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An Existence of Local Minkowski Spaces is Insufficient for Einstein's Equivalence Principle

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Abstract

The equivalence principle, which states the local equivalence between acceleration and gravity, requires that in a physical space a free falling observer must result in a co-moving local Minkowski space. On the other hand, a Lorentz metric which is not locally Minkowski, may not implies the existence of an acceleration to a particle. Thus, a time-like geodesic need not physically lead to a local Minkowski space and thus does not represent a physical free falling. Given that the co-moving local Minkowski space exists mathematically along a time-like geodesic in a Lorentz manifold, a crucial question for the satisfaction of the equivalence principle is whether a geodesic represents a *physical* free falling. Also, Einstein's equivalence principle necessarily restricts general mathematical covariance. Examples are given to illustrate the above statements. Moreover, there are Lorentz manifolds none of which can be diffeomorphic to a physical space. *Thus, defining a light speed in a Lorentz manifold is not only invalid but misleading.*

"As far as the prepositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." -- A. Einstein (in 'Geometry and Experience', 1921).

1. Introduction.

A major problem in general relativity is that any Riemannian geometry with the proper metric signature would be accepted as a solution of Einstein's equation of 1915; and many unphysical solutions were accepted as valid [1]. This clearly suggests that, for a manifold to be a physical space, the mathematical existence of local Minkowski space which is due to a Lorentz metric signature [2], would be insufficient.

On the other hand, Einstein's equivalence principle requires that a free physical falling results in a co-moving local Minkowski space[1)] [3,4]. It seems, many relativists [5-11] believed that a proper metric signature would imply a satisfaction of the equivalence principle. A related question would be whether a satisfaction of the equivalence principle is sufficient for a manifold to be a physical space. Another possibility is that their belief is incorrect and that ***a mathematical existence of local Minkowski spaces is a necessary but insufficient condition for a satisfaction of the equivalence principle*** [12].

Einstein proposed that the equivalence principle is satisfied in a physical space. From his calculation of light bending [3,4], ***Einstein seems to believe that a Riemannian space is physically realizable if and only if the equivalence principle is satisfied.*** If Einstein is right, there must be some differences between the mathematical existence of local Minkowski space and Einstein's equivalence principle. In other words, some physical understandings may have been missed from Einstein's equivalence principle.

It should be noted that a physical "free falling" ***automatically*** results in a local Minkowski space is a physical requirement (§ 2). However, in a Lorentz manifold although there always exists a local Minkowski space for any space-time point, a "free falling" ***may not*** result in a co-moving local Minkowski space (§ 3). Moreover, there may not be acceleration even if the Lorentz metric is not locally Minkowski.

The misconception that, in a Lorentz manifold, a "free falling" would result in a local Minkowski space [10,11], has deep-rooted mathematical and physical misunderstandings from believing in the general mathematical covariance [4,10]. Thus, it would be necessary to demonstrate through detailed examples that diffeomorphic coordinate systems may not be equivalent in physics (§§ 3-5). These illustrations are based on

theoretical inconsistency, and thus is independent of the individual preferences of a theorist.

The physical validity of mathematical general covariance is incompatible with that time has a direction [13] and other established facts. For example, to establish special relativity, it has been shown that the Galilean transformation is physically not realizable by the Michelson–Morley experiment [14] and is incompatible with coordinate relativistic causality[2)] [15,16]. Thus, Einstein's interim proposal of a general covariance among all conceivable mathematical coordinate systems [4], has been proven to be an over–extended demand. (Also, the gauge due to general mathematical covariance, was not accepted by Eddington [17].)

Moreover, there are ***intrinsically*** unphysical Lorentz manifolds none of which is diffeomorphic to a physical space–time (§ 6). (Nevertheless, some mathematically "define" the physical light speed in terms of the local Minkowski space in an unphysical manifold.) Thus, to accept a Lorentz manifold as valid in physics, it is necessary to verify a satisfaction of the equivalence principle in a space–time coordinate system on which physical interpretation can be based. Then, for the purpose of calculation only, any diffeomorphism can be used to obtain new coordinates. In other words, only a mathematical (but not physical) coordinate system can be arbitrary. After this necessary rectification, some currently accepted Lorentz manifolds would be exposed as unphysical (§ 6), although ***general relativity as a physical theory is unaffected.***

2. Two Mathematical Theorems in Riemannian Space and Einstein's Equivalence Principle

Currently, Einstein's equivalence principle is often incorrectly considered as equivalent to the mathematical existence of local Minkowski space. Such a misunderstanding is related to two mathematical theorems of Riemannian space [2] which are often confused with Einstein's equivalence principle by some mathematicians and theorists. These theorems are:

Theorem 1. Given any point P in any Lorentz manifold (whose metric signature is the same as a Minkowski space) there always exist coordinate systems (x^μ) in which $\partial g_{\mu\nu}/\partial x^\alpha = 0$ at P.

