galaxy-galaxy correlation function (Maddox *et al.* 1990) and the QDOT IRAS galaxy redshift survey (Efstathiou *et al.* 1990; Saunders *et al.* 1991) have demonstrated – with an impressive consistency between the two – that the standard version of CDM fails to generate enough large-scale structure compared with the observations. Peacock (1991) has demonstrated that some, at least, of these failings can be remedied by choosing a larger normalisation amplitude for the fluctuation spectrum. However, this resolution is unsatisfactory because it requires there to be a much higher degree of gravitational evolution of the dark matter than is consistent with galaxy peculiar motions and clustering properties on small scales. The CDM model has been very successful at explaining small and intermediate-scale observations but only at the expense of requiring there to be a *bias* in the distribution of dark relative to luminous material (Davis *et al.* 1985). The unbiased CDM model favoured by large-scale clustering observations cannot account for small-scale properties from which direction its support has always been greatest.

The knockout punch for standard CDM appears to have been delivered by the COBE detection of large-scale temperature anisotropy on the CMBR sky (Smoot *et al.* 1992) at a level roughly consistent with Peacock's (1991) proposed normalisation of the mass fluctuations. This result has led some of the major adherents of the CDM model to abandon the standard form and appeal to either a cosmological constant (Efstathiou, Bond & White 1992) or a mixture of Hot and Cold Dark Matter (Taylor & Rowan-Robinson 1992; Davis *et al.* 1992) to solve the large-scale structure and CMBR problems without posing difficulties on small scales.

In this paper, we shall take a different approach to the large-scale structure problem. We shall question the usual assumption that primordial density fluctuations generated by inflation need necessarily be of the scale-invariant Harrison-Zeldovich form invoked in the standard CDM model. Non scale-invariant perturbations have been discussed previously (Salopek, Bond & Bardeen 1989; Liddle, Lyth & Sutherland 1992) but these authors have attempted to fit the excess fluctuations seen in galaxy clustering directly by invoking a primordial power-spectrum with $n < 1$. The prospects for achieving this are limited since the COBE detection constrains $0.6 < n < 1.6$ and we require $n \simeq 0.6$ to fit the extra power seen, for example, in APM (Liddle *et al.* 1992). We shall argue that a specific (and theoretically well-motivated) inflationary model does exist that produces $n < 1$. However, it is not the extra primordial power that is important for structure formation theories. In the model we consider, the generation of non scale-invariant (scalar) density perturbations during inflation is accompanied

by a significant production of gravitational wave (tensor) perturbations. These are not relevant to structure formation at late times, but do produce the same distortion of the CMBR temperature via the Sachs-Wolfe effect as do the scalar gravitational potential fluctuations. If scalar and tensor modes contribute equally to the CMBR anisotropy then a given $\Delta T/T$ implies a $\delta\rho/\rho$ that is a factor of $\sqrt{2}$ lower than if all the $\Delta T/T$ had been produced by scalar perturbations.

We begin by looking at the generation of scalar and tensor perturbations during inflation.

2 Scalar and Tensor Perturbations from Inflation

One of the most important questions cosmology has had to answer is why the observable universe is almost, but not quite exactly, homogeneous and isotropic. The inflationary scenario offers a possible explanation for these observations (Olive 1990). Consider a Friedman universe containing matter with density ρ and pressure p. The evolution is governed by the Einstein acceleration equation

$$6\ddot{a} = -a(\rho + 3p) \tag{1}$$

and the mass conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where the Hubble expansion rate, $H(t)$, is defined by $H = \dot{a}/a$ (the dot denotes differentiation with respect to cosmic time, t).

The horizon, flatness and isotropy problems can be solved if the Universe undergoes a sufficiently long period of accelerated expansion ($\ddot{a} > 0$). From equation (1), this is equivalent to the requirement that $\rho + 3p < 0$. The simplest way to achieve such an anti-gravitational effect is by the presence of a homogeneous scalar field, ϕ, with some self-interaction potential $V(\phi) \geq 0$. In the Friedman universe such a field is equivalent to a fluid with

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{3a}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{3b}$$

Other matter fields play a negligible role in the evolution during the period of inflation so their presence will be ignored. Hence $-\rho \leq p \leq \rho$ and we have the inflationary requirement $\ddot{a} > 0$ as long as $\dot{\phi}^2 < V$. Inflation is thus achieved when the matter sector of the theory applicable at some stage in the early Universe is dominated by decaying vacuum energy (Guth 1981).

