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How to extract the Quark Masses in the Heavy Quark Symmetry

TOSHIAKI ITO

Department of Physics, Nagoya University, Nagoya, 464-01, Japan

TOSHIYUKI MORII

Faculty of Human Development, Division of Sciences for Natural Environment

and Graduate School of Science and Technology, Kobe University, Nada, Kobe 657, Japan

AND

MORIMITSU TANIMOTO

Science Education Laboratory, Ehime University,Matsuyama, 790, Japan

Abstract

The semileptonic decays of the D and B mesons are studied in the heavy quark effective theory(HQET) with leading $1/m_Q$ corrections at $v \cdot v' = 1$. By using the experimental data of form factors f_+^D, A_2^D, V^D, A_2^B and V^B, we determine unknown five parameters in HQET, which are independent of the Isgur-Wise function. We show the allowed regions on the $m_q - \overline{\Lambda}$ plane, which are acceptable ones in HQET. The s-quark mass is fixed by taking account of the QCD corrections in the form factor A_1^D. Accordingly, the c- and b-quark masses are also determined by using the spectroscopic constraints.

The heavy quark effective theory(HQET) is by now a well-established tool to investigate the properties of hadrons containing a single heavy quark Q[1,2,3,4,5]. Most important is its application to the semileptonic transitions of such hadrons. The heavy quark experiences great simplifications when its mass is treated in the formal limit $m_Q \to \infty$. For heavy flavored hadrons with a finite heavy quark mass m_Q, the physical decay amplitudes are expanded in powers of $1/m_Q$. In practice, the effect of the $1/m_Q$ corrections is not negligible in the semileptonic decays of the B meson. In addition, some challenging studies have suggested that HQET is still valid phenomenologically for the s-quark system[6,7,8,9,10]. In these situations, it is very interesting to examine the applicability of HQET to the $\overline{D} \to K^{(*)} l \overline{\nu}$ decays, which possess the rather large $1/m_s$ corrections. So far, these semileptonic decays have been sucessfully analyzed by us[6] and Amundson-Rosner[7]. However, their results depend on the specific form of the Isgur-Wise function. Moreover, HQET might be inapplicable around $q^2 \simeq 0$ for the $\overline{D} \to K^{(*)} l \overline{\nu}$ decays since the validity range of HQET is restricted as[1]

$$\frac{q^2_{max} - q^2}{2m_c m_s} \ll \left(\frac{m_s}{\Lambda_{QCD}}\right)^2 . \tag{1}$$

In this paper, we examine HQET in the $\overline{B} \to D^{(*)} l \overline{\nu}$ and $\overline{D} \to K^{(*)} l \overline{\nu}$ decays at the zero recoil momentum point, $q^2 = q^2_{max}$, where the Isgur-Wise function is normalized to be 1. We search for the parameters being consistent with the observed form factors in the framework of HQET including $1/m_b$, $1/m_c$ and $1/m_s$ corrections. Then, we get the quark masses m_s, m_c and m_b by combining these parameters with the spectroscopic parameters obtained by the B, D and K mesons spectroscopy[7,8,9].

Let us start with describing the current matrix elements for $\overline{B} \to D^{(*)} \ell \overline{\nu}$ as folows[1]:

$$\begin{aligned}
\langle D(v') \mid \bar{c}\gamma_\mu b \mid \overline{B}(v)\rangle &= \sqrt{m_B m_D}\{\tilde{f}_+(y)(v+v')_\mu + \tilde{f}_-(y)(v-v')_\mu\} , \\
\langle D^*(v') \mid \bar{c}\gamma_\mu b \mid \overline{B}(v)\rangle &= i\sqrt{m_B m_{D^*}}\tilde{g}(y)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} v'^\alpha v^\beta , \\
\langle D^*(v') \mid \bar{c}\gamma_\mu\gamma_5 b \mid \overline{B}(v)\rangle &= \sqrt{m_B m_{D^*}}\{\tilde{f}(y)(1+y)\epsilon^*_\mu - (\tilde{a}_+(y) \\
&+ \tilde{a}_-(y))\epsilon^* \cdot v v_\mu - (\tilde{a}_+(y) - \tilde{a}_-(y))\epsilon^* \cdot v v'_\mu\} ,
\end{aligned} \tag{2}$$

