BOUND STATES OF PRIMORDIAL BLACK HOLES AND DARK MATTER

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ABSTRACT

We show that stable bound states of black holes, called holeums, having asymptotic freedom, an exclusion property and a nearly parameter-free gravitational analogue of the hydrogen atom spectrum would be formed in the early universe and in accelerator experiments. They are massive components of dark matter, a new source of gravitational waves and would form haloes around galaxies.

In this letter we consider the problem of the missing mass of the universe. In the early history of the universe large quantities of primordial black holes, henceforth called black holes, of various masses would be created. If the temperature T of the bigbang universe is much higher than $T_b = mc^2/k_B$ where m, c and k_B are the mass of a black hole, the speed of light in vacuum and the Boltzmann constant, respectively, the number density of such black holes would be proportional to $(k_BT)^3$ and they would be in intense fields. These conditions are conducive to the formation of their bound states, henceforth called holeums[1]. Once a part of a stable holeum a black hole is immune to the Hawking radiation just as a neutron in a stable nucleus is immune to decay. Since the holeums give off only gravitational radiation they remain undetected at present. Thus, they would form an important component of the dark matter in the universe. Since no consistent theory of quantum gravity with predictive power exists, we will use the Newtonian gravity and the Schrodinger equation for the purpose. The former is a weak field limit of general relativity and possesses unsurpassed simplicity and transparency[2] and the latter possesses exact solution to the problem at hand. We will deal with submicroscopic black holes of mass less than 100 keV/c^2 having a radius 10^{-58} m or less. They would form holeums of size 10^{-10} m or less. Now using the uncertainty principle it can be shown that if a particle is confined in a cube of side d then its de Broglie wavelength would be $3^{-1/2}$d which is comparable to d. Thus, the use of the quantum theory is mandatory. In this simple treatment we take the r^{-1} type gravitational potential as the effective potential between tow black holes for all r. This is done in view of the relative sizes of the black holes and their bound states mentioned above. It is further supported by the observation that the behaviour of a potential within a bound state does not seem to have a decisive influence on the parametes of the latter. For example, in quarkonia a Coulomb plus a linear, a logarithmic and a small positive power potential having radically different singularity structures at r = o give acceptable description of the charmonium and the bottonium spectra. The relative sizes of the nucleons and their bound state, the deuteron, and the dispensability of the hard core potential in the latter[3] all support but do not prove the observation.

Consider two black holes of masses m_1 and m_2 interacting through the gravitational potential

$$V(r) = -\hbar c\alpha_g / r \qquad (1)$$

$$\alpha_g = m_1 m_2 / m_P^2 \qquad (2)$$

$$m_P = (\hbar c/G)^{1/2} \qquad (3)$$

where m_P is the Planck mass and $\hbar$ and G are the Planck's constant h reduced by 2π and the universal constant of gravity, respectively. The Schrodinger equation is exactly solvable for this potential and the energy eigen values are given by

$$E_2 = -\mu_2 c^2 \alpha_2^{\,2} / (2n_2^{\,2}) \qquad (4)$$

where the reduced mass μ_2 is given by

$$1/\mu_2 = 1/m_1 + 1/m_2 \quad (5)$$

$$\alpha_2 = \alpha_g \quad (6)$$

Formally these are the eigen values of the hydrogen atom. n_2 is the principal quantum number of the holeum. The size r_2 and the mass M_2 of the holeum are given by

$$r_2 = (R_1 + R_2)\, n_2 / (2\alpha_2^{\ 2}) \quad (7)$$

$$M_2 = (m_1 + m_2)(1 - m_1 m_2 \alpha_2^{\ 2}) / (2n_2^2(m_1 + m_2)^2)) \quad (8)$$

where

$$R_i = 2m_i G/ c^2, \qquad i = 1,2 \quad (9)$$

are the radii of the black holes. Since the area $4\pi R^2$ of a black hole of a radius R never decreases and since the black holes must not overlap, therefore we must have

$$r_2 \geq R_1 + R_2 \quad (10)$$

for all n_2 and at all times. From Eqs. (7) and (10) we can prove the following theorem.

