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IMAGING WITH ARRAY DETECTORS USING DIFFERENTIAL DETECTION[1]

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ABSTRACT. To detect faint signals in the presence of a high background differential techniques are often used. In infrared astronomy this has led to the so-called chopping and nodding techniques. The introduction of array detectors especially for imaging in the thermal infrared region of the astronomical spectrum requires an adaption of these techniques which also takes into account the pixel non-uniformity of the array detector. I describe one possible imaging algorithm.

1. INTRODUCTION

In astronomical observations it is often necessary to obtain a weak image in the presence of a very high background. Examples are : (i) observations in the thermal infrared in which the background is dominated by the million times brighter thermal emission of the telescope and/or the sky, (ii) deep imaging at visible wavelengths of galaxies and other objects with magnitudes 100 times or more fainter than the dark time sky background, and (iii) the observation of very faint spectral lines against the 100 times or higher "background" of the spectrum continuum. While using single pixel detectors various techniques were developed to detect the signals against the frequently temporally and spatially variable background. The most sophisticated techniques were probably developed in the thermal IR case where sky chopping by means of rapid tilts (tens of Hz) of the telescope secondary mirror were combined with slower telescope nodding to eliminate both rapid sky brightness variations and sky and telescope spatial variations in a differential detection which discriminated against the very high background.

With the advent of array-detectors at all optical wavelengths the technical approaches have changed. Precise flat-fielding/calibration of the array-detector gain and dark current characteristics have gone a long way to eliminate the need for differential observations. But even so, the astronomical detection requirements always tend to exceed the limits of the flat-fielding techniques in critical applications where the background is very high. Differential detection is therefore still required in those applications. In this paper I describe one possible imaging algorithm to be used in combination with chopping/nodding observations using IR array detectors. I will consider the specific application to 8 meter aperture telescopes. With the imaging algorithm in mind, I will then derive the technical requirements to be put on the telescope chopping/nodding properties. Unless otherwise mentioned the assumed wavelength in this paper is 10 μm.

For 8 meter telescopes the typical spatial and temporal scales for the sky emission variations equal $\theta_{em} \approx D/H$ and $\tau_{em} \approx D/V_{wind}$. With the following assumed parameters : D = the telescope diameter = 8 meter; H = average height of the emitting layers in the atmosphere = 8000 meters, and V_{wind} = wind velocity at height H = 10 m/sec one obtains θ_{em} = 200 arcsec and τ_{em} = 0.8 sec

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(or frequency $f_{em} \equiv 1/\tau_{em} = 1.25$ Hz). Any differential observations should not exceed these values in chopping full amplitude (or throw) θ and period P ($\equiv$ 1/f).

2. DESCRIPTION OF IMAGING ALGORITHM FOR CHOPPING ONLY

2.1 The Algorithm

Let the original astronomical image have a brightness distribution of f(x,y) and let the superposed background signal due to the combination of the telescope,sky and detector be a(x,y). The detector therefore sees a signal

$$s(x,y) = f(x,y)+a(x,y). \tag{1}$$

The signal a(x,y) fluctuates with time, and both a(x,y) and f(x,y) vary spatially. The atmospheric part of a(x,y) varies on angular scales comparable to θ_{em} and temporal scales τ_{em}. Chopping with full amplitudes/throws $\theta \ll \theta_{em}$ and at time scales $\tau \ll \tau_{em}$ will therefore sharply reduce atmospheric noise sources. Detector zero level and gain spatial variations are largely constant with time so that, except for the residual effects of gain variations on f(x,y), they are also removed by rapid chopping. However, the array detector electronics may suffer from 1/f noise which may dominate the Signal-to-Noise of the observations at least at low chopping frequencies (currently estimated to be below $\approx$ 1 Hz). Fast chopping is then necessary to reduce its effect.

I will assume in the following that the chopping occurs in the x direction with a PTV amplitude of $\theta = 2\Delta$, symmetrical around the nominal zero position of the chopping secondary mirror to minimize the off-axis coma effects resulting from the misalignments of the telescope primary and secondary mirrors. I will omit therefore for convenience the y coordinate in the equations below. The difference signal resulting from the chopping then equals :

$$ds(x) = f(x-\Delta) - f(x+\Delta) \tag{2}$$

and its Fourier transform (always expressed in capital letters) :

$$DS(k) = F(k)\exp(ik\Delta) - F(k)\exp(-ik\Delta) = 2i\cdot F(k)\sin(k\Delta). \tag{3}$$

where $k \equiv 2\pi$/spatial wavelength in the x-direction in the image.

