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PARAMETRIZATION OF THERMAL PHOTON EMISSION RATES FROM MESONIC MATTER

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Abstract

Photons have long been considered as a probe of the dynamics of high energy nuclear collisions. Here we present simple analytical expressions which parametrize the thermal production rates of photons for the processes $\pi\pi \to \rho\gamma$, $\pi\rho \to \pi\gamma$, and $\rho \to \pi\pi\gamma$ which are occuring in hot mesonic matter. These parametrizations are good for photon energies greater than 150 MeV and for temperatures between 100 and 200 MeV. They can be used in various thermal and hydrodynamical models of heavy ion collisions.

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Photon production has long been studied in the context of particle and nuclear collisions for the reason that photons probe the dynamics of the collision but generally do not influence the dynamical evolution itself. Much current research in nuclear physics is concerned with the behavior of hot and dense strongly interacting matter. When two nuclei smash into each other at high energy a dynamic plasma of quarks and gluons is formed. As the system blows apart the quarks and gluons must hadronize, and these hadrons may undergo several scatterings and might resemble for a while a hot hadronic gas [1]. If the initial energy is not high enough then only the hadronic degrees of freedom are relevant [2]. In either case one must understand the behavior of hot and dense hadronic matter. In ref. [3] the thermal production rates for various photon producting reactions in hot mesonic matter were calculated. At each temperature and photon energy it was necessary to evaluate a nontrivial four-dimensional integral using Monte Carlo methods. For models of heavy ion collisions such as hydrodynamics it would be most efficient to have simple analytic expressions which parametrize these rates as functions of temperature and energy [4]. That is the purpose of this brief note.

The thermal production rate is the number of reactions per unit volume per unit time to produce a photon. For the process $1 + 2 \rightarrow 3 + \gamma$ the rate is

$$\begin{aligned} R_i &= \mathcal{N} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} f_1(E_1) f_2(E_2) (2\pi)^4 \delta(p_1^\mu + p_2^\mu - p_3^\mu - p^\mu) \\ &\quad \mid \mathcal{M}_i \mid^2 \frac{d^3p_3}{2E_3(2\pi)^3} \frac{d^3p}{2E(2\pi)^3} [1 \pm f_3(E_3)] \ , \end{aligned} \tag{1}$$

where E and p are the energy and momentum of the photon, the f's are the Fermi-Dirac or Bose-Einstein distribution functions as appropriate, there is either a Bose-enhancement or Pauli-suppression of the strongly interacting particle in the final state, $\mathcal{M}_i$ is the reaction amplitude, and $\mathcal{N}$ is the overall degeneracy factor which depends on the specific reaction. Converting the total rate to a differential one, all but four of the integrations can be done without approximation:

$$\begin{aligned} E\frac{dR_i}{d^3p} &= \frac{\mathcal{N}}{(2\pi)^7} \frac{1}{16E} \int ds\, dt \mid \mathcal{M}_i(s,t) \mid^2 \int dE_1\, dE_2\, f_1(E_1) f_2(E_2) \\ &\quad [1 \pm f_3(E_1 + E_2 - E)] \theta(E_1 + E_2 - E) [aE_1^2 + bE_1 + c]^{-1/2} \ , \end{aligned} \tag{2}$$

where

$$
\begin{aligned}
a &= -(s+t)^2 \ , \\
b &= 2(s+t)(Es - E_2 t) \ , \\
c &= st(s+t) - (Es + E_2 t)^2 . \qquad (3)
\end{aligned}
$$

In this expression $s = (p_1 + p_2)^2$ and $t = (p_1 - p)^2$ are the Mandelstam variables. In ref. [3] it was found that the dominant two-body reactions were $\pi\pi \to \rho\gamma$ and $\pi\rho \to \pi\gamma$ and so it is these reactions we focus on here.

In figure 1 we show the results of Monte Carlo integration for the reaction $\pi\pi \to \rho\gamma$ including all isospin degrees of freedom. The data points are shown with an estimate of statistical errors. The rate is given for T = 100, 150 and 200 MeV and for E > 150 MeV. A parametrization was found by expressing the rate as a product of a ubiquitous Boltzmann factor $\exp(-E/T)$ times a more slowly varying function of E. Our suggested parametrization is

$$
E\frac{dR_{\pi\pi\to\rho\gamma}}{d^3p} = 0.1434\, T^{1.866} \exp(-0.7315/T + 1.45/\sqrt{E} - E/T). \qquad (4)
$$

Here temperature and energy are to be given in GeV; the invariant rate then has dimensions of $\mathrm{fm}^{-4}\mathrm{GeV}^{-2}$. This function is plotted in figure 1 for the three temperatures. It is an excellent parametrization of the Monte Carlo points for E > 200 MeV. The rate diverges as E $\to$ 0 because the reaction is endothermic: most of the available energy in the initial state at these temperatures is required to make the rho meson, leaving very little for the photon. As pointed out in ref. [3] an infrared cut-off is necessary to render the rate finite and this affects photons with energy lower than about 200 MeV. This infrared cut-off arises from higher order QED effects.

