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SELECTED PERTURBATIVE RESULTS ON HEAVY QUARK PHYSICS

Valery A. Khoze[1]
Department of Physics, University of Durham,
Durham DH1 3LE, U.K.

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Valery A. Khoze[1]
Department of Physics, University of Durham,
Durham DH1 3LE, U.K.

Abstract

We review some selected results from the application of the analytical perturbative technique to the chromodynamics of heavy quarks. The characteristic interference features of soft gluon radiation in $t\bar{t}$ production are also discussed.

[1]Invited talk at Les Rencontres de Physique, DE LA VALEE D'AOSTE, La Thuile, March 7-13 1993. "Results and Perspectives in Particle Physics"

1. INTRODUCTION

QCD physics of heavy quarks is now extensively studied experimentally at both e^+e^- and hadronic colliders. The recent progress at LEP[1)] and SLC[2)] has led to the availability of new data on the profiles of jets initiated by b and c quarks. The interest to this subject is connected not only with testing the fundamental aspects of QCD but also with its large potential importance for measurements of heavy particle properties: lifetimes, spatial oscillations of flavour, searching for CP violating effects in their decays *etc.* Properties of b-initiated jets are of primary importance for analysis of the final state structure in $t\bar{t}$ production processes. The detailed knowledge of the b-jet profile is also essential for the Higgs searching strategy.

The physics of heavy quarks is usually considered as one of the best testing grounds for QCD. Despite this, not many attempts to present self-consistent perturbative (PT) predictions for the gross features of heavy quark jets have appeared. To our knowledge, except for the long-term project of the Leningrad/St. Petersburg PT QCD group[2], there has only been one successful PT evolution approach applied by B. Mele and P. Nason[5)].

This talk covers some results on PT description of specific properties of particle distributions in heavy quark jets which have recently been discussed in a number of publications[3,4,6)].

Another PT topic selected for this talk aims to illustrate the characteristic interference features of soft gluon radiation in $t\bar{t}$ production and decay, see Refs. 4,7-10). The large top width controls the interferences between the emissions at different stages of the process and as a result, the radiation pattern potentially may provide a way of measuring the decay width of heavy unstable particles.

2. ON THE PROFILE OF JETS INITIATED BY HEAVY QUARKS

The physics of heavy quarks is a particularly good laboratory for applications of PT QCD. The large Q-quark mass $M_Q \gg \Lambda$ provides a natural cut-off, which keeps the relevant space-time region compact enough to avoid the truly strong, non-PT domain of strong interactions. In the case of e^+e^- annihilation at center of mass energies $W \gg M_Q \gg \Lambda$, one can hope for a good description of many inclusive properties of hadronic jets via MLLA-LPHD analytical technique (see e.g. Ref. 11) and references therein). This technique allows one to describe very successfully the gross features of jets in e^+e^- annihilation without involving phenomenological fragmentation schemes, see e.g. Ref. 12).

[2]For recent progress see Refs. [3-4].

The difference in many properties of hadronic jets produced by heavy quarks (excluding the products of their decays), from that of light quarks, originates from the restriction of the phase space available to gluon radiation associated with the effects of the quark mass M_Q, see Refs. 4) and 13). One of the important consequences of the suppression of forward radiation is that the multiplicity of light hadrons accompanying the heavy quark is less than the particle yield in a light quark jet at the same W.

We shall consider here PT QCD results for heavy quark event multiplicities in more detail. In the early days of the coherent cascade picture a QCD prediction for the difference between the particle multiplicities in light and heavy quark jets was made[14)] but has not been noticed by the experimental community. This calculation incorporated only double-logarithmic contributions and was thus limited in accuracy by higher-order corrections of $O((\alpha_s(W^2))^{\frac{1}{2}} N(q\bar{q}, W))$. In ref. 4) the MLLA expression for the multiplicity of light hadrons accompanying the hard production of a heavy quark pair has been derived (see also 3,6)). It was obtained via the convolution of the particle multiplicity N_G initiated by a gluon with the probability of the gluon bremsstrahlung off Q. The result keeps track of the significant single-logarithmic effects as well, provided that the multiplicity factor N_G is calculated within the MLLA accuracy.

The mean multiplicity in $e^+e^- \to Q\bar{Q}$ can be expressed in terms of the multiplicity in light quark production process $e^+e^- \to q\bar{q}$ as

$$\Delta N^{Q\bar{Q}}(W) \;=\; N^{q\bar{q}}(W) - N^{q\bar{q}}(\sqrt{e}M_Q) + O(\alpha_s(M_Q^2)N(M_Q)) \tag{1}$$

with ΔN the *accompanying* hadron multiplicity. The total particle multiplicity in $Q\bar{Q}$ events then reads

$$N^{Q\bar{Q}}(W) \;=\; \Delta N^{Q\bar{Q}}(W) + n_Q^{dk} \tag{2}$$

where n_Q^{dk} stands for the constant *decay* multiplicity of the two heavy quarks ($n_Q^{dk} = 11.0 \pm 0.2$ for b-quarks, see Ref. 6) for details).

The main consequence of (1) is that the *difference* between particle yields from q- and Q-jets at fixed annihilation energy W depends on the heavy quark mass and remains W-independent[14,6)],

$$\begin{aligned} \delta_Q \;&\equiv\; N^{Q\bar{Q}}(W) - N^{q\bar{q}}(W) \;=\; \mathrm{const}(W), \\ \delta_{bc} \;&\equiv\; N^{b\bar{b}}(W) - N^{c\bar{c}}(W) \;=\; \mathrm{const}(W). \end{aligned} \tag{3}$$

