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An Existence of Local Minkowski Spaces Is Insufficient for Einstein's Equivalence Principle

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Abstract

Einstein's equivalence principle, which states that the local equivalence between acceleration and gravity in the physical space, requires a free fall result in a co-moving local Minkowski space. Thus, if acceleration of a static particle does not exist, a non-constant Lorentzian metric would imply inconsistency in physics. Moreover, if a physical requirement that is independent of coordinates (e.g., the principle of causality) is violated, a Lorentz manifold cannot be diffeomorphic to a physical space. Observation, including confirmation of Einstein's three tests, supports the conclusion that space-time coordinates (and thus covariance) are restricted by physical requirements such as Einstein's equivalence principle. A so-called coordinate-free derivation is actually logically self-defeating if a physical formula involves non-scalars. It is pointed out also that Einstein's equivalence principle, different from the inadequate Pauli version, requires a physical meaning of space-time coordinates although this was ambiguous in Einstein's theory. Concurrently, the meaning of space-time coordinates in a physical space and the coordinate relativistic causality are clarified in terms of the Euclidean-like structure, which is actually included in the theoretical framework of general relativity. Then it is clear that the Schwarzschild solution and the isotropic solution are physically different, although they are diffeomorphic manifolds. Thus, it is necessary to make measurements related to local terrestrial space contractions to discern the more realistic model for gravity of the earth.

Key Words: Einstein's Equivalence Principle, Frame of Reference, Euclidean-like structure, Local Light Speed, and Physical Space. (Submitted July 5, 2001)

"As far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." -- A. Einstein (in 'Geometry and Experience', 1921).

1. Introduction.

Einstein's equivalence principle requires that a free falling point-like particle in a gravitational field is along a geodesic and results in a co-moving local Minkowski space[1)] [1,2]. Some relativists [3-14], however, believed that, a mathematical existence of local Minkowski spaces would be sufficient to satisfy Einstein's equivalence principle in a manifold. They seem to care little, in terms of physics, how such a local Minkowski space is formed and what causes it.

Currently, a major problem in general relativity is that many Lorentz manifolds are clearly not valid in physics although they are solutions to Einstein's field equation [15]. Consequently, a related question would be whether the validity of such an interpretation of the equivalence principle is sufficient for a manifold to be a *physical space* (see Appendix A)? Another possibility is, however, that their belief [3-14] is incorrect and that a mathematical existence of local Minkowski spaces is a necessary but insufficient condition for the satisfaction of Einstein's equivalence principle [16].

Einstein proposed that his equivalence principle is satisfied in the physical space, where all *physical requirements* are sufficiently satisfied (see Appendix A). This principle is further clarified in his calculation of light bending [1,2]. It should be noted that, just as in special relativity, a free falling observer in the physical space automatically experiences a local Minkowski space. However, in a *Lorentz manifold* (see Appendix A) although a local Minkowski space exists for any space-time point, a "free fall" may not result in a co-moving local Minkowski space. For example, for a static particle, there may not be spatial acceleration generated from a non-constant Lorentzian metric, and this leads to inconsistency in physics (§ 4).

The idea that Einstein's equivalence principle would always be satisfied in a Lorentz manifold [3-14] is due to Pauli's misinterpretation [6] to which Einstein objected [17]. However, such an idea seems to be supported by Einstein's notion of unrestricted covariance, which suggested no physical meaning for the coordinates. Thus, to show the contrary, it would be necessary to demonstrate through detailed examples that *diffeomorphic* coordinate systems may not be equivalent in physics.

To this end, it is crucial to recognize that the frame of reference necessarily has the Euclidean-like structure that is independent of gravity. Although Einstein discovered this structure first [1,2], he believed incorrectly that coordinates couldn't have a direct physical meaning [1]. Consequently, the meaning of space-time coordinates was ambiguous in Einstein's theory although his principles require otherwise. In view of this, relativists almost had to take refuge in Pauli's version [6] or others[2)] that require no physical meaning of coordinates, and Whitehead [18] rejected general relativity as a physical theory.

On the other hand, the assumption of no physical meaning for space-time coordinates (as will be shown in § 6) is logically self-defeating because physics involves non-scalars. Experiments including the three tests have shown that a space-time coordinate system necessarily has physical meaning (see §§ 4 & 5). Although the tensor equations are covariant, the calculated results are specific tensor components. Therefore, a valid calculation in physics depends on the physical validity of the space-time coordinate system used (see §§ 4-9) since the physics of coordinates is not unrestrictedly covariant.

A physical space consists of a frame of reference and a space-time metric that is obliged "to define time in such a way that the rate of a clock depends upon where the clock may be [2]." Thus, Einstein's interim proposal of covariance among all conceivable mathematical coordinate systems [2] would be an over-extended demand (see Appendix B), depending on what is being "conceived". (Eddington rejected the gauge based on such a covariance in his book [19], which Einstein himself thought to be the finest presentation of this subject ever written [20].) As pointed out by Einstein [2], the mathematical covariance includes, but is not equivalent to, the general postulate of relativity; therefore, there is actually no theoretical basis in physics for unrestricted general covariance. Moreover, since Einstein's equivalence principle is satisfied in a Lorentz manifold conditionally (§ 4), only some of the Gaussian systems can be used as space-time coordinate systems in physics. Thus, the once prevailing extreme view that coordinates have no physical meaning is incorrect.

One may recall that the Galilean transformation, which implies that there is no speed limit, has been proven to be unrealizable and thus invalid in physics by the Michelson-Morley experiment [21]. However, while special relativity is a special case for Einstein's equivalence principle, it has not been clarified that Pauli's version of the equivalence principle is actually inadequate even for the case of special relativity. Thus, an inadequate understanding of Einstein's equivalence principle has its deep root in an inadequate understanding of special relativity (§§ 3-8).

Since Einstein's equivalence principle is beyond a proper metric signature, intrinsically there should be unphysical Lorentz manifolds that are not diffeomorphic to physical spaces. It will be shown that such unphysical Lorentz manifolds do exist and that violate a physical principle that is independent of coordinates (§ 9). Thus, it is misleading to "define" the speed of light in terms of the local Minkowski spaces since a Lorentz manifold could be intrinsically invalid in physics.

Therefore, to accept a Lorentz manifold as valid in physics, it is necessary to verify the physical requirements, in particular the equivalence principle toward its space-time coordinate system on which the physical interpretation is based. To this end, the *Euclidean-like structure* (§ 6) of a frame of reference is crucial, as it clarifies the physical meaning of space coordinates. (For the purposes of calculation only, any diffeomorphism can be used to obtain new mathematical coordinates.) Moreover, this rectification would expose the deficiency of current theories and some accepted Lorentz manifolds as

unphysical (§§ 6-9). Thus, it becomes necessary to clarify Einstein's notion of physical space further. To help decide what the realistic gauge for terrestrial gravity is, it is necessary to make measurements related to the local space contractions.

2. Einstein's Equivalence Principle, Gravity, and the Gravitational Redshift

In 1911, Einstein [21] derived the gravitational redshift from the initial form of his equivalence principle. The experimental basis is that in a given gravitational field all particles would fall with the same acceleration; and thus the passive gravitational mass and inertial mass are considered as equivalent (the weak equivalence principle) [17]. Einstein's equivalence principle asserts the equivalence of an accelerated frame and uniform gravity, but is often misunderstood as a statement on general equivalence of gravity and acceleration [4, p.17]. Some ambiguities in Einstein's paper may have contributed to this.

Einstein assumed that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system (i.e., reference frame) K' accelerated by γ in the opposite direction, can be extended to other physical processes. He considered two material systems S_1 and S_2 which are situated initially at rest on the z-axis of system K and are separated by a distance h so that the gravitational potential in S_2 is greater than that in S_1 by γh. If a definite radiation energy E_2 be emitted from S_2 to S_1 at the moment the system K' has zero velocity relative to an inertial system K_0, the radiation will arrive at S_1 when the time h/c has elapsed (to a first order approximation); and at this moment the velocity of S_1 relative to K_0 is $\gamma h/c = v$. According to special relativity, the radiation arrives at S_1 with a greater energy E_1 which (to a first order approximation) is related to E_2 by

$$E_1 = E_2(1 + v/c) = E_2(1 + \gamma h/c^2) \qquad (1)$$

By assumption, exactly the same relation holds if the same process takes place in the system K, which is not accelerated, but is provided with a gravitational field. Then, we may replace γh by the gravitational potential Φ and obtain

$$E_1 = E_2(1 + \Phi/c^2) = E_2 + \Phi(E_2/c^2). \qquad (2)$$

Thus, the energy increment of radiation due to gravity is resolved by the equivalence of the K and K' systems. Einstein omitted the step of replacing γh by $\Delta\Phi$. This is crucial *because a small enough $\Delta\Phi$ can be considered as due to uniform gravity*. Thus, formula (2) may appear to presume the equivalence of gravity and acceleration. Unfortunately, this misunderstanding was affirmed by Bergmann's [10] "Einstein's elevator", which replaced Einstein's [22] "closed chest".

If the radiation emitted in a gravitational field Φ, then the equivalence principle, to the first order approximation gives

$$\nu_1 = \nu_2(1 + \Phi/c^2). \qquad (3)$$

If on the surface of a star (where S_2 is located) light is emitted toward the Earth (S_1) where the frequency of the arriving light is measured, then Eq. (3) implies $\nu = \nu_0(1 + \Phi/c^2)$, where Φ is the (negative) difference of the gravitational potential.

To establish general relativity, Einstein's equivalence principle is first extended to the case of a uniformly rotating disk [1,2]. Einstein probably thought it was very clear that gravity and acceleration are only locally equivalent, as uniform rotation is obviously not equivalent to linear acceleration. However, this was not what others perceived it.

For instance, Thorne [23] criticized Einstein's principle as follows:

"In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you (and your reference frame) are very small."

