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F/D Ratios of Baryon Vertices for Arbitrary N_c

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Abstract

In this paper we study the spin-flip and non-flip baryon vertices in QCD for arbitrary number of colors N_c by calculating the F/D ratios on the basis of the non-relativistic quark model and chiral soliton model. It is found that the D/F ratio of spin-nonflip vertex varies from 1 at $N_c = 1$ to the limiting values -1 for $N_c \to \infty$ model independently and the F/D ratio of spin-flip vertex varies from 1 to 1/3.
We investigate the $SU(3)$ structures and N_c dependence in order to understand physical meaning of these limiting values of F/D ratios. Applying the Wigner-Eckart theorem to the isoscalar and isovector baryon vertices, we obtain the $I = J$ rule which states that the leading term of the isospin-nonflip($I = 0$) and spin-nonflip($J = 0$) baryon vertex and the isospin-flip($I = 1$) and spin-flip($J = 1$) baryon vertex are of order $O(N_c)$ for the parts independent of isospin and strangeness. We find that the leading terms of the isospin-nonflip and spin-flip vertex and the isospin-flip and spin-nonflip vertex are order $O(1)$ in the $1/N_c$ expansion irrespectively to the isospin and strangeness. We examine and discuss these obtained results comparing with experimental data such as masses and magnetic moments.

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1 Introduction

It is an important subject of QCD to understand the properties of the lowlying baryons and their effective interactions. Two models have been proposed for describing the baryon, one is the non-relativistic quark model (NRQM) [1] and the other is the chiral soliton model (CSM) [2].

In the NRQM it is assumed that baryon consists of the constituent quarks in order to explain the octet and decouplet structures of the lowest lying baryons observed in the experiment. This model has been successful in describing static properties of baryons quantitatively, such as masses and magnetic moments, however it has been shown by Coleman and Mandula [3] that it is impossible to extend the NRQM with $SU(6)$ symmetry to the Lorentz covariant form.

In the CSM it is assumed that baryons are solitons of the non-linear chiral Lagrangian for the chiral fields. The CSM has been successful in describing the baryon sector in the semi-classical approximation from meson sector, though this model predicts too large baryon masses and too small values for axial vector coupling constant g_A.

Though these models have been successful in describing the baryon sector at least in semi-classical sense, we have not yet succeeded in deriving them directly from QCD as the low energy effective theories. As the theoretical development of the non-perturbative approach to QCD, these models have been revived as the low energy effective theory based on the $1/N_c$ expansion in QCD. The latter (CSM) has been investigated by Adkins, Nappi and Witten [4]. They have derived the model independent relations $g_{\pi N\Delta} = 3/2 g_{\pi NN}$ and $\mu_{N\Delta} = 1/\sqrt{2}(\mu_p - \mu_n)$ at the leading order of $1/N_c$ expansion. The former (NRQM) has been studied by Karl and Paton and they have shown that the above relations can be obtained in the NRQM, too [5]. Bardakci [6] and Manohar [7] independently have pointed that the NRQM and CSM coincide in the large N_c limit. Furthermore Mattis and his collaborators have derived the $I = J$ rule for the baryon vertices from the CSM [9].

In recent years, Gervait and Sakita [8] and Dashen, Jenkins and Manohar [10] have derived a model independent result by solving the consistency condition for the axial coupling of baryon vertex from the viewpoint of large N_c. They have extended the consistency approach to the case containing the strange quark by identifying $K = I + J$ with strangeness [10]. In previous papers [11] we have calculated the F/D ratio of spin-flip baryon vertex for arbitrary value of N_c and discussed the baryon vertices given by

CSM and NRQM.

In this paper we calculate the F/D ratios of the spin-flip and nonflip baryon vertices in the non-relativistic quark model as well as the chiral soliton model for arbitrary N_c. It is found that the F/D ratios tend to -1 and $1/3$ for the spin-nonflip and spin-flip baryon vertices, respectively in the limit $N_c \to \infty$. Furthermore we study the physical meaning of these limiting values $1/3$ and -1 for the F/D ratios. According to the Wigner-Eckart theorem, it is shown that the $I = J$ rules for the baryon vertices hold for arbitrary number of N_c. The isospin and strangeness dependences both in the isovector part of baryon mass and the isoscalar part of magnetic moment are connected to the spin-nonflip and spin-flip baryon vertices, respectively and the dependences are not suppressed by the $1/N_c$ factor while they are suppressed both in the isoscalar part of baryon mass and the isovector part of magnetic moment.

In §2, the baryon states with spin 1/2 are defined for arbitrary N_c. In §3, we calculate the F/D ratios of the spin-nonflip and the spin-flip baryon vertex for arbitrary N_c both in the NRQM and the CSM. In §4, we derive the isoscalar and isovector formulas for arbitrary N_c applying the Wigner-Eckart theorem to the flavor $SU(3)$ symmetry. We show there that the Okubo-Gell-Mann mass relation

$$3\Lambda + \Sigma = 2(N + \Xi), \tag{1.1}$$

and its refined version [12]

$$3\Lambda + \Sigma^+ + \Sigma^- - \Sigma^0 = p + n + \Xi^0 + \Xi^-, \tag{1.2}$$

are obtained for arbitrary N_c. On the other hand, however, the Coleman-Glashow mass relation

$$\Sigma^+ - \Sigma^- = p - n + \Xi^0 - \Xi^- \tag{1.3}$$

can be derived only for $N_c = 3$. In §5 we derive the model independent structures of baryon vertices in the $1/N_c$ expansion and we obtain the $I = J$ rule for the isospin-nonflip spin-nonflip and the isospin-flip spin-flip baryon vertices and a systematic N_c dependence for the isospin-nonflip spin-flip and the isospin-flip spin-nonflip vertices. We examine and discuss the obtained results comparing to experimental data. Section 6 is devoted to the summary. In Appendix A the matrix elements of the U-spin and V-spin in the $SU(3)$ symmetry are calculated for the spin 1/2 baryon states with arbitrary N_c.

2 The Large N_c Baryon States

In order to study the properties of baryons which are given by QCD with the $SU(N_c)$ symmetry and arbitrary N_c, we need to extend the baryon states at $N_c = 3$ to those at arbitrary number of N_c. The baryon states are totally antisymmetric color singlet states of the $SU(N_c)$.

In the three flavor case there are ambiguities in extending the octet baryons to "octet" baryons in large N_c, since we have to introduce some unphysical members of the $SU(N_f)$ multiplet. In order to avoid introduction of unphysical members with unwelcome large N_c behavior and to obtain correct large N_c behavior of various baryon matrix elements, it is necessary to make the following special choice of extension for the large N_c baryon.

Because of total symmetry in the spin-flavor states, the ground state spin 1/2 baryons for arbitrary N_c belong to $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation of flavor $SU(3)$ symmetry where k is given by $N_c = 2k+1$ ($k = 0, 1, 2, \cdots$). This representation is specified by the Young tableau with the first row of length $k+1$ and the second row of length k and the root diagram shown in Fig. 1 and Fig. 2, respectively.

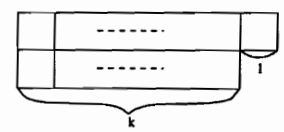

Fig. 1. The Young tableau of the $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation for spin 1/2 baryons.

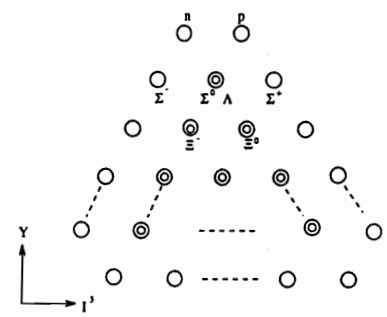

Fig. 2. The root diagram for the $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation of baryon states. The "octet" baryons are located in the upper part of the root diagram.