Theorem : A holeum will have a stable ground state, $n_2 = 1$, only if

$$\alpha_2^{\ 2} < ½ = \alpha_c^{\ 2} \quad (11)$$

otherwise it can only be in excited states given by

$$n_2 > 2\alpha_2^{\ 2} > 1 \quad (12)$$

The latter case will eventually result in coalescence of the black holes and destruction of the holeum[1]. Using Eqs. (2) and (6) we can rewrite Eq. (11) as

$$(m_1 m_2)^{½} < m_c = 2^{-¼}\, m_p \quad (13)$$

Here α_c and m_c are called the cosmic limits for the formation of stable holeums. In reference[1] we considered unstable holeums satisfying Eq. (12). In this paper we consider only the stable holeums satisfying Egrs. (11) and (13). Now a holeum of mass M_2, Eq. (8), picks up a black hole of mass m_3 to form a holeum of mass M_3 and so on. After a holeum has gobbled up k black holes the following situation obtains :

$$E_k = -\mu_k c^2 \alpha_k^{\ 2} / (2n_2^{\ 2}) \quad (14)$$

where the reduced mass μ_k is given by

$$1/\mu_k = 1/m_k + 1/M_{k-1} \quad (15)$$

and

$$\alpha_k = m_k\, M_{k-1} / m_p^{\ 2} \quad (16)$$

Here a bound state of k-1 black holes of mass M_{k-1} has picked up a black hole of mass m_k to form a bound state of mass M_k. Eq. (14) gives a "global view" of a k-holeum. It shows what is formally a hydrogen atom spectrum except that the fine structure constant $\alpha = e^2/hc$ is replaced by the gravitational coupling constant α_k. In a " local view " the following equations reveal all the k black holes it has gobbled up :

$$E_k = -(mc^2 \alpha_g^{\ 2} / 2n_k^2)((k-1)^3 / k)\, g_{k-1}^{\ 3} / f_{k-1} \quad (17)$$

$$g_{k-1} = 1 - \alpha_g^{\ 2} Z_{k-1} / (2(k-1)) \quad (18)$$

$$f_{k-1} = 1 - \alpha_g^{\ 2} Z_{k-1} / (2k) \quad (19)$$

$$Z_{k-1} = \sum_{j=2}^{k-1} ((j-1)^3 / j\, n_j^2)\, g_{j-1}^{\ 3} / f_{j-1} \quad (20)$$

$$r_k = (kRn_k / (2\alpha_g^{\ 2}(k-1)^2))\, f_{k-1} / g_{k-1}^{\ 2} \quad (21)$$

$$M_k = kmg_k \quad (22)$$

$$\alpha_k = (k-1)\, \alpha_g\, g_{k-1} \quad (23)$$

For simplicity here we have specialized to the case of k identical black holes each of mass m and radius R.To produce the most compact and nonoverlapping bound state in Eq. (21) we must have

$$2(k-1)^2 \alpha_g^{\ 2} = 1 \quad (24)$$

Then it can be shown that

$$f_{k-1} \geq g_{k-1}^{\ 2} \quad (25)$$

From Eqs. (21) and (25) we have

$$r_k > k\, R\, n_k \quad (26)$$

for all k and n_k. Eq. (26) is a condition for the stability of a k-holeum. It is an exclusion condition similar to the Pauli exclusion principle.No two black holes can overlap each other at any time. It is amusing to note that Eq. (24) may be rewritten as

