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Henrik Tauber

A Purely Electromagnetic and Spatially Extended Elementary Particle

Henrik Tauber

Abstract
A radically new type of spatially extended elementary particle is sketched. The model, which primarily represents an electron, is purely electromagnetic and has an extended, string-like charge structure, without internal self-repulsion. The proposed structure automatically results in a quantization of spin and electric charge, embodies a 4π rotational symmetry as demanded for fermions, and provides a possible physical mechanism for particle creation and annihilation. With some additional assumptions the structure further allows a direct calculation from first principles of particle properties like electric charge $(\alpha\hbar c)^{1/2}$, spin $(\hbar/2)$, and magnetic moment $(e\hbar/2mc)$. The model contains an internal rotational variable, corresponding to the Zitterbewegung of the Dirac equation, and unaccounted for in the general quantum-mechanical formalism. The predicted charge structure deviates from spherical and axial symmetry, which may give rise to an experimental testing of the model. The model may possibly be extended to include other types of elementary particles and thus lead to a general unification of matter and electromagnetic radiation.

Key words: extended elementary particle, purely electromagnetic electron, elementary charge, quantization of charge, quantization of spin, 4π symmetry, arbitrary phase factor

1. INTRODUCTION

Quantum-theoretical calculation techniques have been developed into an overwhelming and unprecedented success, but the clarification and understanding of the fundamental concepts and interactions have not been developed to the same extent. Quantum theory is still plagued by weird concepts that defy common comprehension, and common physical properties of particles such as electric charge, spin angular momentum, magnetic moment, etc., well defined in the macroscopic world, have degenerated into unexplained and unexplainable quantum numbers or quantum attributes when applied (with unchanged physical dimensions) at the subatomic level.

It is believed that part of the conceptual bewilderment in quantum theory reflects a lack of understanding of the nature and physical structure of elementary particles, which in the standard model are still modeled after the unphysical concept of point particles. This concept was inherited from classical physics as an expedient solution to the difficulties connected with the supposed self-repulsion of an extended elementary charge and with the ban expressed by special relativity against extended rigid bodies.[1] But point particles, in turn, create a different set of difficulties by rendering all internal dynamics of elementary particles incomprehensible and by giving rise to inevitable divergences in mass and charge terms. The concept of an elementary particle has thus ended in an impasse. Numerous attempts have been made throughout the years to model classical, semiclassical, or purely mathematical elementary entities,[2] none so convincing that they have been generally accepted.

In order to escape this conceptual impasse, we will have to find ways to restore some physical reality to elementary entities and to overcome the deep-rooted fear of forming mental and intelligible models in the quantum domain, which has for too long been judged mentally inaccessible.

In the following an attempt is made to follow such a course and to sketch a radically new model of a spatially extended elementary particle. The model is

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purely electromagnetic and has an extended charge structure that precludes internal self-repulsion. The proposed particle structure results automatically in a quantization of spin and electric charge, embodies a 4π rotational symmetry as demanded by quantum mechanics (QM) for particles with half-integral spin, and may provide a possible physical mechanism for particle creation and annihilation, which are still without viable physical explanations. With the assumptions made the model allows a direct calculation of spin, electric charge (equal to $\pm e$), and magnetic moment. It further has an internal periodicity that complies with the de Broglie wavelength and gives a physical significance to the common arbitrary phase factor $e^{i\theta}$, with consequences for the fundamental indeterminism of QM. The model is primarily intended to represent electrons ("You know, it would be sufficient to really understand the electron," Einstein[3]), but may be expanded also to represent other elementary particles.

Only a broad sketch is given below, and with an emphasis on the never-answered question, "What is an elementary charge?" A real validation of the model demands a thorough and careful calculation of its properties and consequences when applied in a large number of physical interactions. This is a lengthy and time-consuming process for which the help and cooperation of the physical community is requested. The main purpose of the present paper is therefore to advance a new idea about elementary particles and to leave the idea free to be exploited.

Only a few hints about a possible spatial extension of the electron can be gained from quantum theory. Bhabha scattering experiments with high-energy electrons are interpreted to indicate a size of electrons $<10^{-16}$ cm corresponding to cutoff levels of about 100 GeV.[4] The Lamb shift, on the other hand, suggests that S-state electrons have their charge distributed over effective distances comparable with a Compton wavelength, i.e., of the order of 10^{-11} cm. A similar extent of the charge distribution may be deduced from the Dirac equation for the electron, as shown already in 1930 by Schrödinger,[5] who deduced the so-called *Zitterbewegung* of the electron charge. For a long time the *Zitterbewegung* was mostly considered a mathematical peculiarity of the Dirac equation, but in recent years it has gained a more prominent place in theoretical discussions of the electron.[6–9] Being a rigorous mathematical consequence of the Dirac equation, the *Zitterbewegung* deserves a central position in any discussion of electron models.

In the following we will consider the Dirac equation as a single-particle equation. If it is written in Hamiltonian form using the matrix representation given by Dirac,[10] we have for a free particle

$$H\psi = (c\alpha p + \beta mc^2)\psi = i\hbar\frac{\partial\psi}{\partial t}. \qquad (1)$$

Working in the Heisenberg picture where all operators are time dependent, we have

$$\frac{dx_k}{dt} = \frac{i}{\hbar}(Hx_k - x_kH) = c\alpha_k, \quad k = 1,2,3, \qquad (2)$$

and

$$\frac{d^2x_k}{dt^2} = \frac{i}{\hbar}c(H\alpha_k - \alpha_kH) = \frac{i}{\hbar}2c(H\alpha_k - cp_k) \neq 0. \qquad (3)$$

