31

FPRINT-02-05

Fermilab Library
0 1160 0092383 3

ON WHAT IS EINSTEIN'S EQUIVALENCE PRINCIPLE AND HOW TO APPLY IT

C. Y. Lo

Applied and Pure Research Institute
17 Newcastle Drive, Nashua, NH 03060

December 2001

Abstract

Einstein's equivalence principle was initially expressed in terms of the equivalence of an accelerated frame and uniform gravity. Due to misunderstanding, erroneous calculations, and misconceptions, this principle was challenged by various theorists as invalid. On the other hand, Einstein insisted on the fundamental importance of the principle to his general theory of relativity. Einstein also objected to Pauli's version of equivalence principle as inadequate in physics. It is pointed out that Einstein's infinitesimal form of equivalence principle exists informally in his 1916 paper and his book. Einstein's version includes physical considerations beyond the mathematical theorems from which Pauli rephrased his version. The main difference from Pauli's version is that a local Minkowski space is obtained through a gravitational acceleration to the frame of reference. Einstein's equivalence principle is proposed to only a physical space where all physical requirements are sufficiently satisfied. Therefore, before Einstein's calculation of light bending, the geodesic equation was checked with perihelion of Mercury and time dilation was compared with earlier formula of gravitational red shifts. Analysis of the case of the uniformly rotating disk clarifies Einstein's claim of general covariance and the need of restriction by Einstein's equivalence principle for a space-time coordinate system of a physical space. The application of Einstein's equivalence principle is presented and discussed. It is pointed out that Pauli's version, which ignores physical requirements beyond the metric signature, leads to theoretical difficulties and claims that disagree with observation. Therefore, it is urged that theories based on Pauli's version be reviewed.

Key Words: Einstein's equivalence principle, frame of reference, physical covariance, Einstein space, physical space-time

FERMILAB
FEB 15 2002
LIBRARY

1. Introduction

It is generally agreed, as pointed out by Einstein [1], Eddington [2], Pauli [3], Weinberg [4], Misner, Thorne & Wheeler [5], Straumann [6], and Yu [7], that Einstein's equivalence principle is the theoretical foundation of general relativity. Einstein explained the initial form of his equivalence principle in terms of the uniform gravity and acceleration clearly in 1911 [1,8]. After his principle of general relativity, Einstein proposed his equivalence principle for the general case of a four dimensional Riemannian physical space-time[1)] in his 1916 paper [8]. However, a surprising fact is, as Einstein [9] saw it, that few like Eddington [2] understand Einstein's equivalence principle in terms of physics adequately.

Einstein's equivalence principle is challenged by Synge's [10] now popular identification of "true" gravitational fields with metrical curvature. Synge [10] "professed" his misunderstandings on Einstein's equivalence principle as follows:

> "...I have never been able to understand this principle...Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's world line...The Principle of Equivalence performed the essential office of midwife at the birth of general relativity...I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced."

Due to also misunderstanding, Thorne [12] criticized Einstein's principle as follows:

> "In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you are (and your reference frame) are very small."

However, these problems have already been explained by Einstein. For instance, the problem of tidal forces has been answered in Einstein's July 12, 1953 letter to A. Rehtz [9] as follows:

> "The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

This makes clear also that this principle is proposed for qualities of physical space (not an arbitrary manifold). Einstein [9] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^l_{ik} (field strength), not the non-vanishing of the R_{iklm}." and no gravity is a special case of gravity. This view is crucial in general relativity because it allows Einstein to conclude that the geodesic equation is also the equation of motion of

a massive particle under gravity. Einstein insisted, throughout his life, on the fundamental importance of the principle to his general theory of relativity [9]. However, Einstein's insistence on this point has created a puzzle for philosophers and historians of science [9]. This shows also, how much was Einstein's equivalence principle being understood in terms of physics.

Now, Pauli's "infinitesimal" - principle of equivalence, is commonly but mistakenly regarded as Einstein's version of the principle, although Einstein had objected this version of misinterpretation [9]. In his article, 'What was Einstein's Principle of Equivalence?' Norton [9] recognized that Einstein's view is very different from Pauli's version. However, Norton [9] could not find Einstein's own version of infinitesimal equivalence principle readily from Einstein's work. Being essentially a historian, Norton did not realize that Einstein presented his infinitesimal version as if just consequences of his earlier version (see Section 6). Perhaps, Einstein did not feel the need of such a labeling, which did not seem to serve any useful purpose in his 1916 paper. Apparently, he did not foresee that a seed of lasting confusion was sown.

Currently, among those who accepted Einstein's equation and formulism, some theorists questioned or even rejected Einstein's physical principles. After accepting Einstein's equation, Fock [11] took the lead of discrediting Einstein's principle of relativity with his own version of "principle of relativity" of his personal preference (see Section 4) and Einstein's equivalence principle with the support of his erroneous calculations on uniform gravity (see Section 7). Based on Fock [11] viewpoints and the problematic unrestricted covariance, Logunov and Mestvirishvili [13] constructed "The Relativistic Theory of Gravitation" which modifies Einstein's formulism and rejects both Einstein's principles. Recently, H. C. Ohanian and R. Ruffini [14] also adapted Fock's viewpoints in their book "Gravitation and Spacetime". Referring to Fock [11] and Synge [10], they also explicitly rejected Einstein's principle of general relativity and criticized Einstein's equivalence principle as confusing instead of examining their own understanding in physics. Nevertheless, J. A. Wheeler [14] evaluated this book as "The best book on the market of 500 pages or less[2)] on gravitation and general relativity." Also, C. J. Goebel [14] commented, "A special strength of this book is the attention paid to experimental gravity and to the physical aspects of general relativity." Thus, misunderstandings have been developed into viewpoints that are explicitly against Einstein's general relativity.

Einstein's theory is severely attacked by many of those who should be his disciples, although most of them taking a somewhat indirect approach. A central issue is that they incorrectly believed Pauli's "equivalence principle" as a valid physical principle. Some theorists such as Fock [11] and Ohanian and Ruffini [14] believed that Einstein's equivalence principle is invalid. Accordingly, they also rejected Einstein's Principle of General Relativity and called their book a theory of gravitation. On the other hand, some theorists such as Wald [15] and Straumann [6] believed Pauli's version and Einstein's equivalence principle

were equivalent. They therefore called their book as theory of general relativity. These manifest that theorists are confused by conceptual errors. Thus, to appreciate Einstein's theory properly, it is necessary to identify the related conceptual as well as mathematical errors. Then, one can clarify Einstein's equivalence principle, which is supported by observations.

In this paper, for clarity, we analyze Pauli's subtle misinterpretation of Einstein's equivalence principle with some crucial details (Section 2). Then, we start with a discussion on Einstein's 1911 paper and the initial form of his equivalence principle [8]. The merits and deficiency of his early approach are deliberated (Section 3). To clarify Einstein's view on gravity and the meaning of his equivalence principle, the problem of uniform rotation is analyzed. In addition, the meaning of Einstein's generally covariance is clarified as for mathematical calculations but is restricted to physical space-time coordinate systems for physical interpretations. In addition, it is pointed out that Einstein's Riemannian space includes a Euclidean structure as a frame of reference (Section 4). Then, Einstein's version of infinitesimal equivalence principle for a curved Riemannian space-time is addressed. The application of Einstein's principle is presented and current theoretical errors are discussed (Sections 5 & 6). To illustrate current misconceptions on Einstein's principle, related calculations by R. C. Tolman [16] and V. A. Fock [11] are analyzed (Section 7). In Section 8, issues related to Einstein's principle are discussed and necessary review of current theories is urged. It is pointed out that space-time coordinates have physical meanings and this is supported by experiments.

2. Pauli's Inadvertent Misinterpretation and the Frame of Reference

It is well known that Pauli, at the age of 21, wrote an article on general relativity for the Mathematical Encyclopedia at the recommendation of A. Summerfeld. But, few other than historians knew that Einstein disagrees with Pauli's interpretation of the equivalence principle [9]. Apparently, Summerfeld, or subsequently many other excellent theoreticians did not detect Pauli's subtle mistake over years. In Pauli's otherwise excellent article, he has indeed demonstrated his understanding of some subtle arguments of this subject. For instance, he pointed out the possibility of an antigravity coupling. So, Summerfeld's judgment on Pauli's understanding on this matter is partially right.

Unfortunately, Pauli's inadvertent misinterpretation on Einstein's principle is very popular. Let us first trace back to the disagreement [9] between Einstein and Pauli. Pauli's [3, p.145] version of the equivalence principle is as follows:

> "For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."

Einstein strongly objected this version [9] since, for some cases, no matter how small the world region, special relativity would not exactly hold (see also metric [11a]). This objection made clear also that Einstein regards the equivalence of the effects of an accelerated frame of reference and gravity being exact only for a special case as he implied in his 1916 paper [8].

However, his critics disregard this and related explanation of Einstein [9]. For instance, the criticism of Ohanian and Ruffini [14, p. 54] to Einstein's equivalence principle was based on misinterpreting the initial version of 1911 as a general equivalence between the acceleration and any gravity. Their inadequate understanding of Einstein's principle is also evident from their another statement, "In order to avoid confusion, we will base our further development of gravitational theory on the very precise and unambiguous equality $m_I = m_G$. This equality is necessary and to a large extent, sufficient for the construction of the relativistic theory." Apparently, they regarded their own confusion (see also Section 6) as originated from Einstein.

