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The Intrinsic Euclidean Structure in General Relativity

and

Einstein's Physical Space

C. Y. Lo

Applied and Pure Research Institute
17 Newcastle Drive, Nashua, NH 03060

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Abstract

In general relativity, Einstein's measuring instruments are resting but in a free falling state, and measurements are performed according to Einstein's equivalence principle. On the other hand, if the measuring instruments are resting and are attached to the frame of reference, since the measuring instruments and the coordinates being measured are under the same influence of gravity, a Euclidean space structure emerges as if gravity did not exist. For example, the Schwarzschild solution has a complementary Euclidean structure. In agreement with observations, this notion of Euclidean structure clarifies the meaning of Einstein's physical space, and explains the previous failures in obtaining a space-time metric for a uniformly accelerated frame. Nevertheless, Pauli's "equivalence principle" that ignores physical requirements beyond metric signature, leads to the incorrect belief that space-time coordinates have no physical meanings. To demonstrate the inadequacy of Pauli's version, it is shown that the local distance formula derived by Landau & Lifshitz is invalid. This illustrates that theories based on merely the existence of local Minkowski space must be reviewed according to Einstein's equivalence principle. Moreover, this analysis shows that once the frame of reference is chosen, the gauge has been determined. Experimental test and related issues are discussed.

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1. Introduction

Some theorists including Pauli [1] believed that "it is necessary to abandon Euclidean geometry" because "Einstein showed for example of a rotating reference system, the time intervals and spatial distances in non-Galilean systems cannot just be determined by means of a clock and rigid standard measuring rod." However, the fact is that Euclidean geometry is abandoned only in the invariant line element [2,3]. However, as shown in the Schwarzschild solution [2], the Euclidean structure is necessarily preserved in Einstein's physical Riemannian space. It will be shown that such a structure is related to measurements, which are different from the measurement of a line element that is related to Einstein's equivalence principle.

For the four-dimensional continuum (x, y, z, t) of physics in special relativity, the invariant line element has the form

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \tag{1}$$

where the units are centimeter and second, and c is the speed of light, 3×10^{10} cm/sec. Thus, invariance of Euclidean geometry has been abandoned already in special relativity, and there are Lorentz contraction and time dilation [3]. However, a Euclidean structure is preserved since the distance d (P_1, P_2) of points P_1 (x_1, y_1, z_1) and P_2 (x_2, y_2, z_2) in the frame of reference is still

$$d(P_1, P_2) = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}. \tag{2}$$

It will be shown that, in a different way, a Euclidean structure is actually preserved even in general relativity (see Section 4). Such a Euclidean structure would make a distinct class of Riemannian spaces. A Riemannian space-time together with its Euclidean structure shall be called the *Einstein Space* named after its creator [3].

In general relativity, the invariant line element is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \tag{3}$$

where $g_{00} > 0$ and $g_{\mu\nu}$ is a general space-time metric in a Riemannian physical space[1)]. Note that form (1) is a special case of (3), and form (1) is used in the infinitesimal form of Einstein's equivalence principle [2,3]. Thus, form (1) is not abandoned at all, and what has been abandoned is that form (1) be considered as an invariant.

However, the transformation from (3) to (1) is generally not global. Thus, it seems that in general it would not be possible to have a simple global distance formula as (2). In general relativity, a local distance formula would be generally

$$dl^2 = -g_{\alpha\beta}\, dx^\alpha\, dx^\beta \qquad \text{where} \quad \alpha, \beta = 1, 2, 3 \tag{4}$$

even if $g_{0\alpha} = 0$. Since metric elements $g_{\alpha\beta}$ are not constants, a global distance formula derived from (4) is not possible. A difficulty related to (4) is that the meanings of spatial coordinates are not clear since *dl* depends on $g_{\alpha\beta}$ that would change according to the distribution of matter. Nevertheless, in Einstein's calculation, it is necessary to choose a frame of reference a priori. It will be shown that this is justified in terms of the notion of Euclidean structure.

For example, consider a solution of metric with coordinates (x, y, z, t) in the isotropic form [4],

$$ds^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]dt^2 - (1 + M\kappa/2r)^4(dx^2 + dy^2 + dz^2) \tag{5}$$

where M is the total mass of a spherical mass distribution with the center at the origin of the frame of reference, (x, y, z) are its coordinates, $r = [x^2 + y^2 + z^2]^{1/2}$, and κ is a coupling constant. Note also that the metric is a function of r, which is defined in terms of the Euclidean characteristics of subspace (x, y, z). Therefore, the Euclidean structure of the frame of reference (x, y, z) is necessarily included in such a Riemannian space-time of Einstein (see also Section 4).

Moreover, this example illustrates that the existence of a Euclidean structure does not necessarily mean the existence of a Euclidean subspace in (5). To understand the physical meaning of the Euclidean structure in connection with the metric, we must first clarify what "measure" means in relation to Einstein's equivalence principle (see Sections 2 and 3).

A popular version of the equivalence principle expressed by Pauli [1] is the following:

> "For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."

But, Einstein strongly objected this version and he argued that, for some cases, no matter how small the world region, special relativity would not exactly hold[2)] as reported in details by J. Norton [5]. Nevertheless, in current literature Pauli's interpretation is incorrectly [5] considered as equivalent to Einstein's equivalence principle (see also Section 7).

However, the fact is Pauli's version cannot be considered as equivalent to Einstein's. Einstein's version requires additionally: i) "the special theory of relativity applies to the case of the absence of a gravitational field [3, p.115]" and ii) a local Minkowski space is obtained by choosing the acceleration. Einstein [3, p.118] wrote, "... we must choose the acceleration of the infinitely small ("local") system of coordinates so that no gravitational field occurs; this is possible for an infinitely small region." Moreover, since physical conditions other than metric signature are ignored in Pauli's version, such a coordinate system

may be physically unrealizable. For instance, if a physical requirement such as the principle of causality[3)] is violated. Then, a particle resting on its frame of reference would also mean that a physical principle is violated (see section 5).

The frame of reference, as pointed out by Fock [6], is crucial in Einstein's general relativity. Einstein chooses the frame of reference, and the time-coordinate is determined by orthogonality. However, some theorists [6,7] considered Einstein's equivalence principle is not well defined on the ground that the frame of reference is ill defined because the notion of distance is not clear. Moreover, Einstein contributed to such misunderstand in 1916 by claiming the over extended physical general covariance with the support of false arguments [3], which he later dropped from his book [2]. This will be pointed out and discussed in Section 3. Understandably, from the viewpoint of general covariance, they do not see the existence of a Euclidean structure.