Theorem 2. Given any time–like geodesic curve Γ there always exist a coordinate system (so–called Fermi coordinates) (x^μ) in which $\partial g_{\mu\nu}/\partial x^\alpha = 0$ along Γ.

From these theorems, it is clear that a local Minkowski metric exists at any given point and that along any time-like geodesic curve Γ, a moving local constant metric exists [2].

However, there is no physical specification for the local coordinate transformation, which transforms a Lorentz metric to a local Minkowski metric along a time-like geodesic curve. Therefore, there is nothing to exclude an unphysical situation, these theorems cannot represent a physical principle. Einstein [18] pointed out, "As far as the prepositions of mathematics refers to reality, they are not certain; and as far as they are certain, they do not refer to reality." Hence, an application of a theorem should be examined for its physical relevance although "one cannot really argue with a mathematical theorem [13]". Note that a mathematical local coordinate transformation may not necessarily be related to the physics of a "free falling" (see §3).

Einstein [4] proposed, as part of the equivalence principle, "special theory of relativity applies to the special case of the absence of a gravitational field." This means that the Minkowski metric is the only constant metric which is valid in physics. In other words, Einstein's equivalence principle is also supported by the Michelson-Morley experiment; and the equivalence principle restricts general mathematical covariance. But, theorists such as Synge [2] did not understand the physical meaning of Einstein's proposal.

3. Free Falling, Special Relativity, and the Equivalence Principle

The equivalence principle is a relation between acceleration and gravity. Therefore, to determine whether the equivalence principle is satisfied, one can directly examine the acceleration of a particle related to the metric. One may also note that there are no accelerations involved in the above theorems.

Initially, based on the observation that the (passive) gravitational mass and inertial mass are equivalent, Einstein proposed the equivalence of uniform acceleration and gravity. Soon, this proposal is changed to the local equivalence of acceleration and gravity because of the obvious fact that gravity is almost never uniform. Thus, if gravity is represented by the space-time metric, the geodesic is the motion of a particle under the influence of gravity. Then, for an observer in free falling, the local metric is locally a constant. To be consistent with special relatvity, such a local metric is required to be locally a Minkowski space.

Since the mathematical existence of local Minkowski space is assured, the central problem in general relativity is ***whether the geodesic represents a physical free falling***. Note, however, that validity of

this global property is realized ***locally*** through a satisfaction of the equivalence principle. Also, Eddington observed [17] that special relativity should apply only to phenomena unrelated to the second order derivatives of the metric. Einstein [19] accepted this criticism and added the crucial phrase, "at least to a first approximation" on the indistinguishability between gravity and acceleration.

To clarify the 1916 paper [4], Einstein wrote ***in his book*** [3], "According to the principle of equivalence, the metrical relation of the Euclidean geometry are valid relative to a Cartesian system of reference of infinitely small dimensions, and in a suitable state of motion (free falling, and without rotation)." Thus, at any point (x,y,z,t) of space-time, a "free falling" observer P must be in a co-moving local ***Minkowski space*** L with the local metric

$$ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2, \tag{1}$$

whose spatial coordinates are attached to P, whose motion is governed by the geodesic,

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \quad \text{where} \quad ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \tag{2}$$

and

$$\Gamma^\mu{}_{\alpha\beta}=(\partial_\alpha g_{\nu\beta}+\partial_\beta g_{\nu\alpha}-\partial_\nu g_{\alpha\beta})g^{\mu\nu}/2 \tag{3}$$

are defined by the space-time metric $g_{\mu\nu}$. (The attachment means that, between P and L, there is no relative motion, nor relative acceleration.) Thus, ***when*** a space-ship is under the influence of gravity only, ***the local space-time is automatically Minkowski***, i.e., no other action required. From the geodesic equation, it is clear that a Lorentz manifold need not imply a satisfaction of the equivalence principle.