Due to the QCD coherence relative accuracy of this statement is

$$\sqrt{\alpha_s(M^2)}\frac{M^2}{W^2}.$$

This QCD prediction contradicts the naive view on the difference of particle multiplicities as due to effective reduction of the energy available for hadroproduction after subtraction of the leading heavy quark share, $W \to W(1 - \langle x_E \rangle)$. Indeed, within this logic one would expect the "companion" multiplicity to be given by the relation

$$\Delta N^{Q\bar{Q}}(W) \;=\; N^{q\bar{q}}(W \cdot [1 - \langle x_E \rangle(W)]), \tag{4}$$

which would result in

$$\begin{aligned} \delta_Q \;&\approx\; n^{dk} - ln[1 - \langle x_E \rangle]^{-1} \frac{\partial}{\partial d \; ln \, W} N^{q\bar{q}}(W) \\ &=\; n^{dk} - ln[1 - \langle x_E \rangle]^{-1} \sqrt{\frac{2N_c \alpha_s(W^2)}{\pi}} N^{q\bar{q}}(W) \end{aligned} \tag{5}$$

where the QCD asymptotic expression for the particle multiplicity has been used for the estimate (see Ref. 11)),

$$N^{q\bar{q}}(W) \;\propto\; \exp\left(\sqrt{\frac{16N_c}{b} ln \, W}\right). \tag{6}$$

The results for the existing experiments on b-quarks are displayed in Fig. 1 (see Refs. 2,6) for details). To the available accuracy, the results are seen to be independent of W, in marked contrast to the steeply rising total multiplicity, and are thus consistent with the MLLA. Fig. 2 addresses the "naive" hypothesis which appears to be disfavoured by the data.

Further measurements with the c-quark events could provide stringent tests of MLLA-LPHD predictions down to the M_c^2 scale. At this lower mass, the question of the relationship between LPHD and QCD confinement becomes particularly interesting.

In Ref. 4) the results were presented for the inclusive energy spectra of leading heavy-flavoured particles (H_Q) based on the perturbative expression for heavy Q distributions that emerge after taking into proper account multiple gluon bremsstrahlung off the $Q\bar{Q}$ pair. This approximation is valid in the relativistic limit and keeps track of the *collinear* logarithms $\alpha_s \, ln \, W$ and $\alpha_s \, ln \, M_Q$, *soft* double-logarithmic $\alpha_s \, ln^2(1 - x_E)$ and essential single-logarithmic $\alpha_s \, ln(1 - x_E)$ contributions in all orders. At the same time it embodies the exact first order result $O(\alpha_s)$ for the inclusive energy distribution.

Motivated by the LPHD concept, the attempt has been performed to derive *pure* PT predictions without invoking the phenomenological fragmentation function. Starting point for such an attempt was the observation that an appearance of the parton model motivated *peak* in the

non-PT fragmentation at large x_E can be attributed to the Sudakov suppression effects *provided* one feels courageous enough to continue the PT description down to the region of gluon transverse momenta, $k_\perp \sim \Lambda \cdot (W/\sqrt{M_Q \Lambda})^{0.2}$, which at present (and foreseeable) energies looks dangerously close to the non-perturbative domain.

When getting rid of the transverse momentum cutoff one faces the problem of the formal "infrared pole" in $\alpha_s(k_\perp^2)$ at $k_\perp = \Lambda$ and is forced to introduce the effective non-singular coupling α_s^{eff} that remains finite at $k_\perp \leq \Lambda$. It is not easy to justify the very notion of $\alpha_s(Q^2)$ at small Q^2 where the PT quark-gluon language seems to be hardly applicable at all. In the problem under consideration it can be related to the effective measure of intensity of accompanying particle production at the confinement stage of the H_Q formation, a finite number of light hadrons produced in addition to the particle yield due to PT-controlled gluon bremsstrahlung at the first stage of the $Q\bar{Q}$ creation process.

The energy losses can be studied to quantify the influence of non-PT effects. The pure perturbative prediction reads[4)]

$$\langle x_E \rangle^{PT} = \exp\left\{ -v^2 \cdot \frac{8C_F}{3} \left(\xi(W^2) - \xi(M^2) - \frac{8}{3} \cdot \frac{\alpha_s(W^2)}{4\pi} + \frac{5}{6} \cdot \frac{\alpha_s(M^2)}{4\pi} \right) \right\}. \tag{7}$$

This expression has been derived in the relativistic approximation and disregarded parametrically narrow integration region $(1-z) \lesssim \frac{\Lambda}{M_Q}$ where non-PT effects may enter the game leading to power corrections $O\left(\frac{\Lambda}{M_Q}\right)$ to (7).

Fixing certain shape of the $\alpha_s(Q^2)$ dependence near the origin makes it possible to quantify the non-PT Λ/M_Q power corrections to the "pure PT" prediction for heavy quark spectra.

As shown in Ref. 4) the W-evolution in quark energy losses $\langle x_E \rangle(W)$ is practically insensitive to the adopted scheme of α_s^{eff} extrapolation. This makes it possible to use the ratios of $\langle x_E \rangle$ at different energies as a way to extract the scale parameter Λ. Another infrared stable quantity found empirically is the PT prediction for the normalized peak position, $\langle x_E^{peak} \rangle(W)/\langle x_E^{peak} \rangle(W_0)$.

At the same time, the absolute values of $\langle x_E \rangle$, peak positions and particle distributions in general are strongly dependent on the value of α_s^{eff} at the origin chosen as an input for calculating the PT-motivated distributions, and on its assumed momentum dependence.

Comparison with the existing experimental information on $\langle x_E \rangle_{b,c}$ results in[4)]

$$\int_0^{1\mathrm{GeV}} d\kappa \frac{\alpha_s^{eff}(\kappa^2)}{\pi} \approx 0.2\mathrm{GeV}, \tag{8}$$

$$\Lambda^{(3)} = 440 \pm^{180}_{120} \mathrm{MeV}. \tag{9}$$

In refs. 3,4,13) the effects of the gluon bremsstrahlung suppression on the spectra of light particles in $Q\bar{Q}$ events were also analysed. The resulting particle spectrum $\bar{D}_Q^n(x, ln(W/\Lambda))$ appears to be *softer* than that in the case of the same energy jet generated by a light quark $\bar{D}_q^h(x, ln(W/\Lambda))$. The "dead cone" phenomenon results in a calculable depopulation of the inclusive energy distribution of hadrons with large $x_F > \Lambda/M_Q$ while the softer part of the spectrum should be identical for light and heavy quark jets.

3. SOFT RADIATION IN TOP PRODUCTION AND DECAY

Heavy unstable charged particles can emit radiation both before and after they decay. The analysis of such radiation is a complex issue, depending sensitively on the timescale of the emission compared to the lifetime of the unstable particle. In particular, the radiation pattern can be very different according to whether the radiation occurs predominantly during the production stage or after the particle has decayed[4,8].