Nevertheless, the inadequacy of Pauli's version for a world region of a physical space seemed not a serious problem until it is incorrectly claimed [8] that the existence of Local Minkowski space had replaced Einstein's equivalence principle such that any Lorentz manifold could be justified as valid in physics. This replacement of Einstein's equivalence principle distorted general relativity. For instance, Einstein's notion of physical space [2,3,5] has been ignored to the point that professional relativists often ask what is a physical space. In this paper, to illustrate the problem of such a distortion, it will be shown that Pauli's version was the source of an invalid formula of distance (see Section 3) derived by Landau & Lifshitz [9].

Based on the misconception that a frame of reference was necessarily associated with a Euclidean subspace, Fock [6] blamed his failure in obtaining a space-time metric for a uniformly accelerated system as an intrinsic problem of Einstein's equivalence principle. Accordingly, Fock claimed also that the principle of general relativity were invalid. To be aware of the seriousness of this problem, one should note that Fock's followers include Wheeler and his students Ohanian and Ruffini [7]. The calculation of the space-time metric corresponding to an accelerated frame will be presented in a separate paper [10].

In view of the fact that Pauli's version was popularly in the literature, a problem such as the invalid formula of Landau & Lifshitz would be just a drop of water in the bucket. Note that their invalid formula was still followed with great faith [11-13] since its invalidity had not been found although their book is well known [14]. The purpose of this example is to demonstrate that it is necessary to review many of the existing theories in terms of Einstein's equivalence principle.

Moreover, it was believed that a gauge condition would be arbitrary although gauge is related to a choice of coordinates. This analysis shows that once the frame of reference is chosen, the space-time coordinates are determined (section3), and therefore the gauge has also been determined (Section 6) To show these problems clearly, it would be necessary to understand first Einstein's equivalence principle starting from the beginning.

2. Einstein's Equivalence Principle, the Principle of General Relativity, and Einstein's Riemannian Space

In general relativity, virtual measurements are performed by utilizing Einstein's equivalence principle, as shown in Einstein's calculation of the time dilation and spatial contraction [2,3]. Thus, we should clarify what Einstein's equivalence principle actually is. In 1911, the initial form of this principle is the assumption [3] that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system K' accelerated by γ in the opposite direction, can be extended to other physical processes. This initial form was further elaborated for a curved space due to additionally the principle of general relativity.

However, Einstein's equivalence principle was often questioned because of inadequate understanding. A noted theorist Synge [15] professed his misunderstandings on Einstein's equivalence principle as follows:

> "...I have never been able to understand this principle...Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's world line...The Principle of Equivalence performed the essential office of midwife at the birth of general relativity...I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced."

Currently, such misunderstandings persist after all these years. For instance, Thorne [14] criticized Einstein,

> "In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you are (and your reference frame) are very small."

Apparently, Thorne paid little attention to Einstein's correspondence on this problem. For instance, the question of tidal forces has been clearly answered by Einstein. For instance, in his July 12, 1953 letter to A. Rehtz [16] Einstein wrote,

> "The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Einstein [5] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^l_{ik} (field strength), not the non-vanishing of the R_{iklm}." and no gravity is a special case of gravity[4]. The viewpoint that gravity must be associated with the non-vanishing of the R_{iklm}, instead of just the non-vanishing of the Γ^l_{ik}, can be traced back at least to Newton's theory of gravity. This difference in philosophy has important consequence in physics. For instance, it is Einstein's viewpoint that leads to the geodesic equation being identified as the equation of motion of gravity, and

subsequently the notion of a curved space-time. It will be shown, their criticisms are due to inadequate understanding of Einstein's equivalence principle, which plays a crucial role in many aspects of general relativity (see also Sections 4-7).

It should be noted that Einstein insisted, throughout his life, on the fundamental importance of the principle to his general theory of relativity [5]. Norton pointed out that Einstein's insistence on this point has created a puzzle for philosophers and historians of science [5]. This shows how much was Einstein's principle being understood in terms of physics.

Moreover, some [6,7] considered Einstein's failure in obtaining a valid formula for light bending in 1911 as a deficiency of Einstein's principle, in spite of his success in 1915. Fock [6] even supported their belief with explicit calculations. However, his calculation must be invalid since Maxwell-Newton Approximation, the linear equation for weak gravity[5)] due to massive sources can be derived directly from Einstein's equivalence principle [17]. A main problem in Fock's calculation is his implicit assumption that the related Riemannian space should have a Euclidean subspace. Apparently, he fails to see that the frame of reference needs to be related to only a Euclidean structure (Section 3).

Einstein was not entirely happy with special relativity. Einstein believes, "The law of physics must be of such a nature that they apply to systems of reference in any kind of motion (principle of general relativity)." From the viewpoint of the principle of general relativity, since the effects of a uniformly rotating cannot be equivalent to the effects of a linear acceleration, *Einstein's principle of equivalence, if exact, is really the equivalence of the effects of an accelerated frame to a related kind of uniform gravity whereas others incorrectly perceived that any gravity is equivalent to a uniformly accelerated frame.* In other words, Einstein's initial equivalence principle must be an example to illustrate an idealized case.

These two principles also lead to [2,3] regarding the geodesic equation,

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta} \frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \tag{6a}$$

where

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \qquad \Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}) g^{\mu\nu}/2, \tag{6b}$$

and $g_{\mu\nu}$ is the space-time metric, as the equation of motion for a particle under the influence of only gravity since the acceleration to a particle under gravity is independent of the mass (or equivalently $m_I = m_G$). Thus, gravity is due to ten metric elements, and this can be used to derive the linear field equation for weak gravity of massive matter [17]. For a resting particle, the acceleration is due to $\Gamma^\mu{}_{tt}$ ($\mu \neq t$), and this is a physical restriction on $g_{\mu\nu}$ a non-constant space-time metric.

On the other hand, if gravity must be associated with the non-vanishing of the R_{iklm} as some argued [6,7,11-15], the justification for the geodesic equation as the equation of motion would be broken, and so far there is no alternative valid justification. One might argue that the geodesic equation would be derived from the field equation. To do this, one must first derive the field equation independent of the geodesic equation. Ohanian and Ruffini [7] tried to derive the 1915 Einstein equation from their linear field equation. Unfortunately, both their derivation and their linear equation, which is based on their notion of gauge, are found to be invalid [17]. In practice, the geodesic also plays an important role because it is used to decide whether the metric is valid in physics. For instance, Einstein used it to obtain the perihelion of Mercury [2,3].

