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Fermion Masses in $SO(10)$: The Case of the Tree-Level Relation $m_b = 3m_\tau$

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By considering the renormalization group flow of the top generation Yukawa couplings in $SO(10)$, we show that models for which the top generation couples to Higgs fields derived from the **126** scalar representation, with the tree level relation $m_b = 3m_\tau$, are not viable. Analytic bounds on the flow show that the bottom quark would necessarily be heavier than $\simeq 8$ GeV in these models.

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The purpose of this note is to show, with a simple analytic bound on Yukawa coupling renormalization group flows, that $SO(10)$ models [1] for which the standard model Higgs field(s) derive from the **126** dimensional scalar representation predict a mass for the bottom quark in excess of $\simeq 8$ GeV, making these models non-viable. In previous work [2], we performed a detailed numerical study of the Yukawa coupling renormalization group flows in $SO(10)$ models for which the standard model Higgs fields derive from the **10** dimensional scalar representation. For this representation, the Yukawa couplings at the unification scale satisfy the relation $h_b = h_\tau$. Almost entirely due to QCD effects, the Yukawa coupling of the bottom quark grows rapidly as we evolve the renormalization scale downward, whereas the Yukawa coupling of the tau does not share this property. This leads to a low energy mass relation which is acceptable, $m_b/m_\tau \simeq 2.2 - 2.7$. On occasion it has been remarked that the scalar **126** leads to a roughly acceptable mass ratio because of the tree-level (unification scale) relation $m_b = 3m_\tau$. However, the same QCD corrections which make the scalar **10** viable will work against the scalar **126**, driving the bottom mass to even higher values relative to the tau mass. We will show that this effect is enough to rule out models where top generation fermion Dirac masses derive from Yukawa couplings to the scalar **126**. The purpose of this note is to show this in a simple analytic way, free from the unnecessary complication of numerical analysis.

First we recall some facts about the family of $SO(10)$ grand unified theories Each generation of fermions in the theory resides in a spinorial **16** of $SO(10)$, with the requisite addition of a right-handed neutrino. Therefore, possible Yukawa coupled Higgs representations must lie in $\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126}$. The **120** is antisymmetric and thus contributes only to intergenerational mixing. We will be concerned here only with the top generation, and so we will ignore the Yukawa couplings of the **120**. It will be clear from the analysis that intergenerational couplings small enouggh to be phenomenologically acceptable would not change the conclusions.

The viable breaking scheme of $SO(10)$ is to the maximal subalgebra $SU(2) \times SU(2) \times SU(4)$, one $SU(2)$ factor being identified as the weak isospin

group, the other being right isospin, breaking to a $U(1)$ of I_{3R}; the $SU(4)$ breaks to $SU(3)_{color} \times U(1)_{B-L}$ [3][1]. The case of intermediate $SU(5)$ symmetry is ruled out by the non-observation of proton decay combined with current measurements of $\sin^2\theta_W$. We use the notation

$$\begin{aligned}
G_{224} &= SU(2)_L \times SU(2)_R \times SU(4)_c, \\
G_{2213} &= SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c, \\
G_{214} &= SU(2)_L \times U(1)_{I_{3R}} \times SU(4)_c, \\
G_{2113} &= SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \times SU(3)_c, \\
G_{13} &= SU(2)_L \times U(1)_Y \times SU(3)_c.
\end{aligned} \tag{1}$$

There are two symmetry breaking chains,

$$\begin{aligned}
G_{224} &\xrightarrow{M_{R_+}} G_{214} \xrightarrow{M_c} G_{2113} \xrightarrow{M_{R_0}} G_{213}, \\
G_{224} &\xrightarrow{M_c} G_{2213} \xrightarrow{M_{R_+}} G_{2113} \xrightarrow{M_{R_0}} G_{213}.
\end{aligned} \tag{2}$$

Recall [4] that the running coupling of an $SU(N)$ gauge theory obeys the (one loop) RG equation

$$2\pi\mu\frac{d}{d\mu}(\alpha^{-1}) = \frac{11}{3}N - \frac{4}{3}n_G - \frac{1}{6}S, \tag{3}$$

where $\alpha = g^2/4\pi$, n_G is the number of generations of Dirac fermions, and S is the quadratic Casimir for the scalar representation, $tr(\theta^a\theta^b) = S\delta^{ab}$. This is the equation we will use for the gauge coupling evolution.