Equation (2) indicates that the velocity components (interpreted as velocity of the charge[7]) of a free Dirac electron invariably have eigenvalues equal to $\pm c$. As a consequence, no rest-frame with a vanishing velocity exists for the motion described by the Dirac position operator. This is analogous to the motion of a photon. Equation (3) further shows that in the Dirac equation the velocity components of a free electron are not constants of the motion. This, however, is the case for the components of the momentum of a free electron:

$$\frac{dp_k}{dt} = \frac{i}{\hbar}[H, p_k] = 0. \qquad (4)$$

The only rest-frame for an electron is, therefore, one in which the momentum is vanishing ($p = 0$), i.e., a rest-frame for the time derivative of a mean position operator. The Dirac equation thus suggests that the electron has a center of mass, the motion of which may vanish, while the charge field performs a forced internal motion with the invariable velocity of c. The same picture is obtained if the Dirac position operator is transformed into a "mean position" operator, e.g. in the transformation of Foldy and Wouthuysen.[11]

By repeated integration of (3) the following is obtained for the position operator:

$$x_k(t) = x_k(0) + c^2 p_k H^{-1} t + i\frac{\hbar c}{2}(\alpha_k(0)H^{-1} - cp_k H^{-2})e^{-i2Ht/\hbar} \quad (5)$$
$$= x_k(0) + x_k^{lin}(t) + x_k^{osc}(t).$$

In the rest-frame with vanishing momentum ($p = 0$), the superimposed oscillatory part of the coordinate operator, the *Zitterbewegung*, becomes

$$x_k^{osc}(t) = i\frac{\hbar}{2mc}\alpha_k(0)\beta\exp(-i\beta 2mc^2 t/\hbar). \quad (6)$$

Equation (6) suggests that the forced internal motion with velocity c is an oscillatory or circular motion with an amplitude and an angular frequency equal to $\hbar/2mc$ and $2mc^2/\hbar$, respectively. In a rest-frame ($p = 0$) this corresponds to half a Compton wavelength and twice a Compton frequency. At increased particle velocity the amplitude decreases and the frequency increases. Analyses of the Dirac equation further indicate that the *Zitterbewegung* is connected with a mixing of positive and negative energy states (negative and positive charge states) at distances comparable with a Compton wavelength.

The Dirac equation thus suggests an internal structure characterized by a circular charge motion with the invariable velocity c, a spatial extension dependent on the external velocity, and a mixing of positive and negative energy (charge) components within a Compton wavelength. These suggestions should be part of a possible electron model that complies with the Dirac equation.

Suggestions for a possible electron structure may also be gained from experimental processes by which electrons are produced. Consider one of the simplest processes of this kind, pair production by a single photon of energy >1.02 MeV in the Coulomb field of a nucleus that may take up excess momentum:

$$\gamma + Ze^+ \rightarrow e^- + e^+ + Ze^+. \quad (7)$$

The process is depicted in Fig. 1, which also shows a possible later annihilation of the created electron and positron. In quantum field theory the process is described by a formalism involving creation and annihilation operators that accounts for the quantitative outcome, but does not provide any insight into the physical mechanisms responsible for particle production.

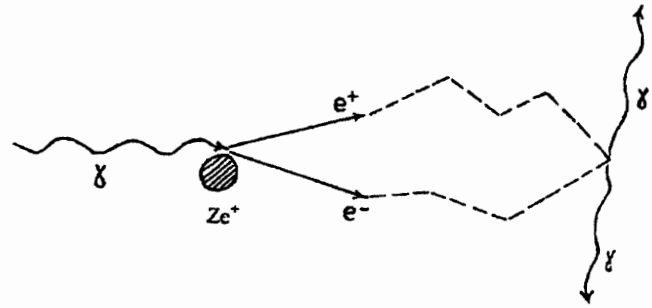

Figure 1. Pair production in the Coulomb field of a nucleus and a later annihilation of the produced electron and positron.

Pair production was originally explained by Dirac, who postulated the existence of an infinite "sea" of occupied negative-energy states from where electrons may be raised to positive energies, leaving behind "holes," which act like positively charged electrons, and later may be filled during the process of annihilation. This explanation of a dual creation and annihilation of twin particles forming matter and antimatter couples proved one of the great triumphs of particle physics, but also constituted a crucial step, which blocked further inquiry into the mechanism of particle creation. The explanation has later been modified and partly reinterpreted, but never fully replaced.

To a simple and uninitiated observer Fig. 1 would seem to suggest a process in which a massless and chargeless photon is transformed in the Coulomb field of a nucleus into two massive and oppositely charged particles, which, in turn, may annihilate each other and return into electromagnetic radiation. This would be a series of purely electromagnetic transformations, where masses and electric charges of the two particles could only come from the electromagnetic field of the original photon, thus suggesting that the mass-energy and the charge of the two elementary particles may originate in some form of curled-up electromagnetic radiation, arranged in such a way that the divergence of the **E** field, div **E**, gives rise to an electric charge.

2. MODEL OF A SPATIALLY EXTENDED ELECTRON

When trying to make an electron model that is consistent with the suggestions both from the Dirac *Zitterbewegung* and from the processes shown in (7) and Fig. 1, we are faced with the difficulty that no established theory of the photon exists. However, if we wish to get out of the conceptual impasse connected

with elementary particles, we will have to take some daring and unconventional steps and try out the consequences. So, as photons are quanta of electromagnetic radiation, as single-photon states show classical wave properties as indicated by tunneling through double prism gaps,[12,13] and as photons add up to form macroscopically observable electromagnetic waves, we will, for the sake of argument, picture a photon as a small (not fully specified), directional, possibly soliton-like,[14–18] wave-train or wave-packet of circularly polarized electromagnetic radiation ($E = \hbar\omega$ and $s = \hbar$) with *unilaterally* directed transverse electric and magnetic fields, and a longitudinal coherence restricted to a few wavelengths.