The consequence of inflation is that any initial inhomogeneities become undetectable on observable scales. On the other hand, short wavelength quantum fluctuations in the matter fields are redshifted beyond the Hubble radius and re-enter during the radiation or matter dominated eras. These fluctuations may provide the primordial density spectrum that leads to galaxy formation (Bardeen, Steinhardt & Turner 1983).

Such a spectrum can be described in terms of the Fourier components of the perturbation: each scale has a wavenumber k/a associated with it, where k is the comoving wavenumber. At any given time microphysical processes are only important on scales $\leq O(H^{-1})$. Hence, the quantity $k/aH > 1$ for a given mode before inflation, but decreases through unity as inflation proceeds. The perturbation is said to 'cross the horizon' when $k/aH = 1$, where the term horizon refers to the Hubble radius at that time. After reheating k/aH grows and the perturbation 're-enters the horizon' when k/aH reaches

unity again. The amplitude of the perturbation remains constant for $k/aH < 1$ and is determined by the particular particle physics model driving inflation.

These perturbations affect the isotropy of the CMBR. Angular scales on the CMBR are related to linear scales l_0 at the present epoch by $\theta(l_0) \approx \frac{1}{2}H_0\Omega_0 l_0$ and the Hubble radius at decoupling subtends an angle of $\theta \approx 1^o$. Hence, detection of anisotropy on scales $\theta \gg 1^o$ yields information on the form of the primordial fluctuation spectrum and also provides a direct window on the physics of the universe during the grand unified era ($t \approx 10^{-35}$s) when these fluctuations were generated.

There exist a number of physical effects that lead to a CMBR anisotropy, but the dominant contribution on large angular scales is from the Sachs-Wolfe effect (Sachs & Wolfe 1967). At decoupling, perturbations in the gravitational potential cause the photons to become redshifted as they climb out of potential wells. The amplitude of the temperature fluctuation in the direction $\hat{\mathbf{n}}$ as seen by an observer at $\mathbf{r}$ is related to the density perturbation $\delta(\mathbf{r})$ by (Sachs & Wolfe 1967; Turner 1991)

$$\frac{\Delta T}{T}(\mathbf{r};\hat{\mathbf{n}}) = -\frac{1}{2}\frac{a^2H^2}{k^2}\delta(\mathbf{r}), \tag{4}$$

where any dipole effects have been removed. During matter domination $a^2H^2\delta$ is time-independent (Turner 1991) and can therefore be identified with the perturbation at horizon crossing, since $k/aH = 1$.

This shows how inflationary density perturbations give rise to anisotropies in the microwave background. But this is just the contribution from *scalar* perturbation modes. Could *tensor* modes give rise to a comparable signal? To answer this, it proves convenient to re-write equations (1) & (2) in the form

$$(H')^2 - \frac{3}{2}\kappa^2H^2(\phi) = -\frac{\kappa^4}{2}V(\phi) \tag{5a}$$

$$\kappa^2\dot{\phi} = -2H', \qquad H = \frac{d\ln a}{dt}, \tag{5b, c}$$

(Muslimov 1990; Salopek & Bond 1990) where a prime denotes $d/d\phi$, $\kappa^2 = 8\pi m_P^{-2}$ and m_P is the Planck mass. Units are chosen such that $\hbar = c = 1$.

The amplitude at horizon crossing arising from scalar fluctuations is (Lucchin & Matarrese 1985a)

$$A_S = \frac{m}{4\pi^{3/2}}\frac{H^2}{|\dot{\phi}|} = \frac{m\kappa^2}{8\pi^{3/2}}\frac{H^2}{|H'|}, \tag{6}$$

where the quantities on the right-hand side are evaluated when the perturbations first cross the Hubble radius approximately 60 e-folds before the end of inflation (Steinhardt & Turner 1984) and A_S is the corresponding amplitude when they re-enter the horizon at $t = t_{HC}$. The constant, m, is 4 or 2/5 if the universe is respectively radiation- or matter-dominated at t_{HC}; perturbations leading to temperature fluctuations on large angular scales re-enter during matter domination.