When gravity (i.e., gravitational acceleration) exists, eq. (2) implies $\Gamma^\mu{}_{\alpha\beta} \neq 0$; and thus the space-time metric $g_{\mu\nu}$ is not a constant. On the other hand, for a given non-constant metric $g_{\mu\nu}$, an acceleration may not exist because $\Gamma^\mu{}_{\alpha\beta}(dx^\alpha/ds)(dx^\beta/ds)$ can still be zero. For instance, when $\Gamma^\mu{}_{tt} = 0$ ($\mu \neq t$), there is no acceleration for an initially static particle. But, $\Gamma^\mu{}_{tt} = 0$ need not implies that $g_{\mu\nu}$ is a constant metric. One may ask would the equivalence principle be satisfied in such a Lorentz manifold? Clearly, the equi-

valence principle is not satisfied. Since there is no acceleration nor motion, the particle remains in the same position (i.e. the particle is not in a local Minkowski space). Although such a coordinate transformation is mathematically possible, there is no ***physical cause*** to make such a transformation physically automatic.

To illustrate this, let us consider a Lorentz manifold with the nonconstant metric,

$$ds^2 = \frac{1}{4}\{\exp(T/C) + \exp(-T/C)\}^2 dT^2 - dx^2 - dy^2 - dz^2, \tag{4}$$

where C is a constant. From metric (4), the Christoffel symbols are zeros except $\Gamma_{t,tt} = \partial_t g_{tt}/2$. If the equivalence principle were valid, the equation of motion for a observer P at (x,y,z,T) is

$$\frac{d^2T}{ds^2} + \Gamma^t{}_{tt}\frac{dT}{ds}\frac{dT}{ds} = 0, \quad \text{and} \quad \frac{d^2x}{ds^2} = \frac{d^2y}{ds^2} = \frac{d^2z}{ds^2} = 0 \tag{5a}$$

where

$$\Gamma^t{}_{tt} = \frac{d}{dT}(\ln\{\exp(T/C) + \exp(-T/C)\}). \tag{5b}$$

Eq. (5) implies there is no spatial acceleration, and thus there is no physical cause to have a physical space-time transformation due to gravity. Then, it follows eq. (5) that

$$\frac{dT}{ds} = k\{\exp(T/C) + \exp(-T/C)\}^{-1} \quad \text{and} \quad \frac{dx^\mu}{ds} = \text{constant}, \quad x^\mu \ (= x,\ y,\ z) \tag{6a}$$

for some constant k. Now, consider the observer P in the state

$$dx/dT = dy/dT = dz/dT = 0; \text{ and thus } dx/ds = dy/ds = dz/ds = 0. \tag{6b}$$

Thus, there is no change in the spatial position nor acceleration. Physically, this means that such an observer P would have the same frame of reference[3)], whether "free falling" or not.

If the equivalence principle were satisfied, according to Einstein's calculation of the light bending [3], $ds^2 = 0$ and the space-time metric imply the measured light speed would be $\{\exp(T/C) + \exp(-T/C)\}/2$..

Then for such an observer, he would absurdly have two different light speeds from the same frame of reference. Thus, the geodesic cannot prepresent a physical free falling because its co-moving local metric is not automatically local Minkowski space.

Moreover, since metric (4) is obtained from the flat metric

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \tag{7a}$$

with

$$ct = C\{\exp(T/C) - \exp(-T/C)\}/2, \tag{7b}$$

a diffeomorphism, this shows that a Gaussian system may not satisfy the equivalence principle. This example supports Einstein's veiwpoint that once the frame of reference is chosen, the time-coordinate is determined.

4. Physics beyond the Metric Signature.

The above example shows that one must consider physics beyond the signature of a metric of a manifold. There are three physical aspects in Einstein's equivalence principle as follows [3]:

1) In a physical space, the motion of a free falling observer is a geodesic.

2) The local space-time of an observer is Minkowski, when 1) is true.

3) A physical local coordinate transformation transforms the metric to local Minkowski, when 2) is true.

Point 3) indicates the fact that this physical local coordinate transformation is due to free falling ***alone***. In other words, the physical validity of the geodesic 1) is a prerequisite for the satisfaction of the equivalence principle, and validity of 3) is an indication of such a satisfaction. Thus, a satisfaction of the equivalence principle involves local physical properties which are beyond the mathematical tangent space [6].

However, if Einstein had greatly emphasized the mathematical existence of local Minkowski spaces, in current theory, the equivalence principle has been incorrectly considered as such.