Within each of the symmetry breaking chains exhibited above, there are two possible low energy Higgs spectra; we can have either one or two light Higgs doublets, each deriving from the **126**. In the case of one light Higgs, some linear combination of the $(\mathbf{2},\mathbf{1})_1$ and $\overline{(\mathbf{2},\mathbf{1})_{-1}}$ fields is assumed to be massive. This introduces an arbitrary mixing angle, which will not concern us here since we will be interested in the ratio m_b/m_τ, which is simply equal to h_b/h_τ, independent of the mixing angle. In the case of two light Higgs, the two extra pieces of information needed to calculate the masses are the two

independent *vev*'s, but again these do not concern us, as the bottom and the tau couple to the same Higgs field and the vev cancels in the ratio.

The renormalization group equations for the Yukawa couplings, as well as the form of the Yukawa interactions, are given in [2], derived as an application of the results in [4]. Reproduced in an appendix are those equations that we will use here. The content of the statement that the scalar representations of the low energy theories derive from the **126** of $SO(10)$ is contained in the unification boundary condition, $h_b = 3h_\tau$ at $\mu = M_c$. This boundary condition is imposed at M_c, since it is actually the $SU(4)_c$ symmetry which enforces it. In this sense, this type of analysis is applicable to any unified theory giving rise to an intermediate Pati-Salam $SU(2) \times SU(2) \times SU(4)$ gauge group[3], provided the Yukawa couplings have the structure detailed in [2]. As is well known, this extra structure gives rise to certain attractive features, such as the natural suppression of flavour-changing neutral currents, so it is not entirely ad hoc out of the context of $SO(10)$.

Combining the RG equations for h_b and h_τ gives the evolution equations for $\xi = (h_b/h_\tau)^2$, both for the one Higgs and two Higgs case. We write $t = \ln \mu$, and then

$$
\begin{aligned}
G^{1H}_{213} &: \frac{d}{dt}\ln\xi = \frac{1}{16\pi^2}\left[3h_b^2 - 3h_t^2 - 3h_\tau^2 - 8\pi(8\alpha_3 - \frac{10}{3}\alpha_Y)\right], \\
G^{1H}_{2113} &: \frac{d}{dt}\ln\xi = \frac{1}{16\pi^2}\left[3h_b^2 - 3h_t^2 - 3h_\tau^2 + 3h_\nu^2 - 8\pi(8\alpha_3 - 2\alpha_{R0} - \frac{4}{3}\alpha_{BL})\right], \\
G^{2H}_{213} &: \frac{d}{dt}\ln\xi = \frac{1}{16\pi^2}\left[3h_b^2 + h_t^2 - 3h_\tau^2 - 8\pi(8\alpha_3 - \frac{10}{3}\alpha_Y)\right], \\
G^{2H}_{2113} &: \frac{d}{dt}\ln\xi = \frac{1}{16\pi^2}\left[3h_b^2 + h_t^2 - 3h_\tau^2 - h_\nu^2 - 8\pi(8\alpha_3 - 2\alpha_{R0} - \frac{4}{3}\alpha_{BL})\right].
\end{aligned}
\tag{4}
$$

In principle we must also study the evolution of the Yukawa couplings in the G_{2213} theory, but in practice the effect of this evolution is at most of order a few percent because it occurs over a limited range of μ and because the smallness of the couplings at large μ makes for smaller beta functions. Thus we ignore

this stage of the evolution. This amounts to assuming that $M_c \leq M_{R_+}$ when dealing with the Yukawa coupling evolution.