There are several important examples of such effects which are directly relevant to present and future high-energy colliders. As a specific example, consider the production and decay of a $t\bar{t}$ pair in high-energy e^+e^- annihilation. With a mass M_t of at least 108 GeV[3] the top quark decays to a *real* W and a b, and the width Γ_t for this decay is quite large. So large that the top weak lifetime can be as short as strong interaction timescales. The resulting interplay between the strong and weak interactions of the top quark gives rise to interesting physical effects. For example, since top is heavier than 100 GeV, then Γ_t is greater than the typical hadronic scale $\mu \sim 1$ fm^{-1} and it may decay *before* it has time to hadronize[15]. In particular, $t\bar{t}$ resonances may never be formed. Here we are interested in the perturbative aspects of the strong-weak interplay: decay versus gluon bremsstrahlung.

Reference 8) discussed soft gluon radiation in $e^+e^- \to t\bar{t}$ and showed that gluons radiated in top production and decay can interfere, and how much they do depends on the top width. This means that top production and decay should not be treated separately – the gluon distribution in top events is not what one might naively guess.

Reference 9) is focussed on soft gluon radiation near the $t\bar{t}$ threshold. The top quarks are produced nearly at rest and essentially do not radiate. The width dependence is a result of interference between gluons radiated in the two decays, which does not play an important role at higher energies. Near the production threshold, the amount of interference between gluons

[3]as we have learned from the talks given by R. Raja and C. Compagnari at this Conference.

from the b and $\bar{b}$ is controlled by the top width, and what matters is the size of Γ_t relative to the gluon energy. Thus when the top width and the gluon energy are more or less the same order of magnitude, the radiation pattern is sensitive to Γ_t.

Consider the process $e^+e^- \to t\bar{t} \to W^+W^-b\bar{b}$ with emission of a gluon, as shown in Fig. 3. Any one of the quarks can radiate the gluon; a gauge-invariant calculation must include emission off the t's and b's. Let the gluon have momentum k. Let the final b, $\bar{b}$, W^+, and W^- momenta be p_1, p_2, P_1, and P_2, respectively, and $q_i \equiv p_i + P_i$. Then, for example, if the b emits the gluon, the momentum of its parent t is $q_1 + k$ and that of the $\bar{t}$ is q_2.

In the soft gluon limit the matrix element for this process factorizes and can be written as a product of the lowest order matrix element and a term associated with the gluon emission. Schematically, $\mathcal{M}^{(1)} \sim \mathcal{M}^{(0)} J \cdot \epsilon$, where ϵ^μ is the gluon polarization. The soft gluon distribution is obtained by taking the absolute square of the current J^μ and integrating over the t and $\bar{t}$ virtualities; the lowest order cross section divides out.

For convenience we include all internal t propagators in the definition of the current J^μ, which can be written as[8)]

$$\begin{aligned} J^\mu &= \left(\frac{q_2^\mu}{q_2 \cdot k} - \frac{q_1^\mu}{q_1 \cdot k}\right) \frac{1}{(q_1^2 - \bar{M}^2)(q_2^2 - \bar{M}^2)} \\ &+ \left(\frac{p_2^\mu}{p_2 \cdot k} - \frac{q_2^\mu}{q_2 \cdot k}\right) \frac{1}{(q_1^2 - \bar{M}^2)((q_2 + k)^2 - \bar{M}^2)} \\ &- \left(\frac{p_1^\mu}{p_1 \cdot k} - \frac{q_1^\mu}{q_1 \cdot k}\right) \frac{1}{((q_1 + k)^2 - \bar{M}^2)(q_2^2 - \bar{M}^2)} \\ &\mathcal{A}^\mu + \mathcal{B}_2^\mu - \mathcal{B}_1^\mu, \end{aligned} \tag{10}$$

where $\bar{M} \equiv M_t - i\Gamma_t/2$. This decomposition is gauge invariant and physical: A, B_2, and B_1 correspond respectively to $t\bar{t}$ production, $\bar{t}$ decay, and t decay. This can be seen from the poles in the q_i. We can then define a gluon emission probability density, which is just the differential cross section for radiating a gluon normalized to the zeroth-order cross section. It is given by

$$dN \equiv 1/\sigma_0 d\sigma_g = \frac{d\omega}{\omega} \frac{d\Omega}{4\pi} \frac{C_F \alpha_s}{\pi} \mathcal{R}, \tag{11}$$

where ω and Ω denote the gluon energy and solid angle. R is obtained by integrating the absolute square of the current over the virtualities of the t and $\bar{t}$.

The important point about the decomposition in Eq. (10) is that when we square the current to compute the cross section, we can identify individual contributions. In particular we can

isolate the production-decay interference term, which comes from $A\cdot(\mathcal{B}_2 - \mathcal{B}_1)^*$. We then find that the corrections to production (from $|\mathcal{A}|^2$) and to decay ($|\mathcal{B}_i|^2$) are independent of the top width Γ_t but the production-decay interference term is proportional to Γ_t^2. Furthermore, for appropriately chosen configurations, this interference is destructive, and so the total distribution can be sensitive to its value.

Let us illustrate the above comments with a specific example. We take $M_t = 140$ GeV, center-of-mass energy 1 TeV, gluon energy 5 GeV, and $\Gamma_t = \Gamma_{SM}^{(0)} = 0.7$ GeV. Figure 4 shows the gluon distribution $dN/d\cos(\theta_g)$ as a function of θ_g, the angle between the gluon and the top quark. We take the $t - b$ angle equal to 90^0 and the $b - \bar{b}$ to be back-to-back, so the $t, b, \bar{t}$ and $\bar{b}$ positions correspond to $\theta_g = 0$, 90, 180 and 270^0, respectively. All particles are coplanar. The distribution $dN/d\cos(\theta_g)$ is shown for various values of the top width. The interference is destructive, and the change in the distribution as Γ_t is increased is quite dramatic.

By far the likeliest possibility, however, is for the t's to decay to *forward* b's in the lab. Unfortunately, for forward b's there is virtually no Γ_t sensitivity whatsoever in the soft gluon distribution (see Ref. 13)).

Near the $t\bar{t}$ threshold, the total cross section is higher and the t's are produced nearly at rest, so that the relative orientations of the t anf b momenta are irrelevant. Emission at the production stage is strongly suppressed, only emission at the decay stage plays the role.

