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Comparing conductance quantization in quantum wires and Quantum Hall systems

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We propose a new calculation of the DC conductance of a 1-dimensional electron system described by the Luttinger model. Our approach is based on the ideas of Landauer and Büttiker on transport in ballistic channels and on the methods of current algebra. We analyse in detail the way in which the system can be coupled to external reservoirs. This determines whether the conductance is renormalized or not. We provide a parallel treatment of a quantum wire and a Fractional Quantum Hall system on a cylinder with two widely separated edges. Although both systems are described by the same effective theory, the physical electrons are identified with different types of excitations, and hence the coupling to external reservoirs is different. As a consequence, the conductance in the wire is quantized in integer units of e^2/h per spin orientation whereas the Hall conductance allows for fractional quantization.

Recent experiments on transport properties of quantum wires [1,2] attracted new interest to the problem of conductance in a 1-dimensional electron gas.

Since the work of Apel and Rice [3], where the authors computed the conductance in the 1-dimensional Luttinger model, it was believed that the effects of interaction in the one-channel quantum wire should lower the conductance with respect to the value

$$\sigma = 2\frac{e^2}{h} \tag{1}$$

predicted by the Landauer-Büttiker formula [4] for the case of non-interacting electrons. However, experimental data suggest that this renormalization of conductance does not actually take place [2].

At the moment there exist a number of theoretical arguments [5–9] intended to explain the non-renormalization of the conductance in a 1-dimensional electron gas. One of them [5,6] is based on the idea that the conductance of the 1-channel quantum wire is completely determined by the structure of the leads to which the wire is attached. So far as the electrons in the leads form a Landau Fermi-liquid one chooses to model the leads by two semi-infinite 1-dimensional 1-channel non-interacting Fermi liquids whereas the wire is modeled by a Luttinger Fermi-liquid. Due to strong non-locality of current-current correlators in 1-dimensional systems the calculation based on the Kubo formula leads to the non-renormalized result (1).

Another approach [8,9] is based on the observation that the dielectric constant in the Luttinger Fermi-liquid increases with increasing strength of interaction. This fact changes the definition of the voltage drop which results in the cancellation of the renormalization factor in the conductance.

Both approaches mentioned above are based on linear response theory and its main tool, the Kubo formula, in their calculation of conductance. Yet there is another, in many ways more intuitive approach based on ideas of Landauer and Büttiker [4]. In this approach one assumes that the ballistic channel (in our case a quantum wire) is attached to two reservoirs of electrons with different chemical potentials. The difference in chemical potentials can be created in some other way than by turning on electric fields. In the derivation of the formula for the conductance one then uses only kinematic properties of the ballistic channel, instead of dynamical properties, as in the Kubo formula. In this paper we show how to apply these ideas to a Luttinger Fermi-liquid type channel.

We study how the Luttinger liquid interacts with external reservoirs. In order to clarify our analysis of quantum wires, we provide a parallel treatment of the Fractional Quantum Hall system which is described by the same effective Hamiltonian [11,12]. In contrast to opinions expressed in [6,10], we claim that these two systems can be analysed in a common framework. However, the electron transfer between the Luttinger liquid and external reservoirs in these two systems is organized in a different way. This difference leads to the integer quantization of conductances in quantum wires, in contrast to the fractional quantization of Hall conductances.

We replace the appealing but clearly unrealistic (one-dimensional) models of the leads [5,6] by a more universal consideration which only relies on the fact that the leads interact with the Luttinger liquid via electron transfer. It is not easy to compare our approach to the one in [8–10], because these authors do not consider interactions of the wires with the reservoirs, at all, while our analysis indicates that the correct definition of the voltage drop in a wire depends on how the physical electrons injected by the reservoirs into the wire are described in the Luttinger model. More precisely, one must correctly identify those conserved quantities in the Luttinger model that, in thermodynamics, are canonically conjugate to the chemical potentials of the two reaservoirs.

It is well known that a 1-dimensional interacting electron system is effectively described by the Luttinger model [13]. For simplicity we consider spinless fermions with short range two-body interactions. This makes the comparison to the spin polarized Quantum Hall system

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more straightforward. Polarization of fermions can be easily recovered in the final expression for conductance.

