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FACTORS AFFECTING THE PERFORMANCE OF STELLAR INTERFEROMETERS.

Application to the VLTI Error Budgets.

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Abstract

This presentation intends to provide a basic and didactic introduction to the various effects affecting the performance of stellar interferometers in terms of fringe visibility. For each effect, we first briefly describe the physical phenomenon involved and give the basic formulae useful to assess the associated visibility loss. Then, we illustrate their respective magnitude for the case of the Very Large Telescope Interferometer (VLTI) using the error budget developed for observation in the K band with fringe tracking.

1. Introduction

The formation of interference fringes in the optical domain is affected by many different physical effects which tend to spoil the quality of the fringe pattern. Optical stellar interferometers present some specific complication with respect to laboratory experiments. First, they tend to be complex systems with a large number of optical elements spread over large distances; second, several of these optical elements need to be moved continuously during the observation to track the stellar object and compensate for the diurnal motion; third the stellar light propagates through the atmosphere and is affected by the atmospheric turbulence. It is therefore of paramount importance to identify, understand, and analyse all potential sources of performance degradation. This is necessary to verify the feasibility of the scientific objectives but also to identify the areas where calibration of systematic errors is the most critically required. This is best achieved by the generation of error budgets which consist in distributing the top-level performance requirements into individual error sources and sub-system requirements as illustrated in figure 1.

We will review, in this paper, the various contributions to the error budget for the instrumental fringe visibility loss shown in figure 2. This error budget has been

developed for the VLTI for observation in the K band with the VLT 8-m telescopes equipped with adaptive optics and with fringe tracking.

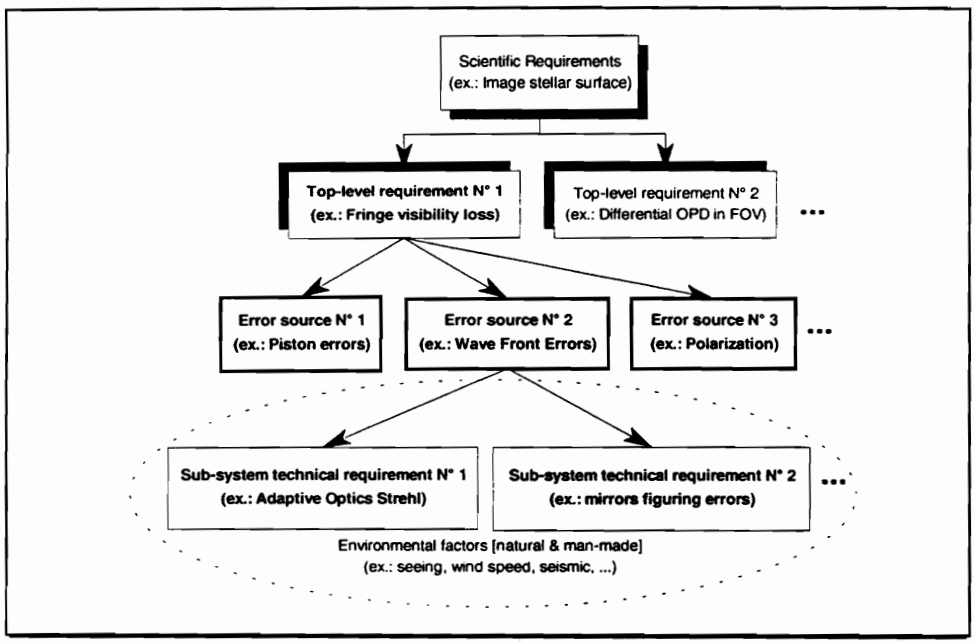

Figure 1: An error budget is a break-down of the top-level requirements into individual error sources and subsystem requirements. It is an essential tool for complex systems like stellar interferometers in order to identify and track the various factors affecting their performance.

The generic equation describing the fringe pattern obtained with a perfect instrument by combining two monochromatic beams of equal intensity is of the form:

$$I(\delta) = 2.I_0\left(1 + V_0.\cos\left(\frac{2.\pi.\delta}{\lambda} + \phi\right)\right) \qquad (1)$$

where
- I_0: Intensity in each beam
- V_0: Degree of mutual coherence of the incoming beams
- δ: Modulation delay between the two beams
- λ: The observing wavelength
- ϕ: Phase difference between the two beams

We recall that the fringe visibility (or fringe contrast), characterises the amplitude of the intensity modulation in the fringe pattern. It is defined by:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

where $I_{\min}$ and $I_{\max}$ are respectively the minimum and maximum intensities.