where $y = v \cdot v'$. In the limit $m_Q \to \infty$, the spin-flavor symmetry leads to

$$\begin{aligned}
\tilde{f}_+(y) &= \tilde{g}(y) = \tilde{f}(y) = \tilde{a}_+(y) - \tilde{a}_-(y) \equiv \xi(y) , \\
\tilde{f}_-(y) &= \tilde{a}_+(y) + \tilde{a}_-(y) = 0 ,
\end{aligned} \tag{3}$$

where $\xi(y)$, the so-called Isgur-Wise function, is independent of the heavy quark mass and normalized at zero recoil ($v = v'$), as $\xi(1) = 1$. Including $1/m_Q$ corrections to hadronic form factors, these form factors come to somewhat complicated expressions, but still can be written down by four additional functions $\chi_1(y), \chi_2(y), \chi_3(y)$ and $\xi_+(y)$ and a constant parameter $\overline{\Lambda}$ due to the binding of light quarks in the heavy flavored hadrons[11]. The functions $\chi_1(y)$ and $\chi_3(y)$ obey the normalization conditions $\chi_1(1) = \chi_3(1) = 0$[11], while there are no such conditions for $\chi_2(y)$ and $\xi_+(y)$.

By taking account of the QCD corrections, these form factors for the processes $\overline{B} \to D^{(*)}l\overline{\nu}$ at $y = 1$ are given as follows:

$$
\begin{aligned}
\tilde{f}_+(1) &= X_{QCD}\{1 + \beta_+ \frac{\alpha_s(\mu)}{\pi}\} , \\
\tilde{f}_-(1) &= X_{QCD}\{\beta_- \frac{\alpha_s(\mu)}{\pi}\} - (\frac{1}{m_c} - \frac{1}{m_b})\{2\xi_+(1) + \frac{1}{2}\overline{\Lambda}\} , \\
\tilde{g}(1) &= X_{QCD}\{1 + \beta_g \frac{\alpha_s(\mu)}{\pi}\} + \frac{1}{2m_c}\overline{\Lambda} + \frac{1}{m_b}\{2\xi_+(1) + \frac{1}{2}\overline{\Lambda}\} , \\
\tilde{f}(1) &= X_{QCD}\{1 + \beta_f \frac{\alpha_s(\mu)}{\pi}\} , \\
\tilde{a}_+(1) + \tilde{a}_-(1) &= X_{QCD}\{\beta_{a_+} \frac{\alpha_s(\mu)}{\pi}\} + \frac{1}{m_c}\{2\chi_2(1) + \xi_+(1) - \frac{\overline{\Lambda}}{2}\} , \\
\tilde{a}_+(1) - \tilde{a}_-(1) &= X_{QCD}\{1 + \beta_{a_-} \frac{\alpha_s(\mu)}{\pi}\} + \frac{1}{m_c}\{-2\chi_2(1) + \xi_+(1)\} + \frac{1}{m_b}\{2\xi_+(1) + \frac{1}{2}\overline{\Lambda}\} ,
\end{aligned}
\tag{4}
$$

where $\beta_i (i = +, -, g, f, a_+, a_-)$ are QCD correction coefficients[12] and

$$
X_{QCD} = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-\frac{6}{(33-2n_f)}} . \tag{5}
$$

Eq.(4) should be also applied for $\overline{D} \to K^{(*)}\ell\overline{\nu}$ if m_b and m_c are replaced by m_c and m_s, respectively and the relevant X_{QCD}, β_i and $\alpha_s(\mu)$ are used.