The property we would like to establish is the non-positivity of the right hand sides of these equations. This would imply that ξ can only increase as we flow toward the infrared, implying at the least that $m_b/m_\tau \geq 3$, which is enough to force the bottom mass to a value two or more σ from its nominal value. In fact we will be able to bound the right hand sides to a sufficient degree that the lower bound on the bottom mass will be $\simeq 10$ GeV for the one Higgs case and $\simeq 8$ GeV for the two Higgs case.

Consider the one Higgs case first. The non-positivity of the right hand side of the G^{1H}_{213} equation is questioned only by the presence of the h_b^2 term, but it is obvious that this term is much smaller than the gauge or top contributions, since in the one Higgs case we must have $h_b << h_t$ within the G_{213} theory. We are assuming here that the "extra" Higgs becomes heavy at the I_{3R} breaking scale, a natural assumption which was invoked in the analysis of [2]. Of course, the renormalization group evolution cannot alter the relation $h_b << h_t$.

Bounding the flow for the G^{1H}_{2113} equations is more difficult than the above, because we do not have a simple approximation which removes the positive contribution of the h_b and h_ν terms to the derivative. In order to bound these contributions, we note that at the upper boundary, M_{R_+}, the unification condition implies that the Yukawa coupling contributions cancel, and so the derivative is negative there. Thus we need only show that the growth of the couplings h_b and h_ν is bounded strongly enough to insure that the contribution of these couplings remains small. This is not difficult; the following is a sketch of the argument.

First, in order to bound the growth of h_b and h_ν, note that their RG equations give

$$
\begin{aligned}
\frac{d}{dt}h_b^2 &\geq -\frac{1}{16\pi^2}h_b^2\, 8\pi\left(\frac{9}{4}\alpha_L + 8\alpha_3\right), \\
\frac{d}{dt}h_\nu^2 &\geq -\frac{1}{16\pi^2}h_\nu^2\left[8\pi\frac{9}{4}\alpha_L + h_\tau^2\right].
\end{aligned}
\tag{5}
$$

In these equations, to ease the algebraic complication, the contributions of the abelian gauge couplings have been removed. The integrated contributions of

these couplings are small, and do not affect the result substantially; in any event, they could easily be re-inserted for completeness. The presence of the h_τ^2 term in the second equation does not cause any difficulty. By integrating both sides of the full set of RG equations and then solving the resulting simultaneous linear equations for the integrals of the h_i^2 functions, we can rewrite these integrals in terms of the Yukawa couplings themselves and integrals of the gauge couplings which can be evaluated explicitly. Thus the integral of h_τ^2, which will appear in the solution of (5), can be replaced by a function of the h_i themselves, and of μ. The resulting solution of (5) shows that the most important effect is the growth of h_b; again the $SU(3)$ coupling proves to be the dominant driving force. We have

$$\begin{aligned} h_b^2(\mu) &\leq f_1(\mu) h_b^2(M_{R_+}), \\ h_\nu^2(\mu) &\leq f_2(\mu) h_\nu^2(M_{R_+}), \end{aligned} \tag{6}$$

for $\mu \leq M_{R_+}$. The function f_2 is not appreciably different from unity, whereas the function f_1 can be of order a few due to its dependence on the strong coupling,

$$f_1(\mu) = \left[\frac{\alpha_L(M_{R_+})}{\alpha_L(\mu)}\right]^{27/40} \left[\frac{\alpha_3(M_{R_+})}{\alpha_3(\mu)}\right]^{8/7}. \tag{7}$$

Therefore we have the bound

$$\frac{d}{dt} \ln \xi \leq \frac{1}{16\pi^2} \left[3 f_1(\mu) h_b^2(M_{R_+}) + 3 f_2(\mu) h_\nu^2(M_{R_+}) - 64\pi\alpha_3(\mu)\right]. \tag{8}$$