The usual octet baryons are located at the top region of this root diagram. The flavor wave functions are represented by the tensors $B^{\beta}_{\alpha_1,\alpha_2,\cdots,\alpha_k}$ with one superscript and k symmetric subscripts and their non-zero components for the usual octet baryons are given by [10]

$$p\ :\ B^1_{3,\cdots,3}=1 \quad n\ :\ B^2_{3,\cdots,3}=1 \tag{2.1}$$

$$\Sigma^-\ :\ B^1_{23,\cdots,3}=\frac{1}{\sqrt{k}} \quad \Sigma^-\ :\ B^2_{13,\cdots,3}=-\frac{1}{\sqrt{k}} \tag{2.2}$$

$$\Sigma^0\ :\ B^1_{13,\cdots,3}=-B^2_{23,\cdots,3}=-\frac{1}{\sqrt{2k}} \tag{2.3}$$

$$\Lambda\ :\ B^1_{13,\cdots,3}=B^2_{23,\cdots,3}=\frac{1}{\sqrt{4+2k}} \quad B^3_{3,\cdots,3}=-\frac{2}{\sqrt{4+2k}} \tag{2.4}$$

$$\Xi^0\ :\ B^3_{23,\cdots,3}=-\frac{3}{2}B^2_{223,\cdots,3}=-3B^1_{123,\cdots,3}=\sqrt{\frac{3}{k(k+2)}} \tag{2.5}$$

$$\Xi^-\ :\ B^3_{13,\cdots,3}=-\frac{3}{2}B^1_{113,\cdots,3}=-3B^1_{123,\cdots,3}=\sqrt{\frac{3}{k(k+2)}} \tag{2.6}$$

For this definition of baryon states, the hypercharge extended to arbitrary N_c is given by

$$Y=\frac{N_cB}{3}+S, \tag{2.7}$$

where B is the baryon number and S is the strangeness which reduces to $Y=B+S$ in the physical case with $N_c=3$.

From now on we consider the $SU(3)$ symmetry breaking effects which are caused by the quark mass differences. We discuss the cases that the $SU(3)$ breaking effects are relevant to the two quark baryon vertices which belong to the octet representation of flavor $SU(3)$ and are expressed as $< B_f|O^{ai}|B_i >$ and $< B_f|\lambda^a\sigma^i|B_i >$ where $a=1,\ 2,\cdots,\ 8$ and $i=0,\ 1,\ 2,\ 3$ and σ^0 is the unit matrix of spin space representing spin-nonflip. These matrix elements correspond to the various physical quantities of baryons such as the mass, the magnetic moment, the axial coupling constant etc..

The most general flavor octet vertex constructed from spin 1/2 baryon is represented by use of the above baryon states as a sum of two independent terms:

$$< B_f \mid O^a \mid B_i >= F\,\mathrm{tr}(\lambda^a[\bar{B}_f,B_i])+D\,\mathrm{tr}(\lambda^a\{\bar{B}_f,B_i\}), \tag{2.8}$$

where λ^a is a flavor octet matrix with $a=1,...,8$, and spin-flavor indices of B_i and $\bar{B}_f$ are dropped. This F/D ratio will be denoted by F_+/D_+. Similarly for the spin-flip vertex

$$< B_f \mid O^{ai} \mid B_i >= F\,\mathrm{tr}(\lambda^a[\bar{B}_f,B_i]\sigma^i)+D\,\mathrm{tr}(\lambda^a\{\bar{B}_f,B_i\}\sigma^i), \tag{2.9}$$

and the F/D ratio is denoted by F_-/D_-. The F/D ratios are useful in studying the large N_c-dependence of QCD independently of details of the specific effective model.

The operators O^{a0} and O^{ai} with $i=1,\ 2,\ 3$ transform as $(\mathbf{8},\mathbf{1})$ and $(\mathbf{8},\mathbf{3})$, respectively under $(SU(3)_f,SU(2)_s)$ transformations.

3 The F/D Ratios for NRQM and CSM

We consider first the baryon vertices in the NRQM which has the $SU(6)$ symmetry. The baryon states with the spin 1/2 are given by the completely symmetric representations with respect to spin-flavor. Then they belong to ${}_6\mathbf{H}_{\mathbf{N_c}}$ dimensional representation of the $SU(6)$, where ${}_nH_r$ is a repeated combination ${}_nH_r={}_{n-r-1}C_r=(n+r-1)!/r!(n-1)!$.

In the NRQM the isoscalar part of spin-nonflip baryon vertex transforms as the 8-th component of the octet of flavor $SU(3)$ symmetry and also belongs to the 35-plet of spin-flavor $SU(6)$ symmetry we have to calculate the F/D ratio of spin-nonflip baryon vertices,

$$(m_B^{(8)})_{NRQM}=\begin{pmatrix}\mathbf{35} & {}_6\mathbf{H}_{\mathbf{N_c}} & {}_6\mathbf{H}_{\mathbf{N_c}}{}^*\\ \sigma^0\lambda^8 & B & \bar{B}\end{pmatrix}m^{(8)}_{NRQM}, \tag{3.1}$$

where $m^{(8)}_{NRQM}$ is a parameter of octet-isoscalar part of spin-nonflip baryon vertex and $\sigma^0\lambda^8$ denotes $J=0$ isoscalar member of $\mathbf{35}$ dimensional representation of $SU(6)$.

From (3.1) we have

$$\left(\frac{m_N^{(8)}}{m_\Sigma^{(8)}}\right)_{NRQM}=\frac{(2k+1)}{2(k-1)}. \tag{3.2}$$

to the ratio of matrix elements for nucleon and Σ particle. In terms of the F/D ratio the ratio of these matrix elements is expressed by

$$\frac{m_N^{(8)}}{m_\Sigma^{(8)}}=\frac{-F_++\frac{1}{3}D_+}{-F_++\frac{1}{3}D_++\frac{1}{k}(F_+-D_+)}, \tag{3.3}$$

where $m^{(8)}$ is a parameter of octet-isoscalar part of spin-nonflip baryon vertex. From (3.2) $\sim$ (3.3) we obtain the F/D ratio of spin-nonflip baryon vertex for the "baryon" with spin 1/2 and arbitrary N_c. The result obtained is

$$\left(\frac{F_+}{D_+}\right)_{NRQM}=-\frac{N_c+1}{N_c-3}. \tag{3.4}$$

The same result can also be derived by the algebraic method. The obtained F_-/D_- ratio of spin-nonflip baryon vertex in the NRQM tends to -1 in the limit of $N_c \to \infty$. For $N_c = 1$ the F_-/D_- ratio becomes 1 reflecting the fact that the baryons are quarks themselves. For $N_c = 3$ the ratio becomes ∞ reflecting the fact that the quark number is conserved implying the pure F-type amplitude [14].

A baryon mass m_B is considered to be given by the spin-nonflip baryon vertices and expressed as

$$m_B = m_B^{(0)} + m_B^{(8)} + m_B^{(3)}, \tag{3.5}$$

where $m_B^{(0)}$ is the flavor independent term and corresponds to the zero-quark vertex and $m_B^{(8)}$ and $m_B^{(3)}$ are attributed respectively to the isoscalar and isovector parts of flavor-octet vertices composed by a quark and an anti-quark. The matrix elements of flavor-octet baryon vertices $(m_N^{(8)})_{NRQM}$ and $(m_\Sigma^{(8)})_{NRQM}$ appeared in (3.2) correspond to the octet part of averaged mass of the isospin-multiplet of nucleon and Σ particle, respectively.

Next we turn to the F_-/D_- ratio of the spin-flip baryon vertex for arbitrary N_c [5, 11]. Similarly, in the NRQM the matrix element of the spin-flip baryon vertex is given by

$$(\mu_B)_{NRQM} = \begin{pmatrix} \mathbf{35} & {}_6\mathbf{H}_{\mathbf{N_c}} & {}_6\mathbf{H}_{\mathbf{N_c}}{}^* \\ \sigma^3 Q & B & \bar{B} \end{pmatrix} \mu_{NRQM}, \tag{3.6}$$

where $\sigma^3 Q$ with $Q = \lambda^3/2 + \lambda^8/2\sqrt{3}$ denotes the spin-flip($J = 1$, $J^3 = 0$) and charge component of **35** dimensional representation and μ_{NRQM} is a parameter of spin-flip charge vertex corresponding to the magnetic moment in NRQM. The isoscalar part and isovector part of the spin-flip vertex belong to the same supermultiplet **35** of $SU(6)$ in the NRQM. The matrix element (3.6) corresponds to the magnetic moment of baryon.

