# ON CRITICISMS OF EINSTEIN'S EQUIVALENCE PRINCIPLE

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**C. Y. Lo**

**Applied and Pure Research Institute**
**17 Newcastle Drive, Nashua, NH 03060**

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## Abstract

Einstein's equivalence principle was initially the equivalence of an accelerated frame and uniform gravity. Moreover, in 1916 the infinitesimal form of Einstein's equivalence principle is obtained for a physical space where all physical requirements are sufficiently satisfied. This principle implies that the local Minkowski space is obtained through a gravitational acceleration to the frame of reference that has the Euclidean-like structure. Although theorists have challenged this principle as invalid, Einstein insisted on the fundamental importance of the principle to general relativity. It is shown that existing criticisms, starting from Synge and Fock, are due to misunderstanding and misconceptions in physics, and/or inconsistent considerations. This includes the misinterpretations of Pauli, Bergmann, Tolman, Fock, Landau & Liftshitz, Logunov & Mestvirishvili, Thorne, Wald, Hawking, and etc. The recent criticism of Hong has the distinction of starting from his intuitive but inadequate observation that "a homogeneous field is characterized by the fact that any part of it is representative of the whole", and his understanding of special relativity. It is pointed out that his notion of uniform gravity is not only against general relativity but also in disagreement with experiment on the gravitational redshift. His arguments toward acceleration, in disagreement with special relativity, repeated the same mistake of Landau & Lifshitz. Moreover, Hong's notion of uniform field is also not supported by experiments of electrodynamics.

**Key Words**: Einstein's equivalence principle, frame of reference, the Euclidean-like structure, physical space-time.

## 1. Introduction

Since Einstein's three predictions are confirmed by observations [1,2,3,4], many theorists have accepted Einstein's field equation [3-16]. Moreover, it is generally agreed that Einstein's equivalence principle is the theoretical foundation of general relativity. However, Einstein's equivalence principle was also a favorite target for theorists such as Synge [5], Fock [6], Thorne [17], and Ohanian & Ruffini [14] who wanted to show a difference from Einstein.

Recently, Hong [18] also claimed that the initial form of Einstein's equivalence principle was invalid. In Hong's distinct approach, he started from his notion of uniform gravity and then showed that uniform gravity is different from an accelerated frame; whereas others started with an accelerated frame and then showed their failure to obtain uniform gravity. Hong's vague notion of uniform gravity seemed to support the belief that Einstein's equivalence principle was invalid. Unfortunately, this seems to be becoming popular since even the Wheeler school has been converted to non-believers of Einstein's principles [14].

In this paper, it will be shown that Hong's claim is also invalid. For convenience of discussions, Einstein's equivalence principle is clarified first. To this end, there are few better ways other than to show what kind of mistakes the previous critics made. The errors can be classified into several types, ranging from misunderstandings, misconceptions, miscalculations, or their combinations. The other well-known cases will be addressed and analyzed, if this is necessary, to show the errors.

## 2. The Initial Form of Einstein's Equivalence Principle and Related Misunderstandings.

In 1911, Einstein explained the initial form of his equivalence principle in terms of uniform gravity and acceleration [1]. In 1916, subsequent to his principle of general relativity, Einstein [1] proposed his equivalence principle in the infinitesimal form for the general case of a four dimensional Riemannian physical space-time[1)]. However, a surprising fact is, as Einstein [19] saw it, that few like Eddington [3] understood Einstein's equivalence principle adequately[2)].

Einstein's equivalence principle was challenged by Synge's [5] now popular identification of "true" gravitational fields with metrical curvature. Synge [5] "professed" his misunderstanding of Einstein's equivalence principle as follows:

> "...I have never been able to understand this principle...Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's world line...The Principle of Equivalence performed the essential office of midwife at the birth of general relativity...I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced."

Currently, misunderstandings persist. For instance, Thorne [17] criticized Einstein's principle as follows:

"In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you (and your reference frame) are very small."