However, tensor fluctuations (i.e fluctuations in the graviton field) will also lead to temperature anisotropies via the Sachs-Wolfe effect. A graviton may be viewed as a massless, minimally coupled scalar field which has two degrees of freedom, $\varphi_{+,\times}$, corresponding to the two modes of the transverse, traceless metric perturbation (Abbott & Wise 1984). For these perturbations, the amplitude at horizon crossing is given by (Abbott & Wise 1984; Lucchin & Matarrese 1985b)

$$A_G = \frac{\kappa}{4\pi^{3/2}}H. \tag{7}$$

The ratio of scalar to tensor perturbation amplitudes is therefore given by

$$\frac{A_S}{A_G} = \frac{\kappa}{5}\frac{H}{|H'|}, \tag{8}$$

and gravitational waves will become important when this ratio is less than unity, i.e. $|H'|/H > \kappa/5$. This simple relation is very useful and is valid for an arbitrary functional form for $H(\phi)$ and hence for the inflaton potential. It should be emphasised that no simplifying assumption concerning the dynamics of the scalar field has been made in our analysis. Both Salopek (1992) and Liddle & Lyth (1992) make use of the the friction-dominated, $|\ddot{\phi}| \ll H|\dot{\phi}|$, and slow-roll, $\dot{\phi}^2 \ll V(\phi)$, assumptions. These assumptions lead to a quasi-exponential expansion during which the condition $|H'|/H \ll \kappa$ holds. In the limit $|H'|/H \to 0$, it is clear from equation (8) that $A_G/A_S \to 0$. However, an accelerated expansion will occur if (Carr & Lidsey 1992)

$$\dot{H} + H^2 > 0 \qquad \Longleftrightarrow \qquad |H'|/H < \kappa/\sqrt{2},$$

(where equation (5b) has been used to rewrite $\dot{H}$ in terms of H'): a condition which does not *necessarily* entail the slow-roll condition. We therefore consider it more appropriate to drop these assumptions whenever possible. This is particularly relevant in the case of power-law inflation (see below), where the use of these assumptions is expected to lead to an underestimate of the production of tensor perturbations.

It is now easy to see that the tensor modes will dominate the scalar modes in the non-negligible region of parameter space defined by $\kappa/5 < |H'|/H < \kappa/\sqrt{2}$. In conclusion, it is clear that gravitational waves become significant whenever the kinetic energy of the field as measured by $(H')^2$ is sufficiently high, *regardless* of the functional form of the inflaton potential.

Furthermore, when the ratio (8) is constant, the two spectra have the same scale dependence and it is easy to see that this is possible *if and only if* $H \propto H'$. Clearly $H(\phi)$ must have an exponential form in this case; eq (5a) then demonstrates that the functional form of the potential is $V(\phi) \propto \exp(-\lambda\kappa\phi)$, for some constant λ. In other words, the scalar and tensor fluctuations have the same scale dependence when the inflation is driven by an exponential potential. This is important because such potentials arise generically in a number of particle physics models. For example, many higher-order and scalar--tensor gravity theories are conformally equivalent to general relativity with a scalar field which self-interacts through such a potential (Maeda 1989; Barrow & Cotsakis 1988). One example is the extended inflationary scenario based on a first-order phase transition in the Brans-Dicke theory (La & Steinhardt 1989). Consequently, we shall consider this model in the following discussion.

It is possible to solve the equations (5) exactly for a potential of this form. The result is that $a(t) \propto t^{2/\lambda^2}$; inflation therefore requires $\lambda^2 < 2$. One can also compute the spectrum of fluctuations which turns out to be $A_S \propto k^{-\lambda^2/(2-\lambda^2)}$ for this potential (Lucchin & Matarrese 1985a). Such a scale dependence is consistent with the COBE analysis; a power spectrum of the form $P(k) \propto A_S^2 k \propto k^n$, with spectral index n, fits the COBE observations if $0.6 < n < 1.6$. Hence, n and λ are related by

$$n = \frac{3\lambda^2 - 2}{\lambda^2 - 2} \qquad \Longleftrightarrow \qquad \lambda^2 = 2\left(\frac{1-n}{3-n}\right), \tag{9}$$

which implies $\lambda^2 < 1/3$ for consistency. For $A_S < A_G$, we require $\lambda^2 > 4/25$ which is equivalent to $n < 0.83$. We note that the scale-invariant spectrum $n = 1$ is recovered in the limit as $\lambda^2 \to 0$.

3. Effect upon the Cosmic Microwave Background

Now let us see what these results mean for the large-scale anisotropy of the CMBR. To proceed, we assume the quadrupole anisotropy observed by COBE is due to both scalar and tensor perturbations which we take to be uncorrelated and obeying gaussian statistics (Barrow & Coles 1990), i.e.

$$\left(\frac{\Delta T}{T}\right)^2_{\rm obs} = \left(\frac{\Delta T}{T}\right)^2_S + \left(\frac{\Delta T}{T}\right)^2_G. \tag{10}$$