Another important point is that, as pointed out by Fock [11], Einstein's equivalence principle is related to acceleration toward a frame of reference [1,8]. The space-time continuum of reality is modeled with a physical space (-time) that includes a frame of reference on which the physical space-time coordinate system together with a space-time metric is based [1,8]. On the other hand, Pauli's version does not consider any physical condition beyond the existence of a local Minkowski space. Consequently, Pauli's version misleadingly suggested that a physical frame of reference does not play any role. Then, the existence of a local Minkowski space is presented as if only a possible mathematical choice but not a physical result as it should be.

This omission inadvertently gives the opportunity of a misinterpretation that any Lorentz manifold could be considered as a physical space because only the signature of the metric of a manifold is examined while other physical requirements are ignored. Nevertheless, Einstein [9] did not point out clearly the relation between the need of a valid space-time coordinate system and a satisfaction of Einstein's equivalence principle. This seems to have an effect that helps the acceptance of Pauli's view.

A physical requirement for Einstein's version of infinitesimal equivalence principle (Section 6) is that it must be applied to a physical space. In a free fall, a local space must be uniquely Minkowskian. For example, when a space ship is under the influence of gravity only, the local space-time is known to be automatically Minkowskian, as determined by the physics of gravity. Moreover, if the manifold under consideration is a physical space (-time), it satisfies all physical principles sufficiently. Thus, the mathematical existence of a local Minkowski space need not mean a satisfaction of Einstein's equivalence principle that depends on the physical validity of the local coordinate transformation (see Sections 5 & 6). If the conditions for a physical space are taken into consideration, Einstein's equivalence principle is not exactly a local principle as Fock [11] misinterpreted.

3. The Initial Form of Einstein's Equivalence Principle and the Gravitational Red Shifts

In 1911, Einstein [8] derived the gravitational red shifts from the initial form of his principle, the equivalence of a uniformly accelerated frame and the uniform gravity. This is independent of the need of a Riemannian space with a Lorentz signature, which is additionally due to the principle of general relativity and special relativity [1]. A known deficiency of his results then is an incorrect formula for light speeds under gravity. Einstein [17] corrected this formula in 1915.

Nevertheless, an unverified belief advocated by Fock [11] and many others [14,18] is that Einstein's equivalence principle could be intrinsically incompatible with the notion of a curved space. Such a belief must be very absurd to Einstein since his argument for a Riemannian space is based on his equivalence principle [1,8].

Recently, such a belief has been proven to be fundamentally incorrect since Maxwell-Newton Approximation, the linear field equation for weak gravity, that produced a valid light bending, has been derived [18] with Einstein's equivalence principle (see Section 4) together with the notion of a Riemannian space if Newtonian theory is taken as a form of first order approximation. Another mistake of Ohanian and Ruffini [14] is that, instead of the Maxwell-Newton Approximation, their linear equation is based on the linearized conservation law, which has been proven invalid for gravity by Wald [15] and Yu [19]. They [14] also claimed, in disagreement with Eddington [2] and Pauli [3], that the covariance principle is a dynamic principle.

To appreciate Einstein's ingenuity, it would be easier to start from his paper of 1911, where he found that his equivalence principle is compatible with the Doppler effects and even the notion of photon. Thus, Einstein's equivalence principle has been firmly established on the ground of universality of physics. Since the notion of curved space would produce a second order effect in his consideration of the effect of gravitational red shifts [1], Einstein's 1911 derivation of the red shifts is valid.

Einstein assumed that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system K' accelerated by γ in the opposite direction, can be extended to other physical processes. He considered two material systems S_1 and S_2 which are situated initially at rest on the z-axis of system K and are separated by a distance h so the gravitation potential in S_2 is greater that S_1 by γh. If a definite radiation energy E_2 is emitted from S_2 to S_1 at the moment that system K' has zero velocity relative to an inertial system K_0, the radiation will arrive at S_1 when the time h/c has elapsed (to a first order approximation); and at this moment the velocity of S_1 relative to K_0 is $\gamma h/c = v$. According to special relativity, the radiation arrives S_1 with a greater energy E_1 which (to a first order approximation) is related to E_2 by

$$E_1 = E_2(1 + v/c) = E_2(1 + \gamma h/c^2) \quad (1)$$

The above is consistent with, $E = mc^2$ in the sense of mass-energy conservation [20]. By assumption, exactly the same relation holds if the same process takes place in the system K, which is not accelerated, but is provided with a gravitational field. Then, gravity must act also on radiation, and we may replace γh by the gravitational potential Φ and obtain

$$E_1 = E_2(1 + \Phi/c^2) = E_2 + \Phi(E_2/c^2). \quad (2)$$

Thus, the energy increment of radiation due to gravity is resolved by the equivalence of the K and K' systems.

If the radiation emitted in the uniformly accelerated system K' in S_2 towards S_1 had the frequency ν_2 relatively to the clock in S_2, then at the arrival of radiation in S_1, it has a greater frequency ν_1 relatively to S_1, such that to a first approximation

$$\nu_1 = \nu_2(1 + \gamma\, h/c^2) \quad (3a)$$

If the radiation is emitted at time that K' has no velocity, S_1 at the time of arrival of the radiation, has relative to K, the velocity $\gamma h/c$. Eq. (3a) is an immediate result of the Doppler's principle.

If γh is substituted by the gravitational potential Φ of S_2 - that of S_1 being taking as zero - then the equivalence principle, to the first order approximation gives

$$\nu_1 = \nu_2(1 + \Phi\, /c^2). \quad (3b)$$

If on the surface of a star (where S_2 is located) the light is emitted to the Earth (S_1) where the frequency of the arriving light is measured, then eq. (3b) implies $\nu = \nu_0(1 + \Phi/c^2)$, where Φ is the (negative) difference of gravitational potential between the surface of the star and the Earth. Also, if (2) and (3) are compared, then one would conjecture that the energy of a photon be

$$E = k\, \nu, \quad (4)$$

where k is a constant. However, this is not the only connection with quantum theory. For instance, Bohr [17] has to consider that the theory of general relativity is a justification for his uncertainty principle in quantum theory.

In the above, the formula for gravitational red shifts is derived from the equivalence principle alone with the Newtonian scalar theory as a first order approximation. This derivation has put Einstein's equivalence principle firmly in the ground of universality of physics and experimental confirmation. However, the effects of a curved space have not been adequately accounted for, and this inadequacy causes the deficiency on the calculated bending of light rays. Currently, the derivation of the Maxwell-Newton Approximation, which shows that the gravitational red shifts are directly related to g_{tt} the time-time component of the space-time metric [1], has removed any remaining doubt on validity of Einstein's equivalence principle [18,21].

4. The Principle of General Relativity, Riemannian Covariance, and Einstein Space

Because there is an inherent epistemological defect for preferring the inertial system, Einstein was not entirely happy with special relativity. Einstein believes, "The law of physics must be of such a nature that they apply to systems of reference in any kind of motion (principle of general relativity)." Along this road, he arrived an extension of the postulate of relativity. In his opinion, this is favored by his previous work on the equivalence of a uniformly accelerated frame of reference K' and uniform gravity. From the viewpoint of the principle of general relativity, *Einstein's principle of equivalence is really the equivalence of the effects of an accelerated frame to a related uniform gravity whereas others incorrectly perceived that any gravity is equivalent to a uniformly accelerated frame.* This is evident since the effects of a uniform rotation cannot be equivalent to the effects of a linear acceleration. In Einstein's view, no gravity is a special case of gravity.

In pursuing the general theory of relativity, according to the equivalence principle, Einstein is able to "produce" a gravitational field merely by changing the system of coordinates. Then, he was able to conclude that the geodesic equation is an equation of motion for gravity. It thus, follows that the space-time in reality has a Riemannian geometry instead of Euclidean geometry. Thus, Einstein's principle of general relativity means that gravitation is fundamentally involved in any theory of physics not only that a unification of gravitation and electromagnetism is needed due to that all charged particles are massive. This analysis also clarifies that the space-time in reality having a Riemannian geometry is also due to the equivalence principle. In other words, the equivalence principle enables an extension of the principle of relativity to accelerated motion. In addition, the frame of reference has a *Euclidean Structure* in the Riemannian space although this structure is not an invariant.

Einstein considered a Galilean (inertial) system of reference K (x, y, z, t) and a system K' (x', y', z', t') in uniform rotation Ω relatively to K. The origins of both systems and their axes of Z permanently coincide. For reason of symmetry, a circle

around the origin in the X, Y plane of K may at the same time be regarded as a circle in the X', Y' plane of K'. Then, according to special relativity, in the X, Y plane and the X', Y' plane, the metrics of K and K' [22] are respectively the following:

$$ds^2 = c^2\,dt^2 - dr^2 - r^2\,d\phi^2 - dz^2 \qquad \text{where} \quad x = r\cos\phi, \quad y = r\sin\phi, \tag{5a}$$

and

$$ds^2 = (c^2 - \Omega^2 r'^2)\,dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\,d\phi'^2 - dz'^2 \tag{5b}$$

Then,

$$\oint ds = (1 - \Omega^2 r'^2/c^2)^{-1/2} r' \int_0^{2\pi} d\phi' \qquad \text{where} \quad x' = r'\cos\phi', \quad y' = r'\sin\phi', \tag{5c}$$

would be the circumstance of a circle of radius r' (= r) for an observer in K'. Thus, Einstein concluded, "With a measuring rod at rest relatively to K', the quotient of circumstances over diameter would be greater than π." and Euclidean geometry therefore breaks down in relation to the system K'. Moreover, Einstein [8] wrote, "An observer at the common origin of co-ordinates, capable of observing the clock at the circumferences by means of light, would therefore see it lagging behind the clock beside him...So, he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be."