However, Einstein has already explained these problems. For instance, the problem of tidal forces had been answered in Einstein's July 12, 1953 letter to A. Rehtz [20] as follows:

"The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Here, Einstein has made clear also that this principle is proposed for a physical space, where all physical requirements are sufficiently satisfied. Einstein [19] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the $\Gamma^l_{ik}$ (field strength), not the non-vanishing of the $R_{iklm}$." and no gravity is a special case of gravity. This view is crucial in general relativity because it allows Einstein to conclude that the geodesic equation is also the equation of motion of a massive particle under gravity, and this made it possible to conceive a field equation for the metric. Einstein insisted, throughout his life, on the fundamental importance of the principle to his general theory of relativity [19]. However, Einstein's insistence on this point has created a puzzle for philosophers and historians of science [19]. This shows also, how much the application of Einstein's equivalence principle was being understood in terms of physics.

Einstein [1] derived the gravitational redshift from the initial form of his principle, the equivalence of a uniformly accelerated frame and uniform gravity. This is independent of the need for a Riemannian space with a Lorentz signature. A known deficiency of his results then is an incorrect formula for light deflection under gravity. Einstein [21] corrected this formula in 1915. The cause of this deficiency has been identified as account not being taken of the effects of a curved space.

Nevertheless, an unverified belief advocated by Fock [6] and others [14,22] is that Einstein's equivalence principle could be intrinsically incompatible with the notion of a curved space. Such a belief must be very absurd to Einstein since his argument for a Riemannian space is based on the application of his equivalence principle to the uniformly rotating disk [1,2]. Recently, such a belief has been proven to be fundamentally incorrect since the Maxwell-Newton Approximation [3)], the linear field equation for weak gravity, that produced a valid light bending, has been derived [22] with Einstein's equivalence principle together with the notion of a Riemannian space if Newtonian theory is taken as a form of first order approximation.

On the other hand, uniform acceleration cannot exist forever; otherwise the resulting speed would exceed the velocity of light. It follows that uniform acceleration must be started at some time, for instance, $t = t_0$, and then decreased some time afterward.

Moreover, uniform gravity must be confined in a finite region since it is equivalent to an accelerated frame; otherwise the light speed as the maximum velocity would be violated. In other words, like an electromagnetic plane wave, uniform gravity also does not really exist in nature. Thus, the equivalence of acceleration and uniform gravity is best described, as Einstein did, in terms of a uniformly accelerated chest[4)].

Einstein assumed that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration $\gamma$ (but, system K is free from acceleration), and a system K' accelerated by $\gamma$ in the opposite direction, can be extended to other physical processes. In Einstein's derivation of gravitational red shift [1], the equivalence principle is used only locally. This was somewhat unclear since the obvious, though crucial, step of replacing the infinitesimal energy $\gamma h$ by $\Delta\Phi$ is omitted. In practice, the uniform gravity is essentially a local idealization of a non-uniform gravity, and a large-scale region of uniform gravity does not really exist. Thus, it is unrealistic to consider observing the bending of light in a field of uniform gravity at different locations as identical. Unfortunately, this is exactly the starting point of Hong's thesis [18].

Einstein assumed that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration $\gamma$ (but, system K is free from acceleration), and a system K' accelerated by $\gamma$ in the opposite direction, can be extended to other physical processes. This initial form was further elaborated to an infinitesimal form for a curved space due additionally to the principle of general relativity [1,2]. Currently, the derivation of the Maxwell-Newton Approximation, which shows that the gravitational red shifts are directly related to $g_{tt}$ the time-time component of the space-time metric [1], should have removed any remaining doubt on the validity of Einstein's equivalence principle [22,23].

There are three important points related to Einstein's equivalence principle: 1) The equivalence principle is proposed for a physical space (-time), in which all physical requirements are sufficiently satisfied. 2) The equivalence principle is defined in terms of acceleration with respect to a frame of reference of Einstein's physical space [1,2]. 3) In a free fall, the resulting local space must be Minkowski. These points were clear in the initial form, but are often neglected in the infinitesimal form, and thus, in current theories Einstein's equivalence principle has been degenerated into Pauli's version [4], which requires nothing beyond the proper metric signature.