An important aspect of the equivalence principle is the automatic physical transfer to a local Minowski space. Einstein [4] proposed "special theory of relativity applies to the case of the absence of a gravitational field". Thus, it is incorrect of claim that the equivalence principle is always satisfied in a Lorentz manifold

because a merely mathematical existence of the orthogonal tetrad is insufficient (see also §§ 5 & 6). In other words, the interpretation of the equivalence principle as identical to the mathematical theorems is not valid in physics. Such a misunderstanding is responsible for accepting many unphysical solutions as valid.

Perhaps, this inadequate understanding is, in part, due to the fact that it is often difficult to see the physical validity of point 2) directly, i.e., how the metric transformed automatically to a local Minkowski space. To this end, examining point 1) and/or point 3) would be useful. Point 1) is a prerequisite of the equivalence principle point 2). For Point 1) to be valid, i.e., the geodesic representing a physical free falling, it is required that the metric of such a manifold should satisfy all physical principles. Needless to say, such a metric must be physically realizable. If point 1) is valid in physics, point 3) should produce valid physical results. Thus, one can check point 3) to determine the validity of point 1) or vice versa.

The mathematical existence of a Local Minkowski space alone implies only that Riemannian geometry is compatible with the equivalence principle. The physics is whether the existence of a physical local transformation which transforms the metric to a local Minkowski space. It is the equivalence principle that ensures the existence of such a physical transformation. Thus, one must carefully distinguish mathematical properties of a Lorentz metric from physical requirements. Apparently, a discussion on the possibility that the equivalence principle can fail in a Lorentz manifold, was ***over-looked*** by Einstein and others (see also §§ 5 & 6).

To illustrate further that the equivalence principle can fail in a Lorentz Manifold, consider the metric,

$$ds^2 = dt^2 - \frac{1}{4}\{\exp(X/C) + \exp(-X/C)\}^2 dX^2 - dy^2 - dz^2 \tag{8}$$

If the equivalence principle were satisfied in manifold (t,X,y,z), the light speed in the X-direction, would be $\pm 2\{\exp(X/C) + \exp(-X/C)\}^{-1}$, and has limit zero as $X \to \infty$ without an identifiable physical cause. In general relativity, a very slow speed of light is due to strong gravity.

To show that metric (8) also fails satisfying the equivalence principle, consider an observer P at (X_0, y_0, z_0, t_0). From metric (8), one has that for P the geodesic equation is,

$$\frac{d^2X}{ds^2} + \Gamma^X{}_{XX}\frac{dX}{ds}\frac{dX}{ds} = 0, \quad \text{and} \qquad \frac{d^2y}{ds^2} = \frac{d^2z}{ds^2} = \frac{d^2t}{ds^2} = 0. \tag{9a}$$

where

$$\Gamma^{X}{}_{XX} = \frac{d}{dT} \ln[\alpha(X)] \ , \quad \text{where} \quad \alpha(X) = \{\exp(X/C) + \exp(-X/C)\}/2$$

Note that d^2X/ds^2 can be zero if $dX/ds = 0$. Thus, the gravity does not really cause an acceleration, and thus no physical changes should be produced by gravity. It follows eq. (9a) one obtains, for some constants k_i

$$\frac{dX}{ds} = k_x \alpha(x)^{-1}, \ \frac{dy}{ds} = k_y \ , \ \frac{dz}{ds} = k_z \ , \ \frac{dt}{ds} = k_t \ , \text{ and } \quad k_t^2 - k_x^2 - k_y^2 - k_z^2 = 1. \qquad (9b)$$

If P is resting, i.e., $dx/dT = dy/dT = dz/dT = 0$; then only $k_t = 1$ is non-zero. Thus, since in such a "free falling", there is no change in the spatial position nor any acceleration on P, the equivalence principle cannot be satisfied. Note also that metric (8) is obtained from the flat metric with the diffeomorphism

$$x = C\{\exp(X/C) - \exp(-X/C)\}/2. \qquad (10)$$

Thus, this example shows also that the space coordinates cannot be arbitrary.

5. Incompatibility of the Galilean Transformation to the Equivalence Principle

It will be shown that a Galilean transformation, which is experimentally unrealizable, is incompatible with the equivalence principle. In current theory, such an physically unrealizable transformation is inconsistently believed as "valid". The equivalence principle was incorrectly considered as satisfied in a Lorentz manifold because the physics of a time-like geodesic has not been adequately considered.