The bosonized Lagrangian density of the Luttinger model looks as follows [13]:

$$\mathcal{L} = \frac{hv_F}{4}\left(\frac{1}{v_F^2}(\partial_t\phi)^2 - (1+g)(\partial_x\phi)^2\right), \qquad (2)$$

where v_F is the Fermi velocity, $n = \partial_x\phi$ is the particle density, $I = e\partial_t\phi$ is the electric current, and g is an effective coupling constant. The corresponding Hamiltonian is given by

$$H_0 = \frac{hv_F}{4}\int dx : \left(\frac{1}{v_F^2}(\partial_t\phi)^2 + (1+g)(\partial_x\phi)^2\right) : , \qquad (3)$$

where :: stands for standard Wick ordering. The model with $g = 0$ describes a non-interacting 1 - dimensional Fermi-liquid.

The Hamiltonian (3) can be expressed in terms of left-moving and right-moving currents:

$$H_0 = \frac{h}{2v_F}\int dx : (I_L^2 + I_R^2) : , \qquad (4)$$

where

$$I_L = \frac{1}{2}(\partial_t\phi + v_F\sqrt{1+g}\,\partial_x\phi),$$
$$I_R = -\frac{1}{2}(\partial_t\phi - v_F\sqrt{1+g}\,\partial_x\phi), \qquad (5)$$

so that

$$I = e(I_L - I_R)\ ,\ n = \frac{1}{v_F\sqrt{1+g}}(I_L + I_R). \qquad (6)$$

Let us remark that the same effective Hamiltonian describes edge excitations of a spin polarized incompressible Quantum Hall fluid on a cylinder [11] with one chiral channel of excitations per edge. Then eI_L and eI_R are the edge currents on the two boundaries of the cylinder. The filling factor of the Quantum Hall fluid is related to the effective coupling of the Luttinger model via:

$$\nu = \frac{1}{\sqrt{1+g}}. \qquad (7)$$

Incompressibility requires that ν^{-1} is an odd integer.

In order to measure conductances in the Luttinger model we should couple it to two reservoirs with different chemical potentials [4]. This can be done in two different ways. We shall see that one way is realized in the Quantum Hall system. The corresponding conductance should be interpreted as the Hall conductance

$$\sigma_H = \nu\,\frac{e^2}{h}. \qquad (8)$$

The other way of coupling the Luttinger model to reservoirs is realized in thin wires and leads to the non-renormalized value

$$\sigma = \frac{e^2}{h}. \qquad (9)$$

The coupling of reservoirs to the Luttinger liquid can only be organized via electron transfer. We therefore must identify excitations in the Luttinger liquid which correspond to electrons and assign to them the chemical potentials of reservoirs according to the conditions of the measurement.

This can be done naturally for incompressible Quantum Hall fluids. In this case the chiral channels are spatially separated by the bulk of the sample and the reservoirs are attached directly to each of them. Mathematically this is expressed by adding an extra term to the Hamiltonian:

$$H_{\rm Hall} = H_0 + \mu_L N_L + \mu_R N_R, \qquad (10)$$

where μ_L and μ_R are the chemical potentials of the reservoirs, $eV = \mu_R - \mu_L$, N_L and N_R are conserved electron numbers in the left-moving and right-moving channels (corresponding to the two different edges of the sample):

$$N_L = \int dx\ n_L\ ,\ n_L = \frac{1}{v_F}\nu\ I_L;$$
$$N_R = \int dx\ n_R\ ,\ n_R = \frac{1}{v_F}\nu\ I_R. \qquad (11)$$

We denote by I_0 the expectation value of the current operator I in the ground state Ψ_0 which minimizes the energy functional $< \Psi, H_{\rm Hall}\Psi >$ (Ψ an arbitrary state of the system). Since H_0 is quadratic in I_L and I_R, the explicit value of I_0 is obtained by minimizing the 'classical' energy functional

$$\mathcal{E}(I_L, I_R) = \frac{h}{2v_F}(I_L^2 + I_R^2) + \frac{\nu}{v_F}(\mu_L I_L + \mu_R I_R) \qquad (12)$$

over I_L and I_R and using (6). The result is

$$I_0 = e\ \nu\left(\frac{\mu_R}{h} - \frac{\mu_L}{h}\right) = \nu\ \frac{e^2}{h}V. \qquad (13)$$

Thus, the Hall conductance is given by formula (8). This result agrees with the renormalized conductance

$$\sigma = \frac{1}{\sqrt{1+g}}\frac{e^2}{h}$$

in the Luttinger model.