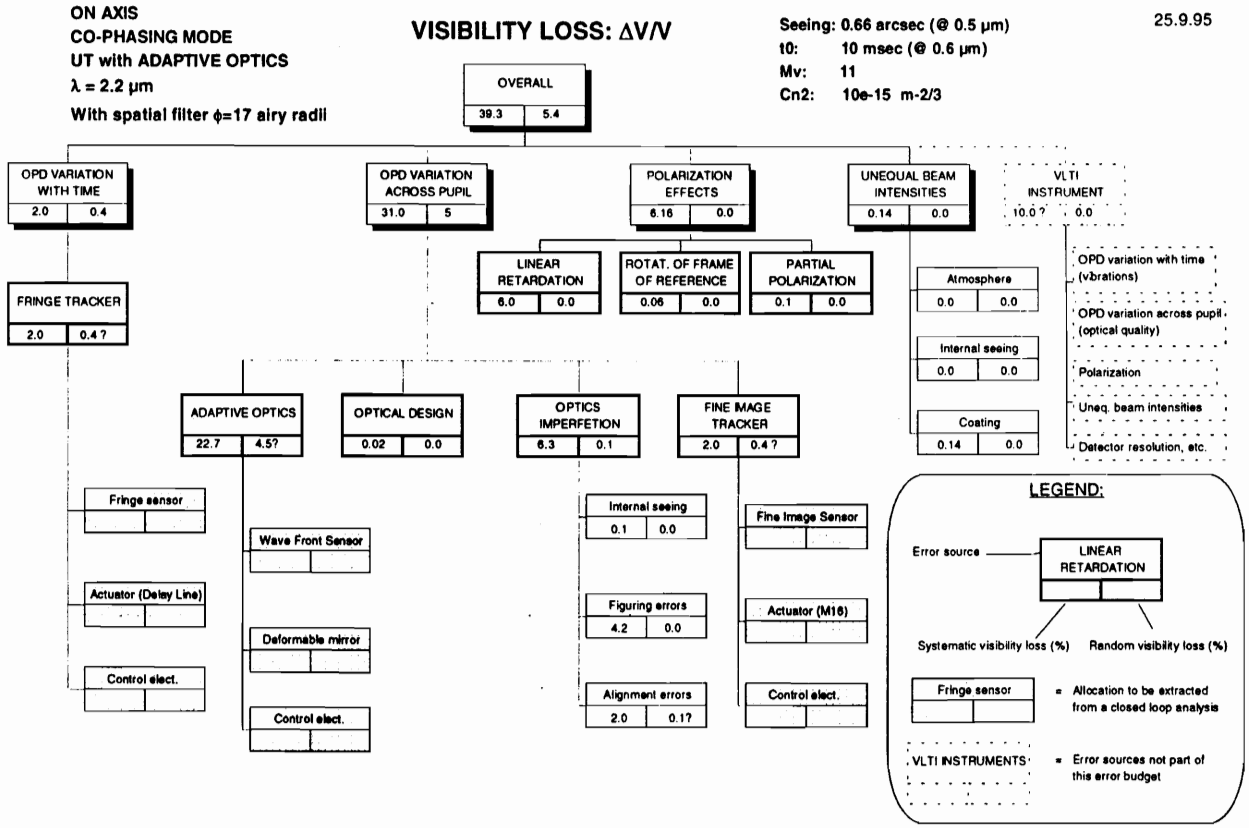

Figure 2: The VLTI Fringe Visibility Error Budget for observation in the K band using the 8-meter telescopes with adaptive optics and fringe tracking.

The fringe visibility is one of the prime observable of a stellar interferometer. If the instrument is perfect, it is a direct measure of V_0, the degree of coherence of the incoming beams which contains the high angular information of the observed object. It may be useful to remember that a visibility of 1.0 implies that the intensity in the fringes goes down to zero (perfect dark).

In the real world, the fringes are affected by instrumental errors. The measured fringe visibility is of the form:

$$V = V_0 \times V_{inst}$$

where V_{inst} ($\leq$1.0) characterises the instrumental visibility degradation.