In figure 2 we show the results of Monte Carlo integration for the reaction $\pi\rho \to \pi\gamma$ including all isospin degrees of freedom. Our suggested parametrization for this reaction is

$$
E\frac{dR_{\pi\rho\to\pi\gamma}}{d^3p} = T^{2.4} \exp[-1/(2TE)^{3/4} - E/T]. \qquad (5)
$$

Again T and E are to be given in GeV and the invariant rate then has the same dimensions as before. This function is plotted in figure 2 for the three chosen

temperatures. It is again an excellent representation of the Monte Carlo points. The reaction in this case is exothermic so high photon energies are preferred. The invariant rate has a maximum at a photon energy that can be estimated from the above parametrization to be

$$E_{peak} = \frac{1}{2}\left(\frac{3}{2}\right)^{\frac{4}{7}} T^{\frac{1}{7}}. \tag{6}$$

This is a very slowly increasing function of T.

In ref. [3] it was found that the vector meson decays $\omega \to \pi\gamma$ and $\rho^0 \to \pi\pi\gamma$ produce photons at finite temperature at a rate comparable to the two-body reactions discussed above. The rate for the first of these can be expressed in terms of a one dimensional integral which can be evaluated very quickly and efficiently.

$$E\frac{dR_{\omega\to\pi^0\gamma}}{d^3p} = \frac{3m_\omega\Gamma_{\omega\to\pi^0\gamma}}{16\pi^3 E_0 E}\int_{E_{min}}^{\infty} dE_\omega\, E_\omega\, f_{BE}(E_\omega)[1 + f_{BE}(E_\omega - E)] \tag{7}$$

Here $E_{min} = m_\omega(E^2 + E_0^2)/2EE_0$ and E_0 is the photon energy in the rest frame of the omega. Therefore it is not worthwhile searching for an analytic parametrization of this thermal decay rate.

The decay of the ρ is a little more complicated since it gives rise to a three body final state. The invariant decay rate distribution is [5]

$$\hat{\Gamma}(E) = E\frac{d\Gamma_{\rho\to\pi\pi\gamma}}{d^3p} = \frac{\alpha\, g_\rho^2}{24\pi^3 m_\rho}\left\{\sqrt{1-\frac{4m_\pi^2}{s}}\right.$$

$$\left. -\frac{m_\rho^2 - 4m_\pi^2}{m_\rho E}\left[\frac{s-2m_\pi^2}{s-m_\rho^2}\ln\left(\frac{\sqrt{s}+\sqrt{s-4m_\pi^2}}{2m_\pi}\right) + \frac{\sqrt{s(s-4m_\pi^2)}}{4m_\rho E}\right]\right\}. \tag{8}$$

Here $s = (p_+ + p_-)^2 = m_\rho(m_\rho - 2E)$ is the square of the energy of the two pion system in the $\pi - \pi$ rest frame. Folding this distribution with the thermal ρ momentum distribution we have

$$E\frac{dR_{\rho\to\pi\pi\gamma}}{d^3p} = \frac{3}{(2\pi)^3}\int \hat{\Gamma}\left(E' = \frac{EE_\rho - \vec{p}\cdot\vec{p}_\rho}{m_\rho}\right) f_{BE}(E_\rho) d^3p_\rho. \tag{9}$$

The integral over the angle ϕ can be done trivially but a two-dimensional integral remains, to be done numerically. Note that this expression does not include the Bose-Einstein enhancement of the final state pions because the decay distribution of eq. (8) already has integrated out the pion degrees of freedom. As in ref. [3] we

can incorporate minimal Bose-Einstein enhancement by sharing the energy $E_\rho - E$ equally between the pions. Unequal sharing increases the enhancement, so we multiply the integrand by $[1 + f_{BE}(\frac{1}{2}(E_\rho - E))]^2$.

The routine used to evaluate the two-dimensional integral in eq. (9) numerically is not very time-consuming. It is available upon request. Nevertheless we did bother to find a simple analytic expression to parametrize this rate also. It is

$$E\frac{dR_{\rho\to\pi\pi\gamma}}{d^3p} = 0.0785\, T^{4.283}\, E^{-2.976+0.1977/T} \exp(-E/T). \tag{10}$$

We have found this to be accurate to about $\pm$ 20% when $T < E < 10T$. It gets worse at smaller and larger values of E.