In deriving his formula for the bending of light rays, Einstein [2,3] used the infinitesimal form of his principle[6)], which is a generalization of the initial form that has a frame of reference [3]. An important but often omitted point is that *Einstein's equivalence principle is applicable only to a physical space*[1)] in which all physical requirements are sufficiently satisfied *since his Riemannian space models the reality*. This will be illustrated in analyzing the case of Einstein's rotating disk.

Einstein considered a Galilean (inertial) system of reference K (x, y, z, t) and a system K' (x', y', z', t') in uniform rotation Ω relatively to K. The origins of both systems and their axes of z and z' permanently coincide. For reason of symmetry, a circle around the origin in the x-y plane of K may at the same time be regarded as a circle in the x'-y' plane of K'. Then, *according to special relativity*, in the x-y plane and the x'-y' plane, the metrics of K and K' [2, 11] are respectively the following:

$$ds^2 = c^2\,dt^2 - dr^2 - r^2\,d\phi^2 - dz^2 \qquad \text{where} \quad x = r\cos\phi, \quad y = r\sin\phi, \tag{7a}$$

and

$$ds^2 = (c^2 - \Omega^2 r'^2)\,dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\,d\phi'^2 - dz'^2, \tag{7b}$$

where

$$x' = r'\cos\phi', \quad \text{and} \quad y' = r'\sin\phi'. \tag{7c}$$

Then,

$$\oint ds = (1 - \Omega^2 r'^2/c^2)^{-1/2} r' \int_0^{2\pi} d\phi' = 2\pi r'(1 - \Omega^2 r'^2/c^2)^{-1/2} \tag{8}$$

would be the circumstance of a circle of radius r' (= r) for an observer in K'. Thus, Einstein concluded that with a measuring rod at rest relatively to K', the quotient of circumstances over diameter would be greater than π, and Euclidean geometry therefore breaks down (in the metric [7b] but preserves in [7c]) in relation to the system K' (see also Sections 3 & 4).

Moreover, as Einstein pointed out, "an observer at the common origin of co-ordinates, capable of observing the clock at the circumferences by means of light, would therefore see it lagging behind the clock beside him". Einstein [3] continued, "*So, he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be.*" Thus, Einstein

concluded, "In general theory of relativity, space and time cannot be defined in such a way that differences of the spatial coordinates can be directly measured (in the way Einstein defined) by the unit measuring-rod, or differences in the time co-ordinate by a stand clock." Concurrently Einstein, in effect, defined a *physical space-time coordinate system together with its metric* that is related to local clock rates and local spatial measurements (see also Section 3). In other words, Einstein has established the notion of a physical space[1)] where all physical requirements are sufficiently satisfied.

According to the principle of equivalence, K' may also be considered as a system at rest, with respect to which there is a gravitational field (field of centrifugal force, and force of Coriolis) [3]. Thus *the equivalence principle enables an extension of the principle of relativity to accelerated motion*. This example illustrates also that Einstein's notion of gravity needs not be related to a source, but can be just related to acceleration (as its cause). For metric (7b), the static acceleration is from $\partial_\mu g_{tt}$, a spatial derivative to the time-time metric component. This suggests that g_{tt} corresponds the gravitational potential in Newtonian theory, and this is confirmed by subsequent calculations [2,3]. In short, the rotating disk case shows not only that the space-time continuum is a Riemannian Space with a Lorentz metric, but also that the equation of motion for gravity is the geodesic equation. Moreover, in Einstein's theory, the principle of general relativity is the physical basis of covariance.

3. Covariance and Physical Space-Time Coordinate Systems

In Einstein's theory, as shown by K (x, y, z, t) and K' (x', y', z', t'), it is clear that the coordinates of a space-time coordinate system have definite physical meanings. Here, it will be shown that the notion that coordinates have no physical meaning comes from confusing an arbitrary coordinate system (which needs not have a physical meaning) for a mathematical calculation with a space-time coordinate system (which does have a physical meaning) in physics.

In a Riemannian space, since the metric $g_{\mu\nu}$ is not restricted as in special relativity, tensor equations are covariant with respect to any substitutions whatever (generally covariant). Moreover if the space-time continuum in physics is a Riemannian space, there are two advantages: i) Physical laws (tensor equation) would satisfy the principle of general relativity. ii) Calculations can be carried out in an arbitrary coordinate system. In the 1916 paper, Einstein was somewhat carried away by this new-found freedom. Instead of recognizing an arbitrary coordinate system as a mathematical tool, he sought to justify this freedom in terms of physics. To argue for unrestricted covariance, he wrote [3]:

> "That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of ma-

terial points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measuring are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences."

However, this seems to be incompatible with his earlier statement [3], "So, he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be."

Moreover, while all verifications amount to a determination of space-time coincidences, to predict such coincidences, one must be able to relate events of different locations in a definite manner. (Examples are the gravitational red shifts and the light bending.) If a space-time coordinate system is related to objective physical measurements, it must have physical meanings. In fact, as early as 1918, unrestricted general covariance was questioned by Lenard [18]. As Eddington [19] pointed out, "space is not a lot of points close together; it is a lot of distances interlocked." For physical considerations, one must have not only just a mathematical coordinate system, but also a physical space-time coordinate system.

Understandably, Einstein [2] dropped the above invalid justification later, and remarked, "As in special theory of relativity, we have to discriminate between time-like and space-like line elements in the four-dimensional continuum; owing to the change of sign introduced, time-like line elements have a real, space-like line elements an imaginary ds. The time-like ds can be measured directly by a suitably chosen clock." Thus, a space-coordinate and the time-coordinates in physics are not exchangeable as Hawking [20] claimed since they have distinct characteristics and physical meanings. Einstein also praised Eddington's book to be the finest presentation of the subject ever written [21].