Note that the QCD corrections at $y = 1$ are simple because the IR-singular terms vanish. For the $\overline{B} \to D^{(*)}\ell\overline{\nu}$ process[12],

$$
\begin{aligned}
\beta_+ &= \tilde{v}_1(1) - \tilde{v}_2(1) - \tilde{v}_3(1) , \quad \beta_- = \tilde{v}_3(1) - \tilde{v}_2(1) , \\
\beta_g &= \tilde{v}_1(1) , \quad \beta_f = \tilde{a}_1(1) , \quad \beta_{a_+} = -\tilde{a}_2(1) , \quad \beta_{a_-} = \tilde{a}_1(1) - \tilde{a}_3(1) ,
\end{aligned}
\tag{6}
$$

where $\tilde{v}_i(y)$ and $\tilde{a}_i(y)$ at $y = 1$ are given by using $z = m_c/m_b$ as follows:

$$
\begin{aligned}
\tilde{v}_1(1) &= v_1(1) + \ln z + dv_1(1) , \quad \tilde{v}_2(1) = v_2(1) + dv_2(1) , \quad \tilde{v}_3(1) = v_3(1) + dv_3(1) , \\
\tilde{a}_1(1) &= a_1(1) + \ln z + da_1(1) , \quad \tilde{a}_2(1) = a_2(1) + da_2(1) , \quad \tilde{a}_3(1) = a_3(1) + da_3(1) ,
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
v_1(1) &= \phi(z) + \frac{2}{3} , \quad v_2(1) = \frac{1}{3}\{1 + \chi(z)\} , \quad v_3(1) = \frac{1}{3}\{1 - \chi(z)\} , \\
a_1(1) &= \phi(z) - \frac{2}{3} , \quad a_2(1) = \chi(z) + 1 - \frac{4}{3}\frac{z}{(1-z)^2}\phi(z) , \\
a_3(1) &= \chi(z) - 1 + \frac{4}{3}\frac{z}{(1-z)^2}\phi(z) ,
\end{aligned}
\tag{8}
$$

with

$$
\phi(z) = \frac{1+z}{1-z}\ln\frac{1}{z} - 2 , \qquad \chi(z) = \frac{1+z}{1-z} - \frac{2z}{(1-z)^2}\ln\frac{1}{z} , \tag{9}
$$

and dv_i and da_i are corrections of the sub-leading logarithms $z \ln z$ as given by Falk and Grinstein[13].

In order to get the numerical values of parameters in eq.(4), we need experimental inputs. The experimentally observed form factors at $q^2 = 0$ are averaged for $\overline{D} \to K^{(*)}\ell\overline{\nu}$ processes[14] as follows:

$$
\begin{aligned}
f_+^D(0) &= 0.77 \pm 0.04 , \qquad A_1^D(0) = 0.48 \pm 0.05 , \\
A_2^D(0) &= 0.27 \pm 0.11 , \qquad V^D(0) = 0.95 \pm 0.20 .
\end{aligned}
\tag{10}
$$

From these data, we can get the magnitudes of the form factors at $y = 1$, which correspond to the maximum value of q^2. These values are given by applying the single-pole fitting to each form factor as follows[14]:

$$
\begin{aligned}
f_+^D(q^2_{max}) &= 1.47 \pm 0.08 , \qquad A_1^D(q^2_{max}) = 0.57 \pm 0.06 , \\
A_2^D(q^2_{max}) &= 0.32 \pm 0.13 , \qquad V^D(q^2_{max}) = 1.21 \pm 0.26 ,
\end{aligned}
\tag{11}
$$

where the pole masses $m_f = 2.0$GeV, $m_A = 2.5$GeV and $m_V = 2.1$GeV are taken[14]. For the $\overline{B} \to D^{(*)}\ell\overline{\nu}$ process, following values have been given by CLEO[15]:

$$
\begin{aligned}
A_1^B(q^2_{max}) &= 0.88 \pm 0.13 , \qquad A_2^B(q^2_{max}) = 0.79 \pm 0.26 , \\
V^B(q^2_{max}) &= 1.05 \pm 0.50 .
\end{aligned}
\tag{12}
$$

Here, these form factors are related to $\tilde{f}_+$, $\tilde{f}_-$, , $\tilde{g}$, $\tilde{f}$, $\tilde{a}_+$ and $\tilde{a}_-$ for $\overline{D} \to K^{(*)}\ell\overline{\nu}$ as follows[16]:

$$
\begin{aligned}
f_+^D &= \frac{1}{2\sqrt{m_D m_K}}\{(m_D + m_K)\tilde{f}_+ - (m_D - m_K)\tilde{f}_-\} , \\
A_1^D &= \frac{\sqrt{m_D m_{K^*}}}{m_D + m_{K^*}}\tilde{f} , \qquad V^D = \frac{m_D + m_{K^*}}{2\sqrt{m_D m_{K^*}}}\tilde{g} , \\
A_2^D &= \frac{m_D + m_{K^*}}{2\sqrt{m_D m_{K^*}}}\Big\{\frac{m_{K^*}}{m_D}(\tilde{a}_+ + \tilde{a}_-) + (\tilde{a}_+ - \tilde{a}_-)\Big\} .
\end{aligned} \tag{13}
$$