In summary, we have presented three simple formulas which parametrize the thermal photon emission rates for the three processes $\pi\pi \to \rho\gamma, \pi\rho \to \pi\gamma, \rho^0 \to \pi\pi\gamma$ in hot hadronic matter. These simple parametrizations should greatly expedite the computation of photon production in various thermal models of high energy collisions, such as relativistic hydrodynamics. We caution that these parametrizations should not be used outside the range we have tested them. We have not bothered with temperatures below 100 MeV because the thermal rates become uninterestingly small. As is well-known, the primary background for thermally produced photons will come from the electromagnetic decays of the π^0 and the η. Since these electromagnetic decays are so slow these two mesons will not decay until long after the hot matter has blown itself apart. We therefore do not treat them here but suppose that the experiment is good enough to subtract them from the total photon yield, leaving an interesting signal [6].

Acknowledgements

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Figure Captions

Fig. 1 The invariant rate to produce photons with energy E via the reaction $\pi\pi \rightarrow \rho\gamma$ in hot hadronic matter. The three curves correspond to T = 200 MeV (top), 150 MeV (middle) and 100 MeV (bottom). The points represent Monte Carlo integration of the four dimensional integral in eq. (2) and the solid curves are the suggested parametrization of eq. (4).

Fig. 2 The invariant rate to produce photons with energy E via the reaction $\pi\rho \rightarrow \pi\gamma$ in hot hadronic matter. The three curves correspond to T = 200 MeV (top), 150 MeV (middle) and 100 MeV (bottom). The points represent Monte Carlo integration of the four dimensional integral in eq. (2) and the solid curves are the suggested parametrization of eq. (5).

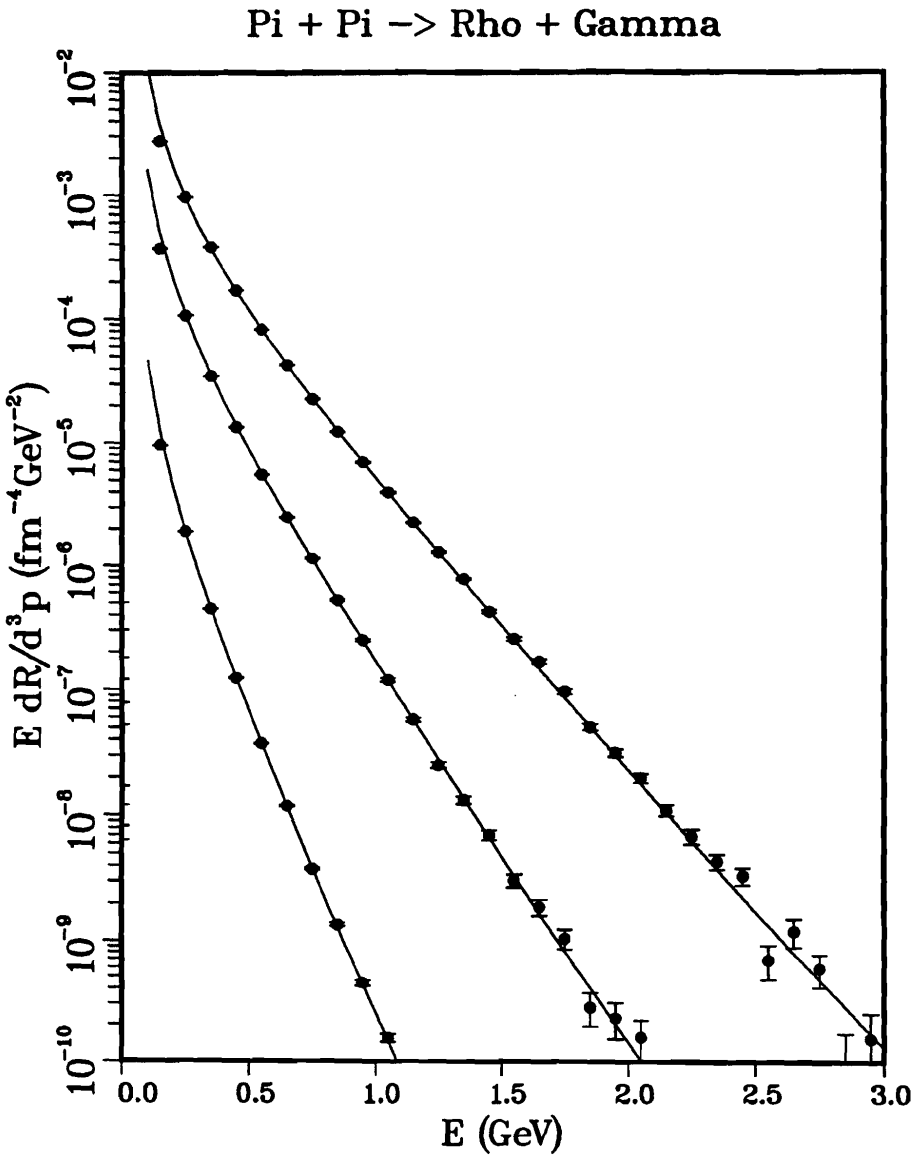
Pi + Pi -> Rho + Gamma
E dR/d³p (fm⁻⁴GeV⁻²)
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
0.0
0.5
1.0
1.5
2.0
2.5
3.0
E (GeV)