This reflects that Pauli's version [6] is mistaken as Einstein's equivalence principle in book by Misner, Thorne, & wheeler [4, p.386]. In fact, the question on tidal forces has been answered in Einstein's letter to A. Rehtz [24]:

"The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Here, Einstein has made it clear that this principle is proposed for a physical space, where all physical requirements are sufficiently satisfied. For the convenience of the reader, Pauli's version is presented here as follows:

"For every infinitely small world region (i.e., a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system $K_0(X_1, X_2, X_3, X_4)$ in which gravitation has no influence either on the motion of particles or any physical processes." –Pauli [6, p.145].

However, there is no frame of reference, and one is not "obliged to define time in such a way that the rate of a clock depends upon where the clock may be [2]." This version also posits the incorrect notion that gravity and acceleration are equivalent since gravitational influence could be transformed away[3]. Nevertheless, Pauli's "infinitesimal" principle of equivalence, to which Einstein objected as inadequate, is commonly but mistakenly regarded as Einstein's version of the principle [17].

Perhaps due to also the influence of Bergmann [10], Synge [11] asserted his criticism to Einstein's theory as follows:

"...I have never been able to understand this principle...Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute

property; it has nothing to do with any observer's world line... The Principle of Equivalence performed the essential office of midwife at the birth of general relativity... I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced."

This is incorrect because the Riemann curvature tensor vanishes in the case of Einstein's rotating disk [2]. Note that Einstein's notion of gravitation is very different from Newtonian gravity. As Einstein explained to Laue [17],

"However, what characterizes the existence of a gravitational field from the empirical standpoint is the non-vanishing of the Γ^{l}_{ik} [coefficients of the affine connection], not the non-vanishing of the R_{iklm}",

and no gravity is a special form of gravity. (A problem is that many theorists did not learn general relativity directly from works of Einstein.) Since the cause of gravity need not be a source, based on the principle of general relativity, the geodesic equation is identified as the equation of motion for a massive particle [1,2].

Nevertheless, theorists such as Fock [25] believed Einstein's equivalence principle inaccurate because of an earlier invalid light bending formula. However, it has been shown that this limitation was not a defect of Einstein's equivalence principle, but rather arises because his 1911 paper failed to account for the effects of a curved space. Moreover, Fock's belief must be incorrect since that the Maxwell-Newton Approximation has been derived from Einstein's equivalence principle [16].

3. Mathematical Foundations of Einstein's Equivalence Principle Versus Pauli's Version

Currently, Einstein's equivalence principle is often incorrectly considered as equivalent to the mathematical existence of local Minkowski spaces (see §§ 3-7). Such a misunderstanding is related to two mathematical theorems [11]:

Theorem 1. Given any point P in any Lorentz manifold (whose metric signature is the same as a Minkowski space) there always exist coordinate systems (x^{μ}) in which $\partial g_{\mu\nu}/\partial x^{\alpha} = 0$ at P.

Theorem 2. Given any time-like geodesic curve Γ there always exists a coordinate system (so-called Fermi coordinates) (x^{μ}) in which $\partial g_{\mu\nu}/\partial x^{\alpha} = 0$ along Γ.

After some algebra, a local Minkowski metric exists at any given point and that along any time-like geodesic curve Γ, a moving local constant metric exists [11]. Accordingly, the Pauli version is essentially a simplified version of these theorems.

As Einstein [17] pointed out, gravity may not be transformed away from the whole region no matter how small, even though special relativity holds approximately. However, there is no physical specification for the local transformation, which transforms a Lorentz metric to a local Minkowski metric. Also, there is nothing relating these two theorems to the

existence of the acceleration of a static particle, or to other physical situations. Thus, Pauli's version is not a physical principle.

Nevertheless, from the geodesic equation, the weak equivalence principle of Galileo implies only that some metric components related to static acceleration are local constants. The mathematical theorems mean that Einstein's equivalence principle requires all the components to be local constants. Thus, these two theorems are the mathematical foundation for the infinitesimal form of Einstein's equivalence principle. Einstein [2] proposed, as a crucial part of his equivalence principle, that the "special theory of relativity applies to the special case of the absence of a gravitational field." This principle implies that the Minkowski metric is the only constant metric in physics, and is supported by the Michelson-Morley experiment. Moreover, since the mathematical theorems show that the local space is locally constant, then Einstein's only additional physical assumption in his equivalence principle is the requirement that such a local constant metric must be Minkowski [1,2]. Thus, the crucial physics of Einstein's principle is that the resulting local Minkowski space is automatic.

Einstein proposed his equivalence principle only to a physical space, where all physical requirements are sufficiently satisfied. A crucial point in Einstein's infinitesimal version is that the physics of gravity requires that a free-falling observer should result in a local Minkowski space [1]. However, Pauli asserted only an existence of a local Minkowski space as stated in the mathematical theorems, while he ignored all other related physical requirements.

It is based on his equivalence principle that Einstein deduced time dilation and space contractions. Logically, however, these have meaning in physics *only if the space-time coordinates have physical meaning*. Unfortunately, Einstein did not give a clear definition of the coordinates in terms of measurements, and thus the related notions such as the frame of reference and acceleration are not clearly defined. Consequently Einstein's equivalence principle, which is defined in terms of acceleration relative to a frame of reference, is also not clear. Understandably, theorists took refuge by accepting Pauli's version, which requires no physical meaning for the coordinates, instead of Einstein's.

Currently, the question of whether Pauli's version is equivalent to Einstein's has not been settled. On one hand, theorists such as Wald [5], Straumann [8] and Liu [13] believed incorrectly that they are equivalent; on the other hand, theorists such as Fock [25], Synge [11] and Wheeler [7] believed they are not and rejected Einstein's principle. The editors of physical journals are divided as well. For instance, John L. Friedman [14,26], Divisional Associate Editor of Physical Review Letters, claimed that Pauli's version has replaced Einstein's. The board of Classical and Quantum Gravity [27] believed also that there are physical differences in the two versions. However, Erick J. Weinberg [28] of Physical Review D believed that the difference between these two versions is only in philosophy but not in physics. Theorists seemed fail to see that, in Ein-

stein's version, the cause and consequences of the local coordinate transformation must be investigated in terms of physics [1,2].

However, if the physical meaning of space-time coordinates is as ambiguous as Einstein [1] believed, and Whitehead [18] objected, what should be blamed for regarding Einstein's principles that require a clear physical meaning of coordinates as heuristic arguments? Understandably, Eddington failed to find a third person, who understands general relativity [29].

4. Free Falling, Special Relativity, and Einstein's Equivalence Principle

Based on observing that the (passive) gravitational mass[4)] and inertial mass are equivalent in high precision, Einstein [21] proposed the equivalence of acceleration with uniform gravity. This proposal can be considered essentially as the local equivalence of acceleration and gravity because uniform gravity can be regarded as an idealization of the local gravity. Then, the principle of general relativity implies that gravity is represented by the space-time metric and the geodesic is the equation of motion of a particle under the influence only of gravity. On the other hand, the mathematical theorems imply that, for an observer in free fall, the local metric is locally a constant. The contribution of Einstein is that such a locally constant metric *is required* to be locally a Minkowski space. Thus, to understand the uniqueness of such a local Minkowski space, one must first understand special relativity and the Michelson-Morley experiment that supports the Minkowski metric [21].

Since an existence of local Minkowski space is assured in mathematics, a central issue in satisfying Einstein's equivalence principle is whether the geodesic represents free fall in a physical space. Physically, the transformation of the metric to a local Minkowski space is required to be unique and automatic although mathematically there are other types of locally constant metric. Also, Eddington observed [19] that special relativity should apply only to phenomena unrelated to the second order derivatives of the metric. Einstein [30] accepted this criticism and added the crucial phrase, "at least to a first approximation" on the indistinguishability between gravity and acceleration.

Einstein clarified [1], "According to the principle of equivalence, the metrical relations of the Euclidean geometry are valid relative to a Cartesian system of reference of infinitely small dimensions, and in a suitable state of motion (free falling, and without rotation)." For example, when a spaceship is under the influence of gravity only, the local space is automatically Minkowski, determined by the physics of gravity. In other words, the transformation to a local Minkowski space is due to a physical cause, i.e., gravity. Thus, the satisfaction of Einstein's equivalence principle can be decided by results so derived.

Now, a "free falling" observer P at a point (x, y, z, t) must be in a co-moving local Minkowski space L with the metric

$$ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2, \tag{4}$$

whose spatial coordinates are attached to P and whose motion is governed by the geodesic,

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \qquad \text{where} \quad ds^2 = g_{\mu\nu}dx^\mu dx^\nu \tag{5a}$$

and

$$\Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})g^{\mu\nu}/2, \tag{5b}$$

are defined by the space-time metric $g_{\mu\nu}$. (The "attachment" means that, between P and L, there is no relative motion or acceleration.) The local coordinates (X, Y, Z, T) and (x, y, z, t) are related by a local coordinate transformation

$$dX^\alpha = \frac{\partial X^\alpha}{\partial y^\mu}dy^\mu \tag{6}$$

The cause and consequence of (6) is the physics of Einstein's equivalence principle. Note that, because of acceleration, the frame of reference (X, Y, Z) attached to L, is in a state different from that of the frame of reference (x, y, z).

A simple reason that the space-time coordinates cannot be arbitrary is the correspondence principle since weak gravity must be approximately Newtonian. Newton's theory requires $\Gamma^\mu{}_{tt} \neq 0$ ($\mu \neq t$), and thus metric $g_{\mu\nu}$ is not a constant. On the other hand, for a given non-constant metric $g_{\mu\nu}$, a spatial acceleration may not necessarily exist for a static observer. For instance, when $\Gamma^\mu{}_{tt} = 0$ ($\mu \neq t$), there is no Newtonian acceleration, but the metric may still be non-constant. For example, if the $g_{tt} = f(t)$ and $g_{t\mu}$ ($\mu \neq t$) is not a function of t, such a metric would have no static acceleration.

Since there is neither acceleration nor motion for an initially static particle, it remains forever in the same position with the same frame of reference. This would mean that the same unit clock and/or the same unit measuring-rod would have two different readings. This non-uniqueness is unacceptable in physics. Mathematically, however, transforming the metric near a point to a local Minkowski space is possible, in spite of no acceleration or any physical cause to make such a transformation necessary. Thus, *Einstein's equivalence principle removes the possibility of gravity without the static acceleration.* However Pauli's version would fail this since the related physics is not considered.