To mimic the process depicted in Fig. 1, we then imagine that such a circularly polarized photon (a superposition of two linearly polarized states with a phase difference of $\pi/2$) interacts with the strong Coulomb field of the nucleus and undergoes a kind of parametric down conversion[19–21] to two linearly polarized photons each of half the frequency. We further imagine that each of the two photons of half frequency will be forced by the Coulomb field to move in circular, hoop-like paths, each with a circumference equal to $\lambda/2$, where they may become self-trapped, having the electric field vectors parallel to the plane of the circumference and the magnetic field vectors normal to that plane (see Fig. 2). In this way each of the new field configurations will consist of one or more double turns of electromagnetic radiation. With a circumference equal to half a wavelength, the negative and positive electric field vectors of each of the new particles become separated by an angle of 2π and thus will alternate in the circular path and produce a 4π rotational symmetry with either a negative **E** field at the outside of the circumference and a positive **E** field on the inside (an electron) or the other way around (a positron). Such field configurations thus result in a divergence of the **E** field equivalent to an electric charge in the form of an extended, axially asymmetric monopole of either negative or positive sign and with no distribution of separate charge elements that may give rise to internal self-repulsion, which has been the general difficulty with extended elementary charges. In this way the simultaneous creation of an electron and a positron as indicated in Fig. 1 may take place.

The magnetic field vectors of these configurations will at the same time form a magnetic dipole that cannot be separated into monopoles, in compliance with the well-known asymmetry of the Maxwell equations. Obviously the configuration also has a spin angular momentum that is directly calculable and a rest mass in a rest-frame where the external momentum is vanishing. Such linearly polarized, hoop-like configurations ("hooptons"), if once formed, may be stable and self-sustained (see later), as they are confined by their own magnetic field and forced to move continuously in a circular path with the invariable velocity of light as a kind of standing electromagnetic wave. The detailed composition of the wave-train (not fully known) making up the photon may decide for which energies such field configurations can settle into fully stable structures with fixed mass energies (for electrons 0.51 MeV), while possible surplus energy is shed in the form of kinetic energy shared by the matter and antimatter couple.

The mass of such a configuration is entirely electromagnetic, which has been a fascinating objective since the time of the old electron theories by Abraham and Lorentz, and has been pursued up to the present, e.g., Ref. 22. The proposed structure may thus provide a simple explanation of the origin of the mass of elementary structures, and the structure may accommodate a mass spectrum, e.g., for a series of charged leptons. We note that the configuration, though spatially extended, does not violate the ban from special relativity against extended rigid bodies, as no message can be carried from one part of the body to another part at velocities faster than the velocity of light.

3. PROPERTIES OF HOOPTON STRUCTURES

The rules of standard QM or quantum electrodynamics are not directly applicable in a calculation of distinct internal properties of elementary structures, as QM has been designed to account for probabilities of interactions between entities represented by wave-functions or state vectors in a Hilbert space. We are therefore referred to the use of classical theory when trying to deduce specific hoopton properties. This somewhat dangerous procedure may be partly justified by the purely electromagnetic nature of the hoopton structure. However, not to stretch the procedure too far, and in order to keep the physical picture as simple as possible, only a simple calculation of basic hoopton properties will be attempted.

3.1 General Properties (1)

The proposed hoopton structure consists of a photon of frequency ω_γ, which is forced to move in a circular path with a circumference equal to $\lambda/2$. In a rest-frame for the hoopton ($p = 0$) we have $E = \hbar\omega_\gamma$ and a rest mass (mass equivalent)

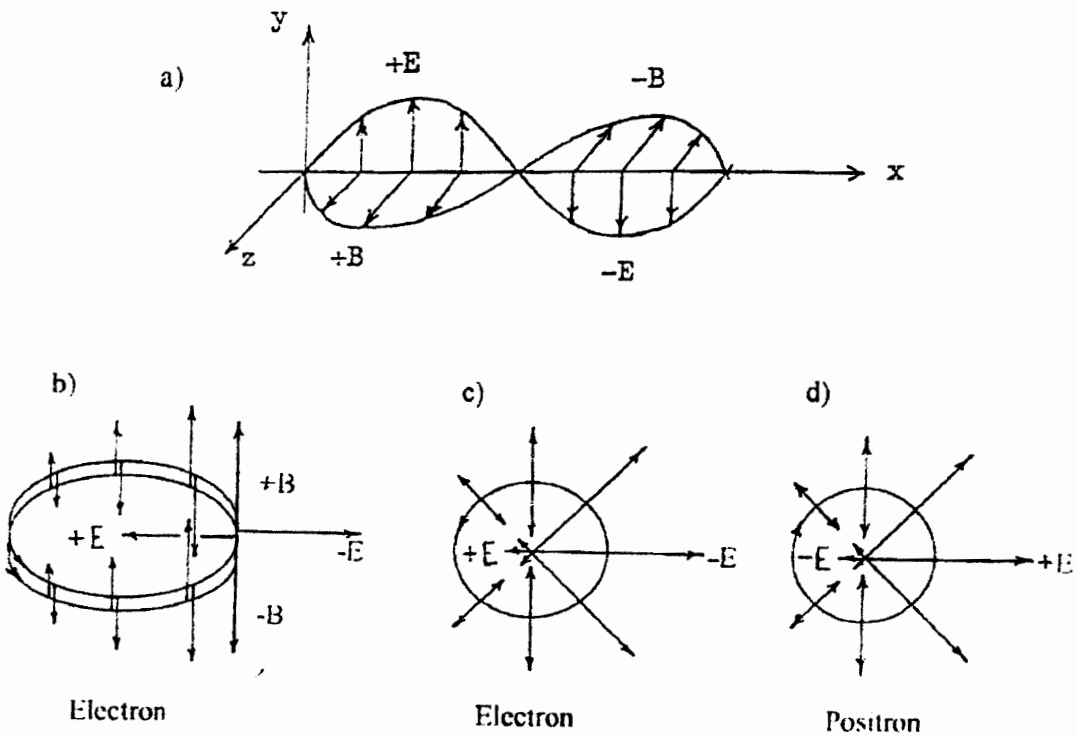

Figure 2. An approximate pictorial representation of the field configuration of an electron and a positron with hooptonic structure. (a) A linearly polarized sinusoidal wave-train with *unilaterally* extended electric and magnetic fields (only a single wavelength λ is shown). If such a wave-train is forced to move in a circular path around an axis parallel to the z axis and with a circumference equal to $\lambda/2$, a hooptonic structure consisting of one or more double turns of electromagnetic radiation is formed. (b) The magnetic fields of such a hoopton structure. (c) The electric fields of the hooptonic double turn seen from above and with the negative **E** field on the outside thus forming an electron. (d) The similar field structure with the positive **E** field on the outside as in a positron.