From (3.6) the ratio of the matrix elements for proton and neutron is given by

$$\left(\frac{\mu_p}{\mu_n}\right)_{NRQM} = -\frac{k+2}{k+1}. \tag{3.7}$$

In terms of the F/D ratio this ratio is expressed by

$$\frac{\mu_p}{\mu_n} = -\frac{F + \frac{1}{3}D}{\frac{2}{3}D}. \tag{3.8}$$

Comparing (3.7) with (3.8) we obtain the F/D ratio of spin-flip baryon vertex for the "baryon" with spin 1/2 and arbitrary N_c. The obtained result is

$$\left(\frac{F_-}{D_-}\right)_{NRQM} = \frac{N_c + 5}{3(N_c + 1)}. \tag{3.9}$$

The F_-/D_- ratio of spin-flip baryon vertex in the NRQM tends to 1/3 in the limit of $N_c \to \infty$. For $N_c = 1$ the F/D ratio becomes 1 reflecting the fact that the baryons are quarks themselves. For physical baryon with $N_c = 3$ the F/D ratio takes the familiar value 2/3 of the NRQM [14].

Here we point out that there is a relation between two F/D ratios of spin-nonflip and spin-flip baryon vertices in the NRQM;

$$(4f_+ + 1)(4f_- - 1) = 3, \tag{3.10}$$

where $f_\pm = F_\pm/(F_\pm + D_\pm)$. This relation holds even when the $SU(6)$ symmetry is broken and the mixing among the baryon states of super-multiplets **56**, **70** and **20** are introduced [15]. This relation is also satisfied by (3.4) and (3.9) independent of N_c.

Next we consider the CSM which has the $SU(3)$ symmetry. In the CSM case the spin 1/2 baryon states are represented by the $SU(3)$ matrix elements of the $(\mathbf{1}, \mathbf{k})$ representation or $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation of $SU(3)$ where $k = (N_c - 1)/2$. $(\mathbf{1}, \mathbf{k})$ denotes the representation of $SU(3)$ with the Young tableau which has the first row of length $k+1$ and the second row of length k. In the case $N_c = 3$ this is the octet or the regular representation of $SU(3)$. In this case there are two different types of the Clebsch-Gordan coefficients, namely F type and D type. This corresponds to a generalization of the well-known relation $\mathbf{8} \times \mathbf{8} = \mathbf{1} + \mathbf{8_F} + \mathbf{8_D} + \mathbf{10} + \mathbf{10^*} + \mathbf{27}$.

The matrix element of octet-isoscalar part of the spin-nonflip baryon vertex in the CSM is given by

$$(m_B^{(8)})_{CSM} = \sum_n \begin{pmatrix} \mathbf{8} & (\mathbf{1},\mathbf{k}) & (\mathbf{1},\mathbf{k})_n^* \\ \lambda^8 & B & \bar{B} \end{pmatrix} \begin{pmatrix} \mathbf{8} & (\mathbf{1},\mathbf{k}) & (\mathbf{1},\mathbf{k})_n^* \\ \lambda^8 & B & \bar{B} \end{pmatrix} m_{CSM}^{(8)} + \cdots, \tag{3.11}$$

where the summation over n means two orthogonal states of baryons of $(\mathbf{1}, \mathbf{k})^*$ representation and $m_{CSM}^{(8)}$ is a parameter. λ^8 in the first parenthesis denotes the isoscalar member of octet and the one in the second does $J = 0$ representing spin-nonflip. The ellipsis in (3.11) denotes corrections from the time derivative of the $SU(3)$ matrix valued dynamical variable describing the "rotations" in spin-flavor space which contain the higher order term of $1/N_c$ expansion.

From (3.11) the ratio of the matrix elements for the nucleon and the Σ particle is

$$\left(\frac{m_N^{(8)}}{m_\Sigma^{(8)}}\right)_{CSM} = \frac{2k^2+6k+1}{2k^2+3k-8} + \cdots. \tag{3.12}$$

From this result and (3.3) we obtain the F_+/D_+ ratio of spin-nonflip baryon vertex in the CSM

$$\left(\frac{F_+}{D_+}\right)_{CSM} = -\frac{N_c^2+4N_c-1}{N_c^2+4N_c-9} + \cdots. \tag{3.13}$$

This F_+/D_+ ratio tends to -1 in the limit $N_c \to \infty$ and is 1 for $N_c = 1$ in the same way as in the NRQM. For the physical case with $N_c = 3$ it is $-5/3$ [16].

The experimental value of the F_+/D_+ ratio is $(F_+/D_+)_{\rm exp} = -3.0 \pm 0.2$ and lies between those of NRQM and CSM. In Fig. 3 we display inverse ratios D_+/F_+ to avoid singularity due to the crossing zero of D_+.

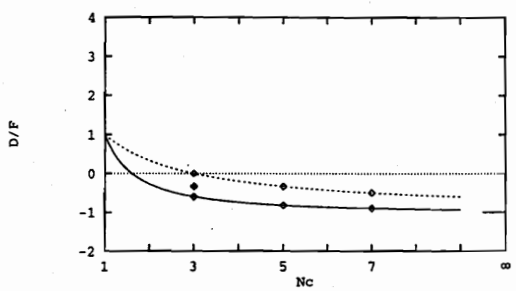

Fig. 3. N_c-dependence of D_+/F_+ ratios of spin-nonflip baryon vertex in the NRQM and CSM. Here D/F ratios are shown instead of F/D in order to avoid singular behaviors of the F/D which appear at $N_c = 3$ and $-2+\sqrt{13}$ in the NRQM and CSM, respectively. The large N_c limiting value -1 and the experimental value -0.33 ± 0.02 which lies also between the curves of NRQM and CSM are shown.

We note here that the $1/N_c$ correction to F/D ratio has two different origins. One is the $1/N_c$ correction from the baryon wave function which are represented by the Clebcsh-Gordan coefficients in expression (3.11). The other comes from the $1/N_c$ correction due to the time derivative of dynamical variables describing spin-isospin rotations.

Next we turn to the F_-/D_- ratio of spin-flip vertex in the chiral soliton model. This vertex corresponds to the magnetic moment of baryon B and is given by [5, 11]

$$(\mu_B)_{CSM} = \sum_n \begin{pmatrix} \mathbf{8} & (\mathbf{1},\mathbf{k}) & (\mathbf{1},\mathbf{k})^*_n \\ Q & B & \bar{B} \end{pmatrix} \begin{pmatrix} \mathbf{8} & (\mathbf{1},\mathbf{k}) & (\mathbf{1},\mathbf{k})^*_n \\ \sigma^3 & B & \bar{B} \end{pmatrix} \mu_{CSM} + \cdots \tag{3.14}$$

where the summation over n means two orthogonal states of baryons of $(1,k)$ and the ellipsis denote corrections from the time derivative and μ_{CSM} is a parameter corresponding to the magnetic moment.

From (3.14) the ratio of matrix elements for proton and neutron is

$$\left(\frac{\mu_p}{\mu_n}\right)_{CSM} = \frac{(k+2)(k+3)}{k^2+5k+3} + \cdots. \tag{3.15}$$

From this result we obtain the F/D ratio of spin-flip baryon vertex in the $SU(3)$ CSM as

$$\left(\frac{F_-}{D_-}\right)_{CSM} = \frac{N_c^2+8N_c+27}{3(N_c^2+8N_c+3)} + \cdots. \tag{3.16}$$

The F_-/D_- ratio becomes 5/9 [14] for $N_c = 3$. This F_-/D_- ratio becomes 1/3 [17] in the large N_c limit and 1 for $N_c = 1$. Thus the F_-/D_- ratios of spin-flip baryon vertex in the NRQM and in the CSM coincide at $N_c = 1$ and $N_c \to \infty$ tending to 1 and 1/3 as in the case of the spin-nonflip baryon vertex. The experimental value of the F_-/D_- ratio is $(F_-/D_-)_{\rm exp} = 0.58 \pm 0.04$ and lies between the two lines nearer to that of $SU(3)$ chiral soliton model. These are displayed in Fig. 4.