To illustrate the above analysis, let us consider a Lorentz manifold with the non-constant metric,

$$ds^2 = c^2ch^2(T/C)dT^2 - dx^2 - dy^2 - dz^2, \qquad \text{where} \quad ch(T/C) = [\exp(T/C) + \exp(-T/C)]/2, \tag{7}$$

and C is a constant. From metric (7), the Christoffel symbols are zeros except $\Gamma_{t,tt} = \partial_t g_{tt}/2$, and thus there is no acceleration. Thus, there is no change in the spatial position and no acceleration, and a static P would have the same frame of reference.

If Einstein's equivalence principle were valid, the local Minkowski space of P at (x_0, y_0, z_0, T_0) is

$$ds^2 = c^2dT'^2 - dx'^2 - dy'^2 - dz'^2. \tag{8a}$$

Since the geodesic of P is (x_0, y_0, z_0, T), the local coordinate transformation would therefore be

$$dx' = dx,\ dy' = dy,\ dz' = dz,\ \text{and}\ dT' = ch(T/C)dT. \tag{8b}$$

Since the frame of reference has not been changed, (8b) would lead to the double reading difficulty of a standard clock. Moreover, it follows from (8b) that the light speed would be *c ch(T/C)* to the frame (x, y, z), whereas according to (8a) the light speed is c. Thus, (7) is not a metric of a physical space, although metric (7) is obtained from the flat metric, $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$, with the diffeomorphism $t = C\ sh(T/C)$, where $sh(T/C) = [\exp(T/C) - \exp(-T/C)]/2$.

This example illustrates that although the geodesic equation (5) is generally covariant, the physics of coordinates is not. In other words, the geodesic equation has no physical meaning unless the physical meaning of the coordinates has been clarified. One might reject Einstein's clarification by claiming that coordinates have no physical meaning. However, as shown in Section 6, such a viewpoint is actually theoretically invalid and logically self-defeating.

5. Einstein's Demonstration of Einstein's Equivalence Principle

To understand Einstein's equivalence principle, Einstein's own application would serve as a good example. To enhance the understanding of the physics, remarks are added to Einstein's calculations.

Einstein [1] illustrated the application of his equivalence principle in his calculations of the gravitational redshifts and the bending of light near the sun. First, he considered a coordinate system S with the sun attached to the spatial origin. Then, based on his 1915 equation, Einstein derived, with his notion of weak gravity, the linear equation,

$$\frac{\partial^2 \gamma_{\mu\nu}}{\partial x_\alpha^2} = 2K(T_{\mu\nu} - g_{\mu\nu}T), \qquad \text{where} \quad \gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}\,,\ T = T_{\alpha\beta}g^{\alpha\beta}. \tag{9a}$$

$\eta_{\mu\nu}$ is the flat metric, $T(m)_{\mu\nu}$ is the energy tensor for massive matter, and K is the coupling constant. (Equation (9a) can be justified, independent of the 1915 equation, directly with Einstein's equivalence principle and other requirements [16].)

Then, from Eq. (9a), Einstein obtained, to a sufficiently close approximation, the metric for system S

$$ds^2 = (1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0})\, dt^2 - (1 + \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0})(dx^2 + dy^2 + dz^2) \tag{10}$$

where

$$r_0 = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}; \tag{11}$$

and $\sigma(x_0, y_0, z_0)$, is the mass density. In the derivation of (10), the asymptotically flat of the metric has been used.

To show that the correspondence principle is satisfied, Einstein derived the Poisson equation with Eq. (9) and Newton's equation of motion with the geodesic equation. Moreover, he supported his geodesic equation by calculating the perihelion of Mercury by a method of successive approximation. Now, since (10) is not a constant metric, according to Einstein's equivalence principle, gravitational acceleration should exist for a static observer. From the geodesic Eq. (5), since $\partial g_{tt}/\partial x^\mu \neq 0$ for x^μ (= x, y, z), the gravitational force is non-zero, and the equivalence principle would be applicable.

Consider an observer P resting at (x, y, z, t) but in a free falling state,

$$dx/ds = dy/ds = dz/ds = 0. \tag{12}$$

Assuming *the equivalence principle* is valid, the local coordinate system attached to the observer P would have a Minkowski metric (4). Thus, metric (11) and state (12) imply the time dt and dT are related by

$$c^2(1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0})dt^2 = ds^2 = c^2\, dT^2 \tag{13}$$

Also, since the space coordinates are orthogonal to dt, at (x, y, z, t), for the same ds^2, Eq. (13) implies [1,3]

$$(dx^2 + dy^2 + dz^2)(1 + \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0}) = (dX^2 + dY^2 + dZ^2). \tag{14}$$

The meaning of eqs. (13) and (14), according to Einstein [1], is that the unit measuring rod has the coordinate length

$$(1 - \frac{K}{8\pi}\int dV_0 \frac{\sigma}{r_0}) \tag{14'}$$

and

$$(1 + \frac{K}{8\pi}\int dV_0 \frac{\sigma}{r_0}) \tag{13'}$$

is the time in the unit used in our system of co-ordinates, and corresponds to the interval between two beats of the unit clock (dT = 1). The rate of a clock is accordingly slower, the greater the mass of the ponderable matters in its neighbourhood.

The gravitational red shifts are due to the acceleration $d^2x/ds^2 \neq 0$. From Eq. (13), one obtains

$$\nu(x, y, z) = \nu_0[1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0}]^{-1/2} \approx \nu_0[1 - \frac{K}{8\pi}\int dV_0 \frac{\sigma}{r_0}] = \nu_0[1 + \Phi c^{-2}]., \tag{15a}$$

where

$$\Phi(x, y, z) = -\frac{K}{8\pi}\int dV_0 \frac{\sigma}{r_0} = \frac{K}{8\pi}\frac{M}{r} \quad \text{where} \quad r = [x^2 + y^2 + z^2]^{1/2}, \quad \text{and} \quad M = \int dV_0 \sigma \tag{15b}$$

is the total mass, $\nu(x, y, z)$ and ν_0 are respectively the frequencies at the sun surface and in the flat space. Since Φ is the Newtonian potential, the gravitational redshift formula is compatible with the one that Einstein derived in 1911.

From eqs. (13) and (14) Einstein is confident in deriving the light speed at (x, y, z),

$$\frac{[dx^2 + dy^2 + dz^2]^{1/2}}{dt} = c(1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0}). \tag{16a}$$

to the first order approximation. It is crucial to note that the light speed (16), for an observer P_1 attached to the system S at (x, y, z), is smaller than c; and this condition satisfies the coordinate relativistic causality (see § 6). Observer P_1 shares the same frame of reference with the sun, and the velocity of light is restricted to be smaller than c. The observer P is in a free falling frame of reference and thus would not experience the same gravitational force as P_1.

From Eq. (16a), a light ray passing at a distance Δ from the origin, will be deflected, in all, by an amount

$$\alpha = \frac{KM}{2\Delta\pi} \tag{16b}$$

towards the sun. For Δ equal to the radius of the sun (in terms of $r = [x^2 + y^2 + z^2]^{1/2}$), α amounts to 1.7".

Einstein wrote [1], "We can therefore draw the conclusion from this, that a ray of light passing near a large mass is deflected." However, the question of energy lost was not addressed due to the limitation of his theory [33-35].

Note that, even without using eqs. (13) and (14), one could have obtained Eq. (16) from the light-cone condition,

$$ds^2 = 0. \tag{17}$$

A reason for deriving eqs. (13) and (14) first is to see whether the validity of the equivalence principle is supported, as otherwise inconsistency in physics could happen. Note that, in spite of the support from his earlier work, Einstein only *assumed*, but did not claim the validity of his principle. Thus, not only Eq. (13), which leads to gravitational redshifts, but also Eq. (16) is a test of the equivalence principle, which can be used to derive Eq. (9) for a massive source [16].

The perihelion precession of Mercury, which requires the second order approximation, will not be reproduced here. The displacement of the perihelion of planetary orbits in the direction of the planet's orbital motion is of magnitude [1]

$$\frac{24\pi^3 a^2}{c^2 T^2 (1-e^2)} \tag{18}$$

where *"a"* is the semi-major axis of the planetary orbit in centimetres, *"T"* is the period of revolution in seconds, and *"e"* is the numerical eccentricity. The semi-major axis is evaluated with "*Euclidean geometry*" (see Eq. [11]). Note that in the above derivations, in effect, Einstein had stated that his measurements are actually based on *"Euclidean-geometry"*.

Note that gravitational redshifts Eq. (13) and Eq. (15a) imply that a diffeomorphic transformation of space-time coordinate systems is restricted by the equivalence principle. To show this, let us present Eq. (15a) alternatively as

$$\nu_1\, [g_{tt}(r_1, t_1)]^{-1/2} = \nu_2\, [g_{tt}(r_2, t_2)]^{-1/2}, \tag{19a}$$

where ν_i is the frequency at point (r_i, t_i). It is known that the expression, on the right-hand side of

$$\nu_1/\nu_2 = [g_{tt}(r_1, t_1)]^{1/2}/[g_{tt}(r_2, t_2)]^{1/2}, \tag{19'a}$$

can be represented as if a coordinate-free scalar [5,7], if $g_{\mu\nu}$ has a Killing vector field K_μ ($\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$) for time independency [5,8]. However, a space-time coordinate system is required such that the frequencies can be represented by scalars.

Consider the diffeomorphism (7), $t = C\ \mathrm{sh}(T/C)$. Then, for the new coordinate system (x, y, z, T), one obtains

$$g_{TT}(r, T) = g_{tt}(r, t)\ \mathrm{ch}^2(T/C) = g_{tt}(r)\ \mathrm{ch}^2(T/C); \tag{19b}$$

and

$$\nu_1[g_{TT}(r_1, T_1)]^{-1/2} = \nu_1[g_{tt}(r_1)\mathrm{ch}^2(T_1/C)]^{-1/2} = \nu_2\, [g_{tt}(r_2)\mathrm{ch}^2(T_2/C)]^{-1/2} \tag{19c}$$

would follow. Obviously, Eq. (19c) does not agree with formula (19a) or (15), which is compatible with the equivalence principle. It should be noted that coordinate systems (x, y, z, t) and (x, y, z, T) share the same frame of reference. Thus, this example illustrates that once the spatial coordinates are chosen, the time coordinate is also determined (see also § 6).