The $\sin(k\Delta)$ term acts like a modulation transfer function (MTF) on F(k). The image f(x) can therefore be recovered from ds(x) by taking the Fourier transform of $\Psi(k) = DS(k)/\sin(k\Delta)$, where it not that this function suffers from singularities at $k_n = n\cdot\pi/\Delta$ values. At these spatial frequencies the image information is irrecoverably lost. Since at the wavelengths where the chopping is most needed ($\lambda \geq 8\ \mu$m) the 8 meter telescopes are diffraction limited the k_1 value should occur at a spatial frequency of $\geq 2\pi\cdot D/\lambda$. Assuming $k_1 = 2\pi\cdot D/\lambda$ gives $\Delta = 0.5\cdot\lambda/D$, or a chopping throw 2Δ equal to the diffraction limit (0.258 arcsec at 10 μm). The only other spatial frequency which is then lost is the zero frequency k_o.

Spatial frequencies below $\approx k_1/6$ (corresponding to spatial structures of 1.6 arcsec and larger) will, however, be seriously attenuated by such a small chopping throw. With future detector arrays of perhaps 1024 x 1024 pixels covering 88 x 88 arcsec with sampling of 3 pixels/resolution element at 10 μm one would often like to see structures at least up to a scale of $\approx$ 80 arcsec.