The $(\Delta T/T)^2_S$ contribution is related to A_S^2 via eqs. (4) and (6). We do not need to calculate exactly the Sachs-Wolfe effect of a gravitational wave background upon the CMBR. Indeed it is sufficient to parameterise it as $(\Delta T/T)^2_G = \beta A_G^2$ for some positive constant $\beta \approx O(1)$ which will depend on l, the order of harmonic considered. Hence, one may write

$$\left(\frac{\Delta T}{T}\right)^2_{\rm obs} = \frac{1}{4}A_S^2 + \beta A_G^2 \tag{11}$$

and gravitational waves are the dominant contribution if the condition $A_S^2/A_G^2 < 4\beta$ is satisfied. In other words, these waves contribute to at least 50 per cent of the quadrupole if

$$25\beta\lambda^2 > 1 \qquad \Longleftrightarrow \qquad n < \frac{50\beta - 3}{50\beta - 1}. \tag{12}$$

It is important to note that this condition is not strongly dependent on β if $\beta \gg 1/50$, which suggests that a small deviation from a scale-invariant spectrum will significantly increase the amplitude of the tensor perturbations.

We can eliminate either A_S or A_G with the use of eqs. (6) and (8) to derive the key result

$$\left(\frac{\Delta T}{T}\right)^2_{\rm obs} = \left(\frac{1}{25\lambda^2} + \beta\right) A_G^2 = \left(1 + 25\lambda^2\beta\right) A_S^2/4. \tag{13}$$

The relative amplitudes of the scalar and tensor perturbations can thus be determined *exactly* for a given λ. In terms of the spectral index these expressions become

$$\left(\frac{\Delta T}{T}\right)^2_{\rm obs} = A_G^2\left[\beta + \frac{1}{50}\left(\frac{n-3}{n-1}\right)\right] = \frac{1}{4}A_S^2\left[1 + 50\beta\left(\frac{n-1}{n-3}\right)\right]. \tag{14}$$

It is possible to predict the contribution of these gravitational waves to the total energy density of the universe. The dimensionless amplitude A_G for a given wavelength l is related to the energy density per octave, $d\rho_G/d\ln l$, by $A_G \approx l\sqrt{8\pi G d\rho_G/d\ln l}$ (Kolb & Turner 1990) and the fraction of the critical energy density contributed per octave by these waves is given by $\Omega_G \equiv (8\pi G d\rho_G/d\ln l)/3H_0^2$. Hence, Ω_G is directly related to the quadrupole anisotropy and spectral index by

$$\Omega_G(l) = \frac{1}{3l^2H_0^2}\left[\beta + \frac{1}{50}\left(\frac{n-3}{n-1}\right)\right]^{-1}\left(\frac{\Delta T}{T}\right)^2_{\rm obs}. \tag{15}$$

A gravitational wave with wavelength $l \approx 3000$ Mpc is re-entering the horizon at the present epoch and leads to the quadrupole anisotropy. Hence, we predict in this model a value of $\Omega_G(l \approx 3000\text{Mpc}) \leq 10^{-11}$.

Thus far in this analysis, we have not needed to place any slow-roll restrictions on the evolution of the scalar field. This is important because such assumptions certainly become less accurate as we move further away from the $n = 1$ spectrum. Furthermore, these results are not sensitive to β in the sense that, whatever value β takes, there will always be a value of λ that makes the tensor contribution large. It is helpful, however, to make an estimate of β. Fabbri & Pollock (1983) have related the anisotropy of the CMBR to the vacuum energy, ϵ_v, in the slow-roll approximation by expanding the temperature fluctuations in spherical harmonics: $\Delta T/T = \sum_{lm} a_{lm} Y_{lm}(\theta, \psi)$. It follows that the l-th harmonic contributes

$$\left(\frac{\Delta T}{T}\right)_l^2 = \frac{1}{4\pi}\sum_m |a_{lm}|^2 = \frac{3\epsilon_v}{4m_P^4} I_l, \tag{16}$$

where I_l is a constant and ϵ_v represents the vacuum energy. Numerical calculations indicate that $I_2 \approx 0.425$. By writing $3H^2 = \kappa^2\rho$, we can express the quadrupole ($l = 2$) in terms of H^2/m_P^2, and hence A_G^2 via eq. (7). This implies

$$\left(\frac{\Delta T}{T}\right)_{l=2}^2 = \frac{9\pi I_2}{16} A_G^2 \tag{17}$$

and we deduce $\beta \approx \frac{3}{4}$. For this value of β, the limit (12) on the spectral index becomes $n < 0.95$.