According to the principle of equivalence, K' may also be considered as a system at rest, with respect to which there is a gravitational field (field of centrifugal force, and force of Coriolis). Thus, Einstein's notion of gravity, though has a cause such as Ω, needs not relate to a source, but just relate to acceleration to a resting massive particle (see also Section 1). This example shows also that the equivalence principle enables an extension of the principle of relativity to accelerated motion.

Thus, Einstein concluded, "In general theory of relativity, space and time cannot be defined in such a way measured by the unit measuring-rod, or difference in the time co-ordinate by a stand clock." (It will be shown later that in such measurements, the instruments are resting, but in a free fall state.) Since a physical space-time is Riemannian, covariance must be done in terms of Riemannian geometry. However, in addition to metric (5b), K' still includes a Euclidean structure for the frame of reference. If a measuring rod is attached to the system K', since both are under the same influence of gravity, a Euclidean structure emerges. In other words, not only is K' a Riemannian space as indicated by (5b), but also K' has a Euclidean structure.

To illustrate the Euclidean structure further, consider the Schwarzschild solution,

$$ds^2 = (1 - 2MG/\rho)dt^2 - (1 - 2MG/\rho)^{-1}d\rho^2 - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \qquad \text{for} \qquad \rho > 2MG \tag{6a}$$

where

$$x' = \rho\sin\theta\cos\varphi, \qquad y' = \rho\sin\theta\sin\varphi, \qquad \text{and} \qquad z' = \rho\cos\theta. \tag{6b}$$

Metric (6a) is a function of ρ (= $[x'^2 + y'^2 + z'^2]^{1/2}$). The radicals ρ is related to Euclidean coordinates (x', y', z'), and thus the metric is defined in terms characteristics of a Euclidean structure, which is corresponding to the metric when M = 0.

Thus, as shown, the notion of a Euclidean space is still implicitly included within a Riemannian space although the Euclidean structure is not an invariant, just as in special relativity. In fact, this is the mathematical basis that the cylindrical coordinate system (r', ϕ', z') is well defined in K'. *Thus, the Euclidean structure is an integral part of Einstein's theory.* For clarity, a Euclid-Riemannian space with a Euclidean structure shall be called an *Einstein space* named after its creator.

Einstein assumed that a frame of reference always has a Euclidean structure. Since such an assumption is implicit, Einstein's equivalence principle was very difficult to be comprehended especially for those who are essentially mathematicians. This might explain that why Pauli's "equivalence principle was so readily accepted in spite of Einstein's objection.

To see the local coordinate transformation between metric (5a) and (5b), let us consider the coordinate transformation [22] to the uniformly rotating disk, in terms of Newton's notion of "absolute time" as follows:

$$x = x' \cos \Omega t - y' \sin \Omega t, \tag{7a}$$

$$y = x' \sin \Omega t + y' \cos \Omega t,$$

$$z = z'$$

where Ω is the angular velocity; or in cylindrical coordinates,

$$r = r', \quad z = z'. \quad \phi = \phi' + \Omega t. \tag{7b}$$

Then, the resulting metric has the following form,

$$ds^2 = (c^2 - \Omega^2 r'^2)\, dt^2 - 2\Omega r'^2\, d\phi' dt - dr'^2 - r'^2\, d\phi'^2 - dz'^2 \tag{5b'}$$

However, the mathematical coordinate system K*(x', y', z', t) is not a physical space-time coordinate system for the uniformly rotating disk K' because the time coordinate t remains associating with the inertial frame of reference K.

In other words, metric (5b') together with its coordinates is not a space-time coordinate system that can be used for physical interpretation. For instance, it follows from $ds^2 = 0$ that the coordinate light speed produced by (5b') could be larger than c (a problem of Newton's notion of absolute time). Since a physical principle is violated, the equivalence principle would not be

applicable in the coordinate system K*(x', y', z', t). This example of Einstein's demonstrates the necessity that, for physical interpretation, you have, not only just a mathematical coordinate system, but also a physical coordinate system.

Nevertheless, as claimed by Zel'dovich & Novikov [22], it is possible that metric (5b') alone can be used to recover metric (5b). This is expected since the metric is transformed from a physical metric. (For an arbitrary Lorentz manifold, however, it has been shown that the hope of finding a valid space-time coordinate system cannot be guaranteed [23].)

To obtain a physical coordinate system including the time t' of the rotating disk, a comparison of (5b) and (5b') leads to,

$$d\phi' = d\phi - \Omega dt \; ; \tag{8a}$$

and

$$cdt' = [cdt - (r\Omega/c)rd\phi][1 - (r\Omega/c)^2]^{-1} \; . \tag{8b}$$

Thus, it is necessary to modify the time coordinate t'. An interesting fact of this local coordinate transformation is that (8a) can be obtained directly from (7b) and looks like a Galilean transformation. The inverse transformation is as follows:

$$d\phi = d\phi'[1 - (r\Omega/c)^2]^{-1} + \Omega dt' \; ; \quad \text{and} \quad cdt = cdt' + (r\Omega/c)rd\phi'[1 - (r\Omega/c)^2]^{-1} \; . \tag{8c}$$

It would be difficult to guess the factor $[1 - (r\Omega/c)^2]^{-1}$, which seems to be incompatible with time dilation and spatial contraction manifested in metric (5b). But, the time dilation and the spatial contraction are results due to comparisons with a clock and a measuring rod in relatively rest at the beginning of a free fall. According to Einstein's equivalence principle (see Sections 5 and 6), such a coordinate system is locally Minkowski. To verify this, consider the Lorentz coordinate transformation,

$$rd\phi = [1 - (r\Omega/c)^2]^{-1/2} [dX + r\Omega dT] \; ; \tag{9a}$$

and

$$cdt = [1 - (r\Omega/c)^2]^{-1/2} [cdT + (r\Omega/c)dX] \; . \tag{9b}$$

Then,

$$rd\phi' = [1 - (r\Omega/c)^2]^{1/2} dX \; ; \quad \text{and} \quad cdt' = [1 - (r\Omega/c)^2]^{-1/2} cdT \tag{9c}$$

These are exactly the time dilation and spatial contraction. This illustrates that a particle resting at K', can attached to a local Minkowski space. This probably was *a starting point of Einstein's version of infinitesimal equivalence principle.*

In addition, from metric (5b), a light speed at r' ($\neq 0$) observed in system K' would be smaller than c because of time dilation effect of gravity. But, space contraction is directional. The light speed is even smaller in the φ-direction, that is, a light

speed can decrease more after a velocity $\Omega r'$ is "added to". However, such relations for the coordinate system K* (x', y', z', t), in spite of $rd\phi' = [1 - (r\Omega/c)^2]^{1/2}\, dX$, are complicated. Since $dt = [1 - (r\Omega/c)^2]^{-1/2}\, [dT + (r\Omega/c^2)\, dX]$, a corresponding time dilation for dt in K* is not there. This illustrates also that the Galilean transformation (7) is invalid in general relativity.

For the frame K' (x', y', z'), Einstein [8] remarked, "So he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be." The time coordinate for K' (x', y', z'), as shown, is severely restricted because the time is related to the local clock rate. Thus, Einstein invented the notion of a space-time coordinate system in physics. On the other hand, Einstein [8] also remarked, "So there is nothing for it but to regards all imaginable systems of coordinates, in principle, as equally suitable for the description of nature." From the above examples, this description of nature by the coordinate system K* (x', y', z', t) includes certain calculations but not physical interpretations.

Thus, although tensor equations may be covariant with respect to any substitutions of whatever (generally covariant), the freedom toward the physical space-time coordinate system, and thus a valid physical interpretation, is severely limited by his equivalence principle. In his book, Einstein [1] remarked, "As in special theory of relativity, we have to discriminate between time-like and space-like line elements in the four-dimensional continuum; owing to the change of sign introduced, time-like line elements have a real, space-like line elements an imaginary ds. The time-like ds can be measured directly by a suitably chosen clock." Special relativity has already taught us [8] that some mathematical coordinate systems are not physically realizable and therefore cannot be used to describe nature. The same has been illustrated for general relativity[3)].

Moreover, from metric (5b), the metric element $(c^2 - \Omega^2 r'^2)$ is zero at a point r'_0, which corresponds to the speed of a particle resting at r'_0 would reach the light speed. When $r' > r'_0$, two metric element change signs. Would this mean that the space coordinate $r'd\phi'$ becomes time-like and the time coordinate becomes space-like? The answer is obviously no because we have $r = r'$, and r is still space-like according to metric (5a). The correct answer is that when $r' \geq r'_0$, the coordinate system K' no longer makes sense in physics. The frame of reference K' cannot go beyond r'_0 because c is the upper limit of any speed for moving matter. This illustrates that a formal validity in mathematics could be inadequate in physics, and also that, for a spatial coordinate system to be meaningful in physics, in principle a massive particle must be able to rest on it. The change of sign for very large r manifests such a restriction in physics. Otherwise, it would be impossible that such a coordinate system can be associated with local physical measurements. For instance, it would be difficult for Einstein to put a clock there! In the above, Einstein has shown also that physical coordinates need not cover the entire space-time as in special relativity.