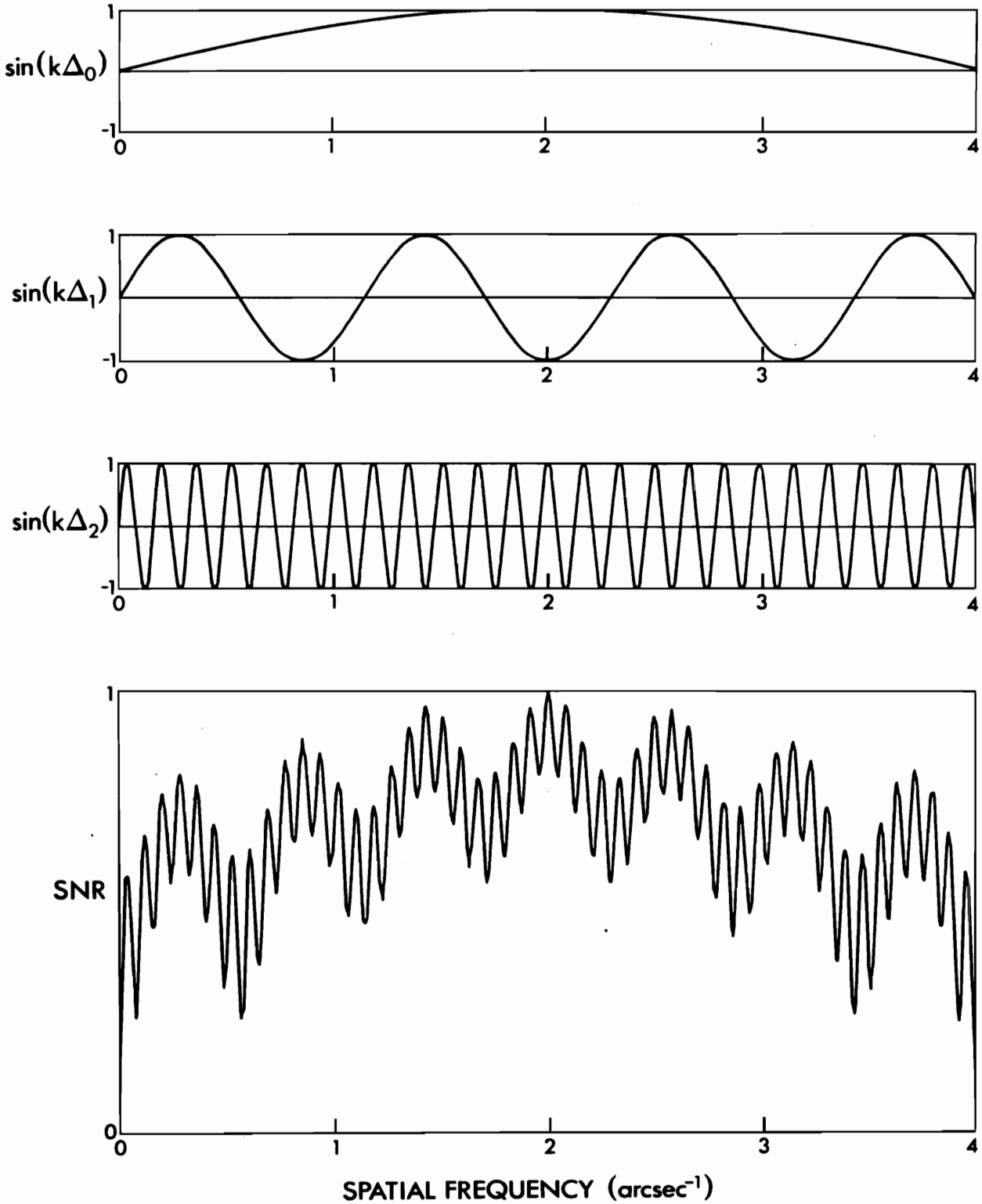

Figure 1 Upper three graphs give the Modulation Transfer for the three differential images chopped with throws of $2\Delta_0$, $2\Delta_1$ and $2\Delta_2$ respectively. The lower curve gives the Signal-to-Noise ratio for the recovered image using equation (4).

I propose that this be done by combining the Fourier transforms DS_1; DS_2 and DS_3 of three successive images chopped with $\Delta_1 = \Delta$; $\Delta_2 = M\cdot\Delta$ and $\Delta_3 = M^2\cdot\Delta$ respectively. Assuming that the noise in DS_i is white, the noise in $DS(k)/\sin(k\Delta_i)$ varies as $1/\sin(k\Delta_i)$. The three DS Fourier transformed images are therefore averaged with weights $\sin^2(k\Delta_i)$ to give a combined :

$$\Psi(k) \equiv \Sigma_i(DS_i(k)\cdot\sin(k\Delta_i))/\Sigma(\sin^2(k\Delta_i)). \tag{4}$$

This procedure results at a good SNR over a broad range of spatial frequencies. Figure 1 shows the relative SNR variation for M = 7 for 2Δ = 0.258 arcsec, where SNR = 1 corresponds to a direct image with a perfect spatially behaved detector/telescope/atmosphere. The SNR averages 70 % and exceeds 25 % at all spatial scales between 90 and 0.257 arcsec, covering all scales of interest for the 88 x 88 arcsec array size. Note that chopping completely off the array to a piece of supposedly blank sky (all structure absent over the entire array) would at best result in a SNR of 50 %. The proposed imaging algorithm chops at throws $\ll \theta_{em}$, whereas chopping off the array involves throws similar to θ_{em}. The latter will therefore suffer from spatial sky noise in addition to the possible mixing in of "nuisance" astronomical signals in the "blank sky" region.

The Fourier transform of $\Psi(k)$ results in the desired image. To avoid edge effects in the Fourier transform procedure it probably will help to repeat the f(x) image a number of times in the x direction.

2.2 Chopping Amplitudes

The chopping amplitudes $2\Delta_i$ equal 0.258; 1.81 and 12.64 arcsec for the case described above in a FOV of 88 x 88 arcsec. The resulting coma blur for the VLT 8 meter telescopes is 0.004; 0.026 and 0.180 arcsec respectively which is just acceptable. Larger chopping throws are only acceptable if one is willing to sacrifice image quality. For longer wavelengths these amplitudes go up proportionally, resulting at amplitudes of, for example, 0.62; 4.33 and 30.34 arcsec at 24 μm and a FOV of 211 x 211 arcsec. The shortest wavelengths for which chopping will be used set therefore the minimum throw for the chopping mechanism, the longest the maximum throw. When using the proposed algorithm for imaging in the 8 to 24 μm wavelength range one thus arrives at the following amplitude criteria : (i) The minimum full chopper amplitude ($2\Delta_1$) shall be 0.20 arcsec in the image plane, and (ii) The maximum full chopper amplitude shall be 30 arcsec in the image plane.

2.3 Effect of Chopper Amplitude Variations

Especially for the smaller chopping throws the throw PTV amplitude 2Δ, as projected on the astronomical image, may vary by a large fraction of 2Δ both because of atmospheric seeing motions and because of opto-mechanical limitations in the chopping mechanism. In addition the image will drift over longer time scales for the same reasons. What are the effects of these variations and what are the resulting requirements to be placed on the secondary mirror chopper design?