2. OPD variation with time (Piston errors)

In most fringe detection schemes the modulation delay (δ) of equation n°1 is created either in the time domain by placing a piezo-driven mirror in one of the beam and observing the modulated flat tint in the combined pupil (pupil plane detection) or in the spatial domain by placing a 2-D detector at the focus of a combining optics (focal plane detection). In both cases, the fringe pattern is measured after integration of the photons during a finite exposure time. If the phase difference (ϕ) is not stable during this exposure time, the resulting fringe pattern is the superposition of phase-shifted instantaneous fringe patterns as illustrated in figure 3. The resulting fringe pattern is obtained by averaging equation n°1 over the exposure time:

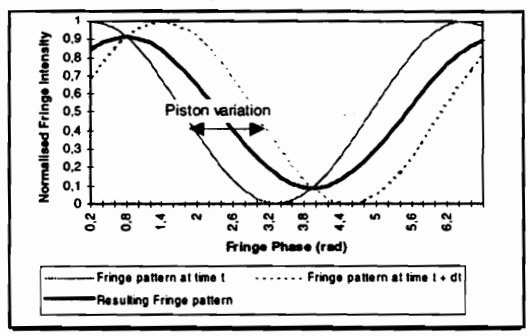

Figure 3: Illustration of the effect of phase fluctuation with time (or piston errors) on the fringe visibility.

$$\langle I(\delta)\rangle = \frac{1}{T}.\int_{t_0}^{t_0+T} I(\delta).dt.$$

It gives, for small phase difference fluctuation,

$$V_{inst} = e^{-\frac{\sigma_\phi^2(T)}{2}}$$

where $\sigma_\phi^2(T)$ is the variance (in radians squared) of the phase difference fluctuation (also called "piston errors") during the exposure time T. This variance can be computed, for any exposure time (T), from the Power Spectral Density of the phase fluctuation (S_ϕ) by:

$$\sigma_\phi^2(T) = \int_0^\infty S_\phi(f).\left(1-\left(\frac{\sin(\pi.T.f)}{\pi.T.f}\right)^2\right).df.$$

The various sources of piston errors in a stellar interferometer are:

- the atmospheric turbulence
- the air turbulence inside the interferometer (internal seeing)
- motion of the optical element due to vibration (wind shaking, natural & man-made seismic motion, acoustics, vibrating equipment installed on or close to the telescope like pumps, etc.)

When the interferometer is equipped with fringe tracking capability, the piston errors are partially compensated and long integration time can be performed. In that case, the fringe contrast is computed as above with the variance of the residual phase fluctuation as delivered by the fringe tracking loop over the integration time.

In the case of the VLTI, a detailed control loop model including the fringe sensor dynamic and noise characteristic, the delay line mechanical model, the control law, and the various piston error sources as listed above has shown that a 2% visibility loss at 2.2 μm can be guaranteed.

3. OPD variation across the pupil (Wave Front Errors)

The effect of the Wave Front Errors (WFE) can best be understood when considering a pupil plane detection scheme. In the absence of WFE, the pupils after combination generate a perfect flat tint alternatively black (zero intensity) and white (full intensity) when the OPD is modulated (see figure 4, *top*). In the presence of WFE, the combined pupil contains spatial interference fringes as indicated schematically in figure 4, *bottom*, the intensity integrated over the pupil area by the monopixel detector never goes to zero and never reaches the full intensity. This means that the fringe contrast is lower that unity. By integration of equation n°1 over the pupil area, it can be easily shown that the instrumental visibility factor is

$$V_{inst} = e^{-\frac{\sigma^2_{(\phi_2-\phi_1)}}{2}}$$

where $\sigma^2_{(\phi_2-\phi_1)}$ is the variance over the pupil area of the differential WFE (in radians squared). When the WFE in each interferometer arm can be considered as statistically uncorrelated and of the same order of magnitude (case of the atmospheric corrugation, and random figuring & alignment errors) the above expression simplifies in

$$V_{inst} = e^{-\sigma^2_\phi},$$

where σ^2_ϕ is the variance of the WFE in *each* individual pupil.

It is worth to note here that the effect of WFE depends very much on the actual detection scheme. Indeed, if we place an array detector in the pupil plane to sample properly the pupil interference pattern shown on figure 4, each pixel of the detector will "see" a perfect fringe pattern of visibility $V = V_0$ when the path is modulated. Similarly, in the image plane, placing a spatial filter in the focal plane (e.g. a monomode fiber)

will remove the high spatial frequency WFE present in the pupil and will greatly improve the instrumental visibility factor. This example shows the importance of considering the actual beam combining instrument when describing the performance of the interferometer. For the VLTI which is designed to feed several instruments, an assumption has therefore to be taken in the error budget. The example shown in figure 2 assumes a spatial filter with a diameter of 17 airy radii. This means that the high spatial frequency WFE which in any case cannot be corrected by the Adaptive Optics system (with 17 actuators across the pupil diameter) are filtered out. A smaller visibility loss, and therefore better calibration accuracy, could be obtained with a smaller filter (e.g. monomode fiber) at the price of some energy loss and more stringent alignment requirements.