In 1916 earlier Einstein seemed to believe that any Gaussian system would be a valid space-time coordinate system. To argue for a belief of the moment, that is, unrestricted covariance, he wrote [8],

> "That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measuring are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences."

However, this argument seems to be incompatible with his equivalence principle and his earlier statement [8], "So he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be."

Moreover, while all verifications indeed amount to a determination of space-time coincidences, in order to predict such coincidences theoretically, one must able to relate events of different locations in a definite manner. Thus, a coordinate system must be related to objective physical measurements. In fact, as early as 1918, unrestricted general covariance was questioned [24]. As Eddington [2] pointed out, "space is not a lot of points close together; it is a lot of distances interlocked." Understandably, Einstein [1] in his lecture of 1921 dropped the above argument and emphasized his equivalence principle first, and remarked, "As in special theory of relativity, we have to discriminate between time-like and space-like line elements in the four-dimensional continuum; owing to the change of sign introduced, time-like line elements have a real, space-like line elements an imaginary ds. The time-like ds can be measured directly by a suitably chosen clock." Thus, a space-coordinate and the time-coordinates in physics are not exchangeable as Hawking [25] claimed since they have distinct characteristics and physical meanings. Einstein also praised Eddington's book of 1923 to be the finest presentation of the subject ever written [26].

However, the damage to general relativity has already been done, and a prevailing conceptual error[3)] is the belief of validity of any Gaussian system as a space-time coordinate system in physics. Consequently, Einstein's equivalence principle is misunderstood and is often incorrectly replaced by the condition for the mathematical existence of a local Minkowski space.

In short, general covariance has no meaning beyond the fact that a tensor calculation must be in terms of Riemannian geometry. Kretschmann [27] pointed out in 1917 that the postulate of general covariance does not make any assertions about the

physical content of the physical laws, but only about their mathematical formulation; and Einstein [28] entirely concurred with this view. As Pauli [3] pointed out "The general covariant formulation of the physical laws acquires a physical content only through the principle of equivalence in consequence of which gravitation is described solely by the (metric) g_{ki} ···."

In Einstein's theory, the principle of general relativity is the physical basis of covariance. However, in order to eliminate the term general relativity form a theory of gravity, Fock [11, p.6] defined a different "principle for relativity" as follows:

> "When speaking of the relativity of a frame of reference or simply of relativity, one usually means that there exist identical physical processes in different frames of reference. According to the generalized Galilean principle of relativity identical processes are possible in all inertial frames of reference related by Lorentz transformations. On the other hand, Lorentz transformations characterized the uniformity of Galilean space-time. Thus, the principle of relativity is directly related to uniformity. This also shows that the nomenclature introduced in Einstein's first papers, by which the theory of uniform Galilean space is named "Theory of Relativity" can to some extent be justified."

Fock's "principle of relativity" is based on identical processes; whereas Einstein's principle is based on the covariance of physical laws. Thus it is, at most, a matter of personal preference (i.e., without any scientific value) that Fock claimed Einstein's principle of relativity to be invalid. But, such a denial of Einstein's principle of relativity would leave the requirement of mathematical covariance without any physical basis. Thus, one may wonder whether Fock's approach[4)] makes sense in physics.

Einstein's principle of relativity is also the theoretical basis for the geodesic equation to be the equation of motion for gravity, whereas Fock's "principle of relativity" does not seem to serve a useful scientific purpose. Since the motion of a particle in an initial system is a straight line, the shortest line between two points, the corresponding equation of motion for system K' is the geodesic equation in Riemann geometry. Thus, gravity is due to ten metric elements that include the velocity-dependent force. But, the time-time metric component g_{tt} still plays the dominating role of a potential that provides the acceleration to a resting particle. In other words, the rotating disk shows not only that general relativity requires a Riemannian Space, but also that the equation of motion involves more than just a potential.

5. Mathematical Theorems on Geodesics, and Physics in a Physical Space

Since the equivalence principle is applicable only in a physical space, where a geodesic representing a free falling particle, it would be useful to discuss the mathematical theorems related to a geodesic. The principle of relativity and the equivalence principle imply that the physical space-time is a Riemannian space with a space-time metric function $g_{\mu\nu}$. For an idealized

point-like classical massive particle (which has no spin, charge, or other attributions), the equation of motion under gravity is independent of the mass, and is the geodesic equation,

$$\frac{d^2x^\mu}{ds^2}+\Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}=0, \tag{10a}$$

where

$$\Gamma^\mu{}_{\alpha\beta}=\frac{1}{2}g^{\mu\nu}\left(\frac{\partial g_{\nu\beta}}{\partial x^\alpha}+\frac{\partial g_{\nu\alpha}}{\partial x^\beta}-\frac{\partial g_{\alpha\beta}}{\partial x^\nu}\right), \tag{10b}$$

is the Christoffel symbols, and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Thus, the gravitational field is zero if $\Gamma^\mu_{\alpha\beta} = 0$.

Currently, some [9,29] have mistaken the geodesic representing a motion or the existence of local Minkowski spaces as Einstein's equivalence principle. Such misunderstandings are related to two mathematical theorems [10] as follows:

Theorem 1. Given any point P in any Lorentz manifold (whose metric signature is the same as a Minkowski space) there always exist coordinate systems (x^μ) in which $\partial g_{\mu\nu}/\partial x^\alpha = 0$ at P.

Theorem 2. Given any time-like geodesic curve Γ there always exist a coordinate system (so-called Fermi coordinates) (x^μ) in which $\partial g_{\mu\nu}/\partial x^\alpha = 0$ along Γ.

From these theorems, it is possible to establish further by simple algebra that a local Minkowski metric exists at any given point and that along any time-like geodesic curve Γ, a moving local constant metric exists [10].

However, there is nothing relating these two theorems to an existence of acceleration to a static particle or other physical situations. Also, there is no physical specification to be the cause of the local coordinate transformation,

$$dx^{\alpha\prime}=\frac{\partial x^{\alpha\prime}}{\partial x^\mu}dx^\mu \tag{10c}$$

such that (10c) transforms the Lorentz metric $g_{\mu\nu}$ to a local Minkowski metric along a time-like geodesic curve. Pauli's version of the equivalence principle is essentially a simplified rephrasing of these theorems. Einstein [30] pointed out, "As far as the prepositions of mathematics refers to reality, they are not certain; and as far as they are certain, they do not refer to reality." An application of a theorem should be examined for its relevance although "one cannot really argue with a mathematical theorem

[25]". The physical validity of a geodesic depends of whether transformation (10c), which transforms from an *Einstein space* to the local Minkowski space, is valid in physics. To this end, we can examine either the cause of or the consequence of (10c).

It will be shown that transformation (10c) in an Einstein space (see Section 4) can be invalid in physics. Consider a manifold K and its orthogonal metric, whose spatial unit is a centimeter and the time unit is a second, as follows:

$$ds^2 = 4c^2dt^2 - dx^2 - dy^2 - dz^2. \qquad (11a)$$

K is obviously a Lorentz manifold with a Euclidean structure and therefore is an Einstein space.

Consider a particle P resting at point (x_0, y_0, z_0) at time t_0. Since $\Gamma^{\mu}{}_{\alpha\beta} = 0$, P stays at the same point (x_0, y_0, z_0) forever. Thus, the coordinates of P are (x_0, y_0, z_0, t) at time t. Then, the local coordinate transformation to a local Minkowski metric is

$$dX = dx, \qquad dY = dy, \qquad dZ = dz\,, \qquad \text{but} \qquad dT = 2\,dt \qquad (11b)$$

The local coordinate transformation (11b), which is not a rescaling of units, is invalid in physics. Since there is no gravity or relative velocity between the frame of its tetrad and the frame K, there is no physical cause that makes a clock rate changes. Thus, the Einstein space (11a) is not a physical space that models reality. Otherwise, there were two standard clocks having different rates, though resting at the same point of a frame of reference. Nevertheless, Pauli's equivalence principle is satisfied.

In next section, it will be shown that a non-constant Lorentz metric may be invalid in physics. For instance, a static observer may not receive any acceleration, and a Lorentz manifold may not be diffeomorphic to a physical space [18,23].

6. Einstein's Infinitesimal Equivalence Principle and its Application.

The mathematical theorems are compatible with Einstein's Infinitesimal equivalence principle, which is applicable only to a physical space (-time) that models reality such that all physical requirements are sufficiently satisfied. Both of Einstein's version and Pauli's version agree on the existence of a local Minkowski space at any point. But, only Einstein specified such a space is obtained through a choice of acceleration. Einstein wrote in the Section 4 of his 1916 paper [8], "For this purpose we must choose the acceleration of the infinitely small ("local") system of the coordinates so that no gravitational field occurs; this is possible for an infinitely small region." Any acceleration to a particle is, of course, relative to a frame of reference, and therefore the physical Riemannian space, in a priori, includes a Euclidean structure (see Section 4).