Einstein had remarked that his frame of reference has a structure " in the sense of Euclidean geometry", but this useful remark was overlooked since Einstein has just proved that space-time is non-Euclidean. Moreover, since the coordinates of a frame of reference are not defined in terms of measurements, acceleration and space contractions would be arbitrary. This explains the rejection of Einstein's equivalence principle and the popularity of Pauli's version even after Einstein objection.

6. Physical Meanings of Space-time Coordinates.

From the above, it is clear that the coordinates in physics have meanings. For instance, redshifts (15) show that a space-time metric is restricted; and the light speed (16) shows that, in principle, a meaning of the coordinates can be verified by

measurements. Here, we will first discuss problems in understanding the physical meaning of coordinates, and then the related problems in coordinate relativistic causality and the question of coordinate-free derivation.

6.1. Space Coordinates and the Euclidean-like structure

According to Eq. (13'), the time unit is related to a clock rate; and therefore should be the same if two coordinate systems are referred to the same frame of reference. Such uniqueness is the theoretical basis that gravitational redshifts are observable. Now, consider a solution of isotropic form and the Schwarzschild solution respectively,

$$ds^2 = [(1 - MG/2r)^2/(1 + MG/2r)^2]dt^2 - (1 + MG/2r)^4 (dx^2 + dy^2 + dz^2), \qquad r > MG/2 \qquad (20a)$$

and

$$ds^2 = (1 - 2MG/\rho)dt^2 - (1 - 2MG/\rho)^{-1}d\rho^2 - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \qquad (20b)$$

where

$$x' = \rho \sin\theta \cos\varphi, \qquad y' = \rho \sin\theta \sin\varphi, \qquad \text{and} \qquad z' = \rho \cos\theta, \qquad (20c)$$

and

$$\rho = r(1 + MG/2r)^2, \qquad \text{for} \qquad \rho > 2MG. \qquad (20d)$$

The isotropic metric is a function of r (= $[x^2 + y^2 + z^2]^{1/2}$), and the Schwarzschild metric is a function of ρ (= $[x'^2 + y'^2 + z'^2]^{1/2}$). Radicals r and ρ are related by (20d). These two systems are related to the Euclidean subspace when M = 0.

However, these similar Euclidean backgrounds are indistinguishable within the limits of the accuracy of the light bending experiments. The clock rates and the spatial contractions of (20a) and (20b) are related by (20d). A problem of metric (20a) is that the measurement of coordinates (x, y, z) is defined against the metric, which depends on r. Metric (20b) has a similar problem. However, since the meanings of space coordinates were not clear, these differences were considered as due to different coordinate systems. Instead, this problem is regarded as an evidence of gauge freedom [1,2] and thus resists investigation. To clarify this, one must understand what measurement means in Einstein's general relativity [1].

In general relativity, Einstein's measuring instruments are at rest but in *a free fall state* [1]. Based on such measurements, Einstein believed, "In the general theory of relativity, space and time cannot be defined in such a way that differences of the spatial coordinates can be directly measured by the unit measuring-rod, or differences in the time coordinates by a standard clock". An implicit assumption is that the measuring instruments are unchangeable as in Riemannian geometry.

However, if the measuring rod is attached to the frame of reference, since the measuring rod and the coordinate being measured are under the same influence of gravity, the measured local distances would emerge as if gravity did not exist. As

shown in Einstein's solutions, the operationally defined local distances comprise a Euclidean-like structure that is a necessary supplement to the space-time metric. This makes clear the physical meaning of spatial coordinates is independent of gravity and thus also of the matter distribution. *Thus, the space coordinates have essentially the same physical meaning as in special relativity* [5]. Then, the time coordinate is determined by orthogonality such that the time rate is related to a local clock. Note that the Euclidean-like structure is a notion in physics and exists only in a physical space.

Thus, under some circumstances, a frame of reference can be validly considered as if Euclidean. This dual characteristic of space-time is the underline reason for Einstein's 1911 successful derivation of the redshift and subsequent confusion.

If metric (20a) represents a physical space, the coordinate system (x, y, z) would have a meaning in terms of physical measurements. Moreover, if metric (20b) is a physical space, the above arguments would be applicable to the system (x', y', z'). If these two metrics have the same frame of reference of a physical space, it follows that, in terms of physics, the two coordinate systems (x, y, z) and (x', y', z') must be the same. Thus, only one of the metrics can be valid for reality.

For instance, form (20b) requires a more complicated correspondence; whereas form (20a) has the advantage that the correspondence depends only upon the place, and not upon the direction. Thus, metric (20a) and metric (20b) are mathematically diffeomorphic but physically different because they have different types of spatial contractions. In principle, for the same frame of reference, only one of the metrics can be physically valid. In fact, Logunov and Mestvirishvili [31] have shown that diffeomorphic coordinate systems could lead to inconsistent formulas for physics.

For Einstein's three well-known tests, the differences of these two metrics would be of the second order, and thus are not distinguishable experimentally. Therefore, it is necessary to design an experiment to help determine the actual space-time metric. An experiment that can distinguish between these two metrics in the first order is measuring the local light speeds. For the Schwarzschild metric, the vertical light speed would be different from the horizontal light speed. This difference can, in principle, be detected with an interferometer of Michelson-Morley type that has a vertical arm and a horizontal arm, and the positions of these arms can be exchanged by turning the whole apparatus around a horizontal axis.

Currently, the simpler Schwarzschild form (20b) is favored for exact calculations. The isotropic form (20a) is used for post-Newtonian approximation [4], which is used to derive the post-Newtonian equation of motion for many-body problems. This equation of motion has been found to be very accurate in tracking the planets and spacecrafts [4]. However, it would be premature to conclude that the local light speed experiment would reject the Schwarzschild solution.

6.2. Coordinate Relativistic Causality

A major conclusion of special relativity is that matter cannot move faster than the speed of light c in a vacuum. Since the space-time coordinates actually have a very clear physical meaning in general relativity, the coordinate light speed cannot be arbitrary. As shown in the case of Einstein's uniform rotating disk, Einstein's equivalence principle implies that time dilation and space contractions would result as a consequence of gravity. Thus, a speed of light is slower under gravity.

The validity of coordinate relativistic causality is deeply rooted in Einstein's equivalence principle right from the start [16]. Einstein [21] showed that, based on special relativity, time dilation is a consequence of the equivalence principle, and subsequently in 1916 that space contraction is also a consequence of this principle. Because of the behavior of rods and clocks under gravity, coordinate relativistic causality is strengthened, as is shown by metrics (20a) and (20b) [1,2]. As a consequence, the bending of a light ray is observed. Thus, the velocity of light cannot be larger than the light speed c. Moreover, the local Minkowski spaces imply that a light velocity must remain to be the local maximum for any speed of matter.

Obviously, coordinate relativistic causality is a simple and useful criterion to reject an unphysical coordinate system as shown in references [16] & [26]. Nevertheless, many ignored this useful tool, because coordinates were incorrectly considered as meaningless, and mathematically diffeomorphic manifolds were incorrectly considered as equivalent in physics [5].

Moreover, confusion arises because physical units are not generally explicit in a theory of gravitation. If a diffeomorphism is only a mathematical transformation such as the Galilean transformation, which is not related to a change of the frame of reference or rescaling, the new coordinate is only a mathematical coordinate system, for which Einstein's equivalence principle may not be satisfied (see § 7). If a diffeomorphism is a coordinate transformation due to a change of the frame of reference, then the resulting manifold should also be a physical space. In fact, even the local Minkowski space is identical. However, if a diffeomorphism is due to a rescaling, the local Minkowski space should be changed accordingly.

Thus, the cause of a transformation being a rescaling or a change of frame of reference, though unimportant in mathematics, is very important in physics. To clarify this, let us consider a simple metric such as the flat metric,

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2. \tag{21a}$$

with the time unit in seconds and the length unit in centimeters, and with $c = 3\text{x}10^{10}$ cm/sec.

Now, consider a rescaling, $t = \alpha t'$, where α (>2) is a constant. Then, metric (21a) becomes

$$ds^2 = (\alpha c)^2\, dt'^2 - dx^2 - dy^2 - dz^2. \tag{21b}$$

According to (21b), $ds^2 = 0$ implies that the numerical value of the light speed is

$$\frac{[dx^2 + dy^2 + dz^2]^{1/2}}{dt'} = c\alpha. \tag{22a}$$

However, because of the rescaling $t = \alpha t'$, the unit of t' is no longer in seconds as t is. Instead, the unit of t' is α-seconds. Thus, the meaning of (22a) is that the light speed is in cm/α-sec, and therefore the light speed is actually

$$c\alpha \text{ (cm/}\alpha\text{-sec)} = c \text{ (cm/sec)}. \tag{22b}$$

On the other hand, unless clearly stated, metric (21b) would be taken as a space-time metric with the time unit in seconds. Then, one would have to conclude that metric (21b) was invalid because there is no gravity to change the rate of a clock.

Thus, it is necessary to identify the form of the local Minkowski metric to determine whether the light speed is c. Also, the confusion is exacerbated, firstly by the fact that the units are usually not clearly identified, and secondly, because they become unclear after a number of transformations. Since a local Minkowski space and the Euclidean-like structure are related to a situation where any physical cause of gravity has been removed, one can consider the following:

1) Consider the case when all the causes of gravity are removed. For metric (20), it is $M = 0$.
2) Take the limit to a vacuum state that is remote from a source of gravity. For metric (20), it is the limit $r \to \infty$.

To use any one of them, or others, depends on the situation (see also § 8). It will be shown that, for some cases, only one method is applicable. For some manifolds, a transformation to a local Minkowski metric is not valid in physics.