The sources of WFE are:

- the atmospheric turbulence
- the air turbulence inside the interferometer (internal seeing)
- the optical design
- the optics misfigure & misalignment

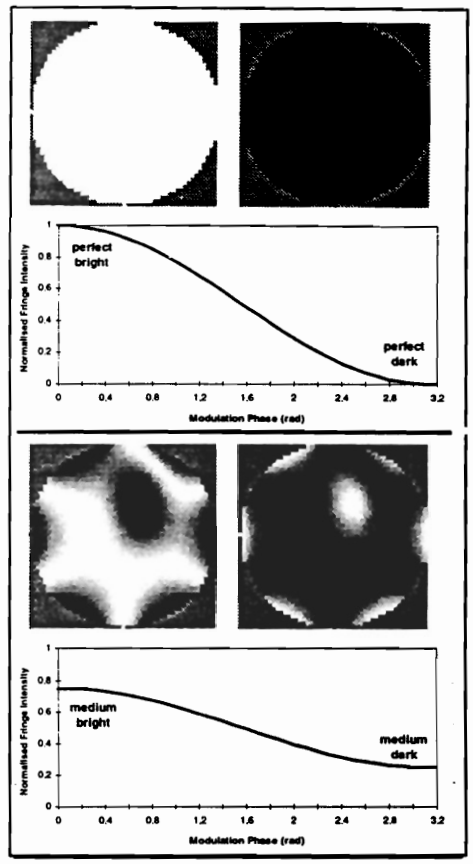

Figure 4: Illustration of the effect of Wave Front Errors on the fringe visibility when using a monopixel detector in the pupil plane.

WFE can be partially compensated by an Adaptive Optics (AO) system. The degree of compensation depends on the performance of the AO system and on its location in the optical train.

For the VLTI, an important part of the WFE will be compensated by the AO system located at the coudé focus of the 8-m telescope. The residual atmospheric WFE after compensation remains nevertheless the dominating source of error in the whole error budget with a corresponding visibility loss of ≈25% including residual image tracking errors. The next most important WFE source is the optics misfigure which, inspite of state-of-the-art optical quality, amounts to about 4% because of the large number of optical elements.

4. Unequal beam intensities

The effect of unequal beam intensities is easily computed considering that the complex amplitude of the combined electro-magnetic field is the sum of the complex amplitudes in each individual aperture: $A = \sqrt{I_1}.e^{j.\varphi_1} + \sqrt{I_2}.e^{j.\varphi 2}$. The combined intensity is therefore $I = A.A^* = (I_1 + I_2).\left[1 + 2.\frac{\sqrt{I_1}.\sqrt{I_2}}{I_1 + I_2}.\cos(\varphi_2 - \varphi_1)\right]$. This evidences the visibility loss factor which, in the general case of partially coherent incoming beams, leads to the formula :

$$V_{inst} = 2.\frac{\sqrt{I_1}.\sqrt{I_2}}{I_1 + I_2}.$$

This expression shows that the visibility is only weakly affected by "reasonable" unequal intensity. As an example, a drastic 50% intensity difference generates a visibility loss of only 6%. Therefore, in most cases, the visibility loss due to unequal intensity remains negligible with respect to the other sources of visibility degradation.

Sources of unequal beam intensities are:

- the atmospheric scintillation
- the scintillation due to internal seeing
- unequal throughput in the interferometer arms (e.g. differential coating efficiency).

Atmospheric scintillation, and internal seeing effects, are generally completely negligible. Scintillation rate, defined as the variance of the intensity fluctuation (normalised by the mean intensity), is at Paranal of the order of 0.1 in average with maximum instantaneous values reaching 0.6. These values, measured on small aperture (e.g. 3 cm) are drastically attenuated by the spatial averaging effect of the large VLT apertures. The resulting visibility loss is virtually zero.

Unequal beam intensity in the VLTI error budget is dominated by the assumption that up to 10% differential throughput can result from differences in the coating efficiency (e.g. ageing).