Note that Einstein's theory is based on his notion of a physical space[1)], which has a frame of reference and local time coordinates that are orthogonal to the frame. To illustrate the difference between the physical space and a manifold in mathematics, consider the coordinate transformation to the uniformly rotating disk, in terms of the time t of K as follows [11]:

$$x = x' \cos \Omega t - y' \sin \Omega t, \quad y = x' \sin \Omega t + y' \cos \Omega t, \quad \text{and} \quad z = z', \tag{9a}$$

or

$$r = r', \quad z = z'. \quad \phi = \phi' + \Omega t, \tag{9b}$$

in cylindrical coordinate systems of K and K', where Ω is the angular velocity. Note that both (x, y, z) and (x', y', z') are Euclidean subspaces. Then substituting the new coordinates (x', y', z') or (r', ϕ', z') to metric (7a), we obtain a metric

$$ds^2 = (c^2 - \Omega^2 r'^2)\, dt^2 - 2\Omega r'^2\, d\phi' dt - dr'^2 - r'^2\, d\phi'^2 - dz'^2 \qquad (7b')$$

for the coordinate system K* (x', y', z', t). However, the mathematical system K* (x', y', z', t) is not a physical space-time coordinate system for the uniformly rotating disk K' because what measured in a resting local clock is time *t'* but not time *t* that remains associating with the inertial frame of reference K. In other words, (7b') failed "to define time in such a way that the rate of a clock depends upon where the clock may be [3]", and thus metric (7b') together with its coordinates K* is not a space-time coordinate system, as Einstein defined, that can be used for physical measurement and therefore physical interpretation.

Moreover, since a physical principle is not satisfied in K*, the equivalence principle is not applicable. It will be shown that this principle is, in fact, not satisfied in K*. Nevertheless, as shown by Zel'dovich & Novikov [11], it is possible to recover metric (7b) that represents local measurements of time and distance from the mathematical metric (7b') alone (see also Section 5). This illustrates that one can start with an arbitrary mathematical coordinate system.

To obtain a physical transformation for the time t' of the rotating disk, a comparison of (7b) and (7b') leads to,

$$d\phi' = d\phi - \Omega dt\ ; \qquad (10a)$$

and

$$cdt' = [cdt - (r\Omega/c) r d\phi][1 - (r\Omega/c)^2]^{-1}\ . \qquad (10b)$$

or

$$cdt = cdt' + (r\Omega/c) r d\phi' [1 - (r\Omega/c)^2]^{-1}\ . \qquad (10c)$$

Note that (10c), which modifies the time coordinate from t to t', transforms (7b') to (7b). Now, (7b) is clearly related to (7a).

The factor $[1 - (r\Omega/c)^2]^{-1}$ in (10) is due to time dilation and spatial contraction manifested in metric (7b). Let us verify that the time dilation and the spatial contraction are results due to comparisons with a clock and a measuring rod in relatively rest at the beginning of a free fall. According to Einstein's equivalence principle such a coordinate system is locally Minkowski. To verify this, consider a particle P resting at (r', ϕ', z'). Then, P has the velocity of Ωr in the ϕ'-direction, which is denoted by dx''. It follows that the Lorentz coordinate transformation is,

$$r d\phi = [1 - (r\Omega/c)^2]^{-1/2}\, [dx'' + r\Omega dt'']\ ; \qquad (11a)$$

and

$$cdt = [1 - (r\Omega/c)^2]^{-1/2}\, [cdt'' + (r\Omega/c) dx'']\ . \qquad (11b)$$

Then,

$$r d\phi' = [1 - (r\Omega/c)^2]^{1/2}\, dx''\ ; \qquad \text{and} \qquad cdt' = [1 - (r\Omega/c)^2]^{-1/2}\, cdt''\ , \qquad (12a)$$

and

$$ds^2 = c^2\, dt''^2 - dx''^2 - dr'^2 - dz'^2 . \qquad (12b)$$

These are exactly the time dilation and spatial contraction as measured. This illustrates that a particle resting at K', can attached to a local Minkowski space.

Thus, this is an example that Einstein's version of infinitesimal equivalence principle is satisfied. In addition, a light speed at r' ($\neq 0$) observed in system K' would be smaller than c due to the time dilation effect of gravity. The light speed is even smaller in the φ-direction. In other words, a light speed can decrease more after a velocity $\Omega r'$ is "added to".

However, for the coordinate system K* (x', y', z', t), the question of time dilation is complicated because Einstein's equivalence principle is not applicable. Nevertheless, let us assume that the Einstein's equivalence principle could be applied to K*. Mathematically, for a particle P resting at K*, the state vector of P is (0, 0, 0, dt). According to (10c), P is also resting at K' with a state vector (0,0,0,dt'). Then the local Minkowski space for P is identical to (12b). It thus follows that

$$dx'' = [1 - (r\Omega/c)^2]^{-1/2}\, r d\phi' , \qquad (13a)$$

and

$$dt'' = [1 - (r\Omega/c)^2]^{1/2}\, dt - [1 - (r\Omega/c)^2]^{-1/2}(r\Omega/c^2)\, r d\phi'. \qquad (13b)$$

Thus,

$$dt = [1 - (r\Omega/c)^2]^{-1/2}\, dt'' \qquad (14)$$

would be considered as the time dilation since a clock rest at K* has $d\phi' = 0$. The problem of this derivation is that the parameter "t" is not the local time for the frame K' (x', y' z'). Thus, a time coordinate alone has no independent physical meaning.

This calculation confirms that Einstein's equivalence principle is applicable only to a physical space where all physical requirements are sufficiently satisfied. This illustrates also that *Pauli's version being satisfied,* is only a statement of mathematical properties (see also Appendix). A related problem is how do we know whether a manifold is a physical space. Then, we must do what Einstein [2,3] did, that is, examining consequences of the metric with physical requirements (see also Section 6).

Einstein [3] had remarked, "So there is nothing for it but to regards all imaginable systems of coordinates, in principle, as equally suitable for the description of nature. This comes to requiring that:- *The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant).*" However, while general covariance is mathematically valid for all tensor equations in a Riemannian space, this description of nature by the coordinate system K* (x', y', z', t) includes certain calculations but not physical interpretations. *Thus, in spite of general covariance, the freedom toward the physical space-time coordinate systems that can be used for physical interpretation is severely limited by his equivalence principle.* Special relativity has already taught us [3] that

some mathematical coordinate systems are not physically realizable and therefore cannot be used to describe nature. The same has been illustrated for general relativity. In short, nature can be described clearly and accurately only in terms of a valid space-time coordinate system with an appropriate space-time metric.

