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Hyperextended Chaotic Inflation

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In this paper, the hyperextended chaotic scenario is proposed for an exponential inflaton potential, $V(\sigma) \propto \exp(-\lambda\sigma)$. In contrast to the model based on general relativity, inflation occurs for $\lambda^2 > 2$ and ends naturally. The predicted scale–dependence of the density spectrum rules out the standard Jordan–Brans–Dicke theory with this potential, and primordial nucleosynthesis arguments are employed to constrain λ^2.

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Introduction : The idea that the universe was dominated by vacuum energy about 10 Gyr ago is attractive because it allows a number of difficulties associated with the hot, big bang model to be resolved through the production of entropy [1]. This process is referred to as *inflation* and in most versions the potential energy of a quantum scalar field, σ, dominates the energy–momentum tensor. This violates the strong energy condition (SEC) and causes the scale factor to *accelerate* with respect to cosmic time. The standard model is recovered when the false vacuum decays and the universe reheats.

However, the origin of the scalar field remains uncertain. The first models were based on general relativity and identified the field with a Higgs boson of some grand unified theory, but failed to reproduce a sufficiently homogeneous universe [2]. This led to the introduction of the 'chaotic' scenario, but this model requires fine–tuning of the coupling constants and is not based on a definite particle physics model [3]. The 'extended' scenarios modify the gravitational sector of the theory by introducing a second scalar field – the dilaton ψ – which is coupled to gravity through a non–minimal interaction $h(\psi)R$. In principle, the evolution of the dilaton during inflation allows a first–order phase transition to complete. However, these models are either ruled out by time–delay tests or are themselves severely constrained [4,5,6].

On the other hand, it is well known that scalar fields which self–interact through an exponential potential, $V(\sigma) = V_0 \exp(-\lambda\kappa\sigma)$, arise generically in higher–dimensional supergravity [7], superstring [8] and higher–order gravity theories [9]. $\{V_0, \lambda\}$ are independent constants, $\kappa^2 \equiv 8\pi m_P^{-2}$ and m_P is a constant of dimension mass, which is identified as the Planck mass in the standard Einstein scenario. Inflation driven by such a potential in general relativity occurs in the limited region $0 < \lambda^2 < 2$, but can never end unless the shape of the potential is altered. This graceful exit problem arises because the ratio of the field's kinetic energy to potential energy is fixed. Furthermore, most particle physics models predict $\lambda^2 > 2$. For example, $\lambda^2 = 8$ when $\sigma \ll 0$ in the six–dimensional Einstein–Maxwell theory, where σ is identified as the degree of freedom of the internal space S^2 [7].

Therefore, it is important to develop a working scenario based on this potential. We proceed to consider these points by proposing a *chaotic* version of the hyperextended scenario [10] with an action

$$S = \int d^4x(-g_4)^{1/2}\left[h(\psi)R - \frac{1}{2}(\nabla\psi)^2 - \frac{1}{2}(\nabla\sigma)^2 - V(\sigma)\right], \tag{1}$$

where $g_4 = \det g_{\mu\nu}$ and $V(\sigma)$ has the exponential form discussed above. Eq. (1) can be expressed in the Jordan–Brans–Dicke (JBD) form through the field redefinition $\Psi \equiv h(\psi)$ and the gravitational

sector becomes $L_g = \Psi R - \omega(\Psi)(\nabla\Psi)^2/\Psi$ where

$$\omega(\Psi) = \frac{h(\psi)}{2(dh/d\psi)^2}. \tag{2}$$

As an ansatz, we shall assume that the functional form of $h(\psi)$ is a truncated Taylor series

$$h(\psi) = \frac{1}{2}\epsilon\psi^2 - \frac{1}{n}\mu\kappa^{n-2}\psi^n, \tag{3}$$

where $n, \epsilon, \mu =$ constants > 0 and we consider initial conditions $(\kappa\psi)^{n-2} \ll n\epsilon/(2\mu)$ with $n > 2$. Hence, the JBD theory is recovered in this regime. The field equations for a spatially flat, Friedmann metric become [11]