$$m = \frac{\hbar\omega_\gamma}{c^2} = \frac{\hbar}{\lambda\!\!\!^{-}c} \quad \left(\lambda\!\!\!^{-} = \frac{\lambda}{2\pi}\right). \tag{8}$$

The radius of the hoopton then becomes

$$r = \frac{\lambda\!\!\!^{-}}{2} = \frac{\hbar}{2mc}, \tag{9}$$

and the internal angular frequency ω_h with which negative and positive charge states oscillate becomes

$$\omega_h = 2\omega_\gamma = \frac{2mc^2}{\hbar}. \tag{10}$$

These values are the same as those inferred from the *Zitterbewegung* in the Dirac equation. In the rest-frame the radius is $r = 1.93 \times 10^{-11}$ cm and the internal frequency is $\omega_h = 1.55 \times 10^{21}$ s^{-1}.

If a hoopton is accelerated to velocity v, the mass equivalent is increased, and the radius r decreased, by a factor $1/(1 - v^2/c^2)^{1/2}$. As $\hbar\omega_\gamma = mc^2$ in all reference frames, this means that both the frequency ω_γ of the electromagnetic radiation making up the hoopton and the internal frequency ω_h in the hoop will be increased by this same factor. The size of a hoopton is thus strongly velocity dependent. At about 100 GeV the size is reduced to $<10^{-16}$ cm. In spite of the large radius in the rest-frame, the magnitude thus may conform with the upper limit derived from high-energy scattering experiments[4] and will continue to conform as long as the hoopton size is decreased by the same factor as the energy is increased. Estimates of limits for electron size have usually presupposed a constant particle size (except for relativistic length contraction); if this is not the case, the whole concept of elementary particle size must be reinterpreted. We note that size and frequency variations may also be a mechanism for storage, and later release, of energy in radiative processes where a hoopton could act as a kind of parametric oscillator.[9]

With a radius equal to (9), it is obvious that there is a fundamental limitation on the sharpness in measurements of the spatial position of an electron

$$\Delta x = \frac{\hbar}{2mc}. \tag{11}$$

This corresponds to an uncertainty of momentum $\Delta p = mc$, which in turn corresponds to the threshold energy for pair production, the limit set by relativistic quantum field theory for probing into spatial extent.

We also note that $\hbar/2mc$ corresponds to the lower limit to spatial extension of an extended rotating body with positive energy density, which is $r = s/mc$,[23] or for a particle with spin one half a radius equal to (9). This spatial extent thus appears as a fundamental limit, which has not found an appropriate place in present theories of elementary particles.

3.2 Hoopton Charge

In order to estimate the electric charge q of the hoopton structure described above, we have to make some specific assumptions about photon representation and the structure of the photon making up the configuration in order to enable calculations of energy densities. This is a field where direct information is lacking. However, a set of plausible assumptions, similar to those made in corresponding estimates of photon dimensions,[14,16,17,24] may show the potential of the hoopton structure as a model for the electron. The plane polarized photon making up the hoopton is then imagined as a small electromagnetic wave-train with a wavelength λ (or very nearly so, as a restricted wave-train requires different frequencies), a longitudinal extent of $n\lambda$, a lateral extent of the plane polarized field equal to $\alpha\lambda$, where α is the fine-structure constant (which may well be related to the field properties of the photon), and a *unilateral* extent of the transverse **E** and **B** fields not exceeding $\lambda\!\!\!^{-}$ (a circularly polarized wave-train must have a finite extent of the **E** and **B** fields in order to have a spin $s = \hbar$,[25] and an extent exceeding $\lambda\!\!\!^{-}$ results in circular velocities greater than c; it is assumed that the plane polarized wave-train has the same transverse extent).

In general a photon may be represented by a solution of the classical Maxwell equations if properly normalized. As a convenient representation we choose a plane sinusoidal wave with a vector potential **A** and a wave vector **k** and use a gauge in which the scalar potential $\varphi = 0$:

$$\mathbf{A} = \mathbf{A}_0 \cos(\omega t - \mathbf{k}x) \tag{12}$$

and

$$\begin{aligned}\mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = \frac{\omega}{c}\mathbf{A}_0 \sin(\omega t - \mathbf{k}x) \\ &= \mathbf{E}_0 \sin(\omega t - \mathbf{k}x).\end{aligned} \tag{13}$$

With these choices the charge of a hoopton may be calculated from first principles using Maxwell's equation (in Gaussian units where $|\mathbf{E}| = |\mathbf{B}|$):

$$4\pi q = \operatorname{div} \mathbf{E}. \tag{14}$$

To find the magnitude of the **E** vectors in the hoopton structure we then normalize **A** to correspond to unit probability of finding the photon that is making up the hoopton in an effective volume V corresponding to the dimensions estimated above. When the photon is curled up into a hoopton structure with a circumference equal to $\lambda/2$, the **E** and **B** fields of the n wavelengths will be added and will act as the effective **E** and **B** fields of the hoopton. The fields will be present within a longitudinal distance corresponding to the length of a double turn, i.e., a wavelength λ that, therefore, is used in the estimate of the effective volume. The lateral extent of the plane polarized fields was supposed to be $\alpha\lambda$, and the maximum extent of the transverse **E** and **B** fields was estimated to be $\lambda\!\!\!^{-}$. However, if the effective transverse extent is proportional to the field strength, the effective mean transverse dimension applicable in the calculation of energy densities is equal to $(2/\pi)\lambda\!\!\!^{-}$. The effective volume to be applied in a calculation of energy density relevant to the field strength in the hoopton structure then becomes $V = \lambda\alpha\lambda(2/\pi)\lambda\!\!\!^{-} = 8\pi\alpha\lambda\!\!\!^{-3}$.