4 Discussion: Implications for a CDM Cosmology

We have shown that a power-law inflationary model can lead to a universe with a near scale-invariant fluctuation spectrum, but with CMBR anisotropies dominated by gravitational waves rather than scalar density perturbations. Gravitational waves have a negligible effect on matter at late times, so we can suppress the amplitude of scalar perturbations required to be compatible with the COBE limit. It is helpful to quantify the various problems for CDM in terms of the ubiquitous – but rather confusingly named – *bias parameter*, b, defined by

$$\left(\frac{\delta\rho}{\rho}\right)_{gals} = b\left(\frac{\delta\rho}{\rho}\right)_{mass}.$$

The CDM model has been successful at reproducing small-scale clustering with a bias parameter in the range $1.5 < b < 2.5$ (Davis *et al.* 1985). The correlations of rich clusters can, just about, be accommodated by this 'standard' value of the bias parameter and these data are, in any case, controversial (e.g. Sutherland 1988). The COBE CMBR detection, however, seems to require a $b \simeq 1$. This is the same order-of-magnitude figure as seems to be required by large-scale galaxy clustering data from APM, QDOT and radio galaxies (Peacock 1991).

Our argument can therefore reconcile small-scale clustering with the COBE results, but the large-scale clustering data are still anomalous. These results may, indeed be irreconcilable with CDM. There is, however, growing realisation that large-scale galaxy clustering may be heavily influenced by large-scale, but very weak, astrophysical effects (Babul & White 1991; Bower *et al.* 1992). Constraints from galaxy clustering alone on the value of b are likely to be controversial for some time. Neither can we expect galaxy clustering studies to tell us directly if the primordial spectral index differs slightly from unity as predicted in our model. On the other hand, observations of large-scale peculiar motions can relate galaxy velocities to the underlying mass fluctuations and so can provide constraints on the bias parameter directly. The analysis of peculiar motion data is by no means straightforward and the results, though extremely important (Lynden-Bell *et al.* 1988; Kaiser 1988; Bertschinger *et al.* 1990)

are not yet conclusive for a high-bias CDM model. For example, analysis of QDOT data seems to indicate the value of b for IRAS galaxies is 1.2 ± 0.2 if $\Omega_0 = 1$ (Rowan-Robinson *et al.* 1990; Kaiser *et al.* 1991). The value of b for optical galaxies may, however, be up to a factor 1.7 larger than this (Lahav *et al.* 1990) and there need not even be a constant relationship between the level of bias appropriate to Infra-red and optical galaxies.

There are, however, two possible direct tests of the picture we are proposing. The first comes from smaller-scale CMBR anisotropy measurements. Although the Sachs-Wolfe effect provides the greatest contribution to CMBR anisotropy on large scales, intermediate and small-scale fluctuations are dominated by streaming motions on the last scattering surface and intrinsic temperature fluctuations in the primordial plasma. Since these latter two mechanisms are essentially related to the scalar perturbations they should appear, in our picture, at the level appropriate to the *biased* CDM model rather than the $b = 1$ version the COBE data would seem to demand. The variation of $\Delta T/T$ with angular scale is therefore very different in our model since we predict a higher amplitude of fluctuations on large scales relative to small scales than would be the case for solely scalar perturbations. Indeed, the observational limits on 1° scales are already close to the COBE level on 10° (Bond *et al.* 1990; Smoot *et al.* 1992). If the COBE fluctuation is all due to scalar perturbations then a detection on 1° scales must be imminent. If, on the other hand, tensor modes contribute to allow a larger value of b, then the $< 1°$ fluctuations can be roughly a factor $1/b$ smaller. The second possible test is by direct detection of the gravitational wave background we predict. Although COBE is only sensitive to gravitational waves on the scale of the horizon at recombination, the waves produced during inflation possess an almost scale-invariant spectrum. There should therefore be other observable astrophysical consequences of a stochastic gravitational wave background on smaller scales. For the level of gravitational radiation we predict, $\Omega_G \sim 10^{-11}$, there is a possibility that some of these consequences, such as the effect on primordial nucleosynthesis, might be observable (e.g. Thorne 1987). Obviously more detailed computations are necessary in both of these cases but they do demonstrate that our model is testable.

Finally, it is interesting to note that the COBE analysis does not rule out the region $n > 1$, corresponding to amplitudes that decrease with scale. However, the amplitude of tensor perturbations always increases with scale in any model based on General Relativity with a flat Friedman metric and a minimally coupled scalar field. This follows because $H(t)$ decreases with respect to cosmic time. Consequently, a value of $n > 1$ would strongly suggest that the quadrupole anisotropy of the CMBR was predominantly due to scalar perturbations. However, gravitational waves could still dominate when $n > 1$ if the Universe underwent a 'super-inflationary' expansion where $\dot{H} > 0$, but this would require a more complicated matter sector than the one considered in this paper.

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