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{V(\sigma)}{3\epsilon\psi^2}, \qquad 3H\dot{\psi} = \frac{1}{1+6\epsilon}\frac{4V}{\psi}, \qquad 3H\dot{\sigma} = -V_\sigma, \tag{4}$$

where the slow–roll conditions $\ddot{\psi} \ll H\dot{\psi}$, $\{\dot{\psi}^2, \dot{\sigma}^2\} \ll V(\sigma)$ and $\ddot{\sigma} \ll H\dot{\sigma}$ are assumed, a subscript σ denotes $d/d\sigma$ and a dot denotes d/dt. In this case, the JBD parameter is given by $\omega_0 = 1/(4\epsilon)$ and observations imply that $\omega_0 > 500$ ($\epsilon < 5 \times 10^{-4}$) [4]. The solution to these equations is

$$\left(\frac{a}{a_i}\right) = \left(\frac{\psi}{\psi_i}\right)^{(1+6\epsilon)/4\epsilon} \tag{5a}$$

$$\psi^2 - \psi_i^2 = -\frac{8}{1+6\epsilon}\int_{\sigma_i}^{\sigma} d\sigma' V(\sigma')\left(\frac{dV}{d\sigma'}\right)^{-1} \tag{5b}$$

and $\dot{\psi} > 0$ for all $\psi > 0$.

The SEC is violated if [12]

$$\psi^2 < \left(\frac{2}{\epsilon}\right)\left(\frac{1+2\epsilon}{1+6\epsilon}\right)\left(\frac{V}{V_\sigma}\right)^2 \tag{6}$$

for arbitrary $V(\sigma)$, which implies that inflation occurs if $\kappa^2\psi^2 < 2/(\epsilon\lambda^2)[(1+2\epsilon)/(1+6\epsilon)]$ for an exponential potential [12]. Hence, inflationary solutions exist for $\lambda^2 > 2$ in theory (1). Furthermore, when the effective gravitational constant is time–dependent, inflation ends *naturally* when ψ reaches ψ_f defined as

$$\kappa^2\psi_f^2 = \left(\frac{2}{\epsilon\lambda^2}\right)\left(\frac{1+2\epsilon}{1+6\epsilon}\right) \tag{7}$$

and the expansion becomes subluminal.

There are a number of constraints that must be satisfied for this analysis to be self–consistent. Firstly, m_P represents the Planck mass today if the present day value of $\psi = \psi_0$ and is normalized such that $2\kappa^2 h(\psi_0) = 1$. The specific value of ψ_0 can be determined by a mechanism similar to

that discussed in the old hyperextended scenario [10]. If $h(\psi)$ contains a maximum at ψ_0, eq. (2) implies that the dilaton's kinetic energy will decouple when $dh/d\psi = 0$, because $\omega(\psi)$ diverges and the theory becomes identical to general relativity. If the dilaton continues to grow when $\psi > \psi_f$, the nth–order terms in the expansion of $h(\psi)$ will become important and ψ decouples at ψ_0 if $\epsilon^{n/2} = \mu[n/(n-2)]^{(n-2)/2}$. We remark that recent constraints requiring $n \geq 6$ do not apply here because no first–order phase transition occurs [5].

It is important that the field equations (4) are valid throughout inflation i.e. that the quadratic part of $h(\psi)$ dominates for $\psi < \psi_f$. This implies that we require $(\kappa\psi_f)^{n-2} < n\epsilon/(2\mu)$ and $\psi_f < \psi_0$. These conditions are equivalent to $\lambda^2 > (n-2)(2/n)^{n/(n-2)}(1+2\epsilon)/(1+6\epsilon)$ and $\lambda^2 > 2[(n-2)/n](1+2\epsilon)/(1+6\epsilon)$ respectively. Finally, the slow–roll conditions are valid if $\epsilon < 1$ and (6) is satisfied. Hence, the analysis is self–consistent for $\lambda^2 > 2$ and all $n > 2$.

There is a simple physical argument that explains this behaviour. The SEC is not violated in general relativity when $\lambda^2 > 2$ because the ratio of kinetic to potential energies is too large. However, the variation of the effective gravitational constant in the JBD theory acts as an additional friction in the field equations. Hence, there is more friction on σ and the field evolves more slowly than in Einstein gravity for a given λ. This implies that the potential can dominate over the kinetic contribution for a wider range of λ. On the other hand, the inflation causes ψ to grow and gravitational friction ($\sim \psi^{-1}$) to weaken. In this sense, inflation brings about its own demise by forcing ψ to reach ψ_f within a finite time interval. Moreover, for a given $\{\epsilon, \lambda\}$, the strength of gravity at $\psi = \psi_f$ is *independent* of any initial conditions.