5. Polarisation effects

5.1. PARTIAL POLARISATION

Partial polarisation is the relative attenuation of one of the polarisation with respect to the other. If partial polarisation is not equal in the two arms, it affects the fringe visibility in a way similar to unequal beam intensities but with a dependence on the polarisation state. For unpolarised incoming light the visibility factor is:

$$V_{inst} = \frac{2(1+p)}{(3+p^2)}$$

where p is the partial polarisation affecting one of the two arms. As for unequal beam intensities, this effect is usually very small. As an example, a 10% polarisation (i.e. $1-p^2 = 0.1$) generates only a 0.07% visibility loss.

5.2. LINEAR RETARDATION

Figure 5 illustrates how linear retardation affects the fringe contrast. After passing through an optics, the two orthogonal polarisations of an incident light beam are phase shifted one with respect to the other by a quantity (P-S), the linear retardation. If the two beams in each arm of the interferometer are affected by exactly the same linear retardation, the fringes produced by the S and P polarisations will be superimposed. If a differential retardation Δ(P-S)=(P2-S2)-(P1-S1) exists, the two fringe patterns will be shifted one with respect to the other. For unpolarised light, these two patterns add incoherently on the detector to produce a fringe pattern with a visibility affected by the factor:

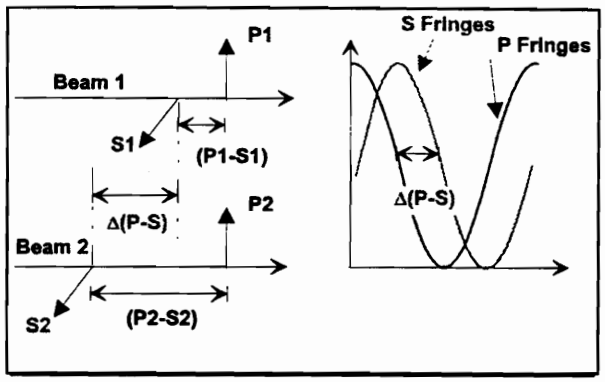

Figure 5: Illustration of the effect of polarisation linear retardation on the fringe visibility.

$$V_{inst} = \left|\cos\left(\frac{\Delta(P-S)}{2}\right)\right|.$$

Differential linear retardation are related to the coatings and occurs in the following cases:

- different coatings are used in the two arms
- different incidence angles are seen by the beam in the two arms
- differential polarisation characteristics due to coating manufacturing errors and ageing

In most interferometers including VLTI, the first two cases are eliminated by a perfectly symmetric design (i.e. same number of P and S reflections and same coating in the various arms). The third error source can be very significant for complex coatings. For the VLTI, it is minimised by the use of simple metallic coatings with a minimum number of protecting layers. As an example, a 4-layers boosted silver coating with less than 2% error in the layer thickness ensures visibility losses below 1% and 5% respectively in the K band and in the visible. Due to the highly coating-dependent

nature of this effect and because the exact coating design is not yet frozen, a 6% visibility loss is allocated in the current VLTI error budget shown on figure 2.

5.3. ROTATION OF POLARISATION FRAME OF REFERENCE

The incident polarisation frame of reference experiences rotation when propagating through the optics of each interferometer arm. If the rotation in each arm is not exactly the same, the situation illustrated in figure 6 occurs after recombination. Each polarisation can be projected on the two reference axes (e.g. P1, S1). If the incoming light is unpolarised (or linearly polarised), the only components generating fringes are P1 with P2.cos(θ) and S1 with S2.cos(θ), where θ is the frame rotation angle. The other two components simply add constant intensities IP2*sin(θ)^2 and IS2*sin(θ)^2. The instrumental visibility factor is

$$V_{inst} = |\cos(\theta)|.$$

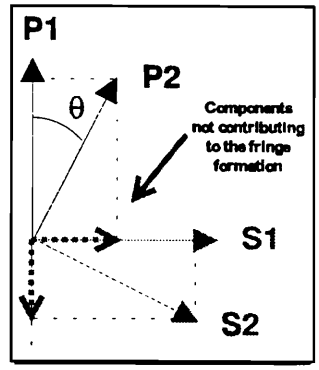

Figure 6: Effect of the rotation of the polarisation frame of reference for unpolarised light.

Rotation of the frame of reference can originate from:

- the optical design (i.e. non symmetric sequence of reflection in the two arms)
- non identical field/pupil rotation in the two arms when the star is tracked
- misalignments

The first two error sources are avoided by design in the VLTI and the third one remains negligible even with a conservative assumption of a 2° pupil alignment error.