In the Quantum Hall system the left- and right-moving edge excitations of charge e are represented by operators

$$\Psi_L(x) =: \exp\{i\pi(\sqrt{1+g}\phi(x) + \frac{1}{v_F}\int^x \partial_t\phi dx)\} : ,$$
$$\Psi_R(x) =: \exp\{i\pi(\sqrt{1+g}\phi(x) - \frac{1}{v_F}\int^x \partial_t\phi dx)\} : . \qquad (14)$$

These operators describe particles with Fermi statistics only when $\sqrt{1+g}$ is an odd integer [11,12]. These values of the filling factor

$$\nu = \frac{1}{2m+1} \tag{15}$$

correspond to Laughlin's plateaux [14] in the theory of the Fractional Quantum Hall Effect.

Next we turn to the analysis of quantum wires. For generic values of the coupling constant g, neither left- nor right-moving excitations can be identified with electrons. Hence, we cannot assign particular values of chemical potentials to these excitation branches. Yet, there always exist other fermionic excitations of charge e that describe the physical electrons in the Luttinger model. They are created by applying the operators Ψ_+, Ψ_- given by

$$\begin{aligned} \Psi_+(x) &=: \exp\{i\pi(\phi(x) + \frac{1}{v_F}\int^x \partial_t\phi dx)\} : , \\ \Psi_-(x) &=: \exp\{i\pi(\phi(x) - \frac{1}{v_F}\int^x \partial_t\phi dx)\} : . \end{aligned} \tag{16}$$

These operators coincide with Ψ_L, Ψ_R only in the non-interacting model ($g = 0$). The particle densities corresponding to the operators Ψ_+ and Ψ_- are given by

$$\begin{aligned} n_+ &= \frac{1+\sqrt{1+g}}{2} n_L + \frac{1-\sqrt{1+g}}{2} n_R, \\ n_- &= \frac{1-\sqrt{1+g}}{2} n_L + \frac{1+\sqrt{1+g}}{2} n_R. \end{aligned} \tag{17}$$

It is natural to view the chemical potentials of the reservoirs as the variables conjugate to the conserved charges

$$N_+ = \int dx\, n_+ \ , \ N_- = \int dx\, n_- \tag{18}$$

which measure the total number of electrons created by powers of Ψ_+ and Ψ_-, respectively. If the quantum channel in the wire is ballistic and adiabatic, the chemical potential of the left reservoir is conjugate to N_+ whereas the chemical potential of the right reservoir is conjugate to N_-. The counterpart of the Hamiltonian (10) is therefore given by

$$H_{\text{Wire}} = H_0 + \mu_L N_+ + \mu_R N_- . \tag{19}$$

A calculation similar to the one in (13) shows that the ground state of H_{Wire} carries a current

$$I_0 = e\left(\frac{\mu_R}{h} - \frac{\mu_L}{h}\right) = \frac{e^2}{h} V \tag{20}$$

which yields formula (9). Note that in our calculation we have assumed that $eV \gg hv_F/L$, where L is the length of the wire. Two spin polarizations of electrons can be included in the final answer for conductance of a single quantum channel by multiplying by a factor of 2 which leads to (1). Our analysis can be easily extended to finite temperatures. This and other generalizations will be considered elsewhere.

We conclude that although the Quantum Hall system and the quantum wire are described by the same effective Hamiltonian, the way in which they are coupled to reservoirs in transport measurements is different. Mathematically, this is reflected in using chemical potentials $\mu_L(\mu_R)$ of left(right) reservoirs conjugate to different conserved charges $N_L(N_R)$, for Quantum Hall fluids, and $N_+(N_-)$ for quantum wires. As a consequence, in the Quantum Hall system the conductance depends on the effective coupling constant related to the filling factor by (7), but this is not the case for the quantum wire.

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