Applying this effective volume V, the energy density and the effective field strengths become

$$\frac{\hbar\omega}{V} = \frac{1}{8\pi}\overline{(\mathbf{E}^2 + \mathbf{B}^2)} = \frac{1}{8\pi}\mathbf{E}_0^2, \tag{15}$$

$$\mathbf{E}_0 = \left(8\pi\frac{\hbar c}{\lambda\!\!\!^{-}V}\right)^{1/2} = \left(\frac{\hbar c}{\alpha}\right)^{1/2}\frac{1}{\lambda\!\!\!^{-2}}. \tag{16}$$

The charge may then be calculated as the divergence of **E** out of a cylinder surface with circumference $\lambda/2$ and height $\alpha\lambda$ (the supposed lateral extent of the plane polarized photon making up the hoopton), where the **E** vectors are normal to the curved surface and parallel to the bases of the cylinder, and where

the mean value of $\mathbf{E} = (2/\pi)\mathbf{E}_0$:

$$|q| = \frac{1}{4\pi}\int v\nabla\cdot\mathbf{E}dv = \frac{1}{4\pi}\int s\mathbf{E}\cdot\mathbf{n}ds = \frac{1}{4\pi}\frac{2}{\pi}\mathbf{E}_0\frac{\lambda}{2}\alpha\lambda = \alpha\lambda^2\mathbf{E}_0, \tag{17}$$

where $\mathbf{n}$ is a unit vector normal to the cylinder surface. Using $\mathbf{E}_0$ from (16), we finally have

$$|q| = (\alpha\hbar c)^{1/2} = |e|. \tag{18}$$

With the assumptions made the proposed geometry of a hoopton thus results in a charge with a magnitude equal to the elementary charge $\pm e$. This charge has no self-repulsion, needs no renormalization, and is, for the first time, an integral and inseparable part of the elementary particle. Moreover, it is independent of the radius of the hoopton; i.e., it is quantized with the same numerical value for all hoopton structures and with opposite signs for a hoopton and its antiparticle. The quantization is a consequence of $E = \hbar\omega$ for all photons, combined with the assumption that photons are electromagnetic wave-trains with effective volumes proportional to λ^3. In that case the divergence of $\mathbf{E}$ will be constant for hooptonic structures with a circumference equal to $\lambda/2$, independent of the magnitude of the wavelength.

The assumptions made above regarding the photon making up a hoopton, of course, can be discussed; it is also possible that part of the external $\mathbf{E}$ field of a hoopton may be countered by the oppositely signed internal $\mathbf{E}$ field, which, in conformity with the demands of the Dirac equation, is present at distances shorter than a Compton wavelength. But the above calculations demonstrate the potential of hoopton structures for giving physical substance to an extended and elementary charge with an effective decoupling of the internal charge velocity and the external momentum as suggested by the *Zitterbewegung* of the Dirac equation.

Maxwell's equation (14) is usually applied to a (supposed) static and spherically symmetric charge distribution. The elementary charge of a hoopton is neither static nor spherically symmetric. The charge calculated in (18) is, therefore, to be considered as the time average over a spherical surface of the divergence of the $\mathbf{E}$ vector produced by a rotating ($\omega \approx 10^{21}$ s^{-1}), precessing, and thermally jiggling hoopton. If the hoopton is cooled to temperatures close to absolute zero in an electron trap, the time average may no longer mimic a spherically distributed charge. This opens a possibility of testing the proposed charge structure (see Section 3.8).

Once formed, the two turns in the hoopton structure may act on each other. The positive magnetic field of one turn may attract the corresponding negative magnetic field of the other turn (see Fig. 2b), and for sinusoidal waves the two fields are in phase along the circumference, each being a half-wavelength of the same wave pattern. With the assumptions made above, the magnitude of the magnetic force acting between the two half-wavelengths will be of the order of

$$F_{mag} = \left(\frac{2B_0}{\pi}\right)^2\frac{(\alpha\lambda^2/2)^2}{(\alpha\lambda)^2}. \tag{19}$$

Using $|\mathbf{E}| = |\mathbf{B}|$ and (9) and (16), we get

$$F_{mag} = \frac{2}{\alpha}\frac{\hbar\omega}{r}. \tag{20}$$

At the same time a charge element dq along an arc length $rd\theta$ (the angle θ calculated from the point on the circumference where $\mathbf{E} = \mathbf{B} = 0$) in a half-wavelength turn may be acted on by the $\mathbf{B}$ vector of the other half-wavelength with a Lorentz force directed toward the center of the hoopton:

$$|dF(\theta)| = dq\left(\frac{v}{c}\times\mathbf{B}(\theta)\right) = dq\,|B(\theta)|, \tag{21}$$

where $dq = (q/2\pi)(\pi/2)\sin(\theta/2)d\theta$ and $B(\theta) = B_0\sin(\theta/2)$.

The Lorentz force applies to a free-moving point charge, and it is not known what the exact relation is for an element of an extended hooptonic charge, which, in the present case, cannot move relative to the magnetic field vector. If the formula for the Lorentz force applies in this case, we have for the integrated force acting on the two turns

$$|F| = 2\int_0^{2\pi}\frac{q}{4}B_0\sin^2\frac{\theta}{2}d\theta = \frac{\pi}{4}\frac{\hbar\omega}{r}, \tag{22}$$

i.e., a force close to the centripetal force required to keep a mass equivalent to $\hbar\omega/c^2$ moving with velocity c in a circular path with radius r. As a result of these forces, the two turns may be firmly locked in a stable position, confined by their own magnetic fields.