Consider the Galilean transformation from (x,y,z,t) to the K' coordinates,

$$t = t', \quad x = x', \quad y = y', \quad \text{and} \quad z = z' - vt', \qquad (11a)$$

where v is a constant. Eq. (11a) transforms metric (7a) to another constant Lorentz metric

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2. \tag{11b}$$

This is a constant metric which is not Minkowski. Thus, point 1) of the equivalence principle is not satisfied.

It follows that (11b) does not give the correct light speed. For light rays in the z'-direction, (11b) and $ds^2 = 0$ would imply the light speeds were

$$\frac{dz'}{dt'} = c + v, \quad \text{or} \quad \frac{dz'}{dt'} = -c + v. \tag{12}$$

Clearly, eq. (12) violates point 1) of the equivalence principle again since (12) also violates ***coordinate relativistic causality*** (i.e., no cause event can propagate faster than the velocity of light in a vacuum). Nevertheless, this problem is currently being ignored by claiming (even against Einstein physical reasoning in his calculation of the light bending) that the coordinate light speed has no meaning by simply "define" invalidly (see next section) a light speed with the local Minkowski space.

Moreover, according to the geodesic equation (2), metric (11b) implies $d^2x'^\mu/ds^2 = 0$, and thus

$$\frac{dx'^\mu}{ds} = \text{constant.} \qquad \text{where } x'^\mu \ (= x', y', z', \text{ or } t') \tag{13}$$

at any point. According to the equivalence principle, no physical changes are caused by gravity. However, according to metric (11b), consider the "free falling" of an observer P' at (x'_0, y'_0, z'_0, t'_0)

$$dx'/ds = dy'/ds = dz'/ds = 0, \quad \text{and} \quad dt'/ds = (c^2 - v^2)^{-1/2}. \tag{14}$$

For such a "free falling" observer P' is at rest and carries with him the frame of reference K'. But, the K' metric (11b) is not a Minkowski space. Since a "free falling" does not automatically obtain a local Minkowski space, point 2) of the equivalence principle is violated. Moreover, for observer P', according eq. (1) the measured light speed is c, but according (11b) the light speed in the x-direction is $(c^2 - v^2)^{1/2}$. This inconsistency also implies that point 3) of the equivalence principle is not satisfied in K'.

Nevertheless, ***mathematics*** ensures the existence of a local Minkowski space. According to condition (14), the time coordinate would remain the same dt'. But, the coordinate dz' is not orthogonal to dt'. In order to have three orthogonal space coordinates to form a local Minkowski space, one may transform dz' by

$$dt'' = dt', \quad dx'' = dx', \quad dy'' = dy', \quad \text{and} \quad dz'' = dz' - vdt'. \tag{15}$$

But, the coordinate system (dx",dy",dz") is not co-moving with the observer P'.

Now, let us work out the local orthogonal tetrad of P', whose direction $v_{P'}$ is (0,0,0,dt'). Then, the orthonormal vectors of the tetrad are

$$a_1 = (1,0,0,0),\ a_2 = (0,1,0,0),\ a_3 = (0,0,\alpha,\beta), \quad \text{and} \quad b_{P'} = (0,0,0,\gamma) \tag{16a}$$

where

$$\alpha = \gamma^{-1},\ \beta = -\gamma v/c^2, \text{ and } \gamma = (1 - v^2/c^2)^{-1/2}.$$

The corresponding transformations is as follows:

$$dt' = \gamma(dT - v/c^2 dZ)\ , \quad dz' = \gamma^{-1}dZ, \quad dx' = dX, \quad \text{and} \quad dy' = dY. \tag{16b}$$

Thus, (dx',dy',dz') and (dX,dY,dZ) share the same frame of reference[3)] because there is no acceleration nor motion. But, there is space measurement changes in the z-direction. Metric (11b) does not satisfy point 2) of the equivalence principle since there is ***no physical cause*** for transformation (16b). In relativity, such a physical transformation happens only when there is relative motion or the presence of gravity. But, P' is rest at K'. Thus, (16b) illustrates also that geodesic (14) does not represent a physical free falling.

A misunderstanding of the equivalence principle, as Yu (p. 42 of [11]) believed, is that at ***any*** space-time point, it is always possible to establish a local Minkowski space which is related to a "free falling". However, ***this is necessary but insufficient.*** For instance, at any space-time point of manifold (11b), there is a local Minkowski space which is co-moving with a "free falling" observer in the manifold. But, in

such Lorentz manifolds, the geodesic does not represent a ***physical free falling***.