For $\overline{B} \to D^{(*)}\ell\overline{\nu}$ processes, the similar relations are satisfied.

Now, we can present numerical analyses using the eqs.(4)∼(13). Let us begin with estimating the magnitudes of the QCD corrections for the $\overline{D} \to K^{(*)}\ell\overline{\nu}$ processes, which are most significant due to the rather large QCD coupling in our analyses. In order to calculate the QCD corrections for each form factor, we have to fix $\alpha_s(\mu)$. To do this, we take notice of the following replacement under the renormalization group in deriving eqs.(5)∼(7) as presented by Neubert[12]:

$$
1 + \frac{\alpha_s(\mu)}{\pi}\ln\frac{m_Q}{m_q} \longrightarrow \left(\frac{\alpha_s(m_Q)}{\alpha_s(m_q)}\right)^{-\frac{6}{(33-2n_f)}} \tag{14}
$$

We use this replacement to choose the scale μ so that the numerical values of these two expressions are equal. Then, the QCD running coupling up to the order $\mathcal{O}(\ln(\ln\mu))$ can be fixed if m_Q and m_q are given. The values of $\alpha_s(\mu)$, X_{QCD} and β_i are shown in table 1 for fixed quark masses and for $\Lambda_{\overline{MS}}^{(4)} = 260$MeV[17] in both B and D semileptonic decays.

table 1

The effect of the QCD corrections on form factors will be tested in the form factors A_1^D and A_1^B at $y = 1$ since these form factors are determined by the QCD corrections alone as follows:

$$
A_1^D(q^2_{max}) = \frac{\sqrt{m_D m_{K^*}}}{m_D + m_{K^*}} X_{QCD}\{1 + \beta_f\frac{\alpha_s(\mu)}{\pi}\} . \tag{15}
$$

This QCD correction depends on the value of m_s significantly while the one of $A_1^B(q^2_{max})$ mildly depends on m_c. We show the allowed region of m_s being consistent with the experimental value of $A_1^D(q^2_{max})$ in table 2, where $\Lambda_{\overline{MS}}^{(4)}$ is taken to be 260^{+54}_{-46}MeV[17].

table 2

Now, we can get the numerical values of unknown parameters in eq.(4) by using the experimental values of the form factors $f_+^D(q^2_{max})$, $V^D(q^2_{max})$, $V^B(q^2_{max})$, $A_2^D(q^2_{max})$ and $A_2^B(q^2_{max})$. In order to get the numerical solutions, we define the following dimensionless five parameters,

$$
\begin{aligned}
\epsilon_s &\equiv \frac{\overline{\Lambda}}{2m_s} , \quad \epsilon_c \equiv \frac{\overline{\Lambda}}{2m_c} , \quad \epsilon_b \equiv \frac{\overline{\Lambda}}{2m_b} , \\
r &\equiv \frac{\xi_+(1)}{\overline{\Lambda}} , \quad s \equiv \frac{\chi_2(1)}{\overline{\Lambda}} .
\end{aligned} \tag{16}
$$

Since the number of the input experimental data are five, these five parameters can be determined definitely under the experimental errors of input data. These equations can be solved easily though the analytic forms of the solutions are very complicated. Since the parameter r is to be given as the solution of a quadratic equation, we obtain two set of the solutions, in which we choose the convenient one.