Let δ_1 and δ_2 be the variations in x direction in the position on the astronomical image for the two chop positions. Equation (3) then changes to :

$$\begin{aligned} DS(k) &= F(k)\exp(ik\Delta+ik\delta_1) - F(k)\exp(-ik\Delta+ik\delta_2) \\ &= 2\cdot F(k)\sin(k\{\Delta+\delta_{diff}\})\exp(ik\delta_{ave}). \\ &\equiv 2\cdot F(k)\cdot A\cdot B \end{aligned} \tag{5}$$

where $\delta_{ave} = (\delta_1+\delta_2)/2$ and $\delta_{diff} = (\delta_1-\delta_2)/2$.

The $A = \sin(k\{\Delta+\delta_{diff}\})$ term in equation (5) changes the chopping transfer function. The $B = \exp(ik\delta_{ave})$ term moves the resulting image, after taking the Fourier transform of $\Psi(k)$, by an amount of δ_{diff} in the x direction.

2.3.1 Effect on the Modulation Transfer Function

The $\sin(k\{\Delta+\delta_{diff}\})$ term equals :

$$A = \sin(k\Delta)\cos(k\delta_{diff}) + \cos(k\Delta)\sin(k\delta_{diff})$$

$$\approx \sin(k\Delta)\cdot(1-0.5\ k^2\delta_{diff}^2) + \cos(k\Delta)\sin(k\delta_{diff}) \qquad (6)$$

For random variations in δ the second term in equation (6) average out to zero, resulting in :

$$\langle A \rangle \approx \sin(k\Delta)\cdot\{1-0.5\ k^2\cdot RMS(\delta_{diff})^2\} \qquad (6')$$

The average MTF is therefore decreased by an amount which increases proportionally with k^2. For the largest k value under consideration (π/Δ_1) the fractional decrease equals (omitting the < > signs) :

$$\Delta A/A = 0.5\cdot\pi^2\cdot\Delta_1^{-2}\cdot RMS(\delta_{diff})^2$$

$$= 4.93\ \Delta_1^{-2}\cdot RMS(\delta_{diff})^2$$

$$= 300\ RMS(\delta_{diff})^2 \text{ at } 10\ \mu m \text{ wavelength} \qquad (7)$$

If I allow a maximum decrease in the MTF of 20 % that implies $RMS(\delta_{diff}) \leq 0.026$ arcsec. The wavelength independent RMS image motions in one direction caused by atmospheric seeing amount to $0.424\cdot(\lambda/D)^{1/6}\cdot(\lambda/r_o)^{5/6}$ radian, or 0.144 arcsec for D = 800 cm and 0.66 arcsec seeing (at 0.5 μm). The $RMS(\delta_{diff})$ could therefore be as much as $0.144/\sqrt{2} = 0.102$ arcsec if very slow chopping were an option. This is well beyond the tolerable value. The δ_{diff} refers, however, to one chopping cycle of 1/f seconds, if chopping occurs at a frequency of f Hz. This will substantially decrease $RMS(\delta_{diff})$, the amount of which will be estimated next.

The Power Spectrum Density (PSD) for image motions equals (se eg Madec et al, 1993) :

$$PSD(f) = C_1\cdot f^{-2/3} \qquad \text{for } f \leq f_o \text{ , and}$$

$$PSD(f) = C_2\cdot f^{-11/3} \qquad \text{for } f \geq f_o \qquad (8)$$

where the breakpoint f_o equals approximately $0.5\ V_{wind}/D$ or 0.625 Hz in the present case. For 0.66 arcsec seeing (at 0.5 μm) this results in $RMS(\delta_{diff})$ values of 0.102, 0.073, 0.056, 0.018, 0.005 and 0.001 arcsec above frequencies f of 0, 0.1, 0.3, 1.0, 3.0 and 10 Hz respectively. To meet the $RMS(\delta_{diff}) = 0.026$ arcsec criterion on has to chop therefore at frequencies above 0.75 Hz. Opto-mechanical inaccuracies which contribute to the δ_{diff} values have to be smaller than 0.026 arcsec as well. If the two effects are given equal contributions to the $\Delta A/A$ error budget one arrives at the following criteria : (i) Opto-mechanical errors in full chopping amplitudes shall not exceed 0.018 arcsec RMS over time intervals of a few seconds. For the positioning at each chopping extreme this means a maximum allowable error of 0.013 arcsec RMS, and (ii) For Image MTF preservation the minimum chopping frequencies shall be 1 Hz