Nevertheless, Pauli's version is still very popular after Einstein objected as a misinterpretation [19]. This is due to the fact the physical meaning of space-time coordinates is ambiguous in Einstein's theory [1,2,6,24]. Some theorists simply regarded Einstein's equivalence principles as merely a heuristic argument [25] because they believed that the space-time coordinates were arbitrary. This is discussed in a separate paper [26]. In the next section, the problem of misunderstanding and misconception in connection with the initial form are discussed.

## 3. Misunderstandings and Misrepresentations of Tolman and Fock.

Einstein concluded that the reality is a curved space-time. However, the connection between an accelerated frame and a space-time metric has not been established as Einstein envisioned. Here, it will be shown that the main reason is conceptual errors. Theorists have incorrectly assumed that an accelerated frame must be related to a Euclidean subspace.

To apply Einstein's equivalence principle, it is crucial that the space-time under consideration must be a physical space. Theorists, both for and against general relativity have made mistakes by ignoring this. For example, Logunov and Mestvirshvili [27] showed that inconsistent results would be obtained through a coordinate transformation. On the other hand, Tolman [12] also ignored this problem in his illustration of Einstein's equivalence principle. Thus, instead of validity of Einstein's theory, Tolman seemed to show the opposite, i.e., arbitrariness and invalidity just as Logunov et al. claimed.

Tolman claimed that his treatment [12] is based on the relation of the principle of equivalence to the fundamental idea of the relativity of all kinds of motion. Tolman is a good example of those theorists claimed to apply Einstein's equivalence principle without adequate understanding. To illustrate the equivalence principle, Tolman started with system $K_0$ with the flat metric,

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \tag{1}$$

for the first observer. Consider a second observer in a system K', which can be taken as moving relative to the first with acceleration $a$ in the x-direction, and use coordinates x', y', z', and t' as given by

$$x' = x - \frac{1}{2}at^2 \qquad y' = y \qquad z' = z \qquad t' = t \tag{2}$$

according to the usual transformation to accelerated axes, which Tolman regards as a reasonable change at least at low velocities. Substituting from (2) into (1), Tolman thought that he obtained the formula for the interval for the second observer as

$$ds^2 = (c^2 - a^2t'^2)\, dt'^2 - 2at'dx'dt' - dx'^2 - dy'^2 - dz'^2. \tag{3}$$

Then, according to the geodesic equation, from metric (3) Tolman obtained

$$\frac{d^2x'}{ds^2} = \frac{-a}{c^2 - a^2t'^2}, \tag{4a}$$

and

$$\frac{d^2y'}{ds^2} = \frac{d^2z'}{ds^2} = \frac{d^2t'}{ds^2} = 0 \tag{4b}$$

as approximately the equations of motion for the case of particles having negligible velocity. Thus, Tolman claimed that the equivalence principle was illustrated by (4).

On the other hand, consider a particle P in K' at the beginning of a free fall. Since the velocity of K' relative to $K_0$ is v = at, for the local Minkowski space (X, Y, Z, T) of P, we have $dx = \gamma[dX + v\,dT]$, $cdt = \gamma[cdT + (v/c)dX]$, where $\gamma = [1 - (v/c)^2]^{-1/2}$. It thus follows from (3) that there is no time dilation although $\gamma dx' = dX$. Thus, Einstein's equivalence principle is not satisfied and K' is not a physical space. In addition, if metric (3) were valid in physics, $ds^2 = 0$ would imply the light speed to be $(at \pm c)$ in the x'-direction, and thus metric (3) violates coordinate relativistic causality. These demonstrate that Tolman does not understand the need of a local time in Einstein's theory [1,2].