Now we must bound the initial values, $h_b(M_{R_+})$ and $h_\nu(M_{R_+})$. In order to do this we invoke the quasi-fixed-point structure of the the RG equations. A brief explanation of this structure is in order. When we imagine the gauge couplings to be fixed constants, the RG equations will have fixed points at zero and at the (constant) values of the Yukawa couplings such that the beta functions vanish. Starting at couplings above these fixed point values will cause a rapid divergence of the solutions as t moves to the ultraviolet, whereas the flows with initial couplings below these fixed point values will be smooth into the ultraviolet. When we allow the gauge couplings to vary with μ, the non-trivial fixed points are no longer true fixed points, and the solution can "wander" around these fixed point values, becoming approximately trapped, but not behaving

monotonically, thus giving a quasi-fixed-point behaviour. Also, it is still true that sufficiently large initial couplings will cause a rapid divergence toward increasing μ, and it is this effect which leads to the so-called triviality bounds on the top quark mass in the standard model [5]. For our analysis, we assume that the Yuakwa couplings are such that they lie within the quasi-fixed-point boundary. This is essentially a statement about the naturalness of the couplings; if we imagined beginning with unified couplings, at scales of the order 10^{15} GeV, which were arbitrarily larger than the quasi-fixed-point boundary, then the evolution toward the infrared would rapidly bring them down toward that boundary, in exponentially fast "time". In the case of $SO(10)$ models, this naturalness is insured because there does exist an RG flow for the Yukawa coupling above the $SU(4)$ breaking scale, and this flow will act to correct any unnatural initial coupling before it reaches the $SU(4)$ breaking scale, the point of application for our boundary condition. Thus we assume that the Yukawa coupling beta functions are not positive at M_{R_+}. In actuality, we need only make the significantly more conservative assumptions

$$\begin{aligned} 9h_b^2(M_{R_+}) &\leq 8\pi \left(\frac{9}{4}\alpha_L(M_{R_+}) 8\alpha_3(M_{R_+})\right), \\ 4h_\nu^2(M_{R_+}) &\leq 8\pi \frac{9}{4}\alpha_L(M_{R_+}), \end{aligned} \tag{9}$$

which is what we use.

Combining equations (8) and (9) gives the bound

$$\begin{aligned} \ln \xi(M_{R_0}) \;\geq\; & \ln \xi(M_{R_+}) + \frac{8}{7} \ln \left[\frac{\alpha_3(M_{R_0})}{\alpha_3(M_{R_+})}\right] \\ & - \frac{3}{4\pi}\left[\frac{13}{8}\alpha_L(M_{R_+}) + \frac{91}{36}\alpha_3(M_{R_+})\right] \int_{M_{R_0}}^{M_{R_+}} \frac{d\mu}{\mu}\, f_1(\mu) + f_2(\mu) \end{aligned} \tag{10}$$

The integral which appears here need not be computed. Instead, the crude bound which dominates it by the integral of the maximum over the whole interval will suffice.

Integrating the $G_{213}^{1\mathrm{H}}$ equation we find

$$\xi(m_b) \geq \xi(M_{R_0}) \left[\frac{\alpha(m_b)}{\alpha(M_{R_0})}\right]^{8/7} \left[\frac{\alpha_Y(M_Z)}{\alpha_Y(M_{R_0})}\right]^{1/2}. \tag{11}$$

A ratio of electromagnetic couplings which should also appear here has been ignored, since it is very nearly unity. Combining equations (11) and (10) gives the final result for the one Higgs case

$$\xi(m_b) \geq \xi(M_{R_+}) \left[\frac{\alpha(m_b)}{\alpha(M_{R_+})}\right]^{8/7} \left[\frac{\alpha_Y(M_Z)}{\alpha_Y(M_{R_0})}\right]^{1/2} \left(\frac{M_{R_+}}{M_{R_0}}\right)^{-p}, \qquad (12)$$

where

$$p = \frac{3}{2\pi}\left[\frac{13}{8}\alpha_L(M_{R_+}) + \frac{91}{36}\alpha_3(M_{R_+})\right] \left[\frac{\alpha(M_{R_+})}{\alpha(M_{R_0})}\right]^{8/7} \left[\frac{\alpha_L(M_{R_+})}{\alpha_L(M_{R_0})}\right]^{27/40}. \qquad (13)$$

This exponent is the manifestation of the small positive contributions which we have controlled. The effect of this contrary contribution is generically a factor of order $.65 - .85$.