That the top width *does* influence the radiation from the b's can be understood by considering the following extreme cases. As $\Gamma_t \to \infty$, the top lifetime becomes very short, the b and $\bar{b}$ appear almost instantaneously, and they radiate coherently, as though produced directly. In particular, gluons from the b and $\bar{b}$ interfere. In the other extreme, for $\Gamma_t \to 0$, top has a long lifetime and the b and $\bar{b}$ appear at very different times and therefore radiate independently, with no interference. Clearly, the top width controls the interference between gluons emitted by the b and $\bar{b}$.

The situation for finite width is between the two extremes. Let v be the b (or $\bar{b}$) velocity, $\theta_{1(2)}$ be the angle between the $b(\bar{b})$ and the gluon, and θ_{12} the angle between the b and $\bar{b}$. Then

$$\mathcal{R} = \frac{v^2\sin^2\theta_1}{(1 - v\cos\theta_1)^2} + \frac{v^2\sin^2\theta_2}{(1 - v\cos\theta_2)^2} + 2\chi\frac{v^2(\cos\theta_1\cos\theta_2 - \cos\theta_{12})}{(1 - v\cos\theta_1)(1 - v\cos\theta_2)}, \tag{12}$$

where

$$\chi \equiv \frac{\Gamma_t^2}{\Gamma_t^2 + \omega^2}. \tag{13}$$

The interference term is proportional to χ. Note that $0 \leq \chi \leq 1$ and $\chi = 0$ for $\Gamma_t = 0$

(independent emission) and $\chi = 1$ for $\Gamma_t \to \infty$ (coherent emission). Thus a finite top width suppresses the interference compared to the "naive" expectation of full coherent emission. And from the form of χ we see that the radiation pattern exhibits maximum sensitivity to Γ_t when Γ_t is comparable to the gluon energy ω.

Note that in the general case the interferences between the emissions at the production and the decay stages are controlled by the "profile functions"[8)]

$$\chi_i = \frac{M_t^2\Gamma_t^2}{(q_i k)^2 + M^2\Gamma_t^2}, \tag{14}$$

$$\chi_{1,2} = \frac{M_t^2\Gamma_t^2((q_1 k)(q_2 \cdot k) + M_t^2\Gamma_t^2)}{((q_1 k)^2 + M_t^2\Gamma_t^2)((q_2 \cdot k)^2 + M_t^2\Gamma_t^2)} \tag{15}$$

which depend on the t mass M_t and decay width Γ_t. They have the (formal) property that $\chi_i, \chi_{12} \to 0$ as $\Gamma_t \to 0$ and $\chi_i, \chi_{12} \to 1$ as $\Gamma_t \to \infty$. Near the threshold $\chi_i, \chi_{12} \to \chi$. Only soft emission with $\omega \lesssim \Gamma_t$ can lead to significant interference contributions: the radiation of energetic gluons (either real or virtual) with $\omega \gg \Gamma_t$ pushes the t-propagators far off their resonant energy and the interference becomes negligible. This is a well-understood phenomenon dating back (at least) to the early days of J/Ψ physics[16)].

Suppression of the interference contribution in the case of radiation with $\omega \gg \Gamma_t$ could be clarified in a semi-classical way describing the colour field formation in terms of Lienard-Wiecherd potentials, see Ref. 9).

The radiation accompanying the process with heavy top quarks decaying at times t_{0i} after the $t\bar{t}$ production ($t_{prod} \sim \frac{1}{W} \ll \frac{1}{\Gamma}$) could be as

$$dN = \frac{d\omega}{\omega}\frac{d\Omega}{4\pi}\frac{C_F\alpha_s}{\pi} \cdot \{\mathcal{R}_{\text{indep}} + 2\text{Re}\, e^{i\omega(t_{01}-t_{02})} \cdot \mathcal{J}\}. \tag{16}$$

Here R_{indep} is the sum of the "independent" radiation contributions and 2J is the interference term:

$$\mathcal{R}_{\text{indep}} = \mathcal{R}_1 + \mathcal{R}_2 = \frac{v^2\sin^2\theta_1}{(1 - v\cos\theta_1)^2} + \frac{v^2\sin^2\theta_2}{(1 - v\cos\theta_2)^2}, \tag{17}$$

$$\mathcal{J} \equiv \frac{v^2(\cos\theta_1\cos\theta_2 - \cos\theta_{12})}{(1 - v\cos\theta_1)(1 - v\cos\theta_2)}. \tag{18}$$

cf. Eq. (12).

The times t_{0i} are not measured but are distributed according to the decay exponentials

$$\left[\Gamma \int_0^\infty dt_{0i} e^{-\Gamma t_{0i}}\right]. \tag{19}$$

Substituting (16) into the decay-time integrals we see that the interference term gives the χ factor,

$$\langle e^{\pm i\omega t_{0i}}\rangle_i \equiv \Gamma \int_0^\infty dt_{0i} e^{-\Gamma t_{0i}} \cdot e^{\pm\omega t_{0i}} = \frac{\Gamma}{\Gamma \mp i\omega}, \tag{20}$$

$$\langle\langle e^{\pm i\omega(t_{01}-t_{02})}\rangle_1\rangle_2 = \frac{\Gamma}{\Gamma + i\omega}\frac{\Gamma}{\Gamma - i\omega} = \chi(\omega), \tag{21}$$

leading to the final expression which is identical to the representation[12].

Thus we conclude that the ω-dependence of the soft radiation is due to incoherence induced by the uncertainty $\Delta t_0 \sim \Gamma_t^{-1}$ in the acceleration times of the two (b-quark) charges. When the relative phases of radiations accompanying two decay processes become large

$$\omega\Delta t_0 \sim \omega/\Gamma_t \gg 1 \tag{22}$$

the interference vanishes as the result of averaging over the instances of these decays.

The width effects in the threshold region are discussed in detail in Ref. 9). Here we present only the example corresponding to the case when one integrates over the azimuthal angle about the direction of quark 1. In Fig. 5(a), $\theta_{12} = 30^0$ and in Fig. 5(b), $\theta_{12} = 150^0$. The azimuthally integrated distributions are shown for $\chi = 0$ (solid line), 0.5 (dotted line), and 1 (dashed line).