6.3. An Example of Examining Coordinate Relativistic Causality

To illustrate a common misunderstanding on coordinate relativistic causality, let us consider the static metric (10),

$$ds^2 = c^2(1 + 2\Phi/c^2)dt^2 - (1 - 2\Phi/c^2)(dx^2 + dy^2 + dz^2), \text{ where } \Phi = -\frac{K}{8\pi}\frac{M}{r}, \tag{23a}$$

Now, take an elevator whose center is located at $(x, y, z) = (0, 0, R)$ (i.e., at a constant height from the Sun). Using local rectangular coordinate attached to the center of the elevator: $(x', y', z') = (x, y, z - R)$, the metric has the form,

$$ds^2 = c^2(1 + 2\Phi/c^2)dt^2 - (1 - 2\Phi/c^2)(dx'^2 + dy'^2 + dz'^2) \tag{23b}$$

where

$$\Phi = -\frac{KM}{8\pi R}\sum_{l=0}^{\infty}\left(\frac{z'}{R}\right)^l P_l(\cos\theta) = -\frac{KM}{8\pi R}\left(1 + \frac{z'}{R}\cos\theta + \cdots\right).$$

Then, let us rescale the local coordinates as follows:

$$T = (1 - KM/8\pi c^2R)t, \qquad \text{and} \qquad (X, Y, Z) = (1 + KM/8\pi c^2R)(x', y', z'). \tag{24a}$$

Thus,

$$ds^2 \approx c^2(1 + 2c^{-2}g\,Z\cos\theta)dT^2 - (1 - 2c^{-2}g\,Z\cos\theta)(dX^2 + dY^2 + dZ^2), \tag{24b}$$

where $g = KM/8\pi R^2$, would be the metric within the elevator approximately. Note that transformation (24a) is a rescaling, but this would not be noticed when one takes $M = 0$.

Now, the local metric (24b) implies the coordinate speed of light at points in the elevator, to sufficient accuracy,

$$\frac{[dX^2 + dY^2 + dZ^2]^{1/2}}{dT} = c(1 - 2c^{-2}g\,Z\cos\theta). \tag{25}$$

Since the right-hand side can be larger than c, coordinate relativistic causality seems to be violated. A deficiency of the above consideration is that the local Minkowski metric has been implicitly assumed to be in the form of (4).

If the equivalence principle were satisfied by (24b) and the free falling local Minkowski space is,

$$ds^2 = c^2dT'^2 - (dX'^2 + dY'^2 + dZ'^2), \tag{26a}$$

then

$$dT'^2 = (1 + 2c^{-2}g\,Z\cos\theta)dT^2$$

and

$$(dX'^2 + dY'^2 + dZ'^2) = (1 - 2c^{-2}g\,Z\cos\theta)(dX^2 + dY^2 + dZ^2). \tag{26b}$$

Form (26a) is obtained from (24b) by considering $M = 0$. These relations reveal that the equivalence principle would determine the meaning of metric (24b). Now, it is also clear that (25) is a consequence of (26b).

However, since metrics (23) and (24b) share the same frame of reference, we have already known [1] that the interval between two beats of the unit clock ($dT' = 1$) corresponds to the time $(1 - \Phi/c^2)$, which is not equal to $(1 - c^{-2}g\,Z\cos\theta)$. The unit measuring-rod has the coordinate length $(1 + \Phi/c^2)$, which is not equal to $(1 + c^{-2}g\,Z\cos\theta)$. The rate of a clock is unique at any place, although it depends upon where the clock may be. Thus, relation (26b) is not valid in physics. Indeed, Einstein's equivalence principle is not satisfied by metric (24b). It follows that (25) is not the light speed.

On the other hand, we would get a very different physical picture if we refer to metric (23) as the space-time metric instead of the rescaled metric (24b). Because of the rescaling (24a), the light speed related to (25) actually is

$$c(1 - 2c^{-2}g\,Z\cos\theta)[(1 + KM/8\pi c^2R)^{-1}\ \text{cm}]/[(1 - KM/8\pi c^2R)^{-1}\text{sec}]. \tag{27a}$$

$$\approx c(1 - 2KM/8\pi c^2R - 2c^{-2}g\,Z\cos\theta)\ \text{cm/sec} \approx c(1 - 2KM/8\pi c^2r)\ \text{cm/sec}.$$

In other words, coordinate relativistic causality is actually not violated. If one takes the limit $r \to \infty$, metric (24) becomes

$$ds^2 = c^2(1 - KM/8\pi c^2R)^{-2}dT^2 - (1 + KM/8\pi c^2R)^{-2}(dX^2 + dY^2 + dZ^2) \quad (27b)$$

Thus, we could overlook that the rescaling since there is no rescaling when M = 0. The numerical light speed is

$$c(1 + KM/8\pi c^2R)/(1 - KM/8\pi c^2R) > c \quad (27c)$$

Thus, when coordinate relativistic causality is examined, one must identify the light speed in a local Minkowski space.

6.4. The Question of Coordinate–free Derivation

If coordinates had no objective meaning, a tensor component would have no physical meaning unless it can be derived in coordinate free notations, i.e., scalars. On the other hand, a tensor component and a scalar can be numerically equal only for some special coordinate systems. If such an equality is meaningful in physics, a related coordinate system must have physical meaning. Thus, the use of a physically meaningful coordinate system is necessary because non-scalars exist in physics.

Thus, the once prevailing viewpoint that coordinates have no meaning in physics is actually logically self-defeating, and therefore cannot possibly be valid. To illustrate the above point, consider a vector $V^\mu = (V^x, V^y, V^z, V^t)$. If u_i is a unit vector in the i-direction in a flat metric (i.e., since $(u_i)^j = 1$ for i = j and is zero otherwise) one has

$$V_i = (V_\alpha u_i^\alpha) = (V, u_i) \quad (28)$$

Thus, the component V_i of a vector in a specific coordinate system can appear as a scalar (V, u_i), but the reference to the coordinate system is just been changed into another form, the unit vector u_i. In a curved space, relation (28) is complicated by the fact that a unit vector must be adjusted by a factor, but the method is the same.

Consider another example, the velocity of light in the x-direction of a flat metric (or a local Minkowski space) is

$$dx/dt = c. \quad (29)$$

The right-hand side is a scalar, but the left-hand side is a ratio of two vector components. These two sides are equal because a local Minkowski space is used; that is, Eq. (29) means that a physical coordinate system has been implicitly employed.

Now, consider the frequency of light. A signal frequency is not a scalar, and due to the Doppler effect, the frequencies are different in coordinate systems with relative velocity. In a flat metric, the light frequency ν is the time component of the propagation vector, and can be presented as a scalar [5]. However, Eq. (13) and Einstein's slower clock interpretation are

crucial to allowing such a scalar to be considered as the frequency under gravity. Thus, the so-called coordinate-free derivation implicitly requires a valid space-time coordinate system, which satisfies Einstein's equivalence principle.

Currently, the light deflection is derived directly without calculating the light speeds [3-5]. However, such a new derivation also requires the frame of reference attached to the sun so that the deflection angle can be defined.

7. Physics Related to Einstein's Equivalence Principle

While Pauli's version ignores physics beyond the metric signature, Einstein's equivalence principle requires investigating also the validity of related physics. In Einstein's calculation, there are three physical aspects, which facilitate a determination on a satisfaction of his principle. They are as follows:

1) The motion of a "free falling" observer is a geodesic, which sufficiently satisfies physical requirements.
2) The local space-time of a "free falling" observer is automatically Minkowski.
3) A physical local coordinate transformation (6) transforms the metric to local Minkowski space.

Point 2) is Einstein's equivalence principle, but a direct proof of its satisfaction is a rare event. It is difficult to show, beyond identifying the physical cause, whether such a transformation (6) is unique and physically valid. To this end, point 1) and point 3) are useful since both go beyond an existence of local Minkowski spaces.

In practice, however, it is easier to use these criteria for rejection than acceptance. For instance, a time-like geodesic of a non-constant metric may not have a spatial acceleration, and thus Einstein's equivalence principle fails. Also, if the metric violates some physical principles, then a time-like geodesic is invalid in physics. However, if the time-like geodesics are valid in physics, this Lorentz manifold is likely to be a physical space, and, according to Einstein, his equivalence principle would be satisfied. Thus, point 1) would be equivalent to a satisfaction of Einstein's equivalence principle.

In Einstein's calculation, the physical validity of his equation and solution are compared against Newton's results. Therefore, gravitational acceleration for a static observer clearly exists such that his principle would apply since the principle of causality[6)] seems satisfied. The perihelion of Mercury supports the geodesic equation. In practice, however, it is difficult to decide whether point 1) is valid since the accuracy of both the calculations and observations are limited.

If Einstein's equivalence principle is satisfied, point 3) is a consequence and local transformation (6) is due to the physics of free fall alone. From the formula of redshifts and the bending of light rays, gravity as the cause is evident. If Einstein's equivalence principle is not satisfied, such a local transformation (6) can lead to unphysical results. On the other hand, if point 3) is valid, there is little reason to doubt the satisfaction of Einstein's equivalence principle. In Einstein's calculation,

compatibility with the equivalence principle is examined with the formula for red shifts derived in 1911. Nevertheless, it would be difficult to decide surely that point 3) is valid, and may need the help of an experiment on local light speeds.

In short, validity of point 3) is an indication of satisfying Einstein's principle. If point 1) is valid, point 3) should produce physical results. Thus, one can check point 3) to determine the validity of point 1) or vice versa. It is safe to check both methods since different physical requirements may be different in sensitivity to different methods of examinations.

It is often difficult to see how a free fall in a gravitational field makes the metric transformed automatically to a local Minkowski space. Such a difficulty is precisely why his principle is needed.

