Hidden Symmetry of the Classical Yang - Mills Theory.

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One of the basic properties of classical electrodynamics and general relativity is that any field and its source may be reconstructed from the corresponding curvature. Although this important property looks like a natural requirement of any gauge thery it actually represents a peculiarity of these two theories. Indeed, none of classical non-Abelian gauge theories possess this property. Evidently, in order to prove this fact it suffices to represent a curvature which can be obtained from non-equivalent fields. The proof will be constructed in the framework of the classical Yang-Mills theory with the following denotions:

$$\vec{\omega} = \vec{A}_i dx^i, \quad \vec{A}_i = A_i^a \tau_a \in su(2)$$

$$\vec{\Omega} = d\vec{\omega} + \frac{1}{2}\vec{\omega}\wedge\vec{\omega} \equiv \vec{F}_{ij}\, dx^i \wedge dx^j$$

with τ_a being Pauli matrices, and the field equation

$$d^*\vec{\Omega} + \vec{\omega}\wedge{}^*\vec{\Omega} = \vec{I}, \quad {}^*\vec{I} = \vec{J}_i dx^i \qquad (1)$$

Consider the class of fields specified by the following 1-form

$$\vec{\omega} = \tau_1 f(u)dv + \tau_2 dP(u) + \tau_3 g(u)dv \qquad (2)$$

on a two-dimensional space endowed with a coordinate system $\{u,v\}$. One can check easily that all such fields with the factors satisfying the equations

$$f' + P'g = G; \quad g' - P'f = 0.$$

correspond to the following curvature form:

$$\vec{\Omega} = -\tau_1 G'(u)\, du\wedge dv$$

Since one of the functions, for example, the function $F(u)$ can be chosen arbitrarily the fields (2) which correspond to this curvature constitute a functional space. Apparently, they are non-equivalent physically because they yield non-equal covariant derivatives.

Thus, unlike classical electrodynamics and general relativity the classical Yang-Mills theory contains some kind of non-gauge transformations, which, nevertheless, preserve the curvature. The first question arising since this fact is do any physical systems possess symmetry with respect to these transformations. Apparently the field itself does because its Lagrangian contains the curvature only. So, in order to clear this matter up it suffices to exemine the interaction term $\int Sp\,(\vec{\omega}\wedge\vec{I})$ in the action functional.

Primarily, it must be pointed out that the current form $\vec{I}$ also undergoes the transformation. Indeed, inserting different connections into the field equation (1) one obtains expressions varying in the term $\vec{\omega}\wedge{}^*\vec{\Omega}$. Meanwhile, any other conserving current also undergoes the transformations because the state of being conserved depends on the shape of connection.

An infinitesimal field transformation

$$\vec{\omega} \to \vec{\omega} + \vec{\varepsilon} \qquad (3)$$

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preserves the curvature form $\vec{\Omega}$ if the field $\vec{\varepsilon}$ satisfies the linearized equation

$$d\vec{\varepsilon} + \frac{1}{2}(\vec{\omega} \wedge \vec{\varepsilon} + \vec{\varepsilon} \wedge \vec{\omega}) = 0 \qquad (4)$$

and, if so, the current form $\vec{I}$ transforms as follows:

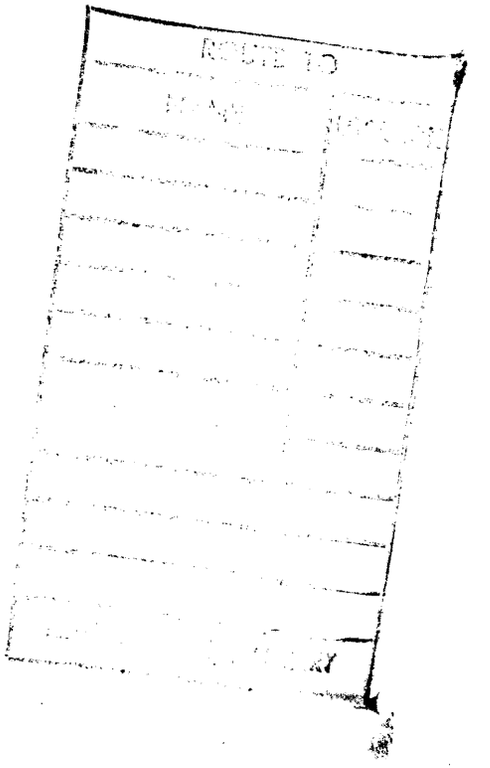
$$\vec{I} \to \vec{I} + \vec{\varepsilon} \wedge {}^*\vec{\Omega}. \qquad (5)$$

Therefore, the transformation of the action integral has the following form:

$$\delta S = \int Sp(\vec{\varepsilon} \wedge \vec{I} + \frac{1}{2}\vec{\omega} \wedge \vec{\varepsilon} \wedge {}^*\vec{\Omega}) =$$

$$= \int Sp[\vec{\varepsilon} \wedge d^*\vec{\Omega} + \frac{1}{2}(\vec{\omega} \wedge \vec{\varepsilon} + \vec{\varepsilon} \wedge \vec{\omega}) \wedge {}^*\vec{\Omega}] =$$

$$= \int Sp\, d(\vec{\varepsilon} \wedge {}^*\vec{\Omega}) = \oint Sp\, (\vec{\varepsilon} \wedge {}^*\vec{\Omega}).$$

The last surface integral is zero since the standard boundary conditions at infinity, and, hence, the action integral remains invariant displaying a hidden symmetry of interacting system.

It must be also pointed out that presence of non-gauge transformations presrving the curvature yields some restriction on the classical theory of gauge fields. The point is that any classical theory admits existance of localized objects i.e. point- like particles. Similarly, in the framework of the classical Yang-Mills theory one can consider point-like particles with su(2)- valued charges coupled to the field. Studying the dynamics of such particles one can proove easily that any influence of a field can be expressed in terms of curvature only. Further,constructing models of continuous media as many-particle systems one obtains some models of non-localized objects and finally comes to a proof that the curvature constitutes the only physical implication of a gauge field. Evidently, such a proof contradicts the properties of conserving currents established above.