Moreover, if Tolman's calculation were valid, he actually showed that Einstein's equivalence principle was invalid. In Einstein's [1] analysis, the effects of an accelerated frame can be related to a gravitational potential $\Phi$, which is a function of spatial variables in Newtonian theory. But, all the metric elements of (3) are functions of time t'. Although $\Gamma^{x'}{}_{t't'} \neq 0$, the non-zero term in (4a) comes from $g_{t'x'}$ but not from $g_{t't'}$ (since $\partial_\mu g_{t't'} = 0$ for $\mu \neq t'$).

Tolman simply ignored that Einstein's later paper [1,2] confirms his 1911 analysis, and one has the relations,

$$g_{t't'} \approx 1 + 2\Phi/c^2, \qquad \text{and} \qquad a_i \approx -\partial\Phi/\partial x^i \tag{5}$$

where $\Phi$ is the negative gravitational potential and a function of x'. Obviously, (5) is not consistent with equation (4). Thus, if Einstein's equivalence principle is valid, metric (3) cannot be a physical space. Since Tolman's calculation is valid in Pauli's version, this illustrates that Pauli's version is actually incompatible with Einstein's theory.

In an attempt to overcome the deficiency of metric (3), in 1958 Fock [6] modified transformation (2) with

$$x = x' - \frac{1}{2}at'^2, \qquad t = t' - at'x'/c^2. \tag{6}$$

Then, he obtained

$$ds^2 = (c^2 - 2ax' - a^2t'^2)\,dt'^2 - dx'^2 - dy'^2 - dz'^2 + a^2(t'dx' + x'dt')^2/c^2. \tag{7}$$

The term 2ax' seems to serve the purpose, and metric (7) would be superficially compatible with relation (5).

However, the equation of motion even for dx'/ds = 0 is very complicated as follows:

$$\frac{d^2x'}{ds^2} = a\,c^{-2}(1 - a^2t'^2/c^2)^{-1}[(1 - ax'/c^2)^2 - a^2t'^2/c^2]^{-1}. \tag{8}$$

It is clear that (8) is not a uniform acceleration for a static particle since the right hand side depends on both space and time. Nevertheless, Fock [6] believed that the problem of time-dependence could be resolved within the speculated metric,

$$ds^2 = (c + ax'/c)^2\, dt'^2 - dx'^2 - dy'^2 - dz'^2\,, \tag{9}$$

Fock [11] proposed the following mathematically ingenious transformation,

$$x = x' \cosh(at'/c) + (c/a)[\cosh(at'/c) - 1] \tag{10a}$$

$$y = y'\,; \qquad z = z' \tag{10b}$$

$$t = (c/a)\sinh(at'/c) + (x'/c)\sinh(at'/c)\,, \tag{10c}$$

although its physics is not clear. Under the condition at'/c << 1, the above equation can be written approximately as

$$x = x' + at'^2/2; \quad y = y'; \quad z = z'; \quad t = t' \tag{11}$$

Substituting (10) into the flat metric, one obtains the metric (9) exactly. Finally, Fock has obtained a metric whose time dilation seems to be compatible with Einstein's paper of 1911. An important difference is that Einstein's is based on physical considerations, whereas Fock gave only a pure mathematical manipulation.

To determine the validity of a manifold as a physical space, the physics must be considered. Apparently, the mathematical requirement, at'/c << 1, instead of just at'/c < 1, is to make (11) approximately valid, but it does not seem to have a physical basis. Moreover, metric (9), in addition to be incompatible with the observed light bending, does not produce a uniform acceleration as claimed. The equation of motion for dx'/ds = 0, though better than (8), is not a uniform gravity as follows:

$$\frac{d^2x'}{ds^2} = -\,a\,c^{-2}[1 + ax'/c^2]^{-1}. \tag{12}$$

As expected, Fock cannot find a valid interpretation for (12). Nevertheless, Fock believed that this is due to an intrinsic deficiency of Einstein's equivalence principle. This is a good example of a failure due to misconception being blamed on Einstein's equivalence principle. Unfortunately, Hong made the same kind of error [18].