In other words, *Einstein's principle is proposed for a physical space that has a frame of reference with a Euclidean structure.* This difference means that Einstein's equivalence principle is a physical principle, whereas Pauli's version is only a rephrasing of mathematical theorems. Moreover, Einstein's equivalence principle explicitly requires the existence of acceleration for a static massive particle (i.e., $\Gamma^{\mu}_{tt} \neq 0$ for some $\mu \neq t$,), and as shown later, theoretical self-consistency demands this.

Consider a local space L (X, Y, Z, CT) whose origin is attached to a particle in free fall. The Galilean weak equivalence principle that all massive matter falling with the same acceleration in the physical space implies only that, at the origin, $\Gamma^{\mu}_{TT} = 0$ for $\mu \neq T$, but the local metric need not have

$$\frac{\partial g_{\alpha\beta}(X,Y,Z,cT)}{\partial X^{\sigma}} = 0 \qquad \text{for any} \qquad \alpha, \beta, \sigma. \tag{12a}$$

But, the mathematical theorems imply that this is always possible. Moreover, the local metric of space L can be chosen as

$$ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2, \tag{12b}$$

i.e., a local Minkowski metric. Metric (12b) is proposed by Einstein [1] due to, "special theory of relativity applies to the special case of the absence of a gravitational field." This proposal is the essence of Einstein's infinitesimal equivalence principle that includes its initial form and later version as the special cases. Now, we shall call it simply as Einstein's equivalence principle.

Einstein [1] clarified, "According to the principle of equivalence, the metric relation of the Euclidean geometry are valid relative to a Cartesian system of reference of infinitely small dimensions and in a suitable state of motion (free falling, and without rotation)." This is the way that Einstein proclaimed the final infinitesimal form of his principle on the local equivalence of gravity and a suitable acceleration in a physical space.

In addition, Eddington [2] observed that special relativity should apply only to phenomena unrelated to the second order derivatives of the metric. Einstein [31] accepted this criticism and added the crucial phrase, "at least to a first approximation" on the indistinguishability between gravity and acceleration.

The real world is modeled by a physical space, where the physical principles are satisfied. The local transformations,

$$g_{\mu\nu}(x, y, z, ct) = \frac{\partial X^{\alpha}}{\partial x^{\mu}}\frac{\partial X^{\beta}}{\partial x^{\nu}} g_{\alpha\beta}(X, Y, Z, cT), \qquad \text{where} \quad g_{\alpha\beta}(0,0,0,0), = \eta_{\alpha\beta} \tag{12c}$$

and

$$dX^{\alpha\,\prime} = \frac{\partial X^{\alpha\,\prime}}{\partial x^{\mu}} dx^{\mu}\,, \tag{12d}$$

by assumption, are due to gravity - the physical cause. Moreover, the results implied by (12d) must be valid in physics. Einstein [1,8] obtained the time dilation and space contraction through (12d). *Thus, the application of Einstein's principle lies on showing that the Euclid-Riemannian manifold under consideration is a physical space.* O the other hand, Pauli's version has no physical requirement for (12d), and thus accepts unphysical Lorentz manifold such as (11a). Thus, the claim of Friedman [29], "The existence of local Minkowski space has replaced the equivalence principle that initially motivated it," is incorrect.

In summary, in addition to a proper metric signature required by Pauli's version, there are physical conditions that Einstein's equivalence principle requires. These additional physical requirements for a physical space are the following:

1) A time-like geodesic represents a physical free fall.
2) The local transformation (12c) in a free fall is automatically due to gravity.
3) The local transformation (12d) is valid in physics.

Since the physical validity of a geodesic cannot be determined at one point, a physical solution must be established in a finite region of the physical space-time. In other words, *when applied to a curved space, Einstein's principle is necessarily considered as non-local.* To establish 1), validity of physical principles or requirements is needed. To verify requirement 1), Einstein [1] has shown an agreement with the Mercury perihelion. Requirement 2) is Einstein's equivalence principle, but it is difficult to prove its validity directly. The usual method is assuming it validity and examine its consequences through requirements 1) and 3). Thus, another crucial point of Einstein's principle is whether the transformation (12d) that leads to local Minkowski space (12b), has a valid physical cause, and this can be found out by examining the physical validity of its consequences. To this end, Einstein [1] has compared his earlier formula for gravitational red shifts[5)], before deriving his formula for the light bending. In addition, one may assume the validity of both 1) and 2) and then examine requirement 3).

Note that Einstein has checked validity of only some physical principles, such as the correspondence principle in connection with Newtonian theory. It is known that many accepted metric solutions are actually unphysical [32,33]. Moreover, for a given metric $g_{\mu\nu}$, acceleration may not necessarily exist for a static observer. When $\Gamma^{\mu}_{tt} = 0$ ($\mu \neq t$), there is no Newtonian acceleration for a resting particle although the metric may still be non-constant. However, since there is no acceleration for an initially static particle, it would remain forever in the same position with the same frame of reference. Thus, the same unit clock and/or the same unit measuring-rod would have two different readings. In other words, static acceleration must exist.

To illustrate this, let us consider a Lorentz manifold K_T with the non-constant metric,

$$ds^2 = c^2 ch^2(T/C)dT^2 - dx^2 - dy^2 - dz^2, \tag{13}$$

where ch(T/C) = [exp(T/C) + exp(-T/C)]/2, and C (> 0) is a constant. Metric (13) can be considered as including a Euclidean structure and metric (13) becomes a flat metric when C = ∞. From metric (13), the Christoffel symbols are zeros except $\Gamma_{t,tt} = \partial_t g_{tt}/2$, and thus no static acceleration. In addition, the physical cause of gravity is not clear.

Let us assume that a time like geodesic were a physical free fall and Einstein's equivalence principle were applicable in this manifold, and examine the consequence. The equation of motion for an observer P at (x_0, y_0, z_0, T_0) would be

$$\frac{d^2T}{ds^2} + \Gamma^t_{tt}\frac{dT}{ds}\frac{dT}{ds} = 0, \qquad \text{where} \quad \Gamma^t_{tt} = \frac{d}{dT} ln\ \{ch\ (T/C)\}. \tag{14a}$$

and

$$\frac{d^2x}{ds^2} = \frac{d^2y}{ds^2} = \frac{d^2z}{ds^2} = 0, \tag{14b}$$

Eq. (14) implies that there is no spatial acceleration to cause a local transformation. Then, it follows eq. (14) that

$$\frac{dT}{ds} = k\{\exp(T/C) + \exp(-T/C)\}^{-1} \quad \text{and} \quad \frac{dx^\mu}{ds} = \text{constant}, \ x^\mu (= x, y, z) \tag{15a}$$

for some constant k. Now, consider the observer P in the state

$$dx/dT = dy/dT = dz/dT = 0; \ \text{and thus} \ \ dx/ds = dy/ds = dz/ds = 0. \tag{15b}$$

Thus, the geodesic of P is (x_0, y_0, z_0, T), and P would have the same frame of reference whether at "free fall" or not.

Consider the local Minkowski space of P at (x_0, y_0, z_0, T_0)

$$ds^2 = c^2dT'^2 - dx'^2 - dy'^2 - dz'^2, \tag{16a}$$

and

$$dx' = dx, \ dy' = dy, \ dz' = dz, \ \text{and} \ dT' = ch(T/C)dT. \tag{16b}$$

is the local transformation to the local Minkowski space if Einstein's equivalence principle were applicable. Thus, K_T is not a physical space since (16b) implies that two standard clocks have different rates, resting at the same point of a frame. Although K_T is diffeomorphic to the Minkowski space K by the transformation $t = C\{\exp(T/C) - \exp(-T/C)\}/2$ and have the same Euclidean structure, T in metric (13) is not the local time for a physical space-time coordinate system.

To show further that Einstein's equivalence principle makes a difference in physics from Pauli's version, let us examine an accepted manifold, which is due to an electromagnetic plane wave [34] as follows:

$$ds^2 = du\, dv + H\, du^2 - dx^i\, dx^j, \quad \text{where} \quad H = h_{ij}(u)x^i x^j, \tag{17}$$

and

$$h_{ii}(u) \geq 0, \quad h_{ij} = h_{ji}, \quad \text{where} \quad u = ct - z, \quad \text{and} \quad v = ct + z.$$

Here t is the time coordinate with the unit in second; and x, y, z are the space coordinates with the unit in centimeter; and $h_{ij}(u)$ is an energy-stress tensor related to the cause of this gravity, an electromagnetic plane-wave. It can be easily shown that metric (17) has the proper signature, and therefore Pauli's "equivalence principle" is satisfied.

When the source of gravity is removed, i.e., $h_{ij}(u) = 0$, metric (17) is reduced to the flat metric, which is also the form of metric (17) when $x = 0$ and $y = 0$. However, metric (17) violates coordinate relativistic causality since $1 \geq (1 + H)/(1 - H)$ may not be valid. Since $H > 0$ implies $g_{tt} > c^2$, it would have an unusual blue shift compared with the situation of no gravity. Independent of the strength of h_{ij}, H can be arbitrarily large. This is incompatible with Einstein's notion of weak gravity. Due to special relativity, the correspondence principle to classical electrodynamics requires H to be negligible. In the light bending calculation, the gravity of the light is implicitly assumed as negligible. Thus, manifold (17) is not valid in physics.