This bound is a somewhat complicated function of M_{R_0} and M_{R_+}, but the minimum over the allowable parameter range can easily be determined. We find $\xi(m_b) \gtrsim 4\xi(M_{R_+})$. Remembering that the boundary condition is $\xi(M_{R_+}) = 9$, this means that $m_b \gtrsim 6m_\tau \simeq 10$ GeV, as claimed above.

The analysis of the equations in the two Higgs case proceeds precisely as for the one Higgs case, the one difference being that neither the G_{213} stage nor the G_{2113} stage admits an obvious monotonicity property. Instead one must, for each stage, perform a detailed analysis analogous to that of the G^{1H}_{2113} case above. In this case the positive contributions to the derivative of ξ can come from growth of h_b and h_t, as opposed to h_b and h_ν in the G^{1H}_{2113} case. There is one minor simplification in that an integrated Yukawa coupling does not appear. The final bound which is derived will be less strict than that for the one Higgs equations because of the existence of small positive contributions to the derivatives in each of the G_{213} and G_{2113} stages, as opposed to just in the G_{2113} stage as in the one Higgs case. The bound for the two Higgs case becomes

$$\xi(m_b) \geq \xi(M_{R_+}) \left[\frac{\alpha(m_b)}{\alpha(M_{R_+})}\right]^{8/7} \left[\frac{\alpha_Y(M_Z)}{\alpha_Y(M_{R_0})}\right]^{1/2} \left(\frac{M_{R_+}}{M_{R_0}}\right)^{-p}, \qquad (14)$$

where

$$p = \frac{2}{9\pi}\left[\frac{9}{4}\alpha_L(M_{R_+}) + 8\alpha_3(M_{R_+})\right] \left[\frac{\alpha(M_{R_+})}{\alpha(M_{R_0})}\right]^{8/7} \left[\frac{\alpha_L(M_{R_+})}{\alpha_L(M_{R_0})}\right]^{27/40}. \qquad (15)$$

The minimum of this bound is $\xi \gtrsim 2.5$. or $m_b \gtrsim 4.6m_\tau \simeq 8$ GeV.

The result of this analysis has been confirmed by numerical analysis of the RG equations. The final result, however, was of such a simple nature that it seemed appropriate to give a straightforward (though perhaps tedious) derivation. In short, the QCD instigated infrared growth of h_b/h_τ operates to make non-viable those $SO(10)$ models with a heavy bottom quark at tree level. In particular, $SO(10)$ models with top generation Dirac masses arising from couplings to the **126** dimensional scalar representation are not viable. The extension to other groups which break through a Pati-Salam [3] intermediate stage is envisioned.

Appendix A. The Yukawa Coupling RG Equations

In this appendix we collect the RG equations for the Yukawa couplings of the intermediate theories in the cases that interest us. The normalization of the Yukawa couplings was chosen such that "mass" = "Yukawa" × 174 GeV . We use the abbreviated notation

$$\mathcal{D} = 16\pi^2\mu\frac{d}{d\mu}. \qquad (A.1)$$

A.1. G_{213}, One Higgs doublet

These are the usual Yukawa coupling RG equations for the standard model with one Higgs doublet [6]. The couplings are as defined in Section 4.