$$(m_p/m)^4 = 2(k-1)^2 = 2, 8, 18, 32 \ldots\ldots\ldots\ldots \quad (27)$$

Eqs. (24) and (27) signify the quantization of the coupling constant and the mass of the black holes, respectively and at the same time Eq. (27) gives the magic number sequence of atomic physics. The latter arises from spin ½ of the electrons and it gives the total number of electrons in a closed shell. Our black holes have no spin. Their nonoverlap and the maximum compactness of their bound states give Eqs. (24) and (26). Now if the size of a compact bound state is at all times greater than kRn_k for arbitrary n_k then it can only be a straight linear chain or a straight string. In an endless, ring-like, structure all the black holes in an identical quantum state are equivalent. By reductio ad absurdum it can be proved that a holeum is not an endless, ring-like, structure. Now we make Taylor series expansion of f_{k-1} and g_{k-1} treating α_g^2 as a small parameter. Keeping only the first two terms we get

$$E_{nn'} = -(R_g/n^2)(1 - 1/6n'^2 + 0(k^{-1})) \quad (28)$$

$$M_{nn'} = M_g(1 - 1/12n'^2 - 1/(4k^{½} n^2) + 0(k^{-1})) \quad (29)$$

$$\alpha_{nn'} = \alpha_c(1 - 1/12n'^2 + 0(k^{-1})) \quad (30)$$

$$r_{nn'} = nr(1 + 1/12n'^2 + o(k^{-1})) \quad (31)$$

$$R_g = m_c c^2 (k-1)^{½}/4k \quad (32)$$

$$M_g = m_c k^{½} \quad (33)$$

$$r_g = R_c k^{½} \quad (34)$$

$$R_c = 2m_c G/c^2 \quad (35)$$

$$\alpha_{jn} = \alpha_c(k-1)^{-1}(j-1)(1 - 1/12n'^2 + 0(k^{-1})) \quad (36)$$

Here $n_2 = n_3 = \ldots\ldots\ldots\ldots\ldots\ldots n_{k-1} = n'$ and $n_k = n$. The R_g in Eq. (32) is the gravitational Rydberg constant. Here we have assumed $k >> 2$ but exact equations can be written down for $k = 2,3,4$ after which they become cumbersome. Every hydrogen atom-like energy level characterized by $n_k = n$ is spread out into a band whose lower and upper bounds correspond to $n' = \infty$ and $n' = 1$, respectively. Here $n' = 1$ corresponds to all the inner subholeums being in their ground states and $n' = \infty$ corresponds to their complete dissociation leaving only the two outermost black holes to define the holeum. For $k = 2$ there is no spreading of energy levels into bands and the spectrum is formally identical with the line spectrum of the hydrogen atom. Spreading begins with $k = 3$ where it is about 3.5 % and reaches an upper limit of 16.7 % for $k>>2$. Normally all the inner subholeums will be in their ground states $n'=1$, and the system will be described by only one quantum number n. Greater excitation may lead to the raising of n_{k-1} to values greater than unity. And the system will be described by n_k and n_{k-1} and all the other n's set equal to unity and so on. From Eq. (36) it is clear that the gravitational coupling α_{jn} of the jth subholeum from the centre goes like k^{-1} for small values of $j << k$ and that it tends to the cosmic limit α_c as j k for $k >> 2$. This means that the black holes near the centre of the holeum display asymptotic freedom while those near the edge, j k, show a saturation of their coupling strengths to the upper bound α_c. In this theory there are no free parameters except k which is related to an appropriate temperature of the bigbang universe. The temperature at which the rate of reaction of holeum with black holes falls below the rate of expansion of the bigbang universe the holeum stops picking up more black holes and its k value freezes. In a treatment similar to that of nucleosynthesis we assume that the k-holeums are in thermal equilibrium at a temperature $T << M_k c^2/k_B$ and their number density N_k is given by

$$N_k = g_k(M_k k_B T/(2\pi\hbar^2))^{3/2} \exp((\mu - M_k c^2))/k_B T) \quad (37)$$

where g_k is the degeneracy factor and $\mu_k = k\mu$ where is μ the chemical potential of a black hole interacting with another one. Eliminating μ between Eq. (37) and a similar one for N, the number density of black holes of mass m, we get