To become unlocked again, a magnetic field of the same geometric pattern and the same strength but with opposite sign would be needed. This is the magnetic field of the hooptonic antiparticle. An encounter between a hoopton and its antihoopton may therefore result in a release of both of the electromagnetic fields that were tied up in the particles, forming two oppositely directed photons each of the same energy. This is what may happen in a low-energy encounter between an electron and a positron, as shown in Fig. 1. With this explanation of the annihilation process, one would expect that collisions between particle and antiparticle with spins opposed would result in a more direct and rapid annihilation than collisions with parallel spins, because opposed spins will bring the particle and antiparticle directly into positions where the magnetic confinement of both particles is directly counteracted. This is in agreement with the known differences in annihilation rates between singlet states (^{1}S, spins opposed) and triplet states (^{3}S, spins parallel). Both in free collisions and in bound positronium states, the mean life of the singlet state is approximately 10^3 times as short as the mean life of the triplet state, and the annihilation of a singlet state is known to result in the emission of two oppositely directed photons that conserve energy, momentum, and angular momentum. No effects due to nonelectromagnetic interactions are observed.

With the proposed hooptonic structure different situations will also arise when similar hooptons approach each other, depending on whether spins are parallel or opposed. With parallel spins both magnetic and electric repulsion will tend to keep particles apart, whereas opposed spins may result in a coupled precession of the two hooptons, which may allow the maintaining of the two independent configurations within the same quantum cell. This is in accordance with the Pauli exclusion principle for fermions.

A wave-train of limited extension will contain some modifications of the sinusoidal wave pattern, and these modifications may decide whether the configuration is stable. This may be the reason why only waves with an energy of 0.51 MeV form stable configurations in the low-energy range. A search for stable magnetic confinement patterns may thus be a key to solving the question of the mass spectrum of charged leptons.

3.3 Hoopton Spin

In a hooptonic structure that consists of a curled-up photon, the electromagnetic field is bound to spin with the constant velocity c. In a rest-frame with vanishing external momentum the angular momentum of this internal motion is

$$|\mathbf{s}| = |\mathbf{r}\times\mathbf{p}_{int}| = \frac{\lambda\!\!\!^{-}}{2}\frac{\hbar\omega}{c} = \frac{\hbar}{2}. \tag{23}$$

If the hoopton is moving, $\mathbf{r}$ is decreased and $\mathbf{p}_{int}$ is increased by the same factor $1/(1 - v^2/c^2)^{1/2}$ (see Section 3.1) and the spin will remain unchanged. The spin of a hoopton is thus quantized and equal to $\hbar/2$. In this way the observed spin of an elementary particle, which is generally considered a definite quantum phenomenon with no possible classical analogue, may find a simple explanation. The agent that rotates is the electromagnetic field. The structure may also explain why there is no specific spin-energy term in the Dirac Hamiltonian for a free electron. The spin-energy is already accounted for in the mass-energy term. Spin and magnetic moment therefore only give rise to additional energy terms in the Hamiltonian if an external electromagnetic field is applied.

3.4 Hoopton Magnetic Moment

A magnetic dipole is an integral part of the hoopton structure. The magnitude of this magnetic moment may be calculated directly from the common formula[1] for a magnetic moment (Gaussian units)

$$|\mu| = \sum \frac{1}{2c} q\mathbf{r}\times\mathbf{v}. \tag{24}$$

If $v = c$ is inserted together with a factor 2 due to the double turn per wavelength, the magnetic moment of a hoopton becomes

$$|\mu| = \frac{e\hbar}{2mc}, \tag{25}$$

which gives the same gyromagnetic ratio $g = 2$ as the Dirac equation

$$|\mu| = \frac{e}{mc}s. \tag{26}$$

This is the correct ratio apart from the second-order correction known from the anomalous moment, which cannot be expected to turn up before the exact wave structure of a hoopton is known and the photon dimensions ($\alpha\lambda$) are taken into account.

The hoopton structure does not admit a magnetic monopole but only a dipole, and this magnetic dipole

is present also when a hoopton has no external motion. A "pure" elementary electric charge, or a "pure" magnetic dipole, thus does not seem to exist, but only a combined electric and magnetic field configuration. The magnetic field produced by an electric current, therefore, should rather be characterized as a polarization effect, with increased polarization of the moving hooptons (and therefore increased magnetic field strength) at increased velocity, thus giving a physical explanation of why a moving electric charge creates a magnetic field.

3.5 Rotational Symmetry

The hoopton structure is not axially symmetric, but has a 4π symmetry corresponding to the quantum-mechanical demand for particles with half-integral spin. The charge state of a hoopton thus changes sign for each rotation of 2π, as does the sign of the QM rotation operator $R_z(\theta) = \exp(-is_z\theta/\hbar)$ with values $R_z(2\pi) = -1$ and $R_z(4\pi) = 1$. Up to now this has been considered an unexplainable quantum effect, but it is a simple hoopton property.

3.6 Dynamical Periodicity

The internal frequency of a full rotation of a hoopton in its rest-frame is half the angular frequency in the hoop, i.e., $\omega = m_0c^2/\hbar$, and the phase of the rotation is

$$\varphi = \frac{m_0c^2}{\hbar}t_0 \quad (t_0 = \text{proper time}). \tag{27}$$

Seen in a frame of reference in which the hoopton moves with velocity v, the phase becomes

$$\varphi = \frac{m_0c^2}{\hbar}\frac{t - vx/c^2}{(1-v^2/c^2)^{1/2}} = -\left(\frac{mv}{\hbar}x - \frac{mc^2}{\hbar}t\right). \tag{28}$$

The dynamical periodicity of a hoopton, therefore, conforms to a wave equation

$$\psi = a\exp(-i/\hbar)(px - Et), \tag{29}$$

i.e., to a de Broglie wavelength $\lambda\!\!\!^{-} = \hbar/p$ and a wave equation that is a plane wave solution of the Schrödinger equation. The hoopton structure thus readily unites particle properties (localization, rest mass, electric charge, etc.) and wave aspects, and with no duality problem, as both sets of properties originate in the same elementary electromagnetic configuration.