Initial conditions : However, it is also necessary to know when inflation starts. Linde has argued that the quantum boundary (QB) yields the most natural initial conditions for extended chaotic inflation [11,13]. This corresponds to the hypersurface at the Planck density and represents the earliest times at which initial conditions can be placed on classical fields. In this scenario, the Planck mass is related to the dilaton by $m_P^2(\psi) = 16\pi h(\psi)$, so the QB is reached when $V(\sigma) \sim 256\pi^2 h^2(\psi)$. Furthermore, the probability that the universe can be created from nothing via quantum tunnelling is $p \sim \exp(-3m_P^4(\psi)/8V(\sigma))$ [13,14], which is only high on the QB. The initial conditions for theory (1) when $(\kappa\psi)^{n-2} \ll n\epsilon/(2\mu)$ are estimated by considering quantum fluctuations in the fields $\{\sigma, \psi\}$. Eternal inflation occurs when the change in the fields due to quantum fluctuations with wavelength larger than H^{-1} dominates the classical change in the time interval H^{-1} due to eq. (4) [11]. When $\epsilon \ll 1$, such a process occurs if

$$12\pi^2\epsilon^3\psi^4(16 + \lambda^2\kappa^2\psi^2) < V(\sigma) < 64\pi^2\epsilon^2\psi^4. \tag{8}$$

and this is easily satisfied for $\kappa^2\psi^2 \ll 1$. Therefore, the self–reproduction of this universe is a natural consequence of the process of quantum creation. A local domain corresponding to our own observable region is formed once the left–hand inequality of (8) is reversed and the initial conditions become

$$12\pi^2\epsilon^3\psi_i^4(16+\lambda^2\kappa^2\psi_i^2) \sim V(\sigma_i). \tag{9}$$

This condition is sufficient to ensure $a_f/a_i \gg e^{60}$ for a wide range of $\{\epsilon, \lambda\}$, thereby solving the horizon and flatness problems.

Density spectrum : The density perturbation spectrum, $\delta\rho/\rho$, can be derived directly from eq. (4) or by expressing the action (1) in the Einstein–Hilbert form by performing a suitable conformal transformaton on the metric [15]. In both cases, we find

$$\frac{\delta\rho}{\rho} = \beta H^2 \left[\frac{|\dot{\psi}| + |\dot{\sigma}|}{\dot{\psi}^2 + \dot{\sigma}^2}\right]_{t=t_{60}}, \tag{10}$$

where $\beta \approx O(1)$. The quantities on the right–hand side are evaluated at the start of the last 60 e–folds of inflation, because scales of cosmological interest lie in the range $1\text{Mpc} \leq l \leq 3000\text{Mpc}$ and crossed the horizon during $\ln 3000 \approx 8$ of these e–folds. Two regions of the power spectrum may be defined in terms of the quantity

$$R_l \equiv \left|\frac{\dot{\psi}_l}{\dot{\sigma}_l}\right| = \left[\frac{8\epsilon}{(1+2\epsilon)(1+6\epsilon)}\right]^{1/2} e^{240\epsilon/(1+6\epsilon)} \tag{11}$$

where $\psi_l = \psi(t = t_{60})$, $\sigma_l = \sigma(t = t_{60})$ and we have used eq. (7) along with the expression $\psi_f/\psi_l = \exp(240\epsilon/(1+6\epsilon))$. If $|\dot{\sigma}| < |\dot{\psi}|$, it has been shown that

$$\frac{\delta\rho}{\rho}(R_l > 1, k) \propto k^{-8\epsilon/(1-2\epsilon)} \tag{12a}$$

where $k(\psi) = a(\psi)H(\psi)$ and $V(\sigma)$ is arbitrary [12]. On the other hand, one may assume that $\psi \approx$ constant relative to σ during these 8 e–folds if $R_l < 1$. Consequently, we recover general relativity to a first approximation, but with the strength of gravity scaled as $\psi_0 \to \psi_l$. This also leads to a power law spectrum given by

$$\frac{\delta\rho}{\rho}(R_l < 1, k) \propto k^{\epsilon\lambda^2\kappa^2\psi_l^2/(\epsilon\lambda^2\kappa^2\psi_l^2-2)} \tag{12b}$$

where $k(\sigma) = a(\sigma)H(\sigma)$ is calculated from eq. (4). By relating ψ_l to ψ_f and using the definition (7) for ψ_f, it is clear that the scale–dependence of the fluctuations is *independent* of λ in both

regions and is uniquely determined by the value of ϵ. Hence, observations of large–scale structure can be employed to constrain the strength of this nonminimal coupling to gravity.