To illustrate further that Einstein's equivalence principle can fail in a Lorentz manifold, consider the metric,

$$ds^2 = c^2dt^2 - ch^2\,(X/C)dX^2 - dy^2 - dz^2 \qquad (30)$$

where C is a constant. If (t, X, y, z) were of a physical space, the light speed in the X-direction, would have the limit zero as $X \to \infty$ without an identifiable physical cause. Consider an observer P at (X_0, y_0, z_0, t_0). From the metric (30), one has

$$\frac{d^2X}{ds^2} + \Gamma^X_{XX}\frac{dX}{ds}\frac{dX}{ds} = 0, \text{ and } \frac{d^2t}{ds^2} = \frac{d^2y}{ds^2} = \frac{d^2z}{ds^2} = 0, \text{ where } \Gamma^X_{XX} = \frac{d}{dX}\ln[ch(X/C)]. \qquad (31a)$$

Note that d^2X/ds^2 can be zero if $dX/ds = 0$. Thus, there may not be any gravitational acceleration, and no physical changes would be produced by gravity. It follows from Eq. (31a), one obtains, for some constants k_i

$$\frac{dX}{ds} = k_x ch(X)^{-1},\ \frac{dy}{ds} = k_y,\ \frac{dz}{ds} = k_z,\ \frac{cdt}{ds} = k_t, \quad \text{and} \quad k_t^2 - k_x^2 - k_y^2 - k_z^2 = 1. \qquad (31b)$$

If P is at rest, i.e., $dx/dT = dy/dT = dz/dT = 0$; then only $k_t = 1$ is non-zero. Consider the local space

$$ds^2 = c^2dt'^2 - dX'^2 - dy'^2 - dz'^2 \qquad (32a)$$

The geodesic of P is (X_0, y_0, z_0, t) and the local coordinate transformation to the local Minkowski space is

$$dX' = ch(X/C)dX, \qquad dy' = dy, \qquad dz' = dz, \qquad \text{and} \qquad dt' = dt. \qquad (32b)$$

Note that, in violation of the principle of causality, there is no acceleration or other physical causes to enforce (32b). Also, since there is no change in the spatial position and no acceleration of P, such an observer P would have two different light speeds in the X-direction from the same frame of reference. Thus, (30) is not a metric of a physical space.

This shows that space coordinates also cannot be arbitrary, although a certain limited freedom could exist according to solution (20a) and (20b), since (30) is obtained from the flat metric (21a) with $x = C\,[\exp(X/C) - \exp(-X/C)]/2$. Moreover, metric (30) illustrates that a coordinate system may not be compatible with the physical notion of Euclidean-like structure.

8. Incompatibility of the Galilean Transformation and Einstein's Equivalence Principle

To comprehend Einstein's equivalence principle, one must first understand the case of special relativity. Unfortunately, what has been experimentally established as invalid in physics such as the Galilean transformation is currently considered as "valid" in general relativity. The acceptance of such an unrealizable transformation as "valid" in physics is based on Pauli's version. It will be shown that a Galilean transformation is incompatible with Einstein's equivalence principle.

Consider the Galilean transformation from (x, y, z, t) to the K' coordinates,

$$t = t', \quad x = x', \quad y = y', \quad \text{and} \quad z = z' - vt', \tag{33}$$

where v is a constant. Eq. (33a) transforms metric (21a) to another constant Lorentz metric

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2. \tag{34}$$

Metric (34) has the proper metric signature, and therefore Pauli's version is satisfied. However, being a constant metric but not Minkowski, according to Einstein, this metric does not satisfy his equivalence principle.

If (34) were a metric of a physical space, for light rays in the z'-direction, $ds^2 = 0$ would imply that the light speeds to be $v \pm c$. Clearly, this violates coordinate relativistic causality. Moreover, metric (34) implies $d^2x'^{\mu}/ds^2 = 0$, and thus

$$\frac{dx'^{\mu}}{ds} = \text{constant.} \qquad \text{where} \qquad x'^{\mu} \ (= x', y', z', \text{ or } t') \tag{35a}$$

at any point. According to metric (34), consider the "free falling" state of an observer P' at (x'_0, y'_0, z'_0, t'_0)

$$dx'/ds = dy'/ds = dz'/ds = 0, \qquad \text{and} \qquad dt'/ds = (c^2 - v^2)^{-1/2} \tag{35b}$$

For such a "free fall", observer P' is at rest and carries with him the frame K'. Thus, metric (34), instead of a local Minkowski space, is automatically obtained for such a "free fall", and point 2) of the equivalence principle is violated.

Nevertheless, mathematics ensures an existence of a local Minkowski space, the local orthogonal tetrad of P', whose direction $v_{p'}$ is (0,0,0,dt'). Then, the orthonormal vectors of the tetrad are

$$a_1 = (1,0,0,0), \quad a_2 = (0,1,0,0), \quad a_3 = (0,0,\alpha,\beta), \quad \text{and} \quad b_{p'} = (0,0,0,\gamma) \tag{36a}$$

where

$$\alpha = \gamma^{-1}, \qquad \beta = -\gamma v/c^2, \qquad \text{and} \qquad \gamma = (1 - v^2/c^2)^{-1/2}$$

The corresponding transformations is as follows:

$$dt' = \gamma(dT - v/c^2 dZ), \quad dz' = \gamma^{-1}dZ, \quad dx' = dX, \quad \text{and} \quad dy' = dY. \tag{36b}$$

Thus, (dx', dy', dz') and (dX, dY, dZ) share the same frame of reference. Metric (34) does not satisfy point 3) of the equivalence principle since there is no physical cause (i.e., relative motion or the presence of gravity.) for transformation (36b). But, P' is at rest in K'. Thus, (36b) illustrates also that geodesic (35b) does not represent a free fall in a physical space.

In the literature, for instance, Yu [9, p. 42] and Misner, Thorne & Wheeler [4, p. 386] believed the equivalence principle is satisfied if at any space-time point, it is always possible to establish a local Minkowski space, which is related to a "free fall". (Obviously, point 3 is simply ignored.) However, this is necessary but insufficient as shown by metric (34).

The Galilean transformation is an unphysical transformation (since it is physically unrealizable), and it simply takes another unphysical transformation to cancel out the unphysical properties so introduced. In fact, (33a), and (36b) imply

$$dt = \gamma(dT - vc^{-2}dZ), \qquad \text{and} \qquad dZ = \gamma dz' = \gamma\,(dz + v\,dt). \tag{36c}$$

Transformation (36c) is just a Lorentz-Poincaré transformation. Transformation (36b) is essentially a complementary transformation that completes the transformation (33a) starting from the flat metric (21a).

This failure to satisfy Einstein's equivalence principle means that this principle is supported by the Michelson-Morley experiment. The possibility that a light speed could be larger than c through a coordinate transformation, is inconsistent with c being the maximum possible speed, but the equivalence principle rules out such a possibility.

9. Einstein's Equivalence Principle and Intrinsically Unphysical Lorentz Manifolds

Since a Lorentz manifold may not be a physical space, it should be possible to show that there are intrinsically unphysical Lorentz manifolds, which cannot be diffeomorphic to physical spaces. Then such a manifold must fail a requirement, which is independent of coordinate systems. For instance, failing the principle of causality can lead to intrinsically unphysical Lorentz manifolds [16]. An accepted solution of a metric for an electromagnetic plane wave [15,32] is

$$ds^2 = du\,dv + H\,du^2 - dx^i\,dx^i, \qquad \text{where} \qquad H = h_{ij}(u)x^ix^j, \tag{37}$$

and

$$h_{ii}(u) \geq 0, \qquad h_{ij} = h_{ji}, \qquad \text{where} \quad u = ct - z, \qquad \text{and} \qquad v = ct + z.$$

Here t is the time coordinate with the unit in seconds; and x, y, z are the space coordinates with the unit in centimeters; and $h_{ij}(u)$ is an energy-stress tensor related to the cause of this gravity, an electromagnetic plane-wave.

It can be shown easily that metric (37) is a Lorentz manifold. However, independent of the strength of h_{ij}, H can be arbitrarily large. This is incompatible with Einstein's notion of weak gravity and the correspondence principle.

The gravitational force related to $\Gamma^z_{tt} = ½(1 + H)c^2\partial H/\partial t$ has parameters, which is chosen at the space origin (0,0) of an x-y plane, otherwise x^i in (37) could have been $(x^i - a^i)$, where a^i (i = 1, 2) are constants. However, the principle of causality[6)] requires that a physical solution is independent of the coordinate origin, which is unrelated to any physical cause [33,34]. A local coordinate transformation with nonphysical parameters cannot be valid in physics. Moreover, metric (37) cannot be diffeomorphic to a physical space, since a diffeomorphism cannot eliminate unphysical parameters.

Note also that the total energy of a plane electromagnetic wave, though infinite, does not play any role in the problem of gravity; otherwise the problem would not be just unboundedness but the metric would be infinite everywhere. An electromagnetic plane-wave is an idealization of a weak wave with a distant source. Such an idealization is suitable to deal with problems due to local influence. The gravitational influence of an electromagnetic wave is essentially local and limited to along the propagation because, according to special relativity, its energy is propagating in the maximum speed [33,34]. This is also the underlining justification of the symmetry used by other authors, for instance, Misner et al. [4].

These examples illustrate that, to prove the validity of a Lorentz manifold in physics, one must show that a time-like geodesic represents a free fall in a physical space as Einstein did (§ 5). In addition, since a Lorentz manifold may be intrinsically unphysical, it is misleading to define the light speed in terms of the local Minkowski spaces.

10. Conclusions and Discussions

Einstein's equivalence principle, which states the local equivalence between acceleration and gravity, requires that a free falling observer must move along a geodesic and result in a co-moving local Minkowski space. Einstein's equivalence principle is proposed for a physical space, where all physical requirements are sufficiently satisfied. However, Einstein's equivalence principle is often criticized because of misunderstanding. Moreover, it is commonly and mistakenly confused with Pauli's version, whose popularity is ironically due to not requiring any physical meaning for the coordinates.

Thus, it is useful to make clarifications as follows:

1) An accelerated frame is equivalent only to a related uniform gravity (§ 2).
2) The equivalence of gravity and acceleration is local because gravity in a region no matter how small, as Einstein [17] pointed out, generally cannot be transformed away (§ 3).
3) Einstein's equivalence principle is not equivalent to the mathematical existence of local Minkowski spaces since a non-constant Lorentz metric may not imply the necessary existence of acceleration for a static particle (§ 4). In

general relativity, the Galilean transformation has been proven to be invalid for a physical situation (§ 8). There are intrinsically unphysical Lorentz manifolds any of which cannot be diffeomorphic to a physical space (§ 9).