6. OPD variation with wavelength

6.1. SPECTRAL BANDPATH EFFECTS

It can be seen from equation n°1 that, as soon as the delay (δ) is not zero, the intensity in the fringe pattern at a given delay depends on the wavelength. When the observing spectral bandpath is not infinitely small, the resulting fringe pattern is the sum of each individual monochromatic pattern and presents therefore a reduced contrast. This is illustrated in figure 7. With the assumption that I_0 is independent of λ, integration of equation n°1 over the spectral bandpath $[\lambda_1 ... \lambda_2]$ shows that the instrumental visibility factor is:

$$V_{inst} = \frac{\sin(\pi.\delta / \Delta l)}{\pi.\delta / \Delta l}$$

where: $\Delta l = \lambda_0^2 / \Delta\lambda$ is the coherence length with $\Delta\lambda = \lambda_2 - \lambda_1$ and $\lambda_0^2 = \lambda_1.\lambda_2$.

In the general case, it can be shown that the function $V_{inst}(\delta)$ is the Fourier transform of the spectral distribution of the light being combined (taking into account the source distribution and the instrumental transmission characteristics).

As an example, the above equation shows that with a spectral resolution $R = \lambda_0 / \Delta\lambda$ of 300, maintaining a visibility loss less than 1% requires to keep $\delta \leq 23 \times \lambda_0$. This is not a difficult requirement as soon as even a poor fringe tracking is performed. When no fringe tracking is available, the atmospheric OPD fluctuation which typically amounts to some 20 µm r.m.s. for long baselines may require to narrow the observing bandpath.

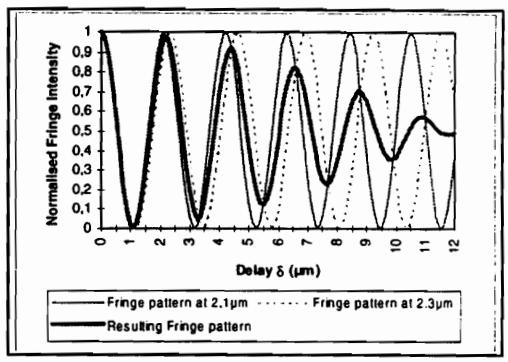

Figure 7: Illustration of the effect of spectral bandpath on the fringe visibility as a function of delay.

The VLTI error budget shown in figure 2 does not include this effect since the fringe tracking loop design ensures tracking around the so-called "white-light fringe" (i.e.δ =0). This effect can nevertheless exist if the OPD seen by the instrument differs from that measured by the fringe sensor because of misalignment. It is considered part of the instrument error budget and will be limited by an internal calibration system enabling to measure and cancel this misalignment.

6.2. LONGITUDINAL DISPERSION

The longitudinal dispersion effect occurs when the external delay due to the zenith angle of the observed star is compensated by delay lines placed in air instead of vaccum. In that case, the external delay (D) which has occured in vaccum above the atmosphere is compensated by placing the delay line at a physical position (X) so that, at a given wavelength (λ_0) the OPD is zero. This means that $OPD(\lambda_0) = n(\lambda_0).X - D = 0$ and therefore $X = D / n(\lambda_0)$ where n is the index of refraction of the air. Because n varies with the wavelength, the OPD at an other wavelength will be different from zero and equal: $OPD(\lambda) = n(\lambda).X - D = \left(n(\lambda) / n(\lambda_0) - 1\right).D$. If the observing spectral bandpath is not infinitely small, the resulting fringe pattern is the sum of each individual monochromatic pattern as described by equation n°1 where ϕ is replaced by $2.\pi.OPD(\lambda)/\lambda$. The resulting fringe visibility can be computed numerically from the dispersion characteristics of the air.

The magnitude of the dispersion effect depends on the observing wavelength and bandpath. It is usually negligible in the infrared but can be very significant in the visible as soon as the spectral bandpath exceeds some Angströms. In that case, partial

compensation can be achieved by inserting in one arm an appropriate glass of adjustable thickness.

This effect does not appear in the error budget shown in figure 2 because it depends essentially on the instrument characteristics and is therefore considered part of the instrument-related error budget.

7. Conclusion

We have described the numerous physical phenomena affecting the fringe visibility in stellar interferometers and have illustrated their respective importance for a specific case of VLTI observation.

Being aware of these sources of performance degradation is certainly a must for engineers building interferometers but we believe it is also important for scientists involved in the definition and/or the use of interferometric facilities. Indeed, they drive the technical feasibility of the scientific objectives and their relative importance can define the calibration needs required to achieve a given measurement accuracy.

It shall also be kept in mind that the exact definition of the combining instrument is required to define the final performance of the interferometer.