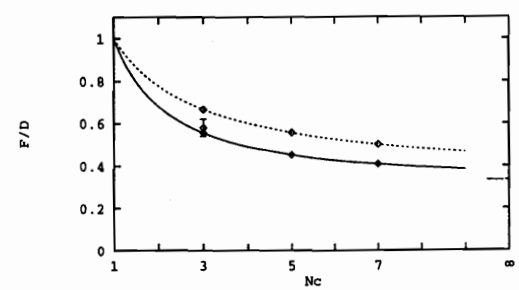

Fig. 4. N_c-dependence of F_-/D_- ratios of spin-flip baryon vertex in the NRQM and CSM. The large N_c limiting value 1/3 and the experimental value 0.58 $\pm$0.04 which lies between

the curves of NRQM and CSM.

Here we note that there are additional contributions of order $1/N_c$ to the F/D ratios in the CSM which are expressed by the ellipsis in (3.16) and come from the soliton rotation in spin-isospin space. In the CSM the relation between F/D ratios given by (3.10) is not satisfied.

Here we compare the F_+/D_+ ratios in the NRQM with that in the CSM on the basis of $1/N_c$ expansion. The $1/N_c$ expansion of F_+/D_+ ratio in NRQM is

$$\left(\frac{F_-}{D_-}\right)_{NRQM} = -\frac{N_c+1}{N_c-3} = -1 - \frac{4}{N_c} + \frac{12}{N_c^2} + \cdots, \tag{3.17}$$

$$\tag{3.18}$$

The expansion of F_+/D_+ in the CSM is

$$\left(\frac{F_+}{D_+}\right)_{CSM} = -\frac{N_c^2+4N_c-1}{N_c^2+4N_c-9} = -1 + \frac{0}{N_c} - \frac{8}{N_c^2} + \cdots. \tag{3.19}$$

The $1/Nc$ expansion of F_-/D_- ratios in NRQM is given by

$$\left(\frac{F_-}{D_-}\right)_{NRQM} = \frac{N_c+5}{3(N_c+1)} = \frac{1}{3} + \frac{4}{3N_c} - \frac{4}{3N_c^2} + \cdots, \tag{3.20}$$

$$\tag{3.21}$$

The expansion of F_-/D_- in CSM is

$$\left(\frac{F_-}{D_-}\right)_{CSM} = \frac{N_c^2+8N_c+27}{3(N_c^2+8N_c+3)} = \frac{1}{3} + \frac{0}{N_c} + \frac{8}{N_c^2} + \cdots. \tag{3.22}$$

In the CSM the terms of order $1/N_c$ are absent both in the expansions of F_-/D_- and F_-/D_-. This absence implies rapid conversion of $1/N_c$ expansion in CSM.

4 The F/D Structure of Baryon Vertices

In this section we consider physical meanings of the limiting values -1 and $1/3$ of F/D ratios in the limit $N_c \to \infty$ in more detail studying the F/D structure of baryon vertices.

From now on we assume that isospin non-flip and isospin flip parts correspond to the operators O^8 and O^3, respectively which transform as two diagonal generators λ^8 and λ^3 of the $SU(3)$ symmetry, since we want to apply Wigner-Eckart Theorem [18] to the matrix elements of octet operators.

First we consider the isospin non-flip part of the baryon vertices which is related to the mass splitting among the baryons due to the mass difference among strange quark and non-strange quarks. The matrix elements of the diagonal operator O^8 for the baryon states given by (2.1) $\sim$ (2.6) are expressed in terms of F and D as

$$< N \mid O^8 \mid N >= -F + \frac{1}{3}D, \tag{4.1}$$

$$< \Sigma \mid O^8 \mid \Sigma >= -F + \frac{1}{3}D + \frac{1}{k}(F-D), \tag{4.2}$$

$$< \Lambda \mid O^8 \mid \Lambda >= -F + \frac{1}{3}D + \frac{1}{k}(F-D) + \frac{2}{k+2}\left(F + D - \frac{1}{k}(F-D)\right), \tag{4.3}$$

$$< \Xi \mid O^8 \mid \Xi >= -F + \frac{1}{3}D + \frac{2}{k}(F-D) + \frac{3}{k+2}\left(F + D - \frac{1}{k}(F-D)\right). \tag{4.4}$$

Using the Wigner-Eckart theorem and the baryon wave function which we have defined in the previous section, the matrix element of the diagonal operator O^8 can generally be expressed as follows.

$$\begin{aligned} &< B_f \mid O^8 \mid B_i > \\ &= \alpha\, Y + \beta\left(I(I+1) - \frac{Y^2}{4} - \frac{\mathbf{F}^a\mathbf{F}^a}{3}\right), \end{aligned} \tag{4.5}$$

where $\mathbf{F}^a$ is the $SU(3)$ generator and $\mathbf{F}^a\mathbf{F}^a$ is the Casimir operator of $SU(3)$ and

$$\mathbf{F}^a\mathbf{F}^a = \frac{(k+2)^2}{3}, \tag{4.6}$$

for $(\mathbf{1}, \mathbf{k})$ representation of $SU(3)$.

Here we note that we *have to* define the hypercharge as $Y = N_cB/3 + S$ which is consistent with the extended baryon states expressed by (2.1) $\sim$ (2.6). The strangeness S and the baryon number B are the quantities of order $O(1)$, thus the extended hypercharge Y is of order $O(N_c)$ so that the center of the root diagram for baryon multiplet in the $(\mathbf{1}, \mathbf{k})$ representation is located at the origin of the (I^3, Y) plane. If we define the hypercharge as $Y = B + S$, we cannot obtain the result that is consistent with the Wigner-Eckart Theorem.

Using the definition $Y = N_cB/3 + S$, we obtain the coefficients α and β in (4.5) as

$$\alpha = -\frac{1}{6k(k+2)}\{(k+5)(2k+1)F + (k-1)(2k+5)D\} \tag{4.7}$$

$$\beta = -\frac{1}{k+2}\left(F + D - \frac{1}{k}(F-D)\right) \tag{4.8}$$

The general expression satisfying (4.1) $\sim$ (4.4) is

$$
\begin{aligned}
&< B_f \mid O^8 \mid B_i > \\
&= -F + \frac{1}{3}D + \frac{2}{k}(F-D)K \\
&-\frac{1}{k+2}\left(F + D - \frac{1}{k}(F-D)\right)\left(I(I+1) - (K+\frac{1}{2})(K+\frac{3}{2})\right),
\end{aligned} \tag{4.9}
$$

where $K = -S/2$, S being the strangeness. This formula is nothing but the Okubo formula[13] for the $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation of $SU(3)$ symmetry. From (4.1) $\sim$ (4.4) and (4.9) we find the following structure among the matrix elements of flavor $SU(3)$ generator λ^8. The first part of (4.9) gives the same contribution of $-F + D/3$ to all states. The second part proportional to $F - D$ which is of the order $O(1/N_c)$ gives the increasing contributions with K. The third part which includes both $O(1/N_c)$ term proportional to $F + D$ and $O(1/N_c^2)$ term proportional to $F - D$ appears only for the states located on the inner triangle of the root diagram because of the factor $I(I+1) - (K+1/2)(K+3/2)$.

From these matrix elements of O^8 we obtain the Okubo-Gell-Mann mass relation (1.1) among the "octet baryons" for arbitrary N_c model independently. This is because the matrix elements of O^8 for the "octet baryon" states satisfy the Okubo formula, though the multiplet belonging to spin 1/2 baryon is no longer an octet representation of $SU(3)$ except for $N_c = 3$.

The generator $\mathbf{F^a}$ of $SU(3)$ flavor symmetry is represented in terms of $(1+k)(3+k) \times (1+k)(3+k)$ matrix, in the $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation of flavor multiplet of spin 1/2 baryon. The general expressions of these matrix elements contain those of the Casimir operator.