4) Einstein's equivalence principle is to ensure, " the special theory of relativity applies to the special case of the absence of a gravitational field." Thus, the Minkowski metric is the only constant metric (§ 4). Starting from Pauli, theorists, who disagree with Einstein, often do not understand special relativity adequately [6,12,25,35,36,47].

5) A crucial physical requirement is that a free falling observer must be *automatically* in a local Minkowski space (§ 4 & § 5). For example, under the influence of gravity only, a spaceship is in a local Minkowski space.

A physical space has a frame of reference, which has the Euclidean-like structure, and a time coordinate related to the local clocks [2]. For the case of rotating disk, the metric is unique and not harmonic although Fock [25] advocated the harmonic gauge as the valid gauge. It is based on the reference frame that Einstein's principle implies time dilation and space contraction, and thus coordinate relativistic causality is maintained in general relativity [16]. Einstein's equivalence principle requires a physical meaning of space-time coordinates, and unrestricted covariance is invalid in physics[9]. Moreover, it has been shown that a geodesic has physical meaning only if the coordinates have meaning in physics.

However, due to inadequate understanding of special relativity, and thus Einstein's equivalence principle, the resulting local Minkowski space was mistaken as just a possibility. And without any physical justification, general mathematical covariance (extended from the principle of general relativity) was considered as adequate in physics. Theorists also failed to see that the correspondence principle on weak gravity implies that the space-time coordinates cannot be arbitrary.

Moreover, Einstein did not describe precisely the physical process of the coordinate transformation in a free fall. To some theorists, this fact would make Einstein's equivalence principle appeared the same as Pauli's. However, in Einstein's version, the cause and consequences of such local coordinate transformation must be investigated in terms of physics [1,2] (see §§ 7-9). However, Pauli asserted an existence of a local Minkowski space only as a mathematical possibility.

Theorists, who insist on gravity being associated with a non-vanishing curvature [7,11,25], will never accept Einstein's equivalence principle, and it must be replaced with just the existence of local Minkowski space [14]. Since experiments confirmed Einstein's predictions, they advocate that the basis of general relativity should be the Einstein equation alone. However, Einstein pointed out that his equation could be invalid for matter of very high density [1]. It turns out that Einstein's equation cannot handle a dynamic situation[7] even for weak gravity [35]. Moreover, the singularity theorems of Hawking and Penrose [5] are actually irrelevant to physics since their assumptions being invalid[8] in physics [35,37].

On the other hand, Einstein's equivalence principle remains indispensable because of its solid experimental foundation [3,4,7] (see also §§ 4 & 5). Moreover, if Einstein's equivalence principle were not understood, the physical meaning of Ein-

stein's measurements in a curved space would be unclear. Thus, the physical meaning of the Euclidean-like structure would be easily over-looked. Then, inevitably it would hardly escape not only Einstein's interim belief that the space-time coordinates are arbitrary, but also Whitehead's conclusion [18] that a curved space would not be meaningful in physics.

The validity of Einstein's equivalence principle can be considered from three aspects: 1) a geodesic representing the motion of a particle; 2) a free falling observer must be in a local Minkowski space; and 3) effects of the transformation which transforms the metric to a local Minkowski space. *Currently, because coordinates was ambiguous, theorists neglected point 3), which Einstein considered as crucial* [1,2]. We must learn how to examine points 1) and 3) since it is difficult to show the automatic formation of local Minkowski spaces directly. It has been shown that the question of a realistic physical space actually requires the assistance of experiments in addition to theoretical considerations.

Einstein [2,37] had made clear that the (unrestricted) covariance principle is an extension of the principle of general relativity. However, some believed that mathematical general covariance to be a consequence of the principle of general relativity [10-13]. Some even advocated that the gravitational redshift could be derived in a coordinate-free manner (§ 6). Now, these are clarified, and as Bonner et al. [38,39] found, a diffeomorphism may not preserve the validity of physical requirements. Thus, the covariance principle must be restricted to what the principle of general relativity allows.

However, although Pauli's version and unrestricted covariance lead to theoretical inconsistency, theorists did not really have any good choice unless the meaning of space-time coordinates is clarified. Although Einstein has mentioned a Euclidean-like structure in his solutions, he did not define the coordinates in terms physics. This was obscure from the viewpoint of mathematics because the Euclidean-like structure is essentially a notion of physics. Now, since space-time coordinates in relativity are well defined in terms of physics, Whitehead's [18] objection [10)] is no longer valid.

Einstein [40] had believed, "...that a law of gravitation invariant with respect to arbitrary transformations of coordinates was inconsistent with the principle of causality." However, he regarded this as among errors of thought, which cost him two years of excessively hard work. Actually, Einstein was essentially right since covariance must be restricted among space-time coordinate systems so that physical requirements are satisfied. This illustrates that nature is subtle [41].

In conclusion, the equivalence principle provides a theoretical framework and the covariance is restricted to what the principle of general relativity allows. General covariance was Einstein's interim assumption due to his failure in obtaining a physical meaning of coordinates. Now, the foundation of general relativity is clarified. However, this is only a beginning because the question of physical requirements requires continuous efforts in physics. Then, it is possible to develop general relativity such that its scope would encompass every fundamental aspect of physics as Einstein [30] envisioned.

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Appendix A: Physical Spaces and Space-Time Coordinates

In mathematics, things are defined as logic allows. In physics, however, things can be defined only as our understanding of physics allows; for instance, the notion of energy initially referred to mechanical energy. Thus, many physical concepts simply cannot be defined as in mathematics, since physical understanding is a process that may not have an end. Nevertheless, they can be defined at the level understanding of the time being, and be refined as physics progresses. The physical meaning of coordinates of a curved space was difficult for Einstein because it involves different types of measurements.

A.1. The Physical Meaning of Space-time Coordinates

Before relativity, the space is characterized by the Euclidean metric,

$$dl^2 = dx^2 + dy^2 + dz^2, \tag{A1}$$

This Cartesian metric has the advantage of also defining the coordinates. For a cylindrical coordinate system, the metric is

$$dl^2 = dr'^2 + r^2\, d\phi'^2 + dz^2, \tag{A2a}$$

where

$$x = r\cos\phi, \qquad y = r\sin\phi. \tag{A2b}$$

Thus, since the coordinates of (A2a) are curves, the meaning of the coordinates must be clarified with the supplementary information (A2b). In a Riemannian space the metric with a positive definite signature is generally,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu . \tag{A3}$$

The meaning of the coordinates of this curved space is not clear from the metric (A3), and must be clarified with additional information if the curved space is not embedded in a higher dimensional flat space.

In special relativity, the flat metric is

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2. \quad \text{(A4)}$$

This is a pseudo-Euclidean space, and the meaning of the coordinates is clear because the metric distance of two events in an axis is essentially the same as the difference of their coordinates. In general relativity, the space is pseudo-Riemannian and the metric also has the general form (A3), but with an indefinite Lorentzian signature [5] as that of the Minkowski space. Thus, it is necessary to have some supplementary conditions to clarify the meaning of coordinates.

Einstein was unable to provide the physical meaning of coordinates clearly[5)], because he tried to understand coordinates of a curved space without the supplementary conditions [30,42]. Consequently, Einstein claimed that the coordinates are arbitrary. However, if his coordinates were really arbitrary, Einstein should not be able to make any predictions.

Einstein [43] once remarked, "If you want to find out anything from the theoretical physicists about the methods they use, I advise you stick closely to one principle: don't listen to their words, fix your attention on their deeds." Following his advice, it is found that, from his metric solutions (no matter exact or approximate) the space coordinates are characterized by

i) the Euclidean-like distance function $d(P_1, P_2) = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}$, and

ii) the frame of reference is chosen before a solution is obtained, and

iii) the time coordinate is orthogonal to the space coordinates.

The coordinates are independent of the gravity, and Einstein's predictions are related to the Euclidean-like distance instead of just the local metric distance (see Section 5). Thus, Einstein's coordinate system has very specific physical meanings.

The next question is whether these characteristics are co-incidents or intrinsic properties of general relativity. First, the Euclidean-like structure is related to the measuring-rod attaching to the frame of reference (as in special relativity [30]). This measurement is different from Einstein's with the measuring-rod at rest but in a free fall state. Second, a frame of reference is part of Einstein's equivalence principle and Einstein's principle of general relativity. Third, as shown in his calculations, the orthogonality of the time coordinate to the space coordinates is a requirement of Einstein's equivalence principle. Thus, the above three characteristics are inextricably included in the theoretical framework of general relativity.

Nevertheless, Einstein was unable to see that this Euclidean-like structure is physically intrinsic to his theory. Based on only one measurement approach to a curved space-time, he believed, "In the general theory of relativity, space and time cannot be defined in such a way that differences of the special coordinates can be directly measured by the unit measuring-rod, or differences in the time co-ordinate by a standard clock." Besides, the necessary existence of the Euclidean-like structure in his curved space-time, though required by physics, cannot be derived from Riemannian geometry.

A.2. Einstein's Physical Space

An important concept in Einstein's theory is his notion of the *physical space* (-time) that models the real world. Currently, however, the modeling for space and time in the presence of gravitational fields is presented only as a (pseudo-) Riemannian space with manifold M and a metric g that has the same signature as the Minkowski metric, which is (+, -, -, -). The pair (M, g) is called a *Lorentz manifold* and g is called a *Lorentzian metric* [8]. For two Lorentz manifolds, if f: (M, g)→(M', g') is C^∞, one-to-one, onto, and has C^∞ inverse, f is called a *diffeomorphism* and M and M' are said to be *diffeomorphic*.

A manifold M is a topological space, which generalizes the topology of a Euclidean space $\Re^n$. More precisely [5], an n-dimensional, C^∞, real manifold is a set together with a collection of subsets $\{O_\alpha\}$ satisfying the following properties:

(1) Each $p \in M$ lies in at least one O_α, i.e., the $\{O_\alpha\}$ cover M.