It is important to comment on the quark mass dependence of the solutions, because the QCD corrections cannot be calculated without fixing the quark masses as seen in eqs.(6)∼(9). The ambiguity of the QCD corrections due to the s-quark mass is not so large, as far as the allowed value of the s-quark mass in table 2 is taken. Therefore, it is enough to take the averaged value of those m_s in order to fix the QCD corrections, since the ambiguity due to large errors of the input experimental form factor is rather serious at present. Using the averaged value of m_s, the calculated form factors of $A_1^D(q^2_{max})$ and $A_1^B(q^2_{max})$ are given for $\Lambda_{\overline{MS}}^{(4)} = 260^{+54}_{-46}$MeV as follows:

$$
A_1^D(q^2_{max}) = 0.58 \sim 0.59 , \qquad A_1^B(q^2_{max}) = 0.82 \sim 0.84 , \tag{17}
$$

which are of course consistent with the experimental values in eqs.(11) and (12). The c- and b-quark mass dependences of the solutions can be neglected safely.

The set of solutions are shown in table 3 for the fixed values of allowed $\Lambda_{\overline{MS}}^{(4)}$[17].

table 3

The obtained values have large errors following from the input experimental errors, but those central values are reasonable ones expected from the $1/m_Q$ expansion. It is useful to comment on the result of $\epsilon_s = \overline{\Lambda}/2m_s$ obtained by Amundson and Rosner[7]. Since they neglected the corrections of the heavier quark mass expansions($1/m_c$ in the D decay), the formulation becomes too simple. So, ϵ_s and ϵ_c were determined only by $V^D(q^2_{max})$ and $V^B(q^2_{max})$. In our analyses, coupled linear equations with five unknowns are solved including the corrections of the heavier quark mass expansions as seen in eq.(4). Due to the rather large error of the input data, our obtained ϵ_s has considerably large error.

The parameter m_Q and $\overline{\Lambda}$ are constrained by the meson spectroscopy as follows[8,9]:

$$m_s + \overline{\Lambda} = \frac{1}{4}(3m_{K^*} + m_K)\ , \quad m_c + \overline{\Lambda} = \frac{1}{4}(3m_{D^*} + m_D)\ , \quad m_b + \overline{\Lambda} = \frac{1}{4}(3m_{B^*} + m_B)\ , \quad (18)$$

which give us additional informations on ϵ_s. Since errors of the experimental masses are very tiny, these constraints are presented almost on lines. We show in fig.1 these lines in addition to ϵ_Q, which are presented by lines corresponding to the upper-bounds, central values and lower bounds, on the $m_Q - \overline{\Lambda}$ plane for $\Lambda^{(4)}_{\overline{MS}} = 260\text{MeV}$.

figure 1

We omit the figures in the cases of $\Lambda^{(4)}_{\overline{MS}} = 214,\ 314\text{MeV}$ since the similar results are obtained.

It is emphasized that the allowed regions of m_Q and $\overline{\Lambda}$ are reasonable though the values of ϵ_Q have large error. Furthermore, in this figure we show the allowed region of m_s given in table 2 coming from the QCD corrections together with the region of $\overline{\Lambda}$, which is given by the constraints in eq.(18). The fixed $\overline{\Lambda}$ gives the values of m_c and m_b as seen in eq.(18) as follows:

$$m_s = (0.48, 0.43)\text{GeV} \rightarrow \overline{\Lambda} = (0.32, 0.36)\text{GeV}, \quad m_c = (1.67, 1.62)\text{GeV}, \quad m_b = (5.01, 4.96)\text{GeV}, \quad (19)$$

for $\Lambda^{(4)}_{\overline{MS}} = 214\text{MeV}$,

$$m_s = (0.57, 0.53)\text{GeV} \rightarrow \overline{\Lambda} = (0.22, 0.26)\text{GeV}, \quad m_c = (1.76, 1.72)\text{GeV}, \quad m_b = (5.10, 5.06)\text{GeV}, \quad (20)$$

for $\Lambda^{(4)}_{\overline{MS}} = 260\text{MeV}$, and

$$m_s = (0.69, 0.62)\text{GeV} \rightarrow \overline{\Lambda} = (0.10, 0.17)\text{GeV}, \quad m_c = (1.88, 1.81)\text{GeV}, \quad m_b = (5.22, 5.15)\text{GeV}, \quad (21)$$