Next we consider isospin-flip part of the baryon vertices that is related to isospin symmetry breaking caused by the mass difference between up and down quark as well as by the isovector part of the electromagnetic correction. In terms of F and D the diagonal matrix elements of operator O^3 for the baryon states given in (2.1) $\sim$ (2.6) are expressed as

$$< p \mid O^3 \mid p >= F + D, \tag{4.10}$$

$$< n \mid O^3 \mid n >= -(F + D), \tag{4.11}$$

$$< \Sigma^- \mid O^3 \mid \Sigma^+ >= F + D + \frac{1}{k}(F - D), \tag{4.12}$$

$$< \Sigma^0 \mid O^3 \mid \Sigma^0 >= 0, \tag{4.13}$$

$$< \Sigma^- \mid O^3 \mid \Sigma^- >= -(F + D) - \frac{1}{k}(F - D), \tag{4.14}$$

$$< \Lambda \mid O^3 \mid \Lambda >= 0, \tag{4.15}$$

$$
\begin{aligned}
&< \Xi^0 \mid O^3 \mid \Xi^0 > \\
&= -\frac{1}{3}(F+D) - \frac{2}{3k}(F-D) - \frac{3}{k+2}\left(-F + \frac{1}{3}D - \frac{1}{k}(F-D)\right),
\end{aligned} \tag{4.16}
$$

$$
\begin{aligned}
&< \Xi^- \mid O^3 \mid \Xi^- > \\
&= \frac{1}{3}(F+D) + \frac{2}{3k}(F-D) + \frac{3}{k+2}\left(-F + \frac{1}{3}D - \frac{1}{k}(F-D)\right).
\end{aligned} \tag{4.17}
$$

Here we also find a systematic structure in the matrix elements of generator λ^3 as noted in the case of the matrix elements of generator λ^8 .

From the Wigner-Eckart theorem, the matrix elements of operator O^3 for the octet baryons are expressed as

$$
\begin{aligned}
&< B_f \mid O^3 \mid B_i > \\
&= \gamma\, I_3 + \delta\left(\frac{1}{2}\{I_3, Y\} + \frac{1}{4} < B_f \mid (\{V_+, V_-\} - \{U_+, U_-\}) \mid B_i >\right),
\end{aligned} \tag{4.18}
$$

where $U_\pm$ and $V_\pm$ are the raising and lowering operators of U-spin and V-spin in the $SU(3)$ symmetry. Using the definition $Y = N_c B/3 + S$, we obtain the coefficients γ and δ of (4.18) as

$$\gamma = \frac{1}{3k(k+2)}\left((2k^2 + 11k + 5)F + (k-1)(2k+5)D\right) \tag{4.19}$$

$$\delta = \frac{2}{k+2}\left(F + D - \frac{1}{k}(F-D)\right). \tag{4.20}$$

We need to rewrite this formula in order to make the N_c counting rule apparent, since both Y and $\{V_+, V_-\} - \{U_+, U_-\}$ are $O(N_c)$. Then we have to calculate $U^\pm \mid B_i >$ and $V^\pm \mid B_i >$, which depend on the representation of baryon. For the $(k+1)(k+3)$ dimensional representation the states having the same hypercharge consist at most of two isospin multiplets and we denote these states as A and B, assuming that the state A has larger isospin than the state B.

After the tedious calculation and by use of (.10) in Appendix A, we can rewrite the diagonal matrix elements of isovector formula (4.18) as

$$
\begin{aligned}
&_A < I, I_3, K \mid O^3 \mid I, I_3, K >_A \\
&= {}_B < I-1, I_3, K-1 \mid O^3 \mid I-1, I_3, K-1 >_B \\
&= \Big[F + D + \frac{2}{k}(F-D)K \\
&-\frac{3}{k+2}\left(-F + \frac{1}{3}D - \frac{1}{k}(F-D)\right)\left(I(I+1) - (K+\frac{1}{2})(K+\frac{3}{2})\right)\Big] W^3,
\end{aligned} \tag{4.21}
$$

where W^3 does not depend on N_c and is given by the Wigner 6-j symbol and the Clebsch-Gordan coefficient representing isospin conservation

$$W^3 = (-1)^{3/2-I-K}\sqrt{2\dim I}\left\{\begin{array}{ccc} 1 & I & I \\ K & \frac{1}{2} & \frac{1}{2}\end{array}\right\}\left(\begin{array}{cc|c} I & 1 & I \\ I^3 & 0 & I^3\end{array}\right). \tag{4.22}$$

The N_c dependence of the isospin and strangeness in this isovector formula (4.21) have the same structure as the isoscalar formula (4.9). The common contribution term $-F + D/3$ in (4.9) is replaced by $F + D$ in (4.21), while in the coefficient of $I(I+1) - (K+1/2)(K+3/2)$ the term $F + D$ in (4.9) is replaced by $-F + 1/3D$ in (4.21). The second terms proportional to K are the same in both (4.21) and (4.5). These are the essential properties that decide the isospin and strangeness dependence of the baryon vertices.

By use of (.11) in Appendix A the off-diagonal matrix elements of O^3 are given as

$$\begin{aligned} &_B < I-1, I_3, K \mid O^3 \mid I, I_3, K >_A = {}_A < I, I_3, K \mid O^3 \mid I-1, I_3, K >_B \\ &= (-1)^{5/2-I-K}\sqrt{2\dim I}\left\{\begin{array}{ccc} 1 & I & I-1 \\ K & \frac{1}{2} & \frac{1}{2}\end{array}\right\}\left(\begin{array}{cc|c} I & 1 & I-1 \\ I_3 & 0 & I_3\end{array}\right) \\ &\times\sqrt{\frac{k-2K+1}{k+2}}\left(F + D - \frac{1}{k}(F-D)\right). \end{aligned} \tag{4.23}$$

This off-diagonal matrix element of O^3 has the same F/D dependence as the third term of matrix element of O^8 in (4.9).

The isovector formula (4.21) has similar structure as the Okubo formula[13] for those of operator O^8. Then by use of these matrix elements and those of O^8 given in (4.1) $\sim$ (4.4) we obtain again independent of the F/D ratio or model independently the refined Okubo-Gell-Mann relation [12]

$$3\Lambda + \Sigma^+ + \Sigma^- - \Sigma^0 = p + n + \Xi^0 + \Xi^-, \tag{4.24}$$

for arbitrary N_c.

On the other hand we can derive from the matrix elements of operator O^3 given in (4.10) $\sim$ (4.17) the Coleman-Glashow mass relation

$$\Sigma^+ - \Sigma^- = p - n + \Xi^0 - \Xi^- \tag{4.25}$$

that holds only for $N_c = 3$.

5 The Model Independent Structures of Baryon Vertices in QCD

Now we extract model independent characteristics of baryon vertices in QCD from the results in the NRQM and CSM obtained in the previous sections.

First we apply the $1/N_c$ expansions of the F_+/D_+ ratio (3.17) or (3.19) to the spin-nonflip baryon vertices (4.9) and (4.21). In the term $F + D$ of (4.9) the leading order of F cancels that of D then $F + D$ becomes $O(1)$ while in the term $F - D/3$ the leading order of F does not cancel that of D and $F - D/3$ remains in the order $O(N_c)$.

With this F_-/D_- ratio, the isoscalar formula (4.9) for the spin-nonflip baryon vertex with spin 1/2 becomes

$$\begin{aligned} &< I, I_3, 1/2, J_3, K \mid O^8 \mid I, I_3, 1/2, J_3, K > \\ &= N_c a + bK + \frac{c}{N_c}\left(I(I+1) - (K+\frac{1}{2})(K+\frac{3}{2})\right), \end{aligned} \tag{5.1}$$

where a, b and c are constants of order $O(1)$ but contain higher orders of $1/N_c$ expansion in general and do not depend on the isospin and strangeness.

Similarly the $1/N_c$ expansions of the F_+/D_+ ratio (3.17) or (3.19) is applied to the isovector formula (4.21) and we obtain the isospin-flip and spin-nonflip baryon vertex given by

$$\begin{aligned} &< I, I_3, 1/2, J_3, K \mid O^3 \mid I, I_3, 1/2, J_3, K > \\ &= \left[d + eK + f\left(I(I+1) - (K+\frac{1}{2})(K+\frac{3}{2})\right)\right] W^3, \end{aligned} \tag{5.2}$$

where d, e and f are constants of order $O(1)$ which do not depend on the isospin and strangeness. This result prevents us to generalize the Coleman-Glashow mass formula to arbitrary N_c other than $N_c = 3$.