The crucial conceptual error of Tolman and Fock is that they seem to believe that a frame of reference corresponds to a Euclidean subspace. This is not true since the metric of Einstein's rotating disk [1,2] with an angular velocity $\Omega$ is

$$ds^2 = (c^2 - \Omega^2 r'^2)\, dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\, d\phi'^2 - dz'^2 \qquad (13a)$$

where

$$x' = r' \cos\phi', \qquad y' = r' \sin\phi'. \qquad (13b)$$

Although metric (13a) does not have a Euclidean subspace, (13b) is a Euclidean-like structure, which is necessary for the angle $\phi'$ to be well defined (see also [26]). Similarly, a Euclidean-like structure is also included in the Schwarzschild solution,

$$ds^2 = (1 - 2MG/\rho)dt^2 - (1 - 2MG/\rho)^{-1} d\rho^2 - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \qquad \text{for} \qquad \rho > 2MG \qquad (14a)$$

where

$$x' = \rho \sin\theta \cos\varphi, \qquad y' = \rho \sin\theta \sin\varphi, \qquad \text{and} \qquad z' = \rho \cos\theta. \qquad (14b)$$

Metric (14a) is a function of $\rho$ ($= [x'^2 + y'^2 + z'^2]^{1/2}$). The radical $\rho$ is related to Euclidean coordinates ($x'$, $y'$, $z'$), and thus the metric is defined in terms characteristic of a Euclidean-like structure, which corresponds to the metric when $M = 0$. The physical reason of the Euclidean-like structure is that a measuring rod, if attached to a frame, would also be under the influence of gravity.

When a frame K', which is linearly accelerated in the x direction, has a relative velocity v towards the inertial coordinate system K, which was initially at rest relative to K', according to special relativity the metric for K' has the form,

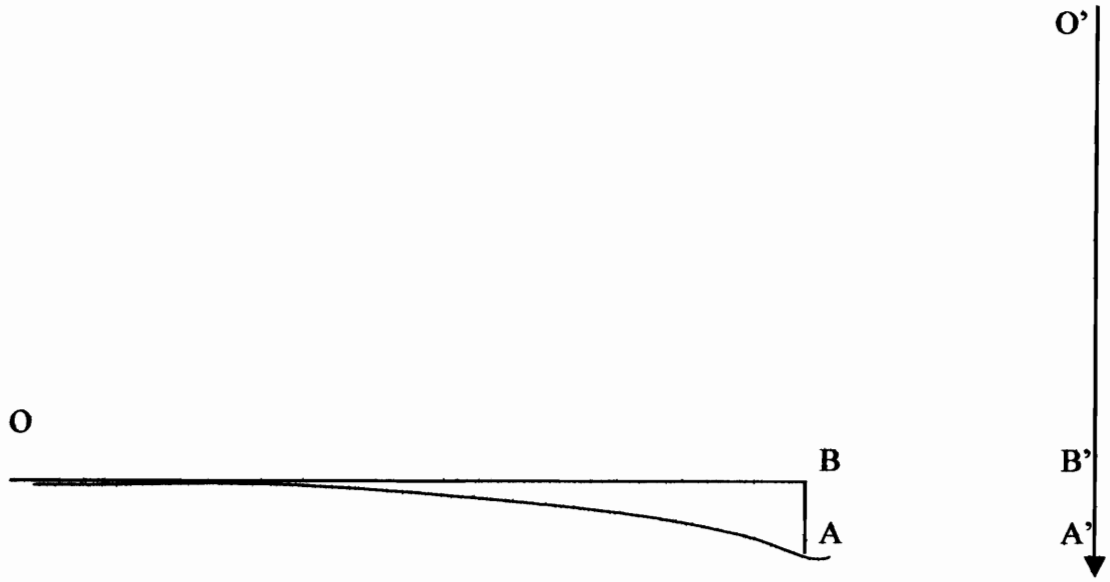
$$ds^2 = (c^2 - v^2)\, dt'^2 - dr'^2 - (1 - v^2/c^2)^{-1} dx^2 - dy'^2 - dz'^2 \qquad (15)$$

by similar arguments used for the uniformly rotating disk. Thus, it is incorrect to assume that the frame of reference of K' corresponds to a Euclidean subspace. The metric for accelerated frames will be presented in a separate paper [28].