$$\begin{aligned}
\mathcal{D}h_t^2 &= h_t^2\left[9h_t^2 + 3h_b^2 + 2h_\tau^2 - 2A_u\right], \\
\mathcal{D}h_b^2 &= h_b^2\left[9h_b^2 + 3h_t^2 + 2h_\tau^2 - 2A_d\right], \\
\mathcal{D}h_\tau^2 &= h_\tau^2\left[5h_\tau^2 + 6h_t^2 + 6h_b^2 - 2A_l\right], \\
A_u &= 4\pi\left(\frac{9}{4}\alpha_L + \frac{17}{12}\alpha_Y + 8\alpha_3\right), \\
A_d &= 4\pi\left(\frac{9}{4}\alpha_L + \frac{5}{12}\alpha_Y + 8\alpha_3\right), \\
A_l &= 4\pi\left(\frac{9}{4}\alpha_L + \frac{15}{4}\alpha_Y\right).
\end{aligned} \qquad (A.2)$$

A.2. G_{2113}, One Higgs doublet

$$
\begin{aligned}
\mathcal{D}h_t^2 &= h_t^2\left[9h_t^2+3h_b^2+2h_\tau^2+2h_\nu^2-2A'_u\right],\\
\mathcal{D}h_b^2 &= h_b^2\left[9h_b^2+3h_t^2+2h_\tau^2+2h_\nu^2-2A'_d\right],\\
\mathcal{D}h_\tau^2 &= h_\tau^2\left[5h_\tau^2-h_\nu^2+6h_t^2+6h_b^2-2A'_l\right],\\
\mathcal{D}h_\nu^2 &= h_\nu^2\left[5h_\nu^2-h_\tau^2+6h_t^2+6h_b^2+2h_\Phi^2-2A'_\nu\right]\\
\mathcal{D}h_\Phi^2 &= h_\Phi^2\left[3h_\Phi^2+4h_\nu^2-2A_\Phi\right],\\
A'_u &= 4\pi\left(\frac{9}{4}\alpha_L+\frac{17}{20}\alpha_{R_0}+\frac{17}{30}\alpha_{BL}+8\alpha_3\right),\\
A'_d &= 4\pi\left(\frac{9}{4}\alpha_L+\frac{1}{4}\alpha_{R_0}+\frac{1}{6}\alpha_{BL}+8\alpha_3\right),\\
A'_l &= 4\pi\left(\frac{9}{4}\alpha_L+\frac{9}{4}\alpha_{R_0}+\frac{3}{2}\alpha_{BL}\right),\\
A'_\nu &= 4\pi\left(\frac{9}{4}\alpha_L+\frac{9}{4}\alpha_{R_0}+\frac{3}{2}\alpha_{BL}\right),\\
A_\Phi &= 4\pi\left(\frac{3}{2}\alpha_{BL}+\frac{9}{4}\alpha_{R_0}\right).
\end{aligned}
\tag{A.3}
$$

A.3. G_{213}, Two Higgs doublets

$$
\begin{aligned}
\mathcal{D}h_t^2 &= h_t^2\left[9h_t^2+h_b^2+2h_\tau^2-2A_u\right],\\
\mathcal{D}h_b^2 &= h_b^2\left[h_t^2+9h_b^2+2h_\tau^2-2A_d\right],\\
\mathcal{D}h_\tau^2 &= h_\tau^2\left[5h_\tau^2+6h_b^2-2A_l\right],
\end{aligned}
\tag{A.4}
$$

Here A_u, A_d, and A_l are as in the G_{213} one Higgs case.

A.4. G_{2113}, Two Higgs doublets

$$
\begin{aligned}
\mathcal{D}h_t^2 &= h_t^2\left[9h_t^2+h_b^2+2h_\nu^2-2A'_u\right],\\
\mathcal{D}h_b^2 &= h_b^2\left[h_t^2+9h_b^2+2h_\tau^2-2A'_d\right],\\
\mathcal{D}h_\tau^2 &= h_\tau^2\left[6h_b^2+5h_\tau^2+h_\nu^2-2A'_l\right],\\
\mathcal{D}h_\nu^2 &= h_\nu^2\left[6h_t^2+h_\tau^2+5h_\nu^2+2h_\Phi^2-2A'_\nu\right]
\end{aligned}
\tag{A.5}
$$

The h_Φ eqn., A'_u, A'_d, A'_l, and A_Φ are as in the G_{2113} one Higgs case.

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