for $\Lambda^{(4)}_{\overline{MS}} = 314\text{MeV}$, where the left and right values in parentheses represent the values corresponding to the upper and lower bound of m_s in table 2, respectively. From eqs.(19)$\sim$(21), we get $0.14 \leq \overline{\Lambda}/m_s \leq 0.84$, which are reasonable values in the $1/m_Q$ expansion of HQET. Thus, our results suggest that HQET is still effective to the semileptonic decay of the D meson at $v \cdot v' = 1$. It is remarked that our obtained $\overline{\Lambda}$ in eqs.(19)$\sim$(21) is rather smaller than the one($\simeq 0.5$GeV) predicted by QCD sum rules[18]. Our value of $\overline{\Lambda}$ has been derived phenomenologically based on the heavy quark symmetry for the s-quark. If the theoretical value of $\overline{\Lambda}$ will be certainly $\simeq 0.5$GeV, HQET might not be available to the s-quark. Thus, the estimate of $\overline{\Lambda}$ is also important to examine whether HQET works well for the s-quark system.

Summary is given as follows. Semileptonic decays of the D and B mesons are studied in HQET with leading $1/m_Q$ corrections at $v \cdot v' = 1$. By using the five experimental form factors $f^D_+(q^2_{max})$, $A^D_2(q^2_{max})$, $V^D(q^2_{max})$, $A^B_2(q^2_{max})$ and $V^B(q^2_{max})$, parameters ϵ_s, ϵ_c, ϵ_b, r and s are determined. Allowed region on the m_Q-$\overline{\Lambda}$ plane is reasonable one. The QCD corrections in the form factor $A^D_1(q^2_{max})$ fix m_s, which is consistent with the constituent s-quark mass. Then, $\overline{\Lambda}$, m_c and m_b are also determined by using the spectroscopic constraints. The ambiguity of the obtained parameters will be reduced if the form factors will be measured more precisely in the semileptonic decays of D and B mesons in the future.

REFERENCES

[1] N. Isgur and M. B. Wise, Phys. Lett. **B232**(1989)113; **B237**(1990)527.

[2] E. Eichten and B. Hill, Phys. Lett. **B234**(1990)511;
B. Grinstein, Nucl. Phys. **B339**(1990)253.

[3] H. Georgi, Phys. Lett. **B240**(1990)447.

[4] A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. **B343**(1990)1.

[5] M. Neubert, SLAC-PUB-6263(1993); other references are therein.

[6] T. Ito, T. Morii and M. Tanimoto, Phys. Lett. **B274**(1992)449;
Prog. Theor. Phys. **88**(1992)561.

[7] J.F. Amundson and J.L. Rosner, Phys. Rev. **D47**(1993)1951.

[8] T. Ito, T. Morii and M. Tanimoto, Prog. Theor. Phys. **90**(1993)419; Z. Phys. **C59**(1993)57.

[9] U. Aglietti, Phys. Lett. **B281**(1992)341.

[10] D. Du and C. Liu, preprint at Beijing IHEP, BIHEP-TH-92-41(1992);
E.J. Eichten, C.T. Hill and C. Quigg, Phys. Rev. Lett. **71**(1993)4116;
P.A. Griffin, M. Masip and M. McGuigan, Preprint at Univ. of Florida, HEP-93-25(1993).

[11] M.E. Luke, Phys. Lett. **B252**(1990)447;
M. Neubert and V. Rieckert, Nucl. Phys. **B382**(1992)97.

[12] M. Neubert, Nucl. Phys. **B371**(1992)149.

[13] A.F. Falk and B. Grinstein, Phys. Lett. **B247**(1990)406.

[14] E691 Collaboration, J.C. Anjos et al., Phys. Rev. Lett. **62**(1989)722, 1587; **65**(1990)2630;
Mark III Collaboration, J. Adler et al. Phys. Rev. Lett. **62**(1989)1821;
CLEO Collaboration, G.Crawford et al., Phys. Rev. **D44**(1991)3394;
A. Freyberger, Preprint at University of Florida, UFIFT-HEP-94-1(1994).

[15] CLEO Collaboration, S. Sanghera et al., Phys. Rev. **D47**(1993)791.

[16] M. Wirbel, B. Stech and M. Bauer, Z. Phys. **C29**(1985)637;
M. Tanimoto, Phys. Rev. **D44**(1991)1449.