4. Riemannian Space-time, the Euclidean Structure, and the Einstein Spaces

The fact that Einstein's equivalence principle plays a crucial role in choosing a space-time coordinate system makes a four-dimension space-time continuum in physics a special kind of Riemannian space in addition to having a Lorenz metric. A feature in Einstein's theory of general relativity is that the Riemannian space-time has a Euclidean structure that serves as a frame of reference. Such a geometry structure shall be called a Euclid-Riemann space (or an Einstein space).

An Einstein space is a physical space if it sufficiently satisfies all physical requirements, including a time coordinate that is related to local time. It should be noted that if a change of the coordinate system were not considered, the frame K' (x', y', z') could be considered as a Euclidean space as if gravity did not exist. Thus, a Euclidean structure in a physical space is independent of gravity. Because of such independence, it becomes possible to make the meanings of coordinates very clear.

If a spatial measurement is performed with a measuring rod which is attached to the frame K' (x', y', z'), it would appear as Euclidean. (Note that the directional spatial contraction in metric (7b) is measured with a resting measuring rod in the state of free fall as shown by equation [11].) The physical reason is that since a measuring rod attached to the system K', would be under the same influence of gravity as what is being measured. In fact, it is based on this implicit assumption that the cylindrical coordinate system (r', ϕ', z') is well defined in K'. One may recall that although K* also has the Euclidean subspace (r', ϕ', z'), the resulting K' (x', y', z', t') no longer has a Euclidean subspace. But, the metric (7b) still retains a Euclidean structure. Its existence would merge as a Euclidean subspace if one reduces the cause of gravity to zero, in case of metric (7b), i.e., $\Omega = 0$.

One of the difficulties in understanding Einstein's theory is that the notion of Euclidean structure has not been explicitly explained. For instance, Fock [6] failed in understanding Einstein's equivalence principle because he believed that an accelerated frame must be related to a Lorentz manifold that has a Euclidean subspace. Perhaps, the popularity of Pauli's [1] version, in spite of Einstein's objection [5], is due to that Pauli omitted the frame of reference and related acceleration.

A valid space-time coordinate system is crucial for a physical space. The system K* (x', y', z', t) also has the same spatial coordinates, but the time t is not associated with the frame K' (x', y', z'). Consequently, K* is not a physical space-time since it fails some physical requirements. Thus, an Einstein space is a Riemannian space with a Euclidean structure, and it is a physical space if it sufficiently satisfies all physical requirements, including Einstein's equivalence principle.

Moreover, the Euclidean structure is meaningful in physics only in a physical space. However, mathematically the existence of a Euclidean structure has to be assumed *a priori*. Then, validity of the existence of a Euclidean structure must be verified because such a structure may be incompatible with the Riemannian space.

For example, consider a manifold L (x, y, z, t) and its metric,

$$ds^2 = 4c^2dt^2 - dx^2 - dy^2 - 9dz^2 , \qquad (15)$$

with time unit in second and length unit centimeter, and $c = 3x10^{10}$ cm/sec. Metric (15) implies that the "light speed" in the x-direction is 2c. Since L is clearly not a physical space, it does not make sense to assume the existence of a Euclidean structure.

Mathematically, one can obtain a local Minkowski space (dX, dY, dZ, dT) with the local coordinate transformation,

$$dX = dx, \qquad dY = dy, \qquad dZ = 3dz , \qquad \text{but} \qquad dT = 2\, dt . \qquad (16)$$

Thus, Pauli's version is satisfied. However, Einstein's equivalence principle is not satisfied since there is no acceleration (since all the Christoffel symbols are zero) to choose from such that a local Minkowski space can be obtained. Moreover, although the gravitational field is absent, special relativity is not applicable in L since L is not a Minkowski space.

Nevertheless, one might argue that a rescaling, 2t = t' and 3z = z' would transform (15) to $ds^2 = c^2dt'^2 - dx^2 - dy^2 - dz'^2$. However, this new metric form does not make L a Minkowski space since the unit of t' is ½ second. Note that a rescaling has no physical content, but the local coordinate transformation (16) means the ratio of two local clocks (see Section 3). For a particle P resting at a point (x_0, y_0, z_0) in L at time t_0, the coordinates of P are (x_0, y_0, z_0, t) at time t. The local coordinate transformation (16) is, of course, not a rescaling of units. However, since there is no gravity or relative velocity, there is no physical cause that makes a clock rate changes. Thus, the principle of causality is violated, and (15) is not valid in physics.

This example illustrates that it is necessary to verify that the existence of a Euclidean structure is valid in physics. A physical Euclidean structure can exist only in a physical space where Einstein's equivalence principle is satisfied.

5. An Invalid Formula for Local Distance and Pauli's Version of Equivalence Principle.

The derivation of Zel'dovich & Novikov [11], though has the same result, is based on Landau and Lifshitz [9]. They stated that the final formula for the square of the spatial distance dl^2 is.

$$dl^2 = -[g_{\alpha\beta} - g_{0\alpha}g_{0\beta}/g_{00}]\, dx^\alpha dx^\beta \qquad \text{where} \quad \alpha, \beta = 1, 2, 3 \tag{17a}$$

if

$$ds^2 = g_{00}(dx^0)^2 + 2g_{0\alpha}\, dx^0 dx^\alpha + g_{\alpha\beta}\, dx^\alpha dx^\beta. \tag{17b}$$

This formula was derived also by Liu and Yu [12,13]. For the uniformly rotating disk, from (7b') one obtains metric (7b) and

$$dl^2 = dr'^2 + (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\, d\phi'^2 + dz'^2 ., \tag{18}$$

which agrees with metric (7b). However, formula (15), based on Pauli's version, is actually not generally valid.

The logic for the derivation of (15) is as follows: For an arbitrary metric $g_{\mu\nu}$, one has

$$ds^2 = g_{00}[dx^0 + g_{0\alpha}dx^\alpha/g_{00}]^2 + [g_{\alpha\beta} - g_{0\alpha}g_{0\beta}/g_{00}]\, dx^\alpha dx^\beta \tag{19a}$$

Then

$$ds^2 = g_{00}(dx')^2 + g'_{\alpha\beta}\, dx^\alpha dx^\beta \tag{19b}$$

where

$$dx'^0 = dx^0 + g_{0\alpha}dx^\alpha/g_{00} \quad (g_{00} > 0), \qquad \text{and} \qquad g'_{\alpha\beta} = g_{\alpha\beta} - g_{0\alpha}g_{0\beta}/g_{00}$$

are a new time coordinate and new metric elements. Thus, one can start with an arbitrary coordinate system[8)], and derive an orthogonal coordinate system. However, this may not always lead to a physical space with the same frame of reference.