## 4. The Problems in Hong's Approach.

First, Hong [18] assumes unrealistically the existence of an extensive field of uniform gravity, whereas Einstein realistically considered an accelerated frame [1,2]. Then, Hong considered a light ray emitted horizontally in a field of vertical uniform gravity. Assuming this light ray travels from O to A and OB is the horizontal component and BA is the vertical component, the time at O is $t_0$ and the light arrives at A at time $t_A$. At time $t_0$ the velocity of a free falling cabin has a velocity $v_0$ that is increased to $v_A$ at time $t_A$. During the time interval $\Delta t = t_A - t_0$, the cabin travels from O' to A' whereas the cabin would have to travel to B' if its

velocity remained unchanged at $v_0$. Then Hong claimed, "it is clear that the validity of this assertion requires the equality of B'A' and BA and the constancy of B'A'." (In general relativity, BA actually depends on the location.)




A crucial point is, however, the rules which govern the motions of the light and the cabin under uniform gravity have not yet been specified [18]. On the other hand, the above claim based on his intuition, requires a proof. This claim seems obvious only for the case $v_0 = 0$. Moreover, the time interval $\Delta t$ may not be constant for traveling the same horizontal distance since in general relativity [1,2] light speed depends on the metric although a gravitational force depends on the differentials of the metric. Thus, in a homogeneous gravitational field, only for some aspects, any part of it is representative of the whole. Moreover, since a homogenous field must have a beginning end and a finishing end, any part of it does not really represent the whole.

Hong argued that B'A' cannot be a constant since $\Delta v$ must decrease in the same time interval. An implicit assumption is that the region of uniform gravity is infinite, but this is not the reality. Uniform gravity exists only as a local idealization in nature. If uniform gravity is no longer valid beyond a region as reality suggests, then there is no reason for $\Delta v$ to decrease.

Hong's other major argument is based on his belief that the accelerations of the cabin, which is an inertial system, and a particle with a velocity relative to the cabin, must be the same[5)] [29]. However, the constant acceleration of the cabin requires only a particle at rest in the cabin to have the same acceleration. Under the same force, special relativity as well as general relativity implies that particles with different velocity have different accelerations. It is only in Newtonian mechanics that particles of different velocity would have the same acceleration. Thus, Hong's belief is due to not accounting for the effects of special relativity. On the other hand, it is based on special relativity that Hong claimed his objection to a uniform acceleration of the frame.

In conclusion, since none of the arguments of Hong is valid, his objection to Einstein's equivalence principle is baseless. Hong seems also a victim of those who copied from Landau & Lifshitz [29] with a blind faith[5)].

## 5. Remarks

It has been shown that misinterpretations of, or objections to, Einstein's equivalence principle are due to inadequate understanding of Einstein's theory and physics in general. In particular, Hong's arguments are based on a number of beliefs, which are unverified or simply invalid in physics. Hong's basic belief that "a homogeneous field is characterized by the fact that any part of it is representative of the whole", implicitly assumes that phenonmena depend on gravitation field but not on the gravitationa potential. This belief is based on his intuition of inadequate understanding rather than all facts of the entire situation in gravity. However, a physical assumption cannot be a just as one would like because of constraints from observations.

A crucial difference between Einstein and Hong is that Einstein realistically treats uniform gravity only as a local idealization, whereas Hong unrealistically considered uniform gravity existing in a region of infinite extent with imagined characteristics. Experimentally, it is known that the gravitational potential, but not the gravitational field, determines gravitational redshifts [1,2,14]. Moreover, this belief is also not true in electrodynamics since experiments [30] show that the electromagnetic potential actually has physical influence just as Aharonov & Bohm predicted [31].