In addition, a time-like geodesic does not represent a free fall. The gravitational force related to $\Gamma^z_{tt} = \frac{1}{2}(1 + H)c^2\partial H/\partial t$ has parameters (the coordinate origin). However, the principle of causality, by the virtue of symmetry, requires that a valid solution is independent of the coordinate origin. This coordinate origin, being unrelated to any physical cause, is a parameter in violation of this principle [23,35]. Thus, physical aspect of point 1) cannot be valid. Moreover, since this violation is independent of the coordinate system used, the manifold is intrinsically unphysical and cannot be diffeomorphic to a physical space. Apparently, Penrose [34] did not consider the principle of causality, and therefore he accepted metric (17) as valid in physics.

8. Misunderstandings and Misleading Calculations of Tolman and Fock.

The case of an accelerated frame was first deliberated by Einstein, and the reality is a curved space -time was concluded. However, in the literature the connection between an accelerated frame and a space-time metric has not been established as Einstein envisioned. Here, it will be shown that a main reason is due to conceptual errors. Theorists have implicitly assumed that an accelerated frame is related to a Euclidean subspace, instead of just a Euclidean structure.

To apply Einstein's equivalence principle, it is crucial that the space-time under consideration must be a physical space. Theorists, both for and against general relativity have made mistakes by ignoring this. For example, Logunov and Mestvirshvili [13] showed that inconsistent results would be obtained through a coordinate transformation. On the other hand, Tolman [16] also ignored this problem in his illustration of Einstein's equivalence principle. Thus, instead of validity of Einstein's theory, Tolman seemed to show its opposite, i.e., arbitrariness and invalidity just as Logunov et al. claimed. Nevertheless, this is different form Fock [11], who had the purpose of establishing his wrong theory.

Tolman claimed that his treatment [16] is based on the relation of the principle of equivalence to the fundamental idea of the relativity of all kind of motion. To illustrate the equivalence principle, he started with system K_0 with the flat metric,

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \tag{18a}$$

for the first observer. Consider a second observer in a system K', which can be taken as moving relative to the first with the acceleration a in the x-direction, uses the coordinates x', y', z', and t' as given by

$$x' = x - \frac{1}{2}at^2 \qquad y' = y \qquad z' = z \qquad t' = t \tag{18b}$$

according to the usual transformation to accelerated axes, which Tolman regards as a reasonable change at least at low velocities. Substituting from (18b) into (18a), Tolman thought that he obtained the formula for interval for the second observer as

$$ds^2 = (c^2 - a^2t'^2)\, dt'^2 - 2at'dx'dt' - dx'^2 - dy'^2 - dz'^2. \tag{19}$$

Then, according to the geodesic equation, from metric (17) Tolman obtained

$$\frac{d^2x}{ds^2}=\frac{d^2y}{ds^2}=\frac{d^2z}{ds^2}=\frac{d^2t}{ds^2}=0\,; \tag{20}$$

and

$$\frac{d^2x'}{ds^2}=\frac{-a}{c^2-a^2t'^2}, \tag{21a}$$

$$\frac{d^2y'}{ds^2}=\frac{d^2z'}{ds^2}=\frac{d^2t'}{ds^2}=0 \tag{21b}$$

are approximately the equations of motion for the case of particles having negligible velocity. Thus, in spite of that (18b) is essentially a Galilean transformation, Tolman claimed that the equivalence principle was illustrated by (21).

On the other hand, consider a particle P in K' at the beginning of a free fall. Since the velocity of K' relative to K_0 is v = at, for the local Minkowski space (X, Y, Z, T) of P, we have $dx = \gamma[dX + v\,dT]$, $cdt = \gamma[cdT + (v/c)dX]$, where $\gamma = [1-(v/c)^2]^{-1/2}$. It thus follows (18) that there is no time dilation although $\gamma dx' = dX$. Thus, K' is not a physical space. This shows Tolman does not understand the need of a local time in Einstein's theory (see Section 4).

Moreover, if Tolman's calculation were valid, he actually showed that Einstein's equivalence principle were invalid. In Einstein's [8] analysis, the effects of an accelerated frame can be related to a gravitational potential Φ, which is a function of spatial variables in Newtonian theory as shown in eq. (3). But, all the metric elements of (19) are functions of time t'. Although $\Gamma^{x'}{}_{t't'} \neq 0$, the non-zero term in (21a) comes from $g_{t'x'}$ but not from $g_{t't'}$ (since $\partial_\mu g_{t't'} = 0$ for $\mu \neq t'$).

Tolman simply ignored that Einstein's later paper [1,8] confirms his 1911 analysis, and one has the relations,

$$g_{t't'} \approx 1 + 2\Phi/c^2, \qquad \text{and } a_i \approx -\partial\Phi/\partial x^i \tag{22}$$

where Φ is the negative gravitational potential and a function of x'. Obviously, (19) is not consistent with equation (22). Thus, if Einstein's equivalence principle is valid, metric (19) cannot be a physical space. Since Tolman believed in physical validity of arbitrary coordinates, he would also disregard that. (19) would imply the light speed in the x-direction to be $-at' \pm c$.

In an attempt to overcome the deficiency of metric (19), in 1958 Fock [11] modified transformation (18) with

$$x = x' - \frac{1}{2}at'^2, \qquad t = t' - at'x'/c^2. \tag{23}$$

However, Fock does not seem to have any justification other than a desire of putting a new term into the metric element g_{tt} so that its spatial derivative of related gravitational potential would generate the required acceleration as (22). Then, he obtained

$$ds^2 = (c^2 - 2ax' - a^2t'^2)\, dt'^2 - dx'^2 - dy'^2 - dz'^2 + a^2(t'dx' + x'dt')^2/c^2. \qquad (24)$$

The term 2ax' seems to serve the purpose, and metric (24) would be superficially compatible with relation (22).

However, the equation of motion even for $dx'/ds = 0$ is very complicated as follows:

$$\frac{d^2x'}{ds^2} = a\, c^{-2}(1 - a^2t'^2/c^2)^{-1}[(1 - ax'/c^2)^2 - a^2t'^2/c^2]^{-1}. \qquad (25a)$$

It is clear that (25a) is not exactly a uniform acceleration. Also, validity of (24) required two more inequalities as follows:

$$1 - a^2t'^2/c^2 > 0; \qquad \text{and} \qquad (1 - ax'/c^2)^2 - a^2t'^2/c^2 > 0. \qquad (25b)$$

However, the second inequality cannot be justified in terms of physics, and can be traced back to the unjustified second transformation in (23). Thus (23) also does not lead to a physical space. Moreover, the velocity of light in the x'-direction would be

$$dx'/dt' = \{a^2x't'/c^2 \pm (c - ax'/c - a^2t'^2/c)\}/(1 - a^2t'^2/c^2). \qquad (26)$$

But, the light bending experiment supports that a speed reduction under gravity. Since metric (24) is time-dependent, this also disagrees with observation. Moreover, metric (24) is in fundamental theoretical disagreement with Einstein's earlier and also subsequent analysis [1,8]. In short, metric (24) still does not represent a physical space.

Nevertheless, Fock [11] still believed that the problem of time-dependence could be resolved within the framework of the speculated metric (4). Fock [11] proposed the following mathematically ingenious transformation,

$$x = x' \cosh(at'/c) + (c/a)[\cosh(at'/c) - 1] \qquad (27a)$$

$$y = y'; \qquad z = z' \qquad (27b)$$

$$t = (c/a) \sinh(at'/c) + (x'/c) \sinh(at'/c), \qquad (27c)$$

although its physics is not clear. Under the condition $at'/c << 1$, the above equation can be written approximately as

$$x = x' + at'^2/2; \quad y = y'; \quad z = z'; \quad t = t' \tag{28}$$

Substituting (27) into the flat metric, one obtains exactly

$$ds^2 = (c + ax'/c)^2\, dt'^2 - dx'^2 - dy'^2 - dz'^2\,, \tag{29a}$$

Finally, Fock has obtained a metric whose time dilation seems to be compatible with Einstein's paper of 1911. An important difference is that Einstein is based on physical considerations; whereas Fock gave only a pure mathematical manipulation.

To determine the validity of a manifold as a physical space, the physics must be considered. Apparently, the mathematical requirement, $at'/c << 1$, instead of just $at'/c < 1$, is to make (28) approximately valid, but it does not seem to have a physical basis. Moreover, metric (29), in addition to be incompatible with the observed light bending, does not produce a uniform acceleration as claimed. The equation of motion for $dx'/ds = 0$, though better than (25a), is not a uniform gravity as follows:

$$\frac{d^2x'}{ds^2} = -\,a\,c^{-2}[1 + ax'/c^2]^{-1}. \tag{29b}$$

As expected, Fock cannot find a valid interpretation for (29a). Nevertheless, Fock believed that this is due to an intrinsic deficiency of Einstein's equivalence principle. Fock [11] believed that the only correct metric form would approximately be,

$$ds^2 = (c^2 - 2U)\, d\,t^2 - (1 + 2U/c^2)(dx^2 + dy^2 + dz^2)\,, \tag{30}$$

where $U = -\,\Phi$. However, metric (30) is not universal because form (30) cannot accommodate the case of a uniform rotation.