In Fig. 5(a) we can just begin to discern the effect of the $\bar{b}$ dead cone. We see that as χ increases from 0 and the interference turns on, emission at angles larger than θ_{12} is suppressed. We get maximum suppression – as much as an order of magnitude – for the coherent case, $\chi = 1$. Between the b and $\bar{b}$, that is, for $\theta < \theta_{12}$, the width makes no visible difference, but at larger angles the interference is destructive.

We consider a nearly back-to-back $b\bar{b}$ pair in Fig. 5(b) where $\theta_{12} = 150^0$. For such a large angular separation there is little room outside the $b\bar{b}$ pair and the entire angular ordering effect amounts to an enhancement of radiation between the b and $\bar{b}$. This is the well-known string/drag effect[11,17].

In Ref. 10) the properties of soft radiation in hadroproduction of $t\bar{t}$ pair are discussed.

4. CONCLUSIONS AND FUTURE DIRECTIONS

The perturbative QCD approach provides an analytical technique for calculation of the gross features of hadronic systems produced in hard processes. Until now the main phenomenological successes of this approach were connected with the description of the inclusive characteristics of jets in e^+e^- annihilation, without distinction between the contributions of light and heavy

primary quarks. Prompted by these successes, and the recent availability of data on heavy quark jets, the Leningrad/St. Petersburg PT QCD group have continued to develop their long-term project on PT studies of heavy quark physics. The inclusive distributions of leading heavy particles and energy spectra and multiplicities of light hadrons from jets initiated by heavy quarks were calculated.

The Durham-St. Petersburg group have studied the characteristic interference between radiation of soft gluons in top production and decay. Potentially, the radiation pattern may provide a way of measuring the decay width of the heavy particles.

Finally, we present a brief "shopping list" of experimental studies of the QCD properties of heavy quark jets. We concentrate mainly on physics of particle flows in c and b-quark events (for the details and discussions of other topics see, e.g., Refs. 3,4,18-23).

The potential of these studies is intimately connected with the following advantages of heavy quark "instrumentation"

i) Discrimination between quark and gluon jets in the multijet events using heavy quark tagging.

ii) "Natural" fixing of the heavy quark direction (for example using vertex detection).

a) With the identified heavy quarks one can study the collimation properties of the multiplicity and energy flows in the quark and gluon jets in three-jet events[19,24].

b) By measuring energy spectra of particles restricted to lie within the particular opening angles with respect to the jet (heavy quark direction) one can explore the specific features of the parton multiplication processes and, in particular, sharpen the influence of angular ordering on the parton branching[18,25].

c) The detailed tests of colour-coherence effects and their discrimination against non-perturbative dynamics require comprehensive studies of the total three-dimensional pattern of particle flows in tagged three-jet events. Comparison with analytical results (accounting for both interjet and intrajet coherence phenomena) should make it possible to distinguish reliably the PT predictions from fragmentation schemes. Of special interest here are the energy dependence of the multiplicity flow and its dependence on m_h and $p_{\rm out}$ [19,23,26].

d) Three-jet events with tagged heavy quarks having "unfriendly" kinematics, when the angle between the quarks is comparatively small, provide a prospective way to "isolate"

an individual gluon jet[23,24]. Thus the unique opportunity appears for studies of the specific properties of a gluon jet.

e) An interesting manifestation of the QCD wave nature of hadronic flows arises from the double-inclusive correlations $d^2N/d\Omega_\alpha d\Omega_\beta$ (α, β denote the interjet valleys) of the interjet flows in $q\bar{q}g$ events. Here one faces such tiny effects as the mutual influence of different $q\bar{q}$ antennae (colour screening phenomenon). As a consequence

$$\frac{d^2N/d\Omega_{(gq)}d\Omega_{g\bar{q})}}{d^2N/d\Omega_{(q\bar{q})}d\Omega_{g\bar{q})}} \equiv r_2 < r_1 \equiv \frac{dN/d\Omega_{(gq)}}{dN/d\Omega_{q\bar{q})}} \tag{23}$$

(the elaborated analytical formulae may be found in Refs. 19,24).

f) Of fundamental importance is the azimuthal asymmetry of QCD jets[19,27]. For example, by studying the asymmetry of a q-jet in the $q\bar{q}g$ events one can observe the colour-suppressed repulsion between particles from q- and $\bar{q}$-jets. Such $O(1/N_c^2)$ collective effects could not be reproduced in a classical probabilistic language in Monte Carlo simulations. An interesting vista on this problem is connected with the $Z^0 \to c\bar{c}g$, $b\bar{b}g$.

To investigate the role of QCD motivated $1/N_c^2$ interference terms, one has to select $q\bar{q}g$ events in which the gluon moves in the direction opposite to quasicollinear $q\bar{q}$ pair. Here comparison with analytical results (accounting for both interjet and intrajet phenomena) should make it possible to distinguish reliably the PT predictions from that of existing fragmentation schemes.

f) Making use of the inclusive properties of heavy hadron decay to statistically remove the heavy hadron decay tracks, it should be possible to study more extensively the properties of radiated hadrons in heavy hadron events. It is expected that the gluonic radiation 'dead cone' will appear as a depopulation in the region $2E_h/W > \Lambda/M_Q$ while the spectrum of soft hadrons with $\frac{2E_h}{W} \ll \frac{\Lambda}{M_Q}$ should be identical to that of light quark jets[3,4]. Finally, PT QCD predicts various aspects of the x_Q distribution itself[4] which can be tested with an accurate measurement of $\langle x_Q \rangle$ and the x_Q spectrum.

