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Coulomb Potential weakened by Self-Field of Electron

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Self-field method suggested by the present author in the previous work is applied to the electron in the Coulomb potential. Effectively weakened Coulomb potential are found due to the self-field of the electron. The form of the weakened Coulomb potential between two electrons is $U(r)=(e^2/4\pi r)[1-\exp(-r/2R_c)\cos(r/2R_c)]$.

§1. Introduction

In recent years, self-field approach was developed by Barut and his coworkers.[1] Most recently, another self-field method has been suggested by the present author.[2] As to the radiation process, self-field approach is rather in favor of the Lorentz-Dirac[3] theory than the standard form of the classical electrodynamics. The Lorentz-Dirac theory strongly suggests that the electron has a non-radiative self-field energy besides the kinetic energy K. The form of which is $-\tau_0 dK/d\tau$, where τ is the proper time, τ_0 is a constant given by eq.(6). This energy is undoubtedly caused by the interaction of the electron with its own self-field. It is well known that the classical Lorentz-Dirac theory contains the several unresolved difficulties. Nevertheless, we put great confidence on the Lorentz-Dirac theory because it is the only theory which is able to account for the origin of the real electromagnetic wave and the radiative energy loss represented by the Larmor power formula.[4] The interesting effects of the self-field which is produced by the charged current are collected into the non-radiative self-field energy

mentioned above. This is a kind of energy which always bound to the electron and never emitted away as radiative energy.[5,6] Obviously, this bound self-field energy cannot be described by the perturbative radiative emission which is always represented by the free photons independent of the motion of the electron. Thus it is natural to ask what is the role of the bound self-field energy in quantum theory. The effect of this suggestive energy in quantum theory was investigated by the present author in the previous works.[2,7] In this paper, we shall show that our self-field method lead us to a Coulomb potential which is possibly weakend by the effect of this bound self-field of the electron.

§2. Form Factor in Energy Shift

The energy shift of the free electron by radiation fields to second order was evaluated in the preceding work.[2] A form factor had appeared in the expression of the energy shift:

$$\Delta E^{(2)} = -\frac{p^2}{2m_0}\left(\frac{4}{3\pi}\right)\left(\frac{e^2}{4\pi\hbar c}\right)\left(\frac{\hbar}{m_0 c}\right)\int_0^\infty |F(k)|^2 dk \quad . \tag{1}$$

It was compared to the famous Bethe's formula[8] calculated by the perturbative method of QED.

$$\Delta E^{(2)} = -\frac{p^2}{2m_0}\left(\frac{4}{3\pi}\right)\left(\frac{e^2}{4\pi\hbar c}\right)\left(\frac{\hbar}{m_0 c}\right)\int_0^\infty dk \quad , \tag{2}$$

which shows the linear divergence of the photon momentum integral. The form factor F(k) in eq.(1) in the preceding work is given by eq.(13). All the notations refer to ref.2. The energy shift $\Delta E^{(2)}$ is renormalized into the mass of the electron as $E+\Delta E^{(2)}=p^2/2(m_0+\delta m)$. Only difference of our result is that the mass sift δm is finite due to the form factor. We may attribute the origin of the form factor to the bound self-field of the electron.

On the other hand, if the electron has a true spread charge in space, and take the form of interaction term as

$$V(x) = -\left(\frac{e}{m_0 c}\right)\int d^3y \; f(x-y)\, p\cdot A(y) \tag{3}$$

instead of $V=-(e/m_0c)\mathbf{p}\cdot\mathbf{A}$, the perturbative method by Bethe leads to the same expression as eq.(1)(see Appendix). The function $f(\mathbf{x})$ in eq.(3) denotes the shape of the charge distribution of the electron, and $F(k)$ in this case is defined as the Fourier transformation of $f(\mathbf{x})$. Therefore, the appearance of the form factor in eq.(1) in our self-field method implies that the electron behaves as if it has a spread charge in space. Namely, electron may have some characteristic size virtually due to its own self-field although the electron is in fact treated as a point particle. Such property was suggested also from the nonlocal nature[14),9)] of the Lorentz-Dirac theory.

In the quantum theory of the electromagnetic field in standard form, the existence of the vacuum fluctuations in empty space was postulated. This causes the broading effect of the charge of the electron over a region of the order of Compton wavelength.[10)] On the other hand, the effect of the self-field of the electron was considered in the absence of the vacuum field.[11)] Consequently, the self-field approach was identified with the vacuum fluctuation method because these two different points of view lead to the same result in many things.[12)] It is for this reason that the broading effect of the charge of the electron by its own self-field is expected. The electron may have then a virtual size mentioned before. It is easily anticipated that the virtual size of the electron make the Coulomb potential effectively weak at the neighborhood of the origin. In the next section, we shall exhibit that this is indeed the case.