3.7 General Properties (2)

The hoopton described above reflects in a remarkable way the elementary particle of the Dirac equation and, being composed entirely of electromagnetic radiation, explains the many similarities between photons and electrons, as expressed, e.g., by Feynman:[26] "electrons behave just like light." The structure also explains why the Dirac position operator apparently violates Newton's second law, and why positive and negative energy states (negative and positive charge states) are mixed at distances shorter than a Compton wavelength. It ensures that the parity of an antihoopton is opposite to that of the hoopton, as it should be if they represent a positron and an electron, and that both observe CP invariance. The hoopton structure may also possibly explain effects like the Lamb shift, which is generally explained by the (energetically doubtful) vacuum polarization effect in which electron-positron pairs of energy mc^2 are created and annihilated around a bare charge for time intervals $\Delta t = \hbar/2mc^2$ and within distances $\Delta x = c\Delta t = \hbar/2mc$, i.e., the reciprocal frequency and the amplitude of the *Zitterbewegung* of a hoopton structure. The possibility of deriving the Lamb shift directly from Maxwell's laws has previously been shown.[27]

However, a marked difference between the point particle of QM and a hoopton is the hooptonic charge distribution, which lacks spherical or axial symmetry. This means that an ensemble of hooptonic electrons prepared in a "pure state," and thus having the same state vector, cannot be considered as identical during interactions, as the momentary internal phases are not accounted for in the state preparation and may vary from hoopton to hoopton at the moment of interaction.

If the charge is rotating with constant velocity, as suggested already by the Dirac equation, the state vectors $|\psi\rangle$ and $e^{i\theta}|\psi\rangle$ will only represent identical states during interactions if the charge is spherically or axially symmetric. For ensembles of hooptons with charge distributions lacking spherical or axial symmetry, individual phase factors will be required for each hoopton, $e^{i\alpha}|\psi\rangle$, $e^{i\beta}|\psi\rangle$, ..., in order to get a complete description. This corresponds to giving the common arbitrary phase factor in QM a definite physical significance as a variable that determines the local field strength during interactions. In this way the common phase factor may have acted as a hidden variable, which, unknowingly, has rendered the standard formalism of QM an incomplete theory. It is a strange irony that, in spite of numerous proofs against the possibility of "hidden variables," such a variable may

deliberately have been built into the quantum formalism from the very beginning and stayed there all the time under a different name, not being recognized as such because elementary charges have been considered spherically symmetric.

This variable is, however, of a subtle kind. With no rest-frame for a photon, a space-time description of the internal motion of a hoopton cannot be given, and the individual phase factor at the moment of interaction cannot be determined. The possibility of a fully deterministic description of hoopton interactions thus remains unattainable even after the discovery of this variable. Though unobservable, an internal variable still has important consequences. The common (Copenhagen) interpretation of quantum theory postulates a fundamental and irreducible indeterminism for individual interactions at the subatomic level. With hooptonic electrons this may be replaced with an irreducible uncertainty in the prediction of elementary interactions due to an unmeasurable internal periodicity. The previous fundamental indeterminism in the quantum domain may thus be changed into an uncertainty caused by ignorance of the exact initial conditions for individual interactions. And so, at least in simple interactions, "God does not play dice," but we still have to, in order to predict the possible outcome.

As a consequence, the probabilistic predictions of quantum theory obviously should be considered as assertions about the behavior of ensembles of particles with similar state preparation, as advocated by a school of physicists, e.g., Ref. 28, rather than allegations about the innate behavior of individual particles, which has been the extreme claim in the Copenhagen interpretation. In this way ambiguities connected with the theory of measurements and with the sudden collapse of wave-packets also become circumvented.

3.8 Experimental Testing

The hooptonic structure deviates markedly from the point-particle electron of QM by its large size, which at low velocities is close to a Compton wavelength, and by its charge distribution, which, in contrast to the point-particle electron, lacks spherical and axial symmetry. These deviations may be tested in various ways.

1. In elastic scattering experiments with atomic nuclei (Mott scattering) the large size of hooptons (and antihooptons) may give rise to reduced wide-angle scattering relative to the Mott scattering theory as proposed by MacGregor.[2] The effect is supposed to be most pronounced at low and moderate energies (approx. 10–100 keV) in thin solid targets of low Z-value (e.g., aluminum). Unexplained discrepancies between experimental results and theoretical angular distributions in the literature may already suggest such effects.[2]
2. Deviations from theoretical scattering cross sections may also occur in elastic electron-electron scattering (Møller scattering) and electron-positron scattering (Bhabha scattering), especially in experiments with longitudinally or transversely polarized beams at moderate energy transfer where the Coulomb contribution to the scattering matrix may deviate relative to the Coulomb contribution from a supposed spherically symmetric charge distribution. (Part of the known spin dependence of the scattering cross sections may actually be due to a nonspherical charge distribution.)
3. Scattering of pulses of low-energy polarized electrons against targets of trapped polarized electrons kept at temperatures close to absolute zero should vary strongly with directions of polarization and demonstrate a spherically asymmetric charge distribution of hooptons.

4. PROSPECTS

The introduction of hooptonic electromagnetic field configurations in elementary particle physics may have ramifications that will pervade large parts of quantum theory. As shown in Section 3, hoopton structures may provide simple explanations of what may be understood by previously unexplained properties of the charged leptons such as mass, spin, and electric charge, and may change the general interpretation of central parts of quantum theory.