2.3.2 Effect due to Image Motion

The B term in equation (5) leads to an image motion of δ_{ave}. This image motion is the same as would be encountered in the non-chopping, staring mode use of an array detector. Since $D/r_o = 1.5$ in the 10 μm case discussed here, rapid guiding is needed to achieve full diffraction limited resolution. A number of ways can be used to obtain δ_{ave} : (i) from the images $f(x-\Delta)$ and $f(x+\Delta)$ themselves before their subtraction. This can be done since the original images are available individually provided that there is a bright reference object is available, (ii) from the difference image $ds(x,y)$ or $DS(k_x,k_y)$. This eliminates much of the detector fixed pattern noise, thus lowering the brightness of the object required to determine the image shift, or (iii) from a field viewing camera signal, possibly at another (e.g. visible) wavelength. Since all of the individual IR images are available rapid guiding correction might be done in software on the difference images $DS(k_x,k_y)$ using expression (5).

2.4 Effects of Spatial Gain Variations

Precise flatfielding to determine the spatial dark level $d(x,y)$ and gain variations $g(x,y)$ in array detectors is critical for array detectors especially in the case where $f((x,y)$ is much less than $a(x,y)$, as is the case for most observations in the thermal IR. Including the effects of $d(x,y)$ and $g(x,y)$ in equations (1) and (2) one obtains :

$$s(x,y) = g(x,y)\cdot f(x,y) + g(x,y)\cdot a_{background}(x,y) + a_{other}(x,y) + d(x,y) \qquad (1'')$$

and

$$ds(x,y) = g(x,y)\cdot\{f(x-\Delta,y) - f(x+\Delta,y)\} \qquad (2')$$

Thus, while differential detection removes the $d(x,y)$ and the effects of $g(x,y)$ on $a(x,y)$, the effects of gain variations remain on the $ds(x,y)$ signal. Since $f(x,y)$ is generally much smaller than $a(x,y)$ the effects of gain variations are much less severe in differential imaging than in direct imaging, they still need correction. Using flatfielding techniques to determine $g(x,y)$, they are removed by dividing the $ds(x,y)$ signals by $g(x,y)$.

3. DESCRIPTION OF IMAGING ALGORITHM FOR CHOPPING AND NODDING

With the addition of "nodding" techniques to the imaging algorithm equations (2) and (3) would change to :

$$ds(x) = 2\cdot f(x) - f(x-\Delta) - f(x+\Delta), \text{ and} \qquad (9)$$

$$DS(k) = 2\cdot F(k)\cdot \{1 - \cos(k\Delta)\} \qquad (10)$$

which has its first zero at $k\Delta = 2\pi$ The chopper throw (now Δ rather than 2Δ) would therefore not change. The amount of nodding for each DS_i exposure would equal Δ_i and the DS_i averaging would change accordingly to :

$$\Psi(k) \equiv \Sigma_i[DS_i(k)\cdot\{1 - \cos(k\Delta_i)\}]/\Sigma[\{1 - \cos(k\Delta_i)\}]^2. \qquad (4')$$

The chopping amplitude ranges, precision and frequencies are identical to the chopping only case.

4. CHOPPING FREQUENCIES DERIVED FROM OTHER REQUIREMENTS

I already derived a minimum chopping frequency for the 10 μm imaging of 1 Hz. This limit was set by atmospheric seeing effects. The noise characteristics of the array detectors and of the atmospheric emission sets additional limits. As long as detector electronics and atmosphere 1/f noise dominates it pays to chop at as high a frequency as possible. Low chopper throw will help to keep this frequency high.

It is at this moment not clear what the minimum acceptable chopping frequency is, although there are indications that a frequency of around 1 to 4 Hz may suffice. More and better measurements are badly needed.

5. CONCLUSION

The imaging algorithm proposed in these notes resulted in estimates for the required chopper amplitudes and frequencies as a function of wavelength. As long as the chopping throw is large compared to the resolution limit of the telescope or instrument/detector combination a number of spatial frequencies are irrecoverably lost. That is even the case when chopping completely off the array unless an area of completely blank sky is available and therefore there are no lost low spatial frequencies available in the object. To allow complete imaging, preserving all spatial frequencies from those corresponding to the diffraction limit of the telescope up to those comparable to the dimension limit of the detector array, I proposed the algorithm described in these notes.

These notes refer to wavelengths of 10 (and 24) μm which cover the wavelength range most likely to require chopping techniques. If applied to shorter wavelengths smaller throws and higher temporal frequencies will be needed.

When applying the imaging algorithm to other cases, it will of course be necessary to "chop" at a frequency consistent with the detector noise contribution. For deep CCD imaging at visible wavelength that means that the chopping frequency has to be very low in order not to become detector read-out noise limited.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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