The Galilean transformation is an unphysical transformation, and it simply takes another unphysical transformation to cancel out the unphysical properties so introduced. In fact, (11a), and (16b) imply

$$dt = \gamma(dT - v/c^2 dZ) \ , \quad \text{and} \quad dZ = \gamma dz' = \gamma\ (dz + v\ dt). \tag{16c}$$

Transformation (16c) is just a Lorentz–Poincaré transformation. Transformation (15) is an inverse Galilean transformation; and (16b) completes the transformation (16c) starting form (11a).

It has been shown in different approaches that metric (11b) is incompatible with physics and in particular the equivalence principle. Since (11a) is a Galilean transformation, it has been shown that the Galilean transformation is also not valid in general relativity. The failure of satisfying the equivalence principle should be expected since the Galilean transformation is experimentally not realizable. This analysis shows also that the Minkowski metric is only valid constant metric in physics. In fact, a general result is that if $\Gamma^{\alpha}{}_{tt} = 0$ for $\alpha = x, y,$ or z, then the equivalence principle is satisfied only if the metric is Minkowski.

Another consequence is the reaffirmation of coordinate relativistic causality in vacuum. The possibility that the speed of light could be larger than c through a coordinate transformation is inconsistent with the notion that c as the maximum possible speed. The equivalence principle rules out such a possibility. It thus follows that physically the speed of light cannot be larger than c at the presence of gravity. In fact, in agreement with the equivalence principle, observation confirms that gravity only leads to a reduction of the light speed.

It has been illustrated that the Galilean transformation is incompatible with the equivalence principle in the absence of gravity. In fact, the incompatibility is also true even when gravity is present.

6. Covariance, Physical Principles, and Intrinsically Unphysical Lorentz Manifolds

Kretschmann [20] pointed out that the postulate of general covariance does not make any assertions about the physical content of the physical laws, but only about their mathematical formulation, and Einstein entirely concurred with his view. However, Einstein [4] argued also that "... there is no immediate reason for preferring certain systems of coordinates to others, that is to say, we arrive at the requirement of general

co-variance." This is, of course, incorrect since the equivalence principle *rejects* some coordinate systems.

The equivalence principle is a requirement for a valid physical space-time coordinate system. In a free falling, the co-moving local coordinate system must be Minkowski -- a non-covariant requirement. Moreover, general covariance must be restricted because a distinction between time and space is inherent. As Hawking [13] pointed out that "something that distinguished the past from the future, giving a direction to time." The principle of general relativity requires only the equivalence of all frames of reference[3].

Einstein proposed that the equivalence principle is satisfied in a physical space-time. This also means that the equivalence principle is satisfied, only if the manifold is physically realizable. The theorems in mathematics support this principle by showing that the local Minkowski spaces always exist for a Lorentz manifold. With the question on the existence of local Minkowski spaces removed, the remaining question is whether the physical motion of a free falling is represented by a geodesic. It follows that, for a Lorentz metric, the metric is not valid in physics, if and only if the local transformation due to a "free falling" produces unphysical results. For instance, since coordinate relativistic causality is not satisfied by metric (11b), assuming the validity of the equivalence principle leads to unphysical results.

On the other hand, all the previous unphysical manifolds in this paper can be considered as diffeomorphic to a physical space. Naturally, one would ask whether a Lorentz manifold is always diffeomorphic to a physical space. If this were true, then the metric signature would be essentially equivalent to the equivalence principle. Thus, it is important to find out whether ***there are intrinsically unphysical Lorentz manifolds any of which cannot be diffeomorphic to a physical space.***

In general, to determine whether a given manifold can be diffeomorphic to a physical space, is a difficult problem because all possible diffeomorphisms must be considered. The fact that a manifold is not a physical space if the equivalence principle is not satisfied at one point, is not very useful because a new manifold may be obtained by modifying the coordinates in a small region. However, if a manifold cannot be diffeomorphic to a physical space, this manifold must fail a physical requirement which is independent of the choice of coordinates. For instance, failing the principle of causality can be such a case.

In fact, there are examples of ***intrinsically*** unphysical Lorentz manifolds in the literature [1,12]. For instance, an accepted solution of metric for an electromagnetic plane wave [21] is

$$ds^2 = du\,dv + H\,du^2 - dx^i\,dx^j, \quad \text{where} \quad H = h_{ij}(u)x^i x^j\,, \tag{17}$$

and

$$h_{ii}(u) \geq 0, \quad h_{ij} = h_{ji}\,, \text{ where } u = t - z, \text{ and } v = t + z\,.$$

Here t is the time coordinate; and x, y, z are the space coordinates; and $h_{ij}(u)$ is related to the cause of this gravity, an electromagnetic plane-wave. It can be easily shown that metric (17) is a Lorentz manifold.