The most far-reaching perspective connected with hoopton structures is, however, the possibility of an application to hadrons and consequently to the strong interaction, which shall here only be discussed qualitatively. This demands some further elaboration and expansion of the hoopton model. However, since most hadrons (quarks) may be produced in e^-e^+ collisions, e.g.,

$$e^- + e^+ \rightarrow (\gamma) \rightarrow (q + \bar{q}) \rightarrow \text{hadron}, \qquad (30)$$

where parentheses indicate supposed intermediaries, and since hadrons may decay again, either directly or in a multistep process, into photons or leptons, e.g.,

$$q + \bar{q} \rightarrow \gamma + \gamma \quad \text{or} \quad q + \bar{q} \rightarrow (\gamma) \rightarrow l + \bar{l}, \qquad (31)$$

it is natural to surmise that hadrons (quarks) may also

have a hooptonic substructure and that hoop-like photons in a variety of configurations may constitute a primordial substratum of the present phenomenological level of leptons and quarks. Such a conjecture is supported by the many unexplained similarities and symmetries between leptons and quarks and by the generic diversity inherent in the hoopton structure and would be in keeping with a general creation of hadrons and leptons out of radiation at a very early stage shortly after the big bang.

In addition to the simple hoopton with a circumference equal to $\lambda/2$, described above, more complex and self-sustained hooptonic structures may also be produced in e^-e^+ collisions or during pair production from photons with longitudinal extents of $n\lambda$, e.g., hooptons with circumferences equal to $\lambda/3$, $\lambda/4$, $\lambda/6$ or to $2\lambda/3$, $3\lambda/4$, etc. These may all have the same electric charge ($\pm e$) as the simple hoopton, and some of the complex hooptons will have two or more internal centers of electric charge (electric field strength) similar to those revealed in high-energy elastic collisions between nucleons and electrons. Due to the internal dynamics of hooptons, such centers with fractional charges could give the impression of freely moving charged entities, which nevertheless are effectively confined by the main hooptonic structure.

A common origin of electric charge due to hooptonic structure would also explain the general quantization of electric charge and the amazing agreement between the numerical values of the net charge of a proton and an electron. Experimentally the two charges are found to agree[29] within $(Q_p + Q_e)/Q_p < 10^{-19}$, which is difficult to account for without assuming a common origin.

If some of the internal substructures in hooptonic hadrons get oppositely directed magnetic moments, corresponding to oppositely directed spin vectors ($q_1^{\uparrow}$, $q_2^{\downarrow}$, $q_3^{\uparrow}$) in the quark model, part of the magnetic dipole field may become internally saturated and only come into play in short-range encounters with similarly organized structures, where the magnetic dipole-dipole interaction may become very strong. In this way part of the long-range magnetic dipole force could be turned into a strong, short-range force acting within distances comparable to the dimensions of nucleons where, due to the special field configuration of hooptons, it may overrule the electric repulsion. A similar magnetic basis for the strong force has previously been suggested, e.g., Refs. 30–33.

Hooptonic structures thus appear to have significant unexplored possibilities as a unifying structure at a primordial level below the present level of leptons and quarks. If these conjectures can be substantiated in quantitative calculations, a large step will have been taken toward a unification not only of the fundamental interactions but also of matter and electromagnetic radiation.

5. CONCLUSIONS

The sketched model of an elementary particle (an electron or a positron), which is based on a "hoopton" structure consisting of electromagnetic radiation forced to move in a circular path with a circumference equal to $\lambda/2$, closely reflects the properties deduced for elementary particles described by the Dirac equation.

The model embodies a spatially extended and repulsion-free elementary charge, which is automatically quantized and has a magnitude that, with additional assumptions, may be directly calculated to $\pm(\alpha\hbar c)^{1/2}$. The size of the spatial extension is not a constant but decreases with particle velocity and thus may conform with the size limits deduced from present scattering experiments. The structure further allows a calculation of other particle properties, such as a quantized spin equal to $\hbar/2$, a magnetic moment $e\hbar/2mc$, a 4π rotational symmetry, and an internal periodicity that accounts for the de Broglie wavelength.

The model provides possible physical explanations of the processes of pair production and pair annihilation and explains why, in compliance with the Maxwell equations, elementary electric charges occur in the form of electric monopoles, whereas magnetic monopoles are not admitted.

The elementary charge structure is neither spherically nor axially symmetric and contains an internal rotational variable not accounted for in QM. The common state vector description for such particles, therefore, will not be a complete description, and calculated probabilities for interactions with such particles cannot be referred to a fundamental indeterminism of quantum interactions, but must be considered due to a lack of knowledge of the exact initial conditions.

The model applies primarily to charged leptons, but may possibly be expanded to cover hadrons, and thus may form the basis of a possible unification of matter and electromagnetic radiation.

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Résumé

L'idée d'une particule élémentaire de type spatialement étendu et radicalement nouveau, est esquissée. Le modèle, qui représente essentiellement un électron, est purement électromagnétique, d'une structure étendue en forme de corde, et sans répulsion interne. Cette structure, qui donne lieu à une quantification automatique de spin et de charge électrique, conduit à une symétrie 4π de rotation, tel que requis pour des fermions, et fournit un mécanisme physique possible pour les réactions de création et d'annihilation de paires de particules. Avec des hypothèses additionnelles la structure proposée permet un calcul direct de la magnitude de propriétés de particule comme la charge élémentaire $(\alpha\hbar c)^{1/2}$, le spin $(\hbar/2)$, et le moment magnétique $(e\hbar/2mc)$. Le modèle contient une variable de rotation interne, répondant au Zitterbewegung de l'équation de Dirac et qui n'est pas contenue dans le formalisme général de la mécanique quantique. La structure de charge proposée dévie de la symétrie sphérique ou axiale, ce qui peut conduire à une preuve expérimentale du modèle. Le modèle peut éventuellement être développé pour comprendre d'autres types de particules élémentaires et ainsi conduire à une unification générale de matière et du rayonnement électromagnétique.

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Henrik Tauber
Natural Science Research Unit
National Museum
Copenhagen, Denmark

For correspondence:
Kollemosevej 2A
Dk-2840 Holte, Denmark

e-mail: h_tauber@yahoo.com