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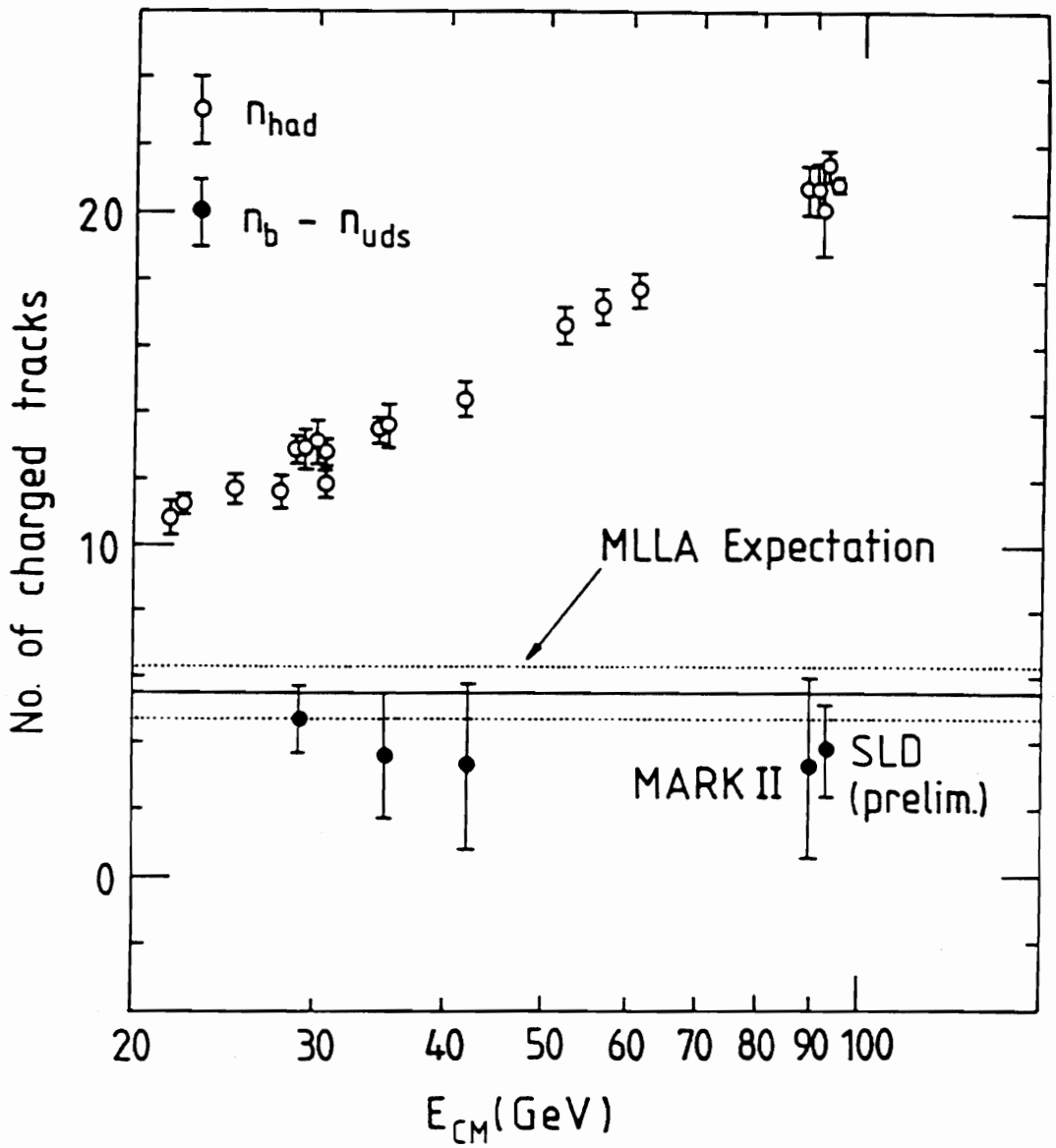

Fig. 1 Energy dependence of total multiplicity (open points) and the multiplicity difference between b and light quark production (filled points) in e^+e^- annihilation. Also shown is the expected value of this multiplicity difference, given by lower energy multiplicity data in accordance with MLLA (see Refs. [2,6] for details).

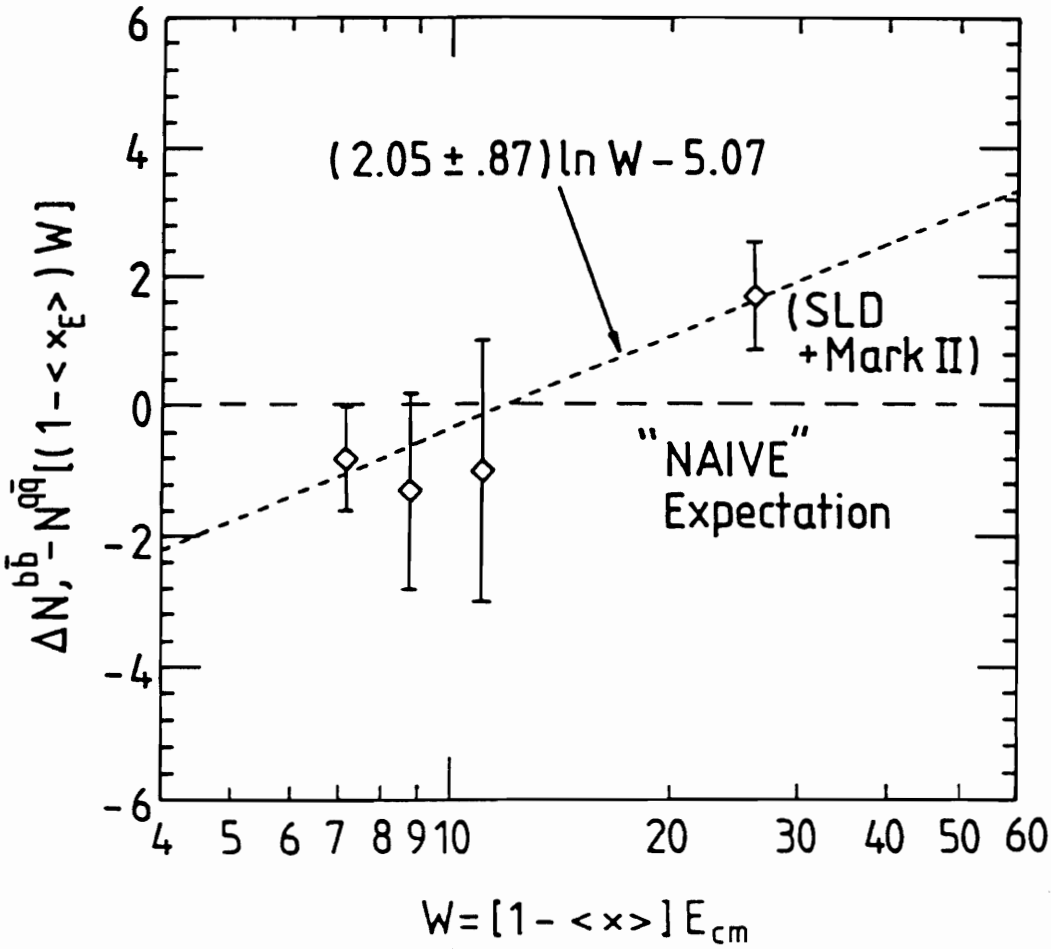

Fig. 2 Comparison with the "naive" expectation (4). The point at the Z^0 is a combined MARK II + SLD result[2]. The best fit line has a slope 2.4σ different from zero.