The mistake comes form Pauli [1] who considered any Lorentz manifold with a proper metric signature as valid in physics. Without realizing that some coordinate systems may not be physically realizable, to examine the passage of time: at a given point, it becomes obvious to him that $dx^1 = dx^2 = dx^3 = 0$ [11-13]. Then, ds will be the time separation between two nearby events; i.e., it will be the interval of time multiplied by the speed of light, $ds = c\, d\tau$. Consequently,

$$d\tau = [(g_{00})^{1/2}/c]\, dx^0 . \tag{20}$$

since in any reference frame formed by real bodies, it is always true that $g_{00} > 0$. Then, following from (19) and (20), (15a) would be obtained. For metric (7b'), $dr' = d\phi' = dz' = 0$ implies the particle is resting at the rotating disk, and from special relativity, we have $d\tau = [1 - (r\Omega/c)^2]^{1/2}\, dt'$, where t' is the local time. Thus, it is valid to consider dx^0/c in (20) as if the local time. However, g_{00} in (20) may not be valid for the local time of the local frame (dx^1, dx^2, dx^3).

To show that (17a) may not be valid, one may consider a metric whose local distance is known. For example, consider the Galilean transformation from an inertial system K (x, y, z, t) to the K' (x', y', z', t') coordinates,

$$x = x', \quad y = y', \quad z = z' - vt', \qquad \text{and} \qquad t = t', \tag{21a}$$

where v is a constant. Eq. (21a) transforms flat metric (1) to another constant Lorentz metric

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2. \tag{21b}$$

Thus, Pauli's version is satisfied in K'. However, since (21b) failed "to define time in such a way that the rate of a clock depends upon where the clock may be [3]", according to Einstein, (21b) as a space-time metric is invalid in general relativity. Thus, this example illustrates also that the notion of local time is crucial for a physical space-time.

Now, consider an observer P' resting at a point in K'. Mathematics ensures the existence of a local Minkowski space [2], the local orthogonal tetrad of P', whose direction $v_{p'}$ is (0,0,0,dt'). Then, the orthonormal vectors of the tetrad are

$$a_1 = (1,0,0,0), \quad a_2 = (0,1,0,0), \quad a_3 = (0,0,\alpha,\beta), \quad \text{and} \quad b_{p'} = (0,0,0,\gamma), \tag{22a}$$

where

$$\alpha = \gamma^{-1}, \ \beta = -\gamma v/c^2, \quad \text{and} \quad \gamma = (1 - v^2/c^2)^{-1/2}.$$

Then

$$dt' = \gamma(dT - v/c^2 dZ), \quad dz' = \gamma^{-1}dZ, \quad dx' = dX, \quad \text{and} \quad dy' = dY, \tag{22b}$$

is the corresponding transformation for the local Minkowski space (dX, dY, dZ, dT). Thus, (dx', dy', dz') and (dX, dY, dZ) share the same frame of reference. It follows that the spatial contraction in z'-direction violates the principle of causality. This failure manifests that the coordinate system K' is not physically realizable. Note, however, that (21a), and (22b) imply

$$dt = \gamma(dT - vc^{-2}dZ), \quad \text{and} \quad dZ = \gamma dz' = \gamma\,(dz + v\,dt). \tag{23}$$

This is just a Lorentz-Poincaré transformation. Transformation (22b) completes the transformation (21a) to (23).

Moreover, although P' is resting at (x', y', z'), as shown by (22b), the coordinate z' has gone through a contraction and the time-time component of the space-time metric has also changed. Thus, a particle resting at (x', y', z') is incompatible with the original g_{00}. This means that formula (15a) is invalid for this case. According to formula (15a), one has the local distance,

$$dl^2 = dx'^2 + dy'^2 + (1 - v^2/c^2)^{-1}dz'^2. \tag{24a}$$

and

$$ds^2 = (1 - v^2/c^2)\, c^2 dt''^2 + dx'^2 + dy'^2 + (1 - v^2/c^2)^{-1} dz'^2. \qquad (24b)$$

(24a) is incorrect for a local distance for the case of no gravity, and t" is not the local time either. From special relativity, we have $d\tau = dt'$, and it is invalid to consider dx^0/c in (20) as if the local time. Thus, the derivation is not generally valid.

Obviously, there are mathematical similarities between (24b) and (7b). In fact, (22b) corresponds to (11b) and the first relation of (12a). But, because of gravitational acceleration, the frame of reference of local Minkowski space is distinct from that of (7b), and (12a) is physically valid. These two examples illustrate that it is necessary to consider physics beyond the metric signature. In conclusion, *Einstein was right, and Pauli's version is inadequate in physics and would lead to incorrect claims.*

6. The Euclidean Structure and the Question of Gauge freedom

It has been shown in the case of a uniform rotating system that once the frame of reference is chosen, the space-time metric of the physical space is determined. Einstein [22] stated in 1919, the body to which events are spatially referred is called the coordinate system. On the other hand, many theorists, except Eddington [19], believed that a gauge can be arbitrarily chosen. Thus, it is necessary to clarify this issue of gauge.

In general relativity, the non-linear Einstein's field equation of 1915 version [2,3] for a space-time metric g_{ab} is

$$G_{ab} \equiv R_{ab} - \tfrac{1}{2} R\, g_{ab} = -KT(m)_{ab}\,, \qquad (25)$$

where R_{ab} is the Ricci curvature tensor, its source $T(m)_{ab}$ is the energy-stress tensor for massive matter and can depend on g_{ab}. However, among these 10 equations of tensor components, only six of them are independent, since

$$\nabla^a G_{ab} \equiv 0. \qquad (26)$$

Thus, to solve Einstein equation (25), four more conditions are needed. These four additional conditions are attributed as due to a certain freedom of choice of coordinates in the physical Riemannian space, and are called the gauge conditions. There are two extreme views on the question of gauge: i) Fock [6] argued that the harmonic gauge is the only physically valid gauge. This has been proven wrong for the case of a uniformly rotating system. ii) A popular view is that the gauge condition can be arbitrarily chosen although such a notion was rejected by Eddington [19]. This invalid notion was popular because it seems consistent with another incorrect notion that a space-time coordinate system can be arbitrary. This is incorrect because it has been shown that this assumption of arbitrary gauge can lead to the acceptance of unphysical solutions [23-25].