(2) For each α, there is a one-to-one, onto, map $\psi_\alpha: O_\alpha \to U_\alpha$, where U_α is an open subset of $\Re^n$.

(3) If any two sets O_α and O_β overlap, $O_\alpha \cap O_\beta \neq \phi$ (where ϕ denotes the empty set), we can consider the map $\psi_\beta o \psi_\alpha^{-1}$ (where odenotes composition) which takes points in $\psi_\alpha [O_\alpha \cap O_\beta] \subset U_\alpha \subset \Re^n$ to points in $\psi_\beta[O_\alpha \cap O_\beta] \subset U_\beta \subset \Re^n$. We require this map to be C^∞, i.e., infinitely continuously differentiable.

Each map ψ_α is generally called a *chart* by mathematicians and a *coordinate system* by physicists. Nevertheless, this mapping in a piecewise continuous 1-1 manner, though suitable for a curved space, is inadequate for a physical space.

However, the space-time coordinate system is a notion of physics. The same mathematical space can be related to different physical spaces due to different frame of references or to an unphysical space if a physical requirement fails. For instance, the flat metric is diffeomorphic to metrics (7), (30), (34), $ds^2 = (c^2 - \Omega^2 r^2)\, dt^2 - 2\Omega r^2\, d\phi dt - dr^2 - r^2\, d\phi^2 - dz^2$ and $ds^2 = (c^2 - \Omega^2 r^2)\, dt'^2 - dr^2 - (1 - \Omega^2 r^2/c^2)^{-1} r^2\, d\phi^2 - dz^2$ (where Ω is an angular velocity), but only the last one is valid in physics.

Physical requirements would be identified more completely as physics progresses. For instance, Einstein thought both the Schwarzschild and the isotropic solutions would be physical spaces. The physical requirements, cited in this paper, are: uniqueness of a deterministic physical value, the principle of causality[6)], and coordinate relativistic causality [16].

Although Einstein's equation and the geodesic equation are covariant, physical understanding are based on the components of a tensor with respect to a frames of reference. In physics, the body to which events are spatially referred is called the coordinate system [44]. According to general relativity, a "frame of reference" must have the Euclidean-like structure (§

6), and its coordinates together with the orthogonal time-coordinate form the space-time coordinate system. For two diffeomorphic manifolds, if they are not equivalent, at least one of them is not a physical space.

Appendix B: Mathematical Covariance and the Physical Restrictions

The principle of covariance states [2] that "The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)." There are two features in the covariance principle: 1) the formulation in terms of the Riemannian geometry; and 2) the general validity of any Gaussian coordinate system as a space-time coordinate system in physics.

Based on his principle of general relativity, Einstein [2] argued, "The law of physics must be of such a nature that they apply to systems of reference in any kind of motion." Then, on a rotating coordinate system, he [1,2] showed that the rate of a clock depends upon where the clock may be and that the length of a measuring-rod can depend on the location and the orientation of the rod. However, these are insufficient to support the validity of feature 2). *Experimentally, there is also no evidence that supports covariance beyond Einstein's principle of general relativity.*

Nevertheless, Einstein believed, as an interim assumption, that any Gaussian co-ordinate system is valid to serve as a space-time coordinate system in physics. His arguments [2], which are absent from his later book [1], are as follows:

> "That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences."

However, this argument is incompatible with his argument for defining time relating to local clocks. Moreover, in order to predict events, one must able to relate events of different locations in a definite manner.

A space-time coordinate system cannot be just any Gaussian coordinate system in mathematics. In practice, we are often interested in things related to a non-local picture such as symmetry, and physical validity of a geodesic. As Eddington [19] pointed out, "space is not a lot of points close together; it is a lot of distances interlocked." Einstein [2] remarked that the

velocity of light is "defined in the sense of Euclidean geometry". Einstein's equivalence principle restores the physical objectivity of space and time, and we observe the gravitational red shifts and the light bending.

Kretschmann [45] pointed out that the postulate of general covariance does not make any assertions about the physical content of the physical laws, but only about their mathematical formulation; and Einstein [46] entirely concurred with this view. As Pauli [6] pointed out, "The general covariant formulation of the physical laws acquires a physical content only through the principle of equivalence in consequence of which gravitation is described solely by the (metric) g_{ki} ···." Note that Einstein's equivalence principle itself is not generally covariant. Thus, the physics of the covariance actually lies on the restriction on feature 2). Physical requirements restrict valid space-time coordinate systems.

Nevertheless, since the physical meaning of space-time coordinates was ambiguous [1,18], the restriction on covariance could not be addressed adequately. Moreover, due to inadequate understanding of special relativity, theorists such a Landau & Lifshitz [47] failed in recognizing that a Lorentz manifold can be physically invalid and that the interim assumption of unrestricted covariance has no theoretical basis in physics, nor any experimental support. (For instance, the Michelson-Morley experiment is against unrestricted covariance, and there are non-scalars.) They probably did not realize that this failure would mean a rejection of physical requirements such as Einstein's equivalence principle, the correspondence principle, Einstein's notion of weak gravity, coordinate relativistic causality, the principle of causality, and so on. Thus, a general problem in current theories would be self-consistency, and Hawking's book [48] provides an example.

Einstein was aware of that there are shortcomings in his theory, and in 1916 he wrote [2,21] the following:

> "It is not my purpose in this discussion to represents the general theory of relativity as a system that is as simple and logical as possible, and with the minimum number of axioms; but my main object is to develop this theory in such a way that the reader will feel that the path we have entered upon is psychologically the natural one, and that the underlying assumptions will seem to have the highest possible degree of security."

Apparently, this psychologically natural approach needs improvements.

ENDNOTES

1) A local Minkowski space is a short hand to express that special relativity is locally valid for phenomena unrelated to the second order derivatives of the space-time metric.
2) Another coordinate-free version [49] is, "the Principle of equivalence states that when gravity is present, as when it is absent, free particles moves along external (geodesic) lines of spacetime -- spacetime now being curved, not flat."

3) Einstein [22] remarked, "We might also think that, regardless of the kind of gravitational field which may be present, we could always choose another reference-body such that no gravitational field exists with reference to it. This is by no means true for all gravitational fields, but only those of quite special form. It is, for instance, impossible to choose a body of reference such that, as judged from it, the gravitational field of the earth (in its entirety) vanishes." Nevertheless, Bergmann [10] illustrated Einstein's equivalence principle with "Einstein's elevator" in the gravity of the earth.

4) A gravitational field acts upon a passive mass, whereas an active mass generates gravity [4,34].

5) A curved two-dimensional space is immersed in Euclidean three-dimensional space. Therefore, one can imagine to have a curved four-dimensional space immersed in a flat space of a large number of dimensions. Dirac [42] believed, "Einstein assumed that physical space is of this nature and thereby laid the foundation for his theory of gravitation. For dealing with curved space one cannot introduce a rectilinear system of axes. One has to use curvilinear coordinates." From a space-time metric that depends on the distribution of matter, the meaning of the coordinates would be very confusing [18]. The fact is, however, Einstein's curved physical space has a rectilinear system of axes.

6) The principle of causality [33-35] assumes that the causes of phenomena are identifiable. This principle is commonly used in symmetry consideration in electrodynamics. In general relativity, Einstein and subsequent theorists have used this principle implicitly on symmetry considerations [1-9]. Other consequences are that parameters unrelated to any physical cause in a solution is not allowed and that a dynamic solution must be related to a dynamic source.

7) A fundamental physical reason is that the gravitational radiation is associated with an anti-gravity coupling [33-37]. However, a time-dependent solution, which could be obtained simply through a coordinate transformation [50], may not be a dynamic solution, which is generally associated with gravitational radiation. General relativity was often presented as being derived in a purely deductive manner [47]. This oversimplifies a great work of science. First, Einstein was assured by his calculation on the perihelion of Mercury only after several earlier attempts in failure [41], and more recently it is proven necessary to modify the Einstein equation because it cannot accommodate the gravitational radiation energy, which is of the fifth order in a post-Newtonian approximation [35,37]. This explains that the post-Newtonian approaches often lead, in higher approximations, to divergent integrals [51]. Thus, Will's [4,52] calculation on the time dependence of the ebb and flow of the ocean tides is invalid as an evidence for the Einstein equation.

8) The case of singularity theorems [5] illustrates that if the conclusion is strange in physics, it is likely that one of the assumptions is invalid even though it may appear to be normal superficially [35,37].

9) In books such as Peng and Xu [53], the question of whether Einstein's equivalence principle or Pauli's version is valid exists in the form of whether the covariance principle is valid. Since an unphysical coordinate system is unrealizable, experimentally one can show only the absence of such a coordinate system as the Michelson-Morley [21] did.

10) Whitehead [18, p. 83] objected, for instance, "By identifying the potential mass impetus of a kinematic element with a spatio-temporal measurement Einstein, in my opinion, leaves the whole antecedent theory of measurement in confusion, when it is confronted with the actual conditions of our perceptual knowledge. The potential impetus shares in the contingency of appearances. It therefore follows that measurement on his theory lacks systematic uniformity and requires a knowledge of the actual contingent physical field before it is possible." Moreover, Whitehead remarked, "But the worst homage we can pay to genius is to accept uncritically formulations of truths which we owe to it."

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6.2. Coordinate Relativistic Causality

6.3. An Example of Examining Coordinate Relativistic Causality

6.4. The Question of Coordinate–free Derivation

7. Physics Related to Einstein's Equivalence Principle

8. Incompatibility of the Galilean Transformation and Einstein's Equivalence Principle

9. Einstein's Equivalence Principle and Intrinsically Unphysical Lorentz Manifolds

10. Conclusions and Discussions

Acknowledgments

Appendix A: Physical Spaces and Space-Time Coordinates

A.1. The Physical Meaning of Space-time Coordinates

A.2. Einstein's Physical Space

Appendix B: Mathematical Covariance and the Physical Restrictions

ENDNOTES

REFERENCES