Currently, there are attempts to substitute Einstein's equivalence principle with Pauli's version [5,6,14,17], which requires only the proper metric signature. However, there are clearly unphysical solutions that have the proper signature [32], in particular when the principle of causality[6)] is violated [33]. Pauli's [4, p.145] version of the equivalence principle is as follows [7)]:

*"For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system $K_0$ $(X_1, X_2, X_3, X_4)$ in which gravitation has no influence either in the motion of particles or any physical process."*

Einstein objected to this version [19] as inadequate in physics. The initial form of Einstein's principle stands against Pauli's version because the equivalence of uniform gravity and acceleration clearly requires considering the frame of reference. Note that Galileo established the equivalence of inertial mass and gravitational mass. The mathematics theorems [5] established the existence of local Minkowski space. Thus, Einstein's contribution is only that in a free fall, the local space *must* be Minkowski.

Moreover, mathematical general covariance, which also does not require any physical meaning of coordinates, had to be used to replace the principle of general relativity [1,2]. It should be noted that unrestricted covariance is also incompatible with Einstein's equivalence principle, in addition to the correspondence principle. For example, Einstein [2] remarked, "As in special the-

ory of relativity, we have to discriminate between time-like and space-like line elements in the four-dimensional continuum; owing to the change of sign introduced, time-like line elements have a real, space-like line elements an imaginary ds. The time-like ds can be measured directly by a suitably chosen clock." On the other hand, Hawking [34] declared, "In relativity, there is no real distinction between the space and time coordinates, just as there is no real difference between any two space coordinates." Nevertheless, Hawking [34] also refers to, "an arrow of time, something that distinguished the past from the future, giving a direction of time". Thus, there is a distinction between a time coordinate and a space coordinate. Apparently, theorists were unaware of that the physical meaning of symmentry in the Schwarzschild solution is also based on the meaning of coordinates. Such logical deficiencies inevitably arise when one's reasoning springs from an inconsistent "theory" masquerading as general relativity.

Attempts to disprove or misinterpret Einstein's equivalence principle will be continued and encouraged unless the physical meaning of coordinates is well understood [26]. For instance, since the notion of black holes requires a presumed validity of the Schwarzschild solution [8-11], the advocates of black holes had to believe Pauli's version as adequate in physics.

Currently a subtle form of criticism toward Einstein's equivalence principle is simply ignoring it [26]. For instance, Wald [10] took the equivalence of inertial mass and the passive gravitational mass as the equivalence principle, but ignored Einstein's equivalence principle. Einstein's equivalence principle is incompatible with his invalid belief that diffeomorphic manifolds are identical in physics. Peng & Xu [35], being unaware of that Pauli's version has been proven as inadequate [22-27], avoided the equivalence principle, but the issue of a valid covariance instead rises. However, they correctly pointed out that general relativity is separated from the rest of physics because the physical meaning of space-time coordinates is not clear[8)]. For this reason, some theorists regarded Einstein's principles as just heuristic arguments [25].

As a result, it is difficult to find a book on general relativity, except those by Einstein [1,2] and Eddington [3] that presents Einstein's equivalence principle correctly in terms of physics. Moreover, according to Fock [6] and Whitehead [36], even Einstein himself seemed to be unable to explain his own principle *precisely.* Their view is obviously supported by the fact that few of Einstein's disciples were able to interpret the physics of Einstein's equivalence principle adequately. A possible exception seemed to be Zhou Pei-Yuan [37,38], who proposed an experiment on local light speeds. Moreover, Einstein did not see the fact that the physical meaning of space-time coordinates has already existed in his theoretical framework [26].

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**ENDNOTES**

1) According to Einstein [1,2], a Riemannian space-time M is characterized by a space-time metric g that can be determined by the distribution of matter. It is in that sense that the metric $g_{ik}$ is subjected to physical considerations, that Einstein considers the four-dimensional space-time continuum of reality as a physical space-time (M, g). Einstein [24] wrote,

> "For the functions $g_{ik}$ describe not only the field, but at the same time also the topological and metrical structural properties of the manifold. ... There is no such thing as an empty space, i.e., a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field."