A suitable limit for the power, r, in the spectum $\delta\rho/\rho \propto k^{-r}$ is $r < 1/6$ [17]. The crossover point $R_l = 1$ occurs when $\epsilon = 6.5 \times 10^{-3}$ and $R_l > 1$ ($R_l < 1$) if $\epsilon > 6.5 \times 10^{-3}$ ($\epsilon < 6.5 \times 10^{-3}$). It follows that $r < 1/6$ when $R_l > 1$ if $6.5 \times 10^{-3} < \epsilon < 0.02$, whereas the corresponding constraint for $R_l < 1$ is $4.2 \times 10^{-3} < \epsilon < 6.5 \times 10^{-3}$. It is important to note that in both cases $\epsilon \gg 5 \times 10^{-4}$ and this scenario is ruled out for the JBD theory. Hence, one must extend the gravitational sector by either considering higher–order terms in $h(\psi)$ or introducing a dilaton self–interaction. On the other hand, a density spectrum consistent with observation is achieved for a wide range of ϵ.

Finally, the constraint arising from the quadrupole anisotropy of the cosmic microwave background must be satisfied [18]. It has been demonstrated that this limit becomes $\kappa^4 V(\sigma_l) < 4 \times 10^{-6}\epsilon^3$ if $R_l > 1$ and $\epsilon \ll 1$. When $R_l < 1$, eqs. (4) and (10) imply that

$$\kappa^4 V(\sigma_l) < \frac{5.8 \times 10^{-8}}{\lambda^4} \left(\frac{1+2\epsilon}{1+6\epsilon}\right)^3 e^{-1440\epsilon/(1+6\epsilon)} \tag{13}$$

As $\epsilon \to +0$, the constraint (13) is independent of ϵ and a similar conclusion has been drawn by McDonald for the polynomial inflaton potential $V(\sigma) \propto \sigma^{2m}$ [16].

Post–inflationary epoch : It is necessary to reheat the universe after the scale factor has stopped accelerating. The field can not oscillate since an exponential potential contains no global minimum. However, the kinetic energy of σ will increase significantly once general relativity is recovered and its couplings to other fields will become important [19]. For simplicity, we consider a toy model where a particle species X, with equation of state $p_X = (\gamma - 1)\rho_X$ for some constant γ, is coupled to the inflaton through an interaction $\rho_X \equiv \Gamma_X \dot{\sigma}^2$. The constant Γ_X represents the decay constant for σ. Hence, matter is produced through the dynamical evolution of the cosmological constant. A more accurate description assumes the coupling varies in direct proportion to the effective mass of the inflaton i.e. $\Gamma_X \propto \sqrt{V_{\sigma\sigma}}$ [19].

The Bianchi identities and Friedmann equation for this system become

$$(1 + 2\Gamma_X)\ddot{\sigma} + 3(1 + \gamma\Gamma_X)H\dot{\sigma} + V_\sigma = 0 \tag{14a}$$

$$3H^2 = \kappa^2(\rho_V + \rho_X) = \kappa^2 \left[\left(\frac{1}{2} + \Gamma_X\right)\dot{\sigma}^2 + V(\sigma)\right], \tag{14b}$$

where ρ_V defines the energy density of σ. Eq. (14b) may be rewritten in the Hamilton–Jacobi form by defining a new time coordinate $t \equiv -\kappa^2(1 + \gamma\Gamma_X)/2 \int^\sigma d\sigma' H_{\sigma'}^{-1}$. We find

$$3\kappa^2(1 + \gamma\Gamma_X)^2 H^2 - 2(1 + 2\Gamma_X)H_\sigma^2 = \kappa^4(1 + \gamma\Gamma_X)^2 V(\sigma), \tag{15}$$

which yields the attractor solution

$$H = \sqrt{A}\exp\left(-\frac{\lambda}{2}\kappa\sigma\right), \qquad \lambda\kappa\sigma = 2\ln\left[\frac{\lambda^2\sqrt{A}}{2(1+\gamma\Gamma_X)}t\right], \tag{16}$$

where $A =$ constant. Hence, the scale factor grows as a power law $a \propto t^p$, where $p \equiv 2\lambda^{-2}(1+\gamma\Gamma_X)$. The contribution of X to the matter content of this universe may be expressed through the quantity $\Omega_X \equiv \kappa^2\rho_X/3H^2 = \rho_X/(\rho_X + \rho_V)$. For the solution (16) we find that