$$N_k = (N/4)g_k k^{3/2} \xi^{k-1} \exp(B_k/k_B T) \quad (38)$$

where

$$\xi = (N/4)(2\pi\hbar^2)/(mk_B T)^{3/2} \quad (39)$$

$$B_k = kmc^2 - M_k c^2 \cong m_c c^2 k^{½}/12n'^2 \quad (40)$$

The smallnes of ξ^{k-1} is compensated for by the largeness of $\exp(B_k/k_B T)$ at a temperature T_k given by

$$T_k = B_k / (k_B(k-1) \ |\ln \ \xi|\ ) \qquad (41)$$

Since no empirical data exists on the reaction rates or the number densities of holeums and the black holes we consider two widely different values : $N = 10^{-10}$ per cm^3 and $N = 10^{10}$ per cm^3 at 3^0 k. This is to be compared with 10^{-6} per cm^3 which is the number density of photons at 3^0 k. For n'=n=1 and $k_BT = 10$ ev, say, we calculate from Eq. (41) k to be 5.4449×10^{47} and 1.5178×10^{48} for the two values of N referred to above, respectively. This is like a galaxy of black holes in a holeum. The former value of k corresponds to $M_k = 1.2391 \times 10^{16}$ kg, $r_k =$ 0.22 A and m = 14.00 keV/c^2. Thus, we have an atom-sized holeum having a mass that is a hundred-millionth that of the earth and containing a galaxy of black holes of mass 14.00 keV/c^2 each. These holeums have only the gravitational interactions and will, therefore, constitute an important component of dark matter in the universe. As opposed to this the particles of ordinary matter have all the four fundamental interactions. Hence clouds of holeums will separate from those of the ordinary matter and will form invisible haloes around the galaxies. The enormous masses of the holeums and their submicroscopic sizes would form impregnable walls around the galaxies. The lower and the upper bounds for the frequencies of gravitational radiation emitted by a holeum in the Lyman series n 1 are $7\nu_0$ /12 and ν_0 respectively, where

$$\nu_0 = R_g / h = 6.22 \times 10^{40} (k-1)^{1/2} k^{-1} \qquad (42)$$

For the atom-sized holeum mentioned above this is about 10^{15} Hz. It can be shown that the maximum population density of holeums with k > > 2 and $k_BT = 10$ eV, say, occurs at n= 5 for n' = ∞ and at n' = 4.56 ≅ 5 for n' = 1. Thus the quantum numbers of subholeums make little difference here. For smaller holeums having k=2,3,4 etc. the maximum population density occurs at about $n = 10^{10}$ and, again the values of n' make little difference. Thus, the smaller holeums are in states of much greater excitation and are more prone to radiate than the larger ones. In the transitions of the type n+1 n with n > > 1 the maximum frequency emitted is given by

$$\nu_{max} \cong 2R_g / n^3 = 1.244 \times 10^{41} (k-1)^{1/2} k^{-1} n^{-3} \qquad (43)$$

For large values of k and n this could well be in the low frequency domain ν = 1500 to 1600 Hz where detectors are being built[4]. Once the gravitational radiation is detected k can be determined, for example, by experimentally finding the lower or the upper limit of the Lyman series. This, in turn, completely determines all the attributes of the holeums emitting this radiation.

In the High energy accelerator experiments involving the collisions of particles and antiparticles highly excited states of vacuum, mini bigbangs, are produced. At sufficiently high energies this may lead to production of micro black holes and their stable holeums which may be manifested in the form of missing mass, momentum, etc.

We have shown that in the early universe large quantities of black holes would have formed stable gravitational bound states called holeums. The latter would be in the form of straight strings. The black holes in these strings obey asymptotic freedom and have an exclusion property. The energy spectrum of a holeum is a gravitational analogue of the hydrogen atom spectrum with lines replaced by bands. The holems would form haloes around the normal galaxies. Their gravitational radiation can be detected when detectors are built.

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