Next we apply the $1/N_c$ expansions of F_-/D_- ratio, (3.20) or (3.22) to the spin-flip vertices which are given also by (4.9) and (4.18) where O^8 and O^3 are replaced by O^{8i} and O^{3i}, respectively. Then the terms $F + D$ and $F - 1/3D$ in (4.9) and (4.21) become $O(N_c)$ and $O(1)$, respectively. Therefore the isoscalar formula (4.9) for the spin-flip baryon vertex can be expressed as

$$\begin{aligned} &< I, I_3, 1/2, J_3, K \mid O^{8i} \mid I, I_3, 1/2, J_3, K > \\ &= \left[a' + b'K + c'\left(I(I+1) - (K+\frac{1}{2})(K+\frac{3}{2})\right)\right] V^i, \end{aligned} \tag{5.3}$$

where a', b' and c' are constants of $O(1)$ which do not depend on the isospin and strangeness and V^i is the Clebsch-Gordan coefficient representing the spin conservation

$$V^i = \left(\begin{array}{cc|c} 1/2 & 1 & 1/2 \\ J_3 & i & J_3 \end{array} \right). \tag{5.4}$$

We can use this result for the isoscalar part of magnetic moment or η-meson baryon coupling.

Similarly the isovector formula (4.21) for the spin-flip baryon vertex is expressed as

$$\begin{aligned} &< I, I_3, 1/2, J_3, K \mid O^{3i} \mid I, I_3, 1/2, J_3, K > \\ &= \left[N_c d' + e' K + \frac{f'}{N_c} \left(I(I+1) - (K + \frac{1}{2})(K + \frac{3}{2}) \right) \right] W^3 V^i, \end{aligned} \tag{5.5}$$

where d', e' and f' are constants of O(1) which do not depend on the isospin and strangeness. We can apply this result to the isovector part of magnetic moment or π-meson baryon coupling.

It will be reasonable to assume that the model-independent results (5.1) $\sim$ (5.5) obtained both from the $1/N_c$ expansions in the NRQM and CSM are those of QCD. Comparing these results on the $1/N_c$ expansions of baryon vertices O^8, O^3, O^{8i} and O^{3i} we find the following systematic N_c dependence among these vertices.

(I) Both matrix elements of O^8 and O^{3i}, (5.1) and (5.5), have the same N_c dependent structure: The leading terms of order $O(N_c)$ are the ones independent of isospin and of strangeness. The strangeness dependent terms are of order $O(1)$. The isospin dependent terms which contribute only to the states B (smaller isospin states) are of order $O(1/N_c)$.

(II) Both matrix elements of O^{8i} and O^3, (5.2) and (5.3), have the same N_c dependent structure: All three terms, independent of isospin and strangeness, strangeness-dependent and isospin-dependent, are of order $O(1)$.

From these systematic N_c dependence of baryon vertices we obtain the $I = J$ rule [9] which states that the isospin-nonflip($I = 0$) spin-nonflip($J = 0$) baryon vertex and the isospin-flip($I = 1$) spin-flip($J = 1$) baryon vertex are dominating. This is a consequence that the order $O(N_c)$ terms are contained only in the matrix elements of O^8 and O^{3i}.

The $I = J$ rule is seen in the low energy hadron phenomena, such as the magnetic moments and masses of baryons. From (I) we can expect that the isovector part of baryon mass is suppressed by a $1/N_c$ factor in comparison to the isoscalar part of baryon mass, while the isoscalar part of the baryon magnetic moment is suppressed by a $1/N_c$

	$I=0$	$I=1$
m_B	$O(N_c)$	$O(1)$
μ_B	$O(1)$	$O(N_c)$

Table 1: The $I = J$ rule for the magnetic moments and masses of baryons

	Vector	Tensor
$g_{\rho NN}$	$O(1/\sqrt{N_c})$	$O(\sqrt{N_c})$
$g_{\omega NN}$	$O(\sqrt{N_c})$	$O(1/\sqrt{N_c})$

Table 2: The $I = J$ rule for $g_{\rho NN}$ and $g_{\omega NN}$ coupling constant

factor compared to the isovector part of the magnetic moment. This expectation is shown in Table 1.

Another example of $I = J$ rule is the vector meson coupling to nucleon as displayed in Table 2. The tensor coupling of of the ρ meson, $g'_{\rho NN}$, which is spin-flip vertex in the non-relativistic limit, is larger than the vector coupling of the ρ meson, $g_{\rho NN}$, which is the spin-nonflip vertex in the non-relativistic limit. In the case of the ω meson the vector coupling, $g_{\omega NN}$, is larger than the tensor coupling, $g'_{\omega NN}$. This is also consistent with the N_c counting rule of $1/N_c$ expansion of baryon-meson coupling [9].

In addition to the $I = J$ rule, the result (I) is concerned to the suppression of isospin and strangeness dependent terms of baryon vertices. The strangeness-dependent term and the isospin-dependent term are suppressed by the $1/N_c$, and $1/N_c^2$ compared to the leading terms in the case of baryon vertices enhanced by the $I = J$ rule.

On the other hand in the case of baryon vertices which are not enhanced by the $I = J$ rule, the isospin and strangeness dependent terms are of the same order in $1/N_c$ expansion with the term independent of isospin and strangeness. Then all three terms of isospin-nonflip spinflip ($I = 0$, $J = 1$) and isospinflip spin-nonflip ($I = 1$, $J = 0$) baryon vertices generally survive even if we take the limit $N_c \to \infty$. These results are summarized in Table 3.

Now we examine whether the obtained results are consistent with experiments. As are seen in Fig. 3 and Fig. 4 both of the experimental ratios of $D_+/F+$ and F_-/D_- are lie between those of NRQM and CSM. Taking this fact into account we substitute the F_+/D_+ ratios and F_-/D_- ratios given by (3.4), (3.13), (3.9) and (3.16) into the

	$\Delta I = 0$			$\Delta I = 1$		
	S, I-indep	S-dep	I-dep	S, I-indep	S-dep	I-dep
$\Delta J = 0$	$O(N_c)$	$O(1)$	$O(1/N_c)$	$O(1)$	$O(1)$	$O(1)$
$\Delta J = 1$	$O(1)$	$O(1)$	$O(1)$	$O(N_c)$	$O(1)$	$O(1/N_c)$

Table 3: The isospin and strangeness dependence of N_c order of baryon vertices

matrix elements of spin-nonflip and spin-flip baryon vertices given by (4.5), (4.18), (4.9) and (4.21), respectively then we obtain the relative values of the coefficients given in (5.1), (5.2), (5.3) and (5.5). We can expect the experimental values of baryon vertex parameters which will also be derived from QCD lie between those of NRQM and CSM.

In Fig's. 5 $\sim$ 8, the N_c dependence of the coefficients are normalized by $b = e$ or $e' = b'$ which are of the order $O(1)$ in $1/N_c$ expansion. For the case $N_c = 3$ the ratio $N_c a : b : c/N_c$ is $-0.5 : 1 : 0$ in NRQM and $-0.375 : 1 : -0.125$ in CSM.

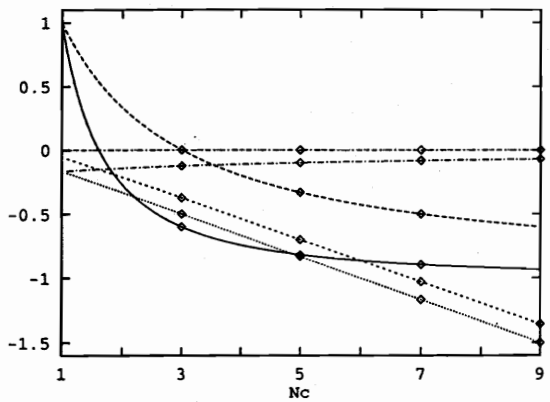

Fig. 5. The N_c dependence of coefficients of spin-nonflip isospin-nonflip baryon vertex in NRCM and CSM are shown in terms of ratios $(aN_c)/b$ and $c/(bN_c)$.