[17] Particle Data Group, Phys. Rev. **D45**(1992)II.1.

[18] M. Neubert, Phys. Rev. **D46**(1992)1076.

Figure Caption

Figure 1: Allowed regions on the m_Q-$\overline{\Lambda}$ plane in the case of $\Lambda^{(4)}_{\overline{MS}} = 260$MeV. The lower- and upper-bounds from ϵ_s, ϵ_c and ϵ_b are shown together with the respective central values. The three lines of eq.(18) are also shown. The dashed vertical lines denote the allowed region of m_s in table 2 and the dashed horizontal lines denote the region of $\overline{\Lambda}$ given in eq.(20).

m_s(GeV)	$\alpha_s(\mu)$	X_{QCD}	β_+	β_-	β_g	β_f	β_{a_+}	β_{a_-}
0.50	1.23	1.43	−1.09	−0.42	−0.35	−1.70	−1.68	−1.31
0.55	1.04	1.33	−0.99	−0.37	−0.26	−1.61	−1.56	−1.18
0.60	0.91	1.26	−0.90	−0.33	−0.18	−1.53	−1.47	−1.06
$B \to D^{(*)}$	0.31	1.12	−1.05	−0.32	−0.35	−1.69	−1.38	−1.28

Table 1: QCD corrections in the D semileptonic decays for fixed m_s. In the bottom row, the QCD corrections for the B semileptonic decays are also shown. Here, $m_c = 1.5$GeV, $m_b = 5.0$GeV and $\Lambda^{(4)}_{\overline{MS}} = 260$MeV are taken.

$\Lambda^{(4)}_{\overline{MS}}$(MeV)	m_s(GeV)
214	$0.43 \sim 0.48$
260	$0.53 \sim 0.57$
314	$0.62 \sim 0.69$

Table 2: Allowed regions of m_s extracted from the form factor $A_1^D(q^2_{max})$ by taking account of the QCD corrections.

$\Lambda^{(4)}_{\overline{MS}}$(MeV)	r	s	ϵ_s	ϵ_c	ϵ_b
214	-1.32 ± 18.6	-3.13 ± 100	-0.08 ± 1.84	-0.002 ± 0.44	0.03 ± 0.56
260	-1.71 ± 31.9	-4.04 ± 39.3	-0.05 ± 1.41	0.006 ± 0.27	0.03 ± 0.55
314	-2.23 ± 48.0	-5.60 ± 25.1	-0.03 ± 0.97	0.009 ± 0.25	0.02 ± 0.49

Table 3: Solutions of the five parameters for fixed $\Lambda^{(4)}_{\overline{MS}}$

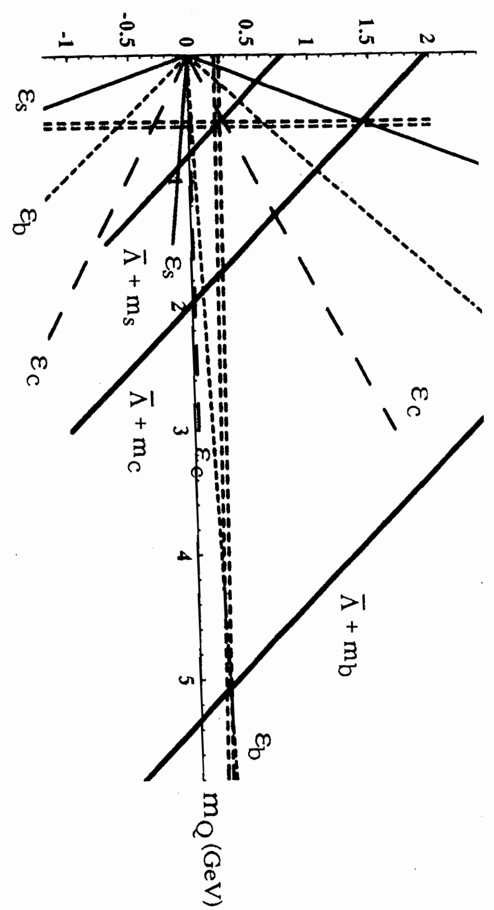

fig. 1