§3. Weakend Coulomb Potential

Let us consider an electron in the Coulomb potential. The non-relativistic spinless electron is supposed here. In order to avoid the extra complication connected with the radiation field, and to see directly the consequence of the bound self-field energy, we neglect the radiation term and the coupling term in the Hamiltonian as in ref 7. The schrödinger equation for the electron in this case is then expressed as

$$[\,-\frac{\hbar^2}{2m_0}\Delta + V(r) + H_{IR}\,]\Psi(r,t) = i\hbar\frac{\partial\Psi(r,t)}{\partial t} \tag{4}$$

with

$$H_{IR} = \sum_{n=1}^{\infty}\left(\frac{i\tau_0}{\hbar}\right)^n [K,[K,\cdots,[K,V(r)]]\cdots] \quad , \tag{5}$$

where $K=p^2/2m_0=-\hbar^2\Delta/2m_0$, and m_0 is the bare electron mass. The self-field energy which is bound to the electron is expressed in terms of Hamiltonian H_{IR}.[2] The square brackets mean the commutator. Constant τ_0 is related to the micro-causality; the Lorentz-Dirac theory gives[13]

$$\tau_0 = \frac{2(e^2/4\pi)}{3m_0c^3} \quad . \tag{6}$$

In order to calculate H_{IR} of eq.(5), the Coulomb potential $V(r)$ is expressed for convenience in the integral form

$$\begin{aligned} V(r) &= \frac{e^2}{4\pi r} \\ &= \frac{e^2}{(2\pi)^3}\int d^3k\ \frac{e^{ikr}}{k^2+\varepsilon^2} \quad . \qquad (\varepsilon>0) \end{aligned} \tag{7}$$

Limit $\varepsilon\to 0$ is taken in the above integration. Each term,

$$\begin{aligned} &(i\tau_0/\hbar)^n[K,[K,\cdots[K,\ e^{ikr}]\cdots]] \\ &= e^{ikr}(i\hbar\tau_0/2m_0)^n(k^2+\frac{2p\cdot k}{\hbar})^n \quad , \end{aligned} \tag{8}$$

appears in the series expansion for H_{IR}, is summed up to get

$$\begin{aligned} V(r) + H_{IR} &= \frac{e^2}{(2\pi)^3}\int d^3k\ \frac{e^{ikr}}{k^2+\varepsilon^2}F(k) \\ &\equiv U(r) \quad , \end{aligned} \tag{9}$$

where $F(k)$ is given by

$$F(k) = \frac{1}{1-(i\hbar\tau_0/2m_0)(k^2+2p\cdot k/\hbar)} \quad . \tag{10}$$

The Schrödinger equation (4) is therefore rewritten as

$$[\,-\frac{\hbar^2}{2m_0}\Delta + U(r)]\Psi(r,t) = i\hbar\frac{\partial\Psi(r,t)}{\partial t} \quad . \tag{11}$$

Namely, the electron is effectively in the potential $U(r)$.

We are able to use the identity for $U(r)$ because $U(r)$ defined

by eq.(9) is Hermitian:

$$U(r) = \frac{U(r) + U^{\dagger}(r)}{2}$$

$$= \frac{e^2}{2(2\pi)^3}\int d^3k \; \frac{e^{ikr}}{k^2+\varepsilon^2}\,[F(k) + F^{\dagger}(-k)] \qquad (12)$$

In order to see the explicit form of U(r) in the coordinate space, we approximate F(k) like in ref. 2 as follows.

$$F(k) = \frac{1}{1-iR_c^2k^2} \; . \qquad (13)$$

Constant R_c is defined by

$$R_c \equiv \sqrt{r_c\lambda_c/2} \; , \qquad (14)$$

where $r_c=\tau_o c$, and $\lambda_c=\hbar/m_o c$ is the Compton wave-length of the bare electron. In the classical Lorentz-Dirac theory, r_c denotes the electron radius[14),6)] associated with the micro-causality. The factor $\sqrt{r_c\lambda_c}$ in R_c coincides with $\Delta x_o=\sqrt{r_o\lambda_o}$ which was first obtained by Caldirola from the finite-difference Schrödinger equation for an electron,[15)] which has physical meaning of the minimal position spread of the wave packet associated to the electron. Indeed, in the preceding work[2)] by the present author, R_c was interpreted as characteristic size of electron. Substituting F(k) of eq.(13) into eq.(12), U(r) is rewritten as

$$U(r) \simeq \left\{ \frac{e^2}{(2\pi)^3}\int d^3k \; \frac{e^{ikr}}{(k^2+\varepsilon^2)(1+R_c^4k^4)} \right\} \; . \qquad (15)$$

The Hermitticity of U(r) which was broken by the approximation of eq.(13) has been recoverd by eq.(12). The integrand in eq.(15) has six poles in the complex k-plane. Closing the integration contour in the upper half k-plane by a large semi-circle, we can perform the calculation of eq.(15) by means of the Cauchy's integration to obtain

$$U(r) = \frac{e^2}{4\pi r}\left[1 - e^{-(r/\sqrt{2}R_c)}\cos\frac{r}{\sqrt{2}R_c}\right] . \qquad (16)$$