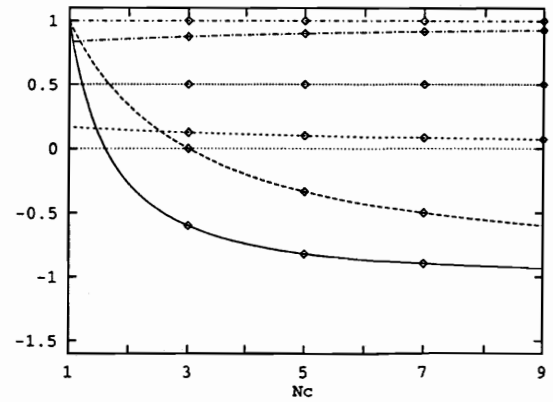

Fig. 6. The N_c dependence of coefficients of spin-nonflip isospin-flip baryon vertex in NRCM and CSM are shown in terms of ratios d/e and f/e.

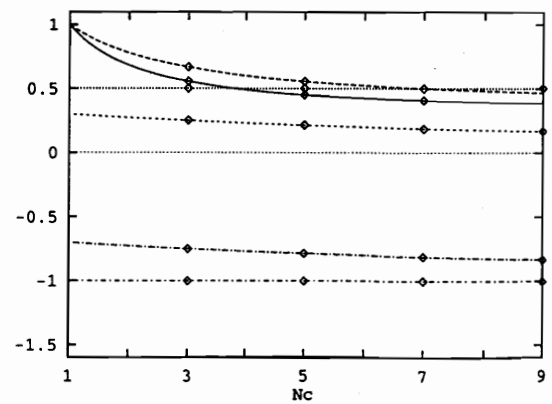

Fig. 7. The N_c dependence of coefficients of spin-flip isospin-nonflip baryon vertex in NRCM and CSM are shown in terms of ratios a'/b' and c'/b'.

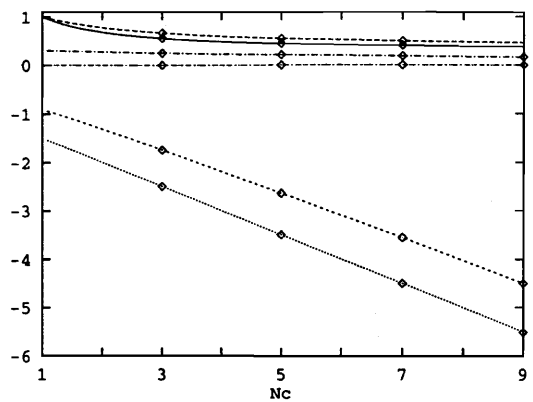

Fig. 8. The N_c dependence of coefficients of spin-flip isospin-flip baryon vertex in NRCM and CSM are shown in terms of ratios $(d'N_c)/e'$ and $f'/(e'N_c)$.

As shown in Fig. 5 the coefficient c/N_c is identically zero in NRQM and very small in CSM compared to b. The coefficient $N_c a$ which is of the leading order is rather small compared to the next leading term b in the region $N_c \leq 5$. Magnitude of $N_c a/b$ increases linearly and magnitude of $N_c a$ exceeds that of b for $N_c \geq 7$ showing that the term $N_c a$ is a quantity of order $O(N_c)$. This means that the contribution to the isoscalar part of spin-nonflip baryon vertex from O^8 is small in the realistic case with $N_c = 3$ though it is a quantity of the order $O(N_c)$.

Here we examine the spin-nonflip vertices (5.1) and (5.2) for the baryon masses. Applying (4.1) $\sim$ (4.4) and (4.10) $\sim$ (4.17) to the experimental masses of octet baryons $m_B = m^{(0)} + m_B^{(8)} + M_B^{(3)}$ we obtain the ratio F_+/D_+. From $m^{(0)} = (m_\Lambda + m_{\Sigma^0})/2$, $D_+ m^{(8)} = 3(m_\Lambda - m_{\Sigma^0})/4$, $F_+ m^{(8)} = (m_{\Xi^0} + m_{\Xi^-} - m_p - m_n)/4$ and the experimental baryon masses we obtain $m^{(0)}$ =1154 MeV, $F_+ m^{(8)} = 189$ MeV and $D_+ m^{(8)} = -58$ MeV. From this results the F/D ratio of the spin-nonflip baryon vertex is $F+/D+ = -3.28$ or $D_+/F_- = -0.30$. If we use relations $m_p + m_n + m_{\Xi^0} + m_{\Xi^-} + m_{\Sigma^0} - m_\Lambda = 4m^{(0)}$, $F_+ m^{(8)} = (m_{\Xi^0} + m_{\Xi^-} - m_p - m_n)/4$ and $m_\Lambda - m^{(0)} = 2D_+ m^{(8)}/3$ we get $m^{(0)} = 1147$MeV, $F_+ m^{(8)} = 189$MeV and $D_+ m^{(8)} = -48$MeV. The F/D ratio of the spin-nonflip baryon vertex becomes $F+/D+ = -3.94$ or $D_+/F_+ = -0.25$. Then the leading contribution to the baryon mass $(-F_+ + D_+/3)m^{(8)}$ which corresponds to $N_c a$

is estimated as -204.5MeV± 0.5MeV. Then the main contribution to the isoscalar part of baryon mass comes from flavor singlet or zero-quark vertex $m^{(0)}$ which is estimated as 1151MeV$\pm$4MeV. This flavor independent mass $m^{(0)}$ corresponds to the classical soliton mass in the CSM which is the quantity of the order $O(N_c)$.

Experimental values of the terms $(F_+ - D_+)m^{(8)}$ and $D_+ m^{(8)}$ which correspond to b and c/N_c are 242MeV $\pm$5MEV and -53MeV$\pm$5MeV, respectively. This result show that the experimental mass values corresponding to the coefficients $N_c a$ are comparable to that of b and that of c/N_c is one forth of those of $N_c a$ and b.

As shown in Fig. 6 the ratios d/e and f/e are almost independent of N_c implying that the coefficients D, e and f are of order $O(1)$. Especially $f = e$ in NRQM for arbitrary value of N_c.

The ratios of coefficients in spin-flip isospin-nonflip vertex, a'/b' and c'/b' are also almost independent of N_c as seen in Fig. 7 and reflecting the fact that they are quantities of the order $O(1)$. Especially $c' = -b'$ in NRQM for arbitrary N_c. The coefficients in the spin-flip isospin-flip vertex, $N_c d'$, e' and f'/N_c are quantities of the order $O(N_c)$, $O(1)$ and $O(1/N_c)$ in $1/N_c$ expansion and Fig. 8 displays clearly these behaviors. In NRQM $f' = 0$ identically.

The large N_c limit of (5.5) for the spin-flip baryon vertices is consistent with the result given by Dashen, Jenkins and Manohar [10]. They have shown that the baryon matrix of the axial vector current $(A^{ia})_{B'B}$ can be determined from the consistency condition approach in the large N_c limit. That is to say, when we expand the baryon matrix element of the axial vector current that is given by

$$(A^{ia})_{B'B} = N_c (A_0^{ia})_{B'B} + (A_1^{ia})_{B'B} + \frac{1}{N_c}(A_2^{ia})_{B'B} + \cdots, \tag{5.6}$$

they have shown that the leading order $(A_0^{ia})_{B'B}$ can be expressed by using the 6-j symbol and Clebsch-Gordan coefficients

$$\begin{aligned} &< I', I_3', J', J_3'; K \mid A_0^{ia} \mid I, I_3, J, J_3; K >= \\ &g(K)(-1)^{2J'-J-I'-K}\sqrt{\dim I \dim J} \begin{Bmatrix} 1 & I & I' \\ K & J & J' \end{Bmatrix} \left(\begin{array}{cc|c} I & 1 & I' \\ I_3 & a & I_3' \end{array} \right) \left(\begin{array}{cc|c} J & 1 & J' \\ J_3 & i & J_3' \end{array} \right) \end{aligned} \tag{5.7}$$

where $g(K)$ is the axial vector coupling constant which depend on the strangeness.