Moreover, since Einstein's Riemannian space-time models reality, all the physical requirements must be sufficiently satisfied by the space-time metric $g_{ik}$. Thus, to apply Einstein's equivalence principle correctly to a Lorentz manifold, one must show, as Einstein did [1,8], that all known physical requirements are sufficiently satisfied.

One might argue that a physical space has not been defined precisely because the physical requirements have not been defined completely. However, physical requirements are almost what physics is all about. Thus, the physical requirements are not complete until the end of physics if it has an end. Einstein himself has indicated the difficulty of presenting general relativity "precisely" as mathematics or even in the degree of Maxwell's theory. For instance, his equivalence principle remains to be clarified probably because the space-time coordinates are still somewhat ambiguous. Einstein wrote in 1916 [1] the following:

> "It is not my purpose in this discussion to represent the general theory of relativity as a system that is as simple and logical as possible, and with the minimum number of axioms; but my main object is to develop this theory in such a way that the reader will feel that the path we have entered upon is psychologically the natural one, and that the underlying assumptions will seem to have the highest possible degree of security."

Another problem is that he has not been able to describe precisely the physical process (due to a free fall), which transforms a metric near a point to a local Minkowski space although he infers the correct result.

2) Einstein also praised Eddington's book of 1923 [3] to be the finest presentation of the subject ever written [39].

3) While Einstein equation was guessed, the Maxwell-Newton Approximation is derived independent of Einstein's equation.

4) This is commonly but mistakenly known as Einstein's elevator [40]. However, to avoid the usual association of an elevator with the gravity of the earth, Einstein actually used the word "chest" [41].

5) Landau & Lifshitz [29] made the same erroneous claim, "A body of arbitrary mass, freely moving in such a system of reference, clearly has relative to this system a constant acceleration, equal and opposite to the acceleration of the system itself." It

is well known that in special relativity the "sum" of two velocities $u$ and $v$ is $(u + v)/(1 + uv/c^2)$. If a particle has a constant velocity $v$ in the direction of the acceleration $a = du/dt$, the acceleration with respect to this particle is $a$, with a factor $[(1 - v^2/c^2)^{1/2}/(1 + uv/c^2)]^3$. The basic problem is that they are confused on the physics of space-time coordinates in general relativity. Consequently, their local distance formula does not necessarily produce the *actually distances* as claimed. Nevertheless, their erroneous claims were accepted as valid by Liu [42], Zel'dovich & Novikov [15], and Yu [16], who copied from or referred to their book [29]. In addition to Landau and Lifshitz, Pauli [3], Tolman [12], and Fock [6], also have made errors in special relativity [43,44]. Nevertheless, their mistakes are not as obvious as Hawking's inconsistent claims (see § 5).

6) The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the *principle of causality*. This principle is the basis of relevance for all scientific investigations. This principle implies that the gravitational radiation must have sources and any parameter in a physical solution must be related to some physical causes. Thus, symmetry is preserved unless some causes break it, and if an unphysical parameter exists in the metric, such a manifold would not be diffeomorphic to a physical space [22,31]. This symmetry consideration is commonly used in electrodynamics. In general relativity, Einstein and subsequent theorists have used this principle implicitly on symmetry considerations [1-16].

7) Pauli [4, p. 145] did remark, "We can think of the physical realization of the local coordinate system $K_0$ in terms of a freely floating, sufficiently small, box which is not subject to any external forces apart from gravity, and which is freely falling under the action of the latter." However, this remark is not part of Pauli's version of the equivalence principle, which is in italic.

8) One can easily imagine a curved two-dimensional space as a surface immersed in Euclidean three-dimensional space. In the same way, Dirac [45] reasoned, one can have a curved four-dimensional space immersed in a flate space of a large number of dimensions. Such a curved space is called a Riemannian space. Dirac believed, "Einstein assumed that physical space is of this nature and thereby laid the foundation for his theory of gravitation." Therefore, Dirac continued, "For dealing with curved space one cannot introduced a rectilinear system of axes. One has to use curvilinear coordinates." The fact is, however, Einstein's curved physical space has a rectilinear system of axes due to physical reasons [26].

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