Figure 1

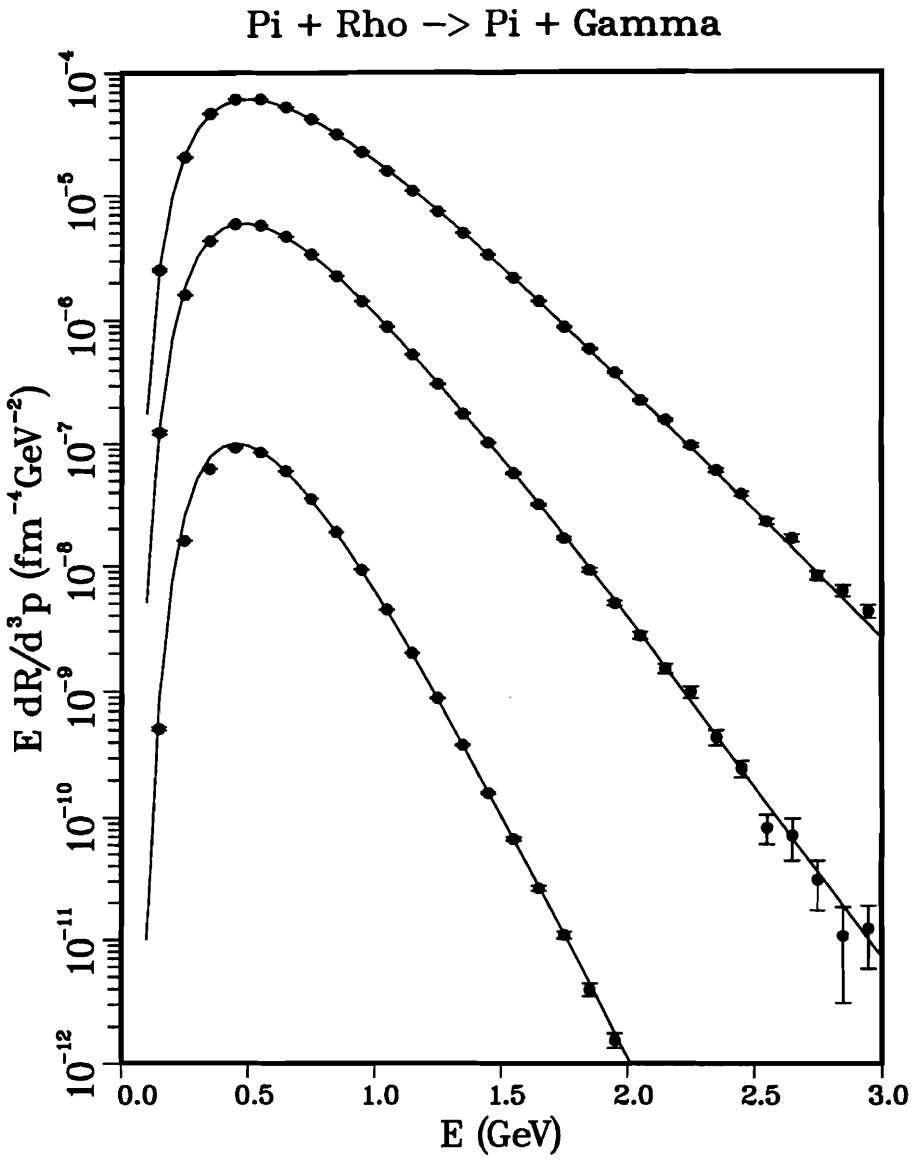
Pi + Rho -> Pi + Gamma
E dR/d^{3}p (fm^{-4}GeV^{-2})
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
0.0
0.5
1.0
1.5
2.0
2.5
3.0
E (GeV)

Figure 2