$$\Omega_X = \frac{\Gamma_X\lambda^2}{3(1+\gamma\Gamma_X)^2} \tag{17}$$

is constant, implying that the matter and vacuum energy densities redshift at the same rate. It follows that $\rho_X = 4\Gamma_X/(\lambda^2\kappa^2t^2) \propto \rho_V \propto a^{-2/p}$. Moreover, by substituting eq. (16) into (15), the condition $V_0 > 0$ implies this attractor only exists if

$$\Gamma_X > \frac{\Omega_X}{2(1-\Omega_X)}. \tag{18}$$

There are a number of constraints on the maximum contribution of the vacuum energy [20]. To proceed, we shall assume that the inflaton is only coupled to radiation ($\gamma = 4/3$) which quickly reaches thermal equilibrium with a temperature $\rho_r = \pi^2 g_* T_r^4/30$. g_* represents the number of relativistic degrees of freedom and $g_* \sim O(10^2)$ in most grand unified theories. In this case, the most important constraints arise from a) sufficient reheating; b) ensuring a matter dominated phase follows a radiation dominated phase; and c) primordial nucleosynthesis arguments at $t \approx 1$ sec.

An upper limit on the reheat temperature, T_{RH}, follows from eq. (13) by specifying $\epsilon = 4.2\times10^{-3}$. If we assume that $\rho_r < \rho_V \sim V(\sigma)$ we find $\rho_r < 2.5\times10^{-13}\lambda^{-4}m_P^4$, which implies $T_{RH} < 3\times10^{15}\lambda^{-1}$ GeV. Hence, it is straightforward to ensure that $T_{RH} > 10^9$ GeV and baryogenesis may proceed via the decay of supermassive Higgs bosons [21].

It is reasonable to assume that the inflaton does not couple to non–relativistic matter, ρ_m. (ρ_V is strongly constrained in the matter dominated era by observations of the gamma–ray flux if such couplings do exist [20]). Hence, $\rho_m \propto a^{-3}$ which implies $\rho_m/\rho_r \propto a^{(2-3p)/p}$. It follows that a matter dominated era ($\rho_m > \rho_r$) will follow radiation domination ($\rho_r > \rho_m$) if $3 + 4\Gamma_r < \lambda^2$ and the vacuum energy will have rapidly redshifted to zero by the present epoch ($\rho_m \gg \rho_r$).

Finally, primordial nucleosynthesis with a decaying cosmological constant produces abundances of He^4 and H consistent with observation if $\Omega_r \geq 0.9$ [20,22]. Hence, eq. (18) implies $\Gamma_r > 9/2$ for the attractor solution (16) and eq. (17) yields the limit $\lambda^2 > 29.4$. The existence of a

lower limit on λ^2 should be expected because $\rho_r \propto \dot{\sigma}^2$ decreases with decreasing λ^2 and primordial nucleosynthesis essentially places a lower limit on the amount of relativistic matter present at $t \approx 1$ sec.

In conclusion, we have presented a new scenario which allows inflation driven by exponential potentials to end naturally by varying the strength of the effective gravitational constant. This occurs when $\lambda^2 > 2$, as predicted by many particle physics models. The scale dependence of the density spectrum was employed to constrain the coupling constant of the dilaton to gravity, and this rules out the standard JBD theory.

By invoking a simple reheating mechanism, it was shown how primordial nucleosynthesis arguments may be used to constrain λ^2. This is an important new feature of the scenario and suggests that current observations of the light element abundances may provide insight into the structure of the inflaton potential. This is encouraging when one considers the successes of primordial nucleosynthesis [22]. Moreover, the parameter λ depends on the dimensionality of the internal space in certain Kaluza–Klein theories [23]. Hence, the cosmological abundance of He^4 may be directly related to the number of internal dimensions in the space–time if this scenario is viable, even though this internal space is thought to have a diameter $O(m_P^{-1})$.

In principle, the arguments presented here can be applied for *any* functional form of $h(\psi)$ which contains a maximum at $\psi_0 > \psi_f$. It can also be generalized to any inflaton potential which exhibits a decaying tail as $\kappa|\sigma| \to \infty$ that does not lead to inflation in general relativity. Finally, the conformal equivalence of theory (1) to Einstein gravity containing two scalar fields implies that we can realise this scenario within the context of general relativity if a second scalar field is coupled to the inflaton Lagrangian in a suitable way [12,15].

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