A common problem of Tolman and Fock is that they tried to impose a Euclidean subspace. This judgment will be confirmed by presenting an accelerated frame in terms of an Einstein space, and this will be presented in a separated paper [36].

8. Discussions and Conclusions

The "Einstein elevator" thought experiment is commonly used in the formulation of Einstein' equivalence principle. This is challenged by theorists based on the notion of a "true" gravitational field characterized by the non-vanishing curvature. Since

the tidal forces cannot be transformed away by a coordinate transformation, Einstein's insistence on the fundamental importance of the principle on general relativity was a puzzle to many theorists. The fact was that the initial form of Einstein's equivalence principle was presented only as an idealization of the realistic situation without sufficient clarification.

Norton found the initial form of Einstein's equivalence principle is misinterpreted and Einstein objected, with good reason, Pauli's version as a misinterpretation, and thus did an excellent job as a historian. A related question is that if Einstein strongly objected Pauli's version of infinitesimal equivalence principle, in view of the importance of the infinitesimal case, should not Einstein have his own? Obviously, the answer must be yes, especially Pauli got his from Einstein's work, but Northon failed in identifying Einstein's own version. However, in view of that many professional physicists specialized in general relativity had failed, why should a historian succeed in this task?

Norton found that a problem appears to be different notions for gravity. Although many theorists remain in the area of Newtonian theory of gravity due to a source, Einstein [9] believes[6)], "what characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^{l}_{ik} (field strength), not the non-vanishing of the R_{iklm}" and that no gravity is a kind of gravity. *Thus, Einstein's principle of equivalence is really the equivalence of the effects of an accelerated frame to a related uniform gravity whereas others incorrectly perceived that any gravity is equivalent to a uniformly accelerated frame.* This is evident if one considers also the case of uniform rotation. From this, it is also evident that Einstein's equivalence principle is incompatible with Newtonian notion of gravity. Apparently, this point was not aware of by many theorists. For instance, Bergmann [33] "illustrated" the initial version of Einstein's principle with Einstein's elevator under the gravity of the globe, and thus inadvertently supported misleading criticisms.

A fact contributes to the current confusion is that Einstein's version of infinitesimal equivalence principle was not presented clearly in one location of his paper. Einstein put his version only as if consequences of his earlier version without the expected further labeling (see Section 1). There are two differences from Pauli's version: 1) As Einstein observed, according to the mathematical theorems, gravity cannot be transformed away for some cases; 2) The manifold under consideration must be a physical space-time[1)], which is very clear for the cases of uniform acceleration and uniform rotation. This is also clearly implies by specifying that a local Minkowski space is obtained by choosing the appropriate acceleration. Einstein demonstrated the need of verifying a physical space by checking the perihelion of Mercury and the formula for gravitational red shifts with his space-time metric before the calculation of light bending. However, the notion of a physical space was implicitly assumed as self-evident, and the characteristics of a frame of reference were not explained clearly in terms of Euclidean structure.

Another confusing point is that Einstein predicted definitive time dilation and space contraction measurements with a coordinate system of a physical space while claiming general covariance with respect to arbitrary coordinate systems. As shown in the case of uniform rotation, any arbitrary coordinate system can be used for calculations because of general covariance of Riemannian geometry while a space-time coordinate system (of the physical space) that can be used for physical interpretation, is restricted by his equivalence principle. Related to this is the question of criterion for a physical space. For a given Lorentz metric, it would be non-trivial to determine that such a manifold is a physical space. Although there is a theoretical framework for such an examination (Section 6), it seems, further work on this area may be needed to make such a process more definitive.

A central issue for the applicability of Einstein's equivalence principle is whether the Lorentz manifold under consideration is a physical space (time). However, both Tolman [16] and Fock [11] and more recently Ohanian and Ruffini [14] did not understand the need of Einstein's notion of a physical space (see Sections 4 and 7). Logunov and Mestvirishvili [13] did not see the implicit notion of physical space, and thus turned again Einstein. In addition, based on Pauli's version, theorists like Wald [15] believed in unrestricted covariance without being aware of the physical space. Thus, as pointed out by Bonner, Griffiths, MacCallum [38,39], a consistent physical interpretation is a problem in current theory.

Due to Pauli's version, it became necessary to believe that in general relativity space-time coordinates have no physical meaning, and thus runs against the fact that there are phenomena such as gravitational red shifts related to non-scalars in physics. Moreover, in disagreement with Einstein, the coordinate light speed was considered as meaningless. It is based on such a belief that Hawking [25] declared, "In relativity, there is no real distinction between the space and time coordinates, just as there is no real difference between any two space coordinates." On the other hand, Hawking [25] also believes, "an arrow of time, something that distinguished the past from the future, giving a direction of time". Thus, there is a distinction between a time coordinate and a space coordinate. Nevertheless, Hawking puts these two contradictory statements in the same book [25].

A related problem in current theory is that physical requirements are often ignored. As pointed out by Kramer, Stephani, Herlt and MacCallum [32], unphysical solutions are often accepted as if valid in physics. Violating the principle of causality is a major problem. The Maxwell-Newton Approximation implies that a gravitational wave solution is bounded in magnitude [18], and thus boundedness is needed to be associated with a source. However, many "wave" solutions are unbounded [33,35].

It is invalid to replace Einstein's equivalence principle, which has been firmly established on the theoretical ground of universality of physics and experiments, with Pauli's "equivalence principle" which has proven to be inadequate in physics because it merely requires the existence of local Minkowski spaces. Moreover, such a replacement demands that coordinates have

no physical meanings [40]. On the other hand, Einstein's implicit notion of a physical space requires definitive physical meanings for coordinates of a space-time coordinate system in physics. Such a replacement will also be incompatible with physical principles such as the correspondence principle, the principle of causality, and etc [18].

Observations support, as Einstein's equivalence principle requires, that the space-time coordinates have physical meaning. In the deflection of light calculation[7)], Einstein [2] has made clear that the velocity of light is defined in the sense of Euclidean geometry, and so is the deflection. Although the angle of light deflection can be measured at infinity by explicit comparisons using physical measures (millimetres on a photographic plate, for instance), such an angle is not independent of a physically meaningful coordinate system." To transform two dots in a photographic plate to an angle, one must refer to the coordinate system used to take the photos. For example, such transformations are first based on the Schwarzschild coordinate system [8]. Moreover, a deflection is against the straight line [1-7], which is defined implicitly as the trajectory of a light ray when the sun was absent (and as the approximate trajectory when the light ray is far away from the sun). However, for a uniformly rotating coordinate system, the light ray is not a straight line, and a deflection angle cannot be defined with respect to such a coordinate system. Thus, Pauli's version that ignored Einstein's notion of physical space is not supported by observation.

It has been judged that theories of the Wheeler-Hawking School are different from Einstein's general relativity [29,33]. They often based on Pauli's version but not Einstein's equivalence principle. It would be very beneficial for general relativity, if previous works were reviewed in terms of Einstein's equivalence principle. In fact, some works of Wheeler and Hawking have already been proven invalid in physics [33]. It is hope this paper would add an impetus for restoring the confidence on Einstein's principles[6)] and thus accelerate the further development of Einstein's general relativity.

9. Acknowledgments

This paper is dedicated to Professor P. Morrison of Massachusetts Institute of Technology, who suggested this problem, for fruitful discussions over years. The author gratefully acknowledges stimulating discussions with Professors C. Au, C. L. Cao, Mauro Francaviglia, Li-Zhi Fang, J. L. Friedman, L. Ford, R. Geroch, A. H. Guth, J. E. Hogarth, H. Nicolai, A. Napier, Liu Liao, H. C. Ohanian, Ethan T. Vishniac, R. M. Wald, Erick J. Weinberg, J. A. Wheeler, H. Yilmaz, Yu Yun-qiang, and Y. Z. Zhang. Special thanks are to Dr. H. C. Chan for reading through this manuscript and useful suggestions, and the referees for valuable comments and pointing out related literature. This work is supported in part by Innotec Design, Inc., U. S. A.

ENDNOTES

1) A Riemannian physical space-time (M, g), according to Einstein [1,8], is a Riemannian space in mathematics. Such a Riemannian space M is characterized by a space-time metric g that can be determined by the distribution of matter. It is in the sense of that the metric g_{ik} is subjected to physical considerations, that Einstein considers the four-dimensional space-time continuum of reality as a physical space-time. In "Relativity and the problem of space", Einstein [31] wrote,

> "For the functions g_{ik} describe not only the field, but at the same time also the topological and metrical structural properties of the manifold. ... There is no such thing as an empty space, i.e., a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field."

Moreover, since Einstein's Riemannian space-time models reality, all the physical requirements must be sufficiently satisfied by the space-time metric g_{ik}. This is another requirement for Einstein's physical space (-time). Since Einstein's equivalence principle is a physical requirement proposed for the real world, to apply this principle correctly to a Lorentz manifold, one must show, as Einstein did [1,8], that all known physical requirements are sufficiently satisfied.

One might argue that a physical space has not been defined precisely because the physical requirements have not been defined completely. However, physical requirements are almost what physics is all about, and can be defined more complete as physics progress. Thus, the physical requirements are not complete until the end of physics if it has an end. Einstein himself has indicated the difficulty of defining general relativity "precisely" as mathematics or even in the degree of Maxwell's theory. Einstein wrote in 1916 [8] the following:

> "It is not my purpose in this discussion to represents the general theory of relativity as a system that is as simple and logical as possible, and with the minimum number of axioms; but my main object is to develop this theory in such a way that the reader will feel that the path we have entered upon is psychologically the natural one, and that the underlying assumptions will seem to have the highest possible degree of security."