For a spherical static distribution of mass, due to the principle of causality [26], the exterior gravitational field has the central symmetry, which requires [4]

$$ds^2 = F(r)c^2dt^2 - D(r)\,dr^2 - C(r)(r^2d\theta^2 + r^2 \sin^2\theta\, d\varphi^2)\,. \qquad (27)$$

If D(r) = C(r) is assumed, one obtains the isotropic solution (5). However, this is not the only solution. If C(r) = 1 is assumed, one obtains another well-known solution, the Schwarzschild solution [3] with coordinate (x, y, z, t),

$$ds^2 = (1 - 2M\kappa/\rho)dt^2 - (1 - 2M\kappa/\rho)^{-1}d\rho^2 - \rho^2d\theta^2 - \rho^2 \sin^2\theta\, d\varphi^2 \qquad (28a)$$

where

$$\rho^2 = x^2 + y^2 + z^2\,, \qquad x = \rho \sin\theta \cos\varphi, \quad y = \rho \sin\theta \sin\varphi, \quad \text{and} \quad z = \rho \cos\theta \qquad (28b)$$

are defined in terms of Euclidean structure, and M is the total mass and κ is the coupling constant. Since the metric is defined in terms of coordinates ρ, θ, and φ, the Riemannian space is actually defined in terms of Euclidean characteristics of subspace (x, y, z). Moreover, the local time can also be considered as the rate of a local clock attached to that point.

Einstein [3] stated that the velocity of light is defined in the sense of Euclidean geometry. In spite of Einstein's declaration, some theorists were not aware of such a Euclidean structure, and incorrectly claimed the coordinate velocity of light has no physical meaning. Nevertheless, theorists [6,7,11-13] accept the deflection of light – a fact related to coordinates[8)].

For clarity of this analysis, let us follow some steps of Einstein's derivation in section 22 of the 1916 paper [3].

> "For a unit-measure of length laid "parallel" to the axis x, for example we should have to set $ds^2 = -1$; $dx_2 = dx_3 = dx_4 = 0$. Therefore $-1 = g_{11}\, dx_1^2$. If in addition, the unit-measure lies on the axis of x, the first of equations (70) gives $g_{11} = -(1 + a/r)$. From these two relations it follows that, correct to a first order of small quantities $dx = 1 - a/2r$."

In the above, the key words are "should have to set", and they actually mean applying his equivalence principle if it is valid. However, unlike the case of metric (7b) (to which validity of this principle is proven by relation [12]), validity of Schwarzschild metric (28) for a physical space and therefore validity of the equivalence principle has not been proven. In a similar situation for an isotropic metric, Einstein [2, p.91] used the phrase, "the possibility of getting" to indicate the uncertainty on the assumed validity of his equivalence principle.

Einstein relied on his 1911 formula for gravitational red shifts and the perihelion of Mercury to justify the validity of a space-time metric. However, these did not lead to a unique metric since both metrics (5) and (28) gave the same result for the

same order of approximation. Moreover, both metrics give the same first order approximation for the bending of light. Thus, these two metrics, though related to the same frame of reference, are indistinguishable by his three tests.

On the other hand, these two metrics give distinct space contractions, which is measurable according to Einstein. Therefore at most only one metric gives the realistic space contraction since these two metrics have the same frame of reference. (The uniqueness of gauge in connection with a frame, has been demonstrated in the case of a uniformly rotating system.) To this end, in principle, one can measure the local light speeds to decide which metric is more realistic. It is suggested that a Michelson-Morley type interferometer [27] with a vertical arm and a horizontal arm can do the job [28].

7. Discussions and Conclusions

In general relativity, Einstein [2,3,5] models the reality with a physical space (-time) that has a frame of reference and its time coordinate is related to the local time rate for the descriptions of the physics. Since such a physical space models reality, all physical requirements must be sufficiently satisfied. Einstein proposed his equivalence principle to a physical space.

Thus, a physical space is a Riemannian space with a Lorentz space-time metric that together with the space-time coordinates forms a physically valid space-time coordinate system. Although the Euclidean geometry breaks down in the invariant line element, a Euclidean structure of the frame of reference is necessarily in place. Since such an intrinsic Euclidean structure is independent of the gravity in the physical space, the physical meanings of space-time coordinates can be clearly defined in terms of measurements based on measuring instruments attached to coordinate system.

Once the intrinsic nature of the Euclidean structure is recognized, the physical meanings of space-time coordinates are clarified. It follows that the frame of reference, acceleration, and Einstein's equivalence principle are also clearly defined in terms of physics. Thus, the objections of Fock [6] and his followers [7] on general relativity are clearly based on misinterpretation only.

From Einstein's simple example of uniform rotation, we have learned also, the difference between a space-time coordinate system and an arbitrary mathematical coordinate system is distinguishable. As shown in Section 3, K and K' are physical space-time coordinate systems, whereas K* is only a mathematical coordinate system. Einstein's equivalence principle is applicable, as shown, only in a physical space, otherwise the so-calculated local time rate and local spatial contraction would be incompatible with physics. This also illustrates the inadequacy of Pauli's version.

Thus, it becomes clear that a manifold may not be a physical space even though it is diffeomorphic [29] to a physical space. In other words, for a Lorentz metric to be valid in physics, there are physical conditions to be considered. It has been shown that for a non-constant metric $g_{\mu\nu}$, the existence of acceleration to a static observer (i.e., $\Gamma^{\mu}_{tt} \neq 0$ for some $\mu \neq t$) is necessary for

a physical space [12]. The invalid derivation of a local distance formula by Landau & Lifshitz [9] demonstrates that an inadequate understanding of physics at the fundamental level could happen to even otherwise very competent theorists[9)].

Now, it is clear that Pauli's version of the equivalence principle is essentially a mathematical statement, but is not a physical principle because it does not contain adequate physical requirements for a situation in reality (see also Appendix). The physical inadequacy of Pauli's version is because physical requirements beyond metric signature are ignored. Since a physical principle is replaced by merely the existence of the local Minkowski space, Landau & Lifshitz [9] derived an invalid formula for a local distance. However, it was followed with a blind faith by others [11-13].