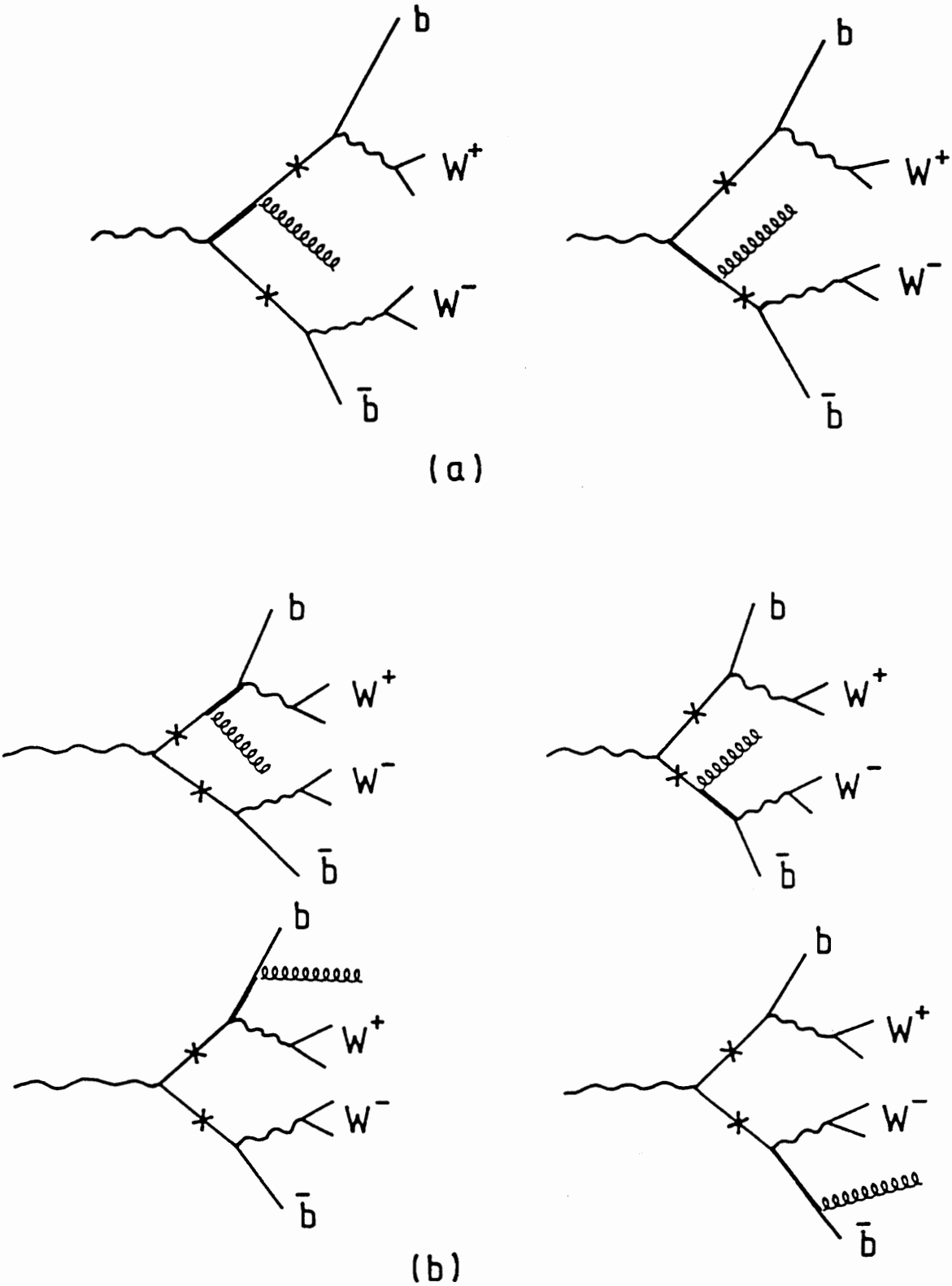

Fig. 3 Feynman diagrams for gluon emission in $e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$; the decomposition corresponds to that in Eq. (10). Heavy lines denote off-shell quarks, and x represents the (artificial) boundary between top production and decay. (a) Correction to top production. (b) Correction to top decay.