The second term in the parenthesis is the form known as the exponential cosine-screened Coulomb potential[16)] which appears in various fields of physics such as plasma physics, nuclear

physics, and solid-state physics. As anticipated previously, the singularity of the Coulomb potential at origin has been weakened. Thus our self-field method based on the Lorentz-Dirac theory leads to a finite value for U(0). It should be noted that we treated fully the bound self-field energy contribution whereas the method by Barut[1] is still perturbative because the self-field was expressed by series expansion in powers of the charge of the electron, and only terms of finite order are adopted in his works. Therefore, the property of weakened singularity of the Coulomb potential may be caused by unperturbative effects of the self-field of the electron. The Lorentz-Dirac theory suggests $R_C \simeq 1.9\times10^{-12}$cm for the non-relativistic electron.[2] We are not able immediately to apply this results to the high energy physics because our result obtained here comes from nonrelativistic treatment. Even though the low energy electron can hardly arrive at the origin of the repulsive Coulomb potential in most cases, effectively weakened potential such as eq.(16) is consistent with the nonlocal nature[14),9)] of the Lorentz-Dirac theory, and with the concept of broading charge of the electron by Weisskopf.[10]

Finally, we give the effectively weakened Coulomb potential between nonrelativistic two spinless electrons:

$$H_{IR} = \sum_{n=1}^{\infty} \left(\frac{i\tau_0}{\hbar}\right)^n [K_1+K_2,[K_1+K_2,\cdots,[K_1+K_2,V(|r_1-r_2|)]]\cdots], \quad (17)$$

where $K_1=p_1^2/2m_0$, $K_2=p_2^2/2m_0$, their coordinates are r_1 and r_2 respectively, and we obtain

$$U(|r_1-r_2|) \equiv V(|r_1-r_2|) + H_{IR}$$
$$= \frac{e^2}{4\pi|r_1-r_2|}\left[1-e^{(-|r_1-r_2|/2R_C)}\times\cos\frac{|r_1-r_2|}{2R_C}\right]. \quad (18)$$

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Appendix

The electromagnetic field is usually quantized and expanded in terms of the creation and annihilation operators, c and c^+ respectively:

$$A(r) = \left(\frac{\hbar c}{2V}\right)^{\frac{1}{2}} \sum_{k} \sum_{\lambda=1}^{2} \frac{1}{\sqrt{\omega_k}} \varepsilon(\lambda,k) \{ c(\lambda,k) e^{ikr} + c^{\dagger}(\lambda,k) e^{-ikr} \}, \qquad (19)$$

where V is the normalization volume, and the orthogonality $k \cdot \varepsilon(\lambda,k)=0$ $(\lambda=1,2)$ is provided. The energy shift of the electron for the interaction term (3) to second order is given by

$$\Delta E^{(2)} = \sum_{\substack{n \\ (n \neq m)}} \frac{|\langle m|V(r)|n;\lambda,k\rangle|^2}{E_m - E_{n,k}}$$

$$= \left(\frac{e}{m_0 c}\right)^2 \left(\frac{\hbar c}{2V}\right) \sum_{\lambda k p'} \frac{1}{\omega_k} \frac{\left|\int d^3y \langle p| f(x-y) p\varepsilon e^{iky} |p'\rangle\right|^2}{p^2/2m_0 - (p-\hbar k)^2/2m_0 - \hbar ck}, \qquad (20)$$

where n means the intermediate state of the electron plus one photon with momentums $\mathbf{p}-\hbar\mathbf{k}$ and $\hbar\mathbf{k}$, respectively. If the Fourier component of f(r) is defined by

$$f(x) = \frac{1}{(2\pi)^3} \int d^3u \, F(u) e^{ixu}, \qquad (21)$$

we obtain an energy shift as follows:

$$\Delta E^{(2)} =$$

$$\left(\frac{e}{m_0 c}\right)^2 \left(\frac{\hbar c}{2V}\right) \sum_{\lambda k p'} \frac{1}{\omega_k} \frac{\left|\int d^3u F(u) \delta^3(u-k) \langle p| e^{iux} p\varepsilon(\lambda,k) |p'\rangle\right|^2}{p^2/2m_0 - (p-\hbar k)^2/2m_0 - \hbar ck}$$

$$= \left(\frac{e}{m_0 c}\right)^2 \left(\frac{\hbar c}{2V}\right) \sum_{\lambda k} \frac{|F(k)|^2}{\omega_k} \frac{|p\varepsilon(\lambda,k)|^2}{p^2/2m_0 - (p-\hbar k)^2/2m_0 - \hbar ck}. \qquad (22)$$

This is exactly equal to Eq.(3.20) in ref.2. The spherical symmetry assumption of the charge distribution of the electron allows to replace F(k) in eq.(22) by F(k). Hereafter the same calculation is carried out to get eq.(1).

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