Moreover, if the existence of the local Minkowski space were the only physical condition as in Pauli's version, it became necessary that the space-time coordinates have no physical meaning. This is in direct conflict with the fact that non-scalars exist in physics. Thus, one cannot help concluding that such theorists have inadequate understanding in mathematics and physics at the fundamental level. Moreover, as specified by Einstein [2,3], the coordinate system used for the calculation of Einstein's three predictions has very clear physical meanings.

As the notion of Euclidean structure is clarified, it becomes obvious also that once the frame of reference has been chosen; the gauge is determined since, according to Einstein [2,3], the space contractions are measurable. Thus, although the isotropic solution and the Schwarzschild solution produce experimentally indistinguishable predictions for the three tests, they cannot be both valid since they produce different space contractions. It should be noted that the validity of the equivalence principle for these metrics is still an unverified assumption (see also Appendix), whose justification was based on of the 1911 formula for gravitational red shifts and the perihelion of Mercury. It remains from the experiment to find the hinds that can be used to determine the appropriate gauge.

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Appendix: The Physics of Geodesic and Mathematical Theorems

The equivalence principle is applicable only in a physical space, where a geodesic representing a free falling particle [2,3]. Therefore, it would be useful to discuss the mathematical theorems related to a geodesic. The principle of relativity and the equivalence principle imply that the physical space-time is a Riemannian space with a space-time metric function $g_{\mu\nu}$. For an

idealized point-like classical massive particle (which has no spin, charge, or other attributions), the equation of motion under gravity is the geodesic equation. The gravitational field is zero if the Christoffel symbols are zero.

Currently, Einstein's equivalence principle is often incorrectly considered as equivalent to an existence of local Minkowski spaces. Such a misunderstanding is related to two mathematical theorems [15] as follows:

Theorem 1. Given any point P in any Lorentz manifold (whose metric signature is the same as a Minkowski space) there always exist coordinate systems (x^{μ}) in which $\partial g_{\mu\nu}/\partial x^{\alpha} = 0$ at P.

Theorem 2. Given any time-like geodesic curve Γ there always exist a coordinate system (so-called Fermi coordinates) (x^{μ}) in which $\partial g_{\mu\nu}/\partial x^{\alpha} = 0$ along Γ.

From these theorems, it is possible to establish further that a local Minkowski metric exists at any given point and that along any time-like geodesic curve Γ, a moving local constant metric exists [15].

However, there is no physical specification on what is the cause of the local coordinate transformation,

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial y^{\mu}} dy^{\mu} \qquad \text{(A1)}$$

such that (A1) transforms the Lorentz metric $g_{\alpha\beta}(y^{\mu})$ to a local Minkowski metric along a time-like geodesic curve. In particular, there is nothing relating these two theorems to an existence of acceleration to a static particle or other physical situations. Thus, they are just mathematical theorems. Pauli's version of equivalence principle is essentially a simplified rephrasing of these theorems. No wonder Einstein strongly objected. Einstein [30] pointed out, "As far as the prepositions of mathematics refers to reality, they are not certain; and as far as they are certain, they do not refer to reality."

Einstein's equivalence principle gives crucial specific descriptions such as physical acceleration with respect to a frame of reference. Nevertheless, it is still insufficient to decide whether the geodesic or the transformation is valid in physics. To justify the physical validity of a space-time metric, Einstein examine the geodesic and both the cause of and the consequence of (A1).

Endnotes

1) In general relativity, a Riemannian space-time (M, g) is a physical space-time, according to Einstein [2,3]. Such a Riemannian space M is characterized by a space-time metric g that can be determined by the distribution of matter. It

is in the sense of that the metric g_{ik} as well as the space-time, is subjected to physical considerations. Moreover, since Einstein's Riemannian space-time models reality, all the physical requirements must be sufficiently satisfied by the space-time metric g_{ik}. As demonstrated by Einstein [2,3], it is necessary that a geodesic represents a free fall. The nomenclature, "physical space" was in fact used by Einstein, for instance, in his correspondence to A. Reht [16].

2) Einstein's objection made clear that he has his own version of infinitesimal equivalence principle. This should have been obvious because Pauli claimed that he did not invent his version but got it from the work of Einstein. Nevertheless, Norton [5], being a historian, failed in identifying Einstein's version because it was not labeled as such.

3) The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the *principle of causality*. This principle is the basis of relevance for all scientific investigations. This principle implies that the gravitational radiation must have sources and any parameter in a physical solution must be related to physical causes [8,26].

4) Einstein's viewpoint is supported by Weinberg [4, p.3] who stated, "In my view, it is much more useful to regard general relativity above all as a theory of *gravitation*, whose connection with geometry arises from the peculiar empirical properties of gravitation, properties summarized by Einstein's Principle of the Equivalence of Gravitation and Inertia."

5) The 1915 equation was guessed by Einstein, and the role of his equivalence principle in arriving his equation was not explicit. In addition, his equation does not have a dynamic solution as conjectured by Hogarth in 1953 [8,31].

6) For the infinitesimal form of Einstein's equivalence principle, the local metric may have the Minkowski form at only one point. Thus special relativity is only approximately valid even in an infinitesimal region. Moreover, as pointed out by Eddington [19], such an approximation would be valid only if the problem is unrelated to curvature.

7) The conditions: $g_{00} > 0$, $\begin{vmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{vmatrix}(-1) > 0$, $\begin{bmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & g_{22} \end{bmatrix} > 0$, and $\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}(-1) > 0$ were called physical condition for a physical coordinate system [9,12,13]. Nevertheless, they are actually insufficient in physics.

8) The deflection of light is an angle that can be measured at infinity by explicit comparisons using physical measures (millimetres on a photographic plate, for instance). However, this depends on a coordinate system because, to transform two dots in a photographic plate to an angle, one must refer to the coordinate system used to take the photos. For instance, such transformations are first based on the Schwarzschild coordinate system [3]. Moreover, one cannot define a defection angle in terms of a uniformly rotating system.

9) By no means, this glorifies incompetence; rather it shows possible vulnerability of an icon. Moreover, well-known theorists such as Penrose [32] Bondi [33] and Wheeler [34] would ignore physical principles. Consequently, although Hogarth [31] conjectured that there is no gravitational wave solution from the 1915 equation, they accepted unphysical gravitational "waves" [35], because they did not consider the principle of causality. Moreover, Einstein and Feynman [36] had claimed incorrectly the existence of dynamic solutions for the 1915 Einstein equation [35]. Christodoulou and Klainerman [37] even claimed incorrectly that they had constructed dynamic solutions [8].

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