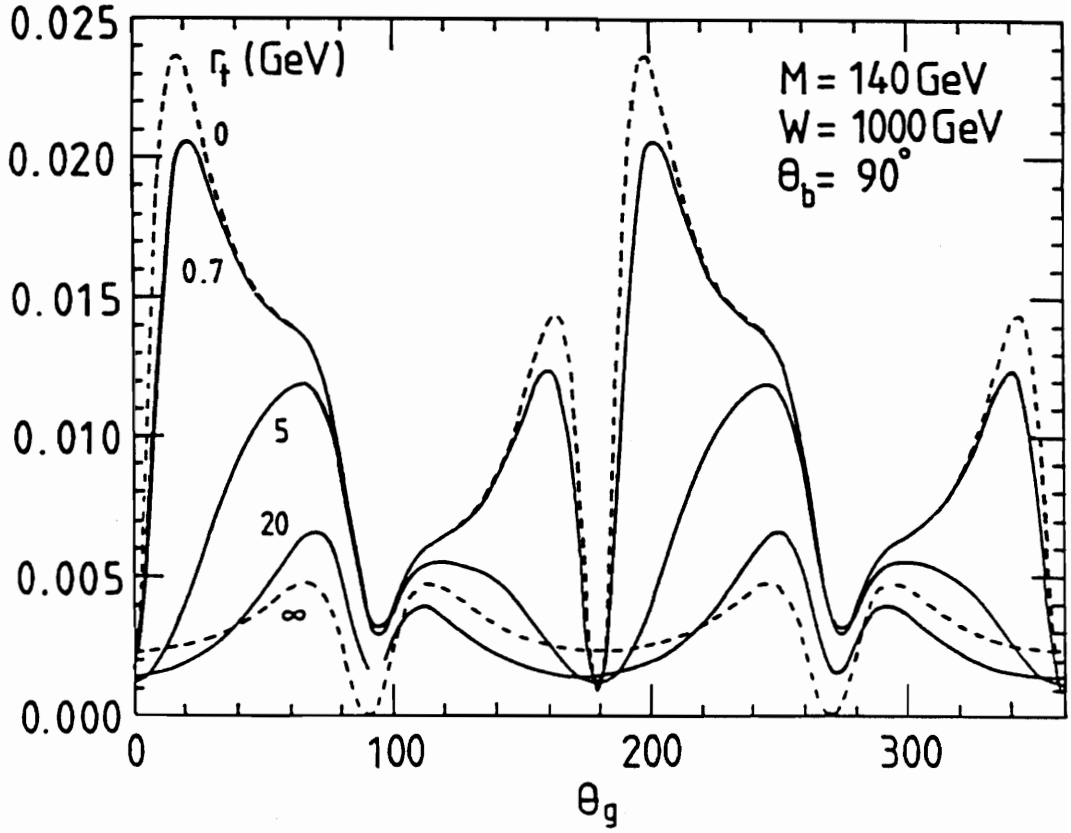

Fig. 4 Soft gluon distribution in $e^+e^- \to t\bar{t}$[8]. The t and b are at 90^0. $M_t = 140$ GeV. $W = 1$ TeV, $\omega = 5$ GeV and Γ_t as marked.

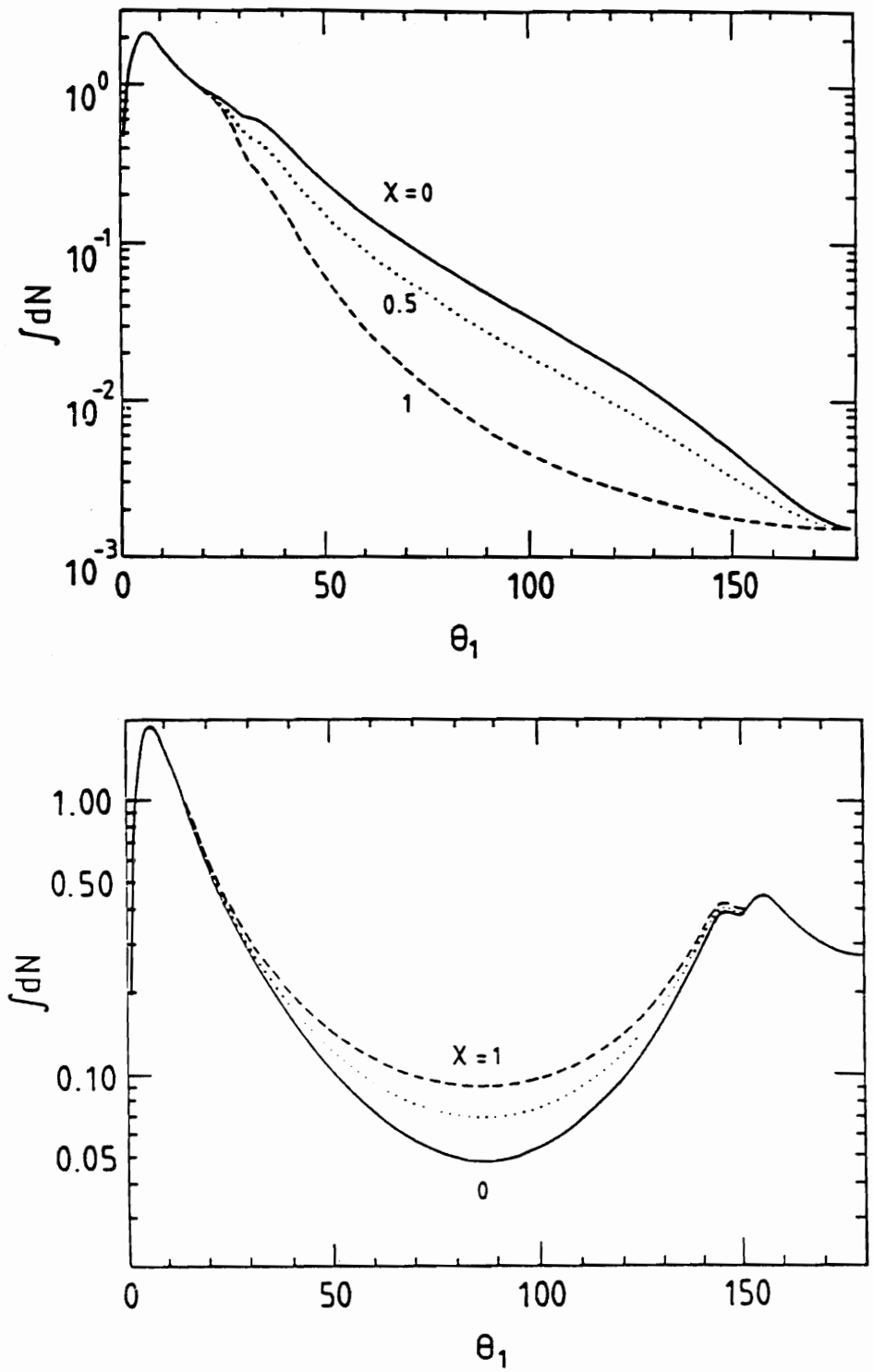

Fig. 5 Angular ordering effects in the soft gluon distribution with azimuthal integration:

$$\int dN \equiv \omega(dN/d\omega d\cos\theta) = \int_0^{2\pi} d\phi\omega(dN/d\omega d\Omega).$$

The angles between the b and $\bar{b}$ are (a) $\theta_{12} = 30^0$ and (b) $\theta_{12} = 150^0$. Solid lines: $\chi = 0$; dotted lines: $\chi = 0.5$; dashed lines: $\chi = 1$.