Nevertheless, since H can be ***arbitrarily*** large, this is incompatible with Einstein's notion of weak gravity and the correspondence principle[4]. In the light bending experiment, the gravitational effect of the light is implicitly assumed to be negligible. Thus, metric (17) cannot be valid in physics. Moreover, because the condition $1 \geq (1 + H)/(1 - H)$ may not be valid, metric (17) does not satisfy coordinate relativistic causality. Therefore, the equivalence principle is not satisfied because physical requirements are violated. Although a space with an electromagnetic plane-wave is not a vacuum, it does not seem to have any physical reason to exclude this consideration, since the scattering between lights is extremely small.

However, to show that metric (17) cannot be diffeomorphic to a physical space, needs another approach since boundedness and coordinate relativistic causality is subjected to the choice of coordinates.

The gravitational force related to $\Gamma^z{}_{tt} = \frac{1}{2}(1+H)\partial H/\partial t$ has arbitrary parameters (the coordinate origin). This arbitrariness in the metric violates the principle of causality (i.e., the causes of phenomena are identifiable) [16]. It follows that the geodesic does not represent a physical free falling. Also, as shown in eq. (16), a tetrad leads to a local coordinate transformation which is derived from the metric of a manifold. A transformation with arbitrary parameters cannot be valid in physics. Thus, the Lorentz manifold (17) cannot be diffeomorphic to a physical space since a diffeomorphism cannot eliminate the arbitrary parameters.

This example illustrates further that a crucial question for a satisfaction of the equivalence principle in a Lorentz manifold is whether a geodesic represents a physical free falling. Moreover, the existence of intrinsically unphysical Lorentz manifold implies that to define the light speed in terms of the local Minkowski space is invalid. One might argue that since the local Minkowski space always exists, one can surely define anything from it. But, the argument has one problem that the light speed is not a mathematical but a physical object. A physical object must be consistent with physics and therefore cannot be defined just mathematically.

7. Conclusions and Discussions

The equivalence principle, which states the local equivalence between acceleration and gravity, requires that a free falling observer must result in a co-moving local Minkowski space. Physically, the equivalence principle, requires that ***a free falling observer must be automatically*** (i.e., without additional efforts) ***in a co-moving local Minkowski space.*** For a long time, many theorists (outstanding physicists such as Eddington [17] are the exceptions) had mistaken that a mathematical existence of the local Minkowski space as the satisfaction of Einstein's equivalence principle. An obvious problem is, however, that the geodesic equation indicates that an acceleration may not exist even for non-constant metrics.

As stated by Einstein, the equivalence principle is necessary to ensure that [4] "special theory of relativity applies to the case of the absence of a gravitational field." This is supported by the fact that, ***under the influence of only gravity, the local space-time of a spaceship is a local Minkowski space.*** Thus, nature unequivocally defeats any attempt to misinterpret the equivalence principle.

The Minkowski metric in special relativity is a special case of the metric in general relativity. Nevertheless, this special case is very useful in clarifying whether all the principles which lead to general relativity are compatible with each other. The equivalence principle can be considered as a generalization of the Minkowski metric. Thus, this principle may not always be compatible with the covariance principle. For instance, due to the equivalence principle, the Minkowski metric is the ***only*** valid constant space-time metric.

The Galilean transformation implies that there is no maximum limit for the light speed and therefore is also in conflict with general relativity. To establish special relativity, the Galilean transformation is proven to be ***unrealizable*** by experiments. Thus, a Galilean transformation cannot be compatible with the equivalence principle which is applicable to only a physical space. Thus, it is physically invalid to extend the space-time physical coordinate system to an ***arbitrary*** Gaussian system. This means that the equivalence of all frames of reference does not require the physical equivalence of all mathematical coordinate systems.