When we apply our result (4.21) to the spin-flip baryon vertices of the baryon with the spin 1/2, the leading order of our result is consistent with their results

$$\begin{aligned} &\lim_{N_c \to \infty} < I, I_3, 1/2, J_3, K \mid O^{i3} \mid I, I_3, 1/2, J_3, K > /N_c \\ &=< I, I_3, 1/2, J_3, K \mid A_0^{i3} \mid I, I_3, 1/2, J_3, K >= g(K) W^3 V^i, \end{aligned} \tag{5.8}$$

where $g(K)$ corresponds to the leading contribution of F/D ratios.

6 Summary

In the preceding sections we have calculated the F/D ratios of spin-nonflip and spin-flip baryon vertices for arbitrary N_c both in the NRQM and in the CSM. We find that the spin-nonflip and spin-flip F/D ratios of baryon vertices starting from 1 at $N_c = 1$ go to -1 and $1/3$ in the limit of $N_c \to \infty$, respectively independent of the model.

By use of $1/N_c$ expansions in the NRQM and CSM and applying the Wigner-Eckart theorem to the baryon vertices O^8, O^3, O^{8i} and O^{3i} we find the model independent and systematic N_c dependence among the baryon vertices: (I) In the isoscalar part of spin-nonflip vertex and the isovector part of spin-flip vertex the terms independent of isospin and of strangeness are $O(N_c)$, the strangeness dependent terms are of order $O(1)$ and the isospin dependent terms are of order $O(1/N_c)$. (II) In the isoscalar part of spin-flip vertex and the isovector part of spin-nonflip vertex all three terms, independent of isospin and of strangeness, strangeness-dependent and isospin-dependent, are of order $O(1)$. These results are summarized in Table 3. From these systematic N_c dependences of baryon vertices we obtain the $I = J$ rule [9].

We examine the results of the spin-nonflip baryon vertex through the $SU(3)$ breaking terms of baryon mass based on the assumption that the octet isoscalar part is caused by the mass difference between strange and non-strange quarks and the isovector part is due to the mass difference between up and down quarks. We can understand that the same Okubo-Gell-Mann mass relation and its refined formula among the octet baryons in the case $N_c = 3$, can be derived model independently for arbitrary N_c while the Coleman-Glashow relation holds only for $N_c = 3$.

The results on the N_c structures of spin-nonflip and spin-flip baryon vertices are examined on the assumption that the experimental ratios D_+/F_+ and F_-/D_- lie between those of NRQM and CSM. We find that the obtained N_c structures are well satisfied even in the actual case with $N_c = 3$.

Appendix A

In this appendix we give the properties of eigenstates for the $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation of $SU(3)$ group. This representation is specified by the Young tableau with the first row of length $k+1$ and the second row of length k shown in Fig. 1 and the root diagram given in Fig. 2.

For this representation there are at most two states that have the same hypercharge and we denote these states to A and B states, and the state A has larger isospin than the state B. Operating the U-spin raising and lowering operators on the eigen states of $(\mathbf{k+1})(\mathbf{k+3})$ dimensional representation, we obtain

$$U_+ \mid I, I_3, K >_A$$
$$= \sqrt{\frac{2K}{2K+1}(2K+1-I+I_3)(k-2K+1)} \mid I-\frac{1}{2}, I_3-\frac{1}{2}, K-\frac{1}{2} >_A, \quad (.1)$$

$$U_- \mid I, I_3, K >_A$$
$$= \sqrt{\frac{2K+1}{2K+2}(2K+2-I+I_3)(k-2K)} \mid I+\frac{1}{2}, I_3+\frac{1}{2}, K+\frac{1}{2} >_A$$
$$+\sqrt{\frac{(I-I_3)(k+2)}{(2K+1)(2K+2)}} \mid I-\frac{1}{2}, I_3+\frac{1}{2}, K+\frac{1}{2} >_B, \quad (.2)$$

$$U_+ \mid I, I_3, K >_B$$
$$= \sqrt{\frac{2K+1}{2K}(2K-1-I+I_3)(k-2K+2)} \mid I-\frac{1}{2}, I_3-\frac{1}{2}, K-\frac{1}{2} >_B$$
$$+\sqrt{\frac{(I-I_3+1)(k+2)}{2K(2K+1)}} \mid I+\frac{1}{2}, I_3-\frac{1}{2}, K-\frac{1}{2} >_A, \quad (.3)$$

$$U_- \mid I, I_3, K >_B$$
$$= \sqrt{\frac{2K+2}{2K+1}(2K-I+I_3)(k-2K+1)} \mid I+\frac{1}{2}, I_3+\frac{1}{2}, K+\frac{1}{2} >_B . \quad (.4)$$

Similarly by operating the V-spin raising and lowering operators we obtain

$$V_+ \mid I, I_3, K >_A$$
$$= -\sqrt{\frac{2K}{2K+1}(I-I_3)(k-2K+1)} \mid I-\frac{1}{2}, I_3+\frac{1}{2}, K-\frac{1}{2} >_A, \quad (.5)$$

$$V_- \mid I, I_3, K >_A$$

$$= -\sqrt{\frac{2K+1}{2K+2}(I-I_3+1)(k-2K)}\,|\,I+\frac{1}{2}, I_3-\frac{1}{2}, K+\frac{1}{2}>_A$$
$$+\sqrt{\frac{(2K+1-I+I_3)(k+2)}{(2K+1)(2K+2)}}\,|\,I-\frac{1}{2}, I_3-\frac{1}{2}, K+\frac{1}{2}>_B, \qquad (.6)$$

$$V_-\,|\,I, I_3, K>_B$$
$$= -\sqrt{\frac{2K+1}{2K}(I-I_3)(k-2K+2)}\,|\,I-\frac{1}{2}, I_3+\frac{1}{2}, K-\frac{1}{2}>_B$$
$$+\sqrt{\frac{(2K-I+I_3)(k+2)}{2K(2K+1)}}\,|\,I+\frac{1}{2}, I_3+\frac{1}{2}, K-\frac{1}{2}>_A, \qquad (.7)$$

$$V_-\,|\,I, I_3, K>_B$$
$$= -\sqrt{\frac{2K+2}{2K+1}(I-I_3+1)(k-2K+1)}\,|\,I+\frac{1}{2}, I_3-\frac{1}{2}, K+\frac{1}{2}>_B\,. \qquad (.8)$$

The factors $2K+1-I+I_3$ and $I-I_3$ in the square roots in (.1) and (.5) assure that the edge states of the root diagram are not degenerate.

Moreover the formulas (.5) and (.6) are obtained from (.1) and (.2) by replacing $2K+1-I+I_3$ by $I-I_3$ and the formulas (.7) and (.8) are obtained from (.3) and (.4) by replacing $2K-I+I_3$ by $I-I_3+1$. This is a consequence of symmetry between the U-spin and V-spin.

We can easily check the above formulas by using the identities

$$= F_a F_a - I(I+1) - \frac{3}{4}Y^2 = \frac{1}{2}\{U_+, U_-\} + \frac{1}{2}\{V_+, V_-\}$$
$$= F_4^2 + F_5^2 + F_6^2 + F_7^2 \qquad (.9)$$

Finally from (.1) $\sim$ (.8) we obtain the formulas with which we calculate the matrix element of diagonal operator O_3 as follows

$$_A< I, I_3, K\,|\,\{V_+, V_-\} - \{U_+, U_-\}\,|\,I, I_3, K>_A$$
$$= {}_B< I-1, I_3, K-1\,|\,\{V_+, V_-\} - \{U_+, U_-\}\,|\,I-1, I_3, K-1>_B$$
$$= -(2K+1-2I+2I_3)\left(\frac{4K(k+2)}{2K+1} - 4K - 1\right) \qquad (.10)$$
$$_B< I-1, I_3, K\,|\,\{V_+, V_-\} - \{U_+, U_-\}\,|\,I, I_3, K>_A$$
$$= {}_A< I, I_3, K\,|\,\{V_+, V_-\} - \{U_+, U_-\}\,|\,I-1, I_3, K>_B$$
$$= -\frac{4}{2K+1}\sqrt{(I-I_3)(2K+1-I+I_3)(k-2K+1)(k+2)} \qquad (.11)$$

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