Another problem is that he has not been able to describe precisely the physical process (due to a free fall), which transforms a metric near a point to a local Minkowski space although he infers the correct result.

2) There are only a few books on gravitation and general relativity, which exceed 500 pages, on the market. This author finds only four. They are: a) Gravitation and Cosmology: principles and applications of the general theory of relativity (John Wiley Inc., 667 pages) by. S. Weinberg; b) ***Gravitation*** (Freeman, 1279 pages) by C. W. Misner, K. S. Thorne, & J. A. Wheeler; c)

Stars and Relativity (Dover, 522 pages) by Ya. B. Zel'dovich & I. D. Novikov, d) ***Relativity Thermodynamics and Cosmology*** (Dover, 501 pages) by R. C. Tolman.

3) Some theorists, for example Bergmann [37] and Liu [41], believed incorrectly that the equivalence of all frames of reference must be represented by the equivalence of all coordinate systems. For instance, the exchange of the time coordinate and a spatial coordinate would form a new coordinate system. But such a new coordinate system is not related to any frame of reference. Moreover, it is based on unrestricted covariance that Logunov and Mestvirishvili [13] rejected general relativity.

4) Fock's problem actually started from special relativity. Fock [11] claimed, "We saw that to any energy E one should ascribe a mass $m = E/c^2$ and to every mass one should ascribe an energy $E = mc^2$." However, this is inconsistent with general relativity that has a tensor source for gravity [20]. According to Einstein [42], only the latter is valid. Tolman [16] also made the same kind of mistake in the form of an assumption, "Hence in what follows we shall postulate in general that a quantity of energy E always has immediately associated with a mass m of the amount $m = E/c^2$."

5) Currently, the bending of light is calculated directly without going through all Einstein's steps, because the checking has already been done. By so doing, one may be misled to believe that Einstein's equivalence principle is equivalent to Pauli's version. Consequently, in the literature, unphysical metric is often accepted as valid [32,33].

6) A traditional viewpoint of the Physics Department of MIT is that general relativity must be understood in terms of physics. Weinberg [4] advocated, "In my view, it is much more useful to regard general relativity above all as a theory of *gravitation*, whose connection with geometry arises from the peculiar empirical properties of gravitation, properties summarized by Einstein's Principle of the Equivalence of Gravitation and Inertia." Thus, Einstein's viewpoint of gravity is supported. However, many [5,10,11,14,22,25] believe that true gravity is related to the non-vanishing of the curvature R_{iklm}.

7) The defection of light can be calculated directly from the Maxwell-Newton Approximation [1,21] for massive matter, a linear equation for weak gravity, which has been derived from physical principles including Einstein's equivalence principle [18,29]. This would supplement Einstein's 1916 derivation [8] and also his 1921 derivation [1] from the 1915 Einstein equation. It has been found that the 1915 Einstein equation does not have a dynamic solution and is actually inconsistent with the linear equation for a dynamic situation [21,33]. Gullstrand [43] was first to question the existence of dynamic solutions for the 1915 equation in his 1921 report to the Nobel Committee [17]. Einstein [44] himself also recognized the deficiency of the 1915 equation when he was working with Rosen [45,46] on the question of gravitational radiation.

REFERENCES

1. A. Einstein, ***The Meaning of Relativity*** (1921) (Princeton Univ. Press, 1954), pp 90-93.

2. A. S. Eddington, ***The Mathematical Theory of Relativity*** (1923) (Chelsa, New York, 1975), p. 10.

3. W. Pauli, ***Theory of Relativity*** (1921) (Pergamon, London, 1958), p. 145.

4. S. Weinberg, ***Gravitation and Cosmology:*** (John Wiley Inc., New York, 1972), p. 3.

5. C. W. Misner, K. S. Thorne, & J. A. Wheeler, ***Gravitation*** (Freeman, San Francisco, 1973), p. 386.

6. N. Straumann, ***General Relativity and Relativistic Astrophysics*** (Springer, New York, 1984), pp 82-84.

7. Yu Yun-qiang, ***An Introduction to General Relativity*** (Peking Univ. Press, Beijing, 1997), pp 41-43 .

8. A. Einstein, Analen der Physik, **49**, 769-822 (1916); A. Einstein, H. A. Lorentz, H. Minkowski, H. Weyl, ***The Principle of Relativity*** (Dover, New York, 1952), p. 118 & p. 162.

9. J. Norton, "What was Einstein's Principle of Equivalence?" in Einstein's Studies Vol. 1: ***Einstein and the History of General Relativity,*** Eds. D. Howard & J. Stachel (Birkhäuser, 1989).

10. J. L. Synge, ***Relativity*** (North-Holland, Amsterdam, 1956), pp IX-X.

11. V. A. Fock, ***The Theory of Space Time and Gravitation***, trans. N. Kemmer (Pergamon Press, 1964), pp 6, 111, 119, 231.

12. K. S. Thorne, ***Black Holes & Time Warps*** (Northon, New York, 1994), p. 105, p.456.

13. A. Logunov and M. Mestvirishvili, ***The Relativistic Theory of Gravitation*** (Mir Publishers, Moscow, 1989), pp 24 & 106.

14. H. C. Ohanian & R. Ruffini, ***Gravitation and Spacetime*** (Norton, New York, 1994), p. xi, p. 54, and back cover.

15. R. M. Wald, ***General Relativity*** (The Univ. of Chicago Press, Chicago, 1984), p. 78 & p. 438.

16. R. C. Tolman, ***Relativity, Thermodynamics, and Cosmology*** (Dover, New York, 1987), p. 49, pp 175-178.

17. A. Pais, ***Subtle is the Lord ...*** (Oxford University Press, New York, 1996), pp.255-261, 509.

18. C. Y. Lo, Phys. Essays, **12** (3), 508-526 (Sept. 1999).

19. X. Yu, Ap & SS **194**, 159 (1992).

20. C. Y. Lo, Astrophys. J., **477**: 700-704 (1997).

21. C. Y. Lo, Astrophys. J., **455**: 421-428 (Dec. 20, 1995).

22. Ya. B. Zel'dovich & I. D. Novikov, ***Stars and Relativity*** (Dover, New York 1996), pp 7-16.

23. C. Y. Lo, Phys. Essays, **11** (2), 264-272 (June. 1998).

24. P. Lenard, Über das Relativitätsprinzip, Äther, Gravitation (Leipzig, 1918).

25. S. W. Hawking, ***A Brief History of Time*** (Bantam, New York, 1988), pp 24, 50, 87, & 143-152.

26. L. Motz & J. H. Weaver, ***The Story of Physics*** (Avon, New York, 1989), p. 366.

27. Kretschmann, Ann. Phys., Lpz., **53**, 575 (1917).

28. A. Einstein, Ann. Phys.(Leipig) 55, 241 (1918).

29. C. Y. Lo, Phys. Essays, **13** (1) 109-120 (March 2000).

30. A. Einstein, 'Geometry and Experience (1921)' in ***Ideas and Opinions*** (Crown, New York, 1982), p. 233.

31. A. Einstein, 'Relativity and the Problem of Space (1954)' in ***Ideas and Opinions*** (Crown, New York, 1982), p. 372, p. 375.

32. D. Kramer, H. Stephani, E. Herlt and M. MacCallum, ***Exaction Solutions of Einstein's Field Equations, ed. E. Schmutzer*** (Cambridge Univ. Press, Cambridge, 1980), pp 19-24.

33. C. Y. Lo, Phys. Essays, **13** (4), 527-539 (Dec. 2000).

34. R. Penrose, Rev. Mod. Phys. **37** (1) 215-220 (1965).

35. C. Y. Lo, Phys. Essays, **12** (2), 226 (June, 1999).

36. C. Y. Lo, "The Einstein Space of an Accelerated Frame and Uniform Gravity", in preparation.

37. P. G. Bergmann, ***Introduction to the Theory of Relativity*** (Dover, New York, 1976), p. 156 & p. 159.

38. W. B. Bonnor, Gen. Rel. & Grav., **24,** 551-574 (1994).

39. W. B. Bonnor, J. B. Griffiths, & M. MacCallum, Gen, Rel. & Grav., **26** (7), 687 (1994).

40. H. Nicolai, Max-Planck-Institut für Gravitationsphysik, private communications (Aug. 2000).

41. Liu Liao, ***General Relativity*** (High Education Press, Shanghai, China, 1987), pp 13-16.

42. A. Einstein, '$E = mc^2$ (1946)' in ***Ideas and Opinions*** (Crown, New York, 1954).

43. A. Gullstrand, Ark. Mat. Astr. Fys. **16**, No. 8 (1921); ibid. Ark. Mat. Astr. Fys. **17**, No. 3 (1922).

44. ***The Born-Einstein Letters***, commentary by Max Born (Walker, New York, 1968), p. 125.

45. A. Einstein & N. Rosen, J. Franklin Inst. **223**, 43 (1937).

46. N. Rosen, in Jubilee of Relativity Theory, edited by A. Mercier & M. Kervaire (Cirkh user Verlag, Basel, 1956).