The fact that there is a distinction between the equivalence principle and the proper metric signature would imply also that the covariance principle must be restricted. No wonder, unrestricted covariance has been found to be a problem for a consistent physical interpretation [22,23]. The fact that metric (17) is intrinsically unphysical resolves its seemingly paradox with the light bending calculation in which the gravity

due to the light is implicitly assumed to be negligible [3,4]. This is another example that ***a misunderstanding of the equivalence principle can leads to disagreements with experiments.***

Perhaps, due to confusing mathematics with physics, Einstein's equivalence principle is often not explained adequately in some books [2,7,9,11]. Theorists such as Synge [2] and more recently Friedman [24] incorrectly believed that the existence of local Minkowski space has replaced the equivalence principle that initially motivated it. Such theorists advocated explicitly that the basis of general relativity should be the Einstein field equation alone rather than the equivalence principle. Because they did not see the physical meaning of the equivalence principle beyond the theorems in mathematics, they incorrectly considered the equivalence principle as merely a useful midwife which has helped gestating general relativity and thus have done its duty. Therefore, they can suggest only the field equation as the foundation of general relativity.

However, there is no satisfactory proof of rigorous validity of Einstein's field equation [25] (e.g., the inadequate source term, as mentioned in §1, is the cause of the unphysical solution (17) [16]). Experimentally, the validity of Einstein's equation has not yet been established beyond doubt [26]. In fact, the invalidity of Einstein's equation for two-body problems was conjectured by Hogarth [27]; and Einstein himself had pointed out that his equation may not be valid for matter of very high density [3]. Moreover, it has been proven experimentally that, for dynamic problems[5)], Einstein's equation of 1915 must be modified [28] and Yilmaz [29] also pointed out that Einstein's equation of 1915 is only a test particle theory.

But, the equivalence principle remains indispensable because of its solid experimental foundation [9,10, 28]. Moreover, as illustrated, its failure is always accompanied with a violation of another physical requirement. Thus, as Weinberg [5] points out, "it is much more useful to regard general relativity above all as a theory of ***gravitation***, whose connection with geometry arises from the peculiar empirical properties of gravitation, properties summarized by Einstein's Principle of the Equivalence of Gravitation and Inertia."

Nevertheless, since a Lorentz manifold may not satisfy the equivalence principle, further considerations must be made for its valid applicability. This analysis clarifies that the equivalence principle consists of three integral parts: 1) the validity of a geodesic representing the motion of a particle; 2) a free falling observer automatically in a local Minkowski space; and 3) validity of the local coordinate transformation which transform the metric to a local Minkowski space. On the equivalence principle, the crucial feature is not the

mathematical existence of a local Minkowski space, but its ***automatic*** physical formation in a free falling. To ensure this, point 1) must be valid and validity of point 3) would follow. On the other hand, it would not be practical to consider validity of all metrics, discussions in a paper must be limited to the essentials.

In conclusion, the equivalence principle is a theoretical framework and it restricts the covariance. The mathematical theorems show only that Riemannian geometry is compatible with general relativity. Due to the over-extended covariance, a criterion for a satisfaction of the equivalence principle in a Lorentz manifold was not considered. This analysis shows that any physical requirement is a necessary condition for a satisfaction of this principle. Also, since unphysical solutions can be eliminated by this restriction, physical understanding in general relativity is enhanced [16]. Then it is possible to extend the theory of general relativity such that its scope would encompassing every fundamental aspects of physics as Einstein [30] envisioned.

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ENDNOTES

1) A local Minkowski space is a short hand to express that special relativity is locally valid in the co-moving local Minkowski space, except for phenomena involving the space-time curvature.
2) In some text books, the coordinate relativistic causality (i.e., the light speed c is the maximum velocity of propagation for any event) was considered to be equivalent to the existence of local Minkowski spaces. This is not true for a Lorentz manifold may not satisfy the requirement of coordinate relativistic causality.
3) According to Einstein, the body to which events are spatially referred, is called the coordinate system [31]. To be more precise, a spatial coordinate system attached to a body (i.e., no relative motion nor acceleration) is its "frame of reference" [3,4]. Then, these coordinates together with the time-co-

ordinate form the space–time coordinate system. Thus, a frame of reference can be chosen physically and, due to the equivalence principle, the time–coordinate is determined accordingly. Thus, one may call loosely the frame of reference as a coordinate system. In this paper, for the purpose of considering a satisfaction of the equivalence principle, a frame of reference and a related space–time coordinate system, are distinguished as above.

4) Some theorists, R. Penrose [21] for example, believe that a space–time metric could be unbounded even if the source is weak.

5) A dynamic solution in gravity is related to the dynamics of its source matter. A dynamic source, according to relativity, would generate gravitational radiation. For a weak source, according to Einstein, such a solution is bounded. However, as pointed out by F. E. Low [32], a time–dependent solution can be obtained simply through a coordinate transformation and therefore may not relate to dynamics.

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