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DO-TH 97/25
November 1997

Spin Physics : Summary and Outlook

APR 3 1998

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Contents

1. Preface
2. The spin structure function $g_1(x,Q^2)$ and polarized parton densities
3. Sum rules (g_1-related)
   3.1 Ellis–Jaffe 'sum rule'
   3.2 Bjorken sum rule
   3.3 Drell–Hearn–Gerasimov sum rule
   3.4 Helicity sum rule: The Spin of the Proton
4. How to measure $\delta g(x,Q^2)$ [and the photonic δg^γ]
5. Semi-inclusive asymmetries
6. Cosmological spin-offs
7. Non-perturbative approaches
8. The transverse spin structure function $g_2(x,Q^2)$
9. Single spin asymmetries
10. Transverse chiral-odd ('transversity') structure functions
11. 'Shopping List'

Invited talk presented at the 2nd Topical Workshop on 'DIS Off Polarized Targets: Theory Meets Experiment', DESY, Zeuthen, Sept. 1-5, 1997.

1 Preface

The spin renaissance started '87 almost exactly ten years ago with the spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + (L_z^q + L_z^g) \tag{1.1}$$

and the surprising EMC observation [1] that $\Delta\Sigma \ll 1$ as inferred from $\int_0^1 g_1^p(x,Q^2 \simeq 10\ \mathrm{GeV}^2)\,dx = 0.126 \pm 0.018$. To most of us this came as a surprise since it implies that the fermionic degrees (quarks and antiquarks) contribute very little to the proton's spin (sometimes also coined 'spin crisis'), and thus the total helicity of gluons (Δg) and the orbital components ($L_z^{q,g}$) are mainly responsible for the spin of the nucleon. Furthermore it also contradicts naive quark model expectations $\Delta\Sigma^{SU(6)} = 1$ or, alternatively, $\Delta\Sigma \simeq \Delta q_8 \simeq 0.6$ as suggested by Ellis and Jaffe [2] by assuming a vanishing strange sea contribution $\Delta s = \Delta\bar{s} = 0$. For definiteness, the total helicity of a specific parton $f = q, \bar{q}, g$ is defined by

$$\Delta f(Q^2) \equiv \int_0^1 \delta f(x,Q^2)\,dx \tag{1.2}$$

with the density of longitudinally polarized partons being given by $\delta f = f_+ - f_-$, and $\Delta\Sigma \equiv \Sigma_f(\Delta q + \Delta\bar{q})$.

Today, '97, more than ten years later we have a better knowledge of $\Delta\Sigma(Q^2)$ due to several and far more accurate measurements of $g_1^{p,n,d}(x,Q^2)$. On the other hand, however, $\Delta g(Q^2)$, i.e. $\delta g(x,Q^2)$ is experimentally still entirely unknown, but there is a plethora of theoretical developments and studies. Similarly, the orbital components $L_z^{q,g}(Q^2)$ in (1.1) are terra incognita and it is not even known how to relate them to realistic measurable observables (apart from a very recent attempt to which I turn later). This is a very curious and unusual situation indeed!

2 The Spin Structure Function $g_1(x,Q^2)$ and Polarized Parton Densities

The (longitudinally) polarized structure function g_1 is, up to NLO, related to the quark and gluon densities (δf) in the following well known way

$$g_1(x,Q^2) = \frac{1}{2}\Sigma_q e_q^2 \left\{ \delta q(x,Q^2) + \delta\bar{q}(x,Q^2) + \frac{\alpha_s(Q^2)}{2\pi}\left[\delta C_q \otimes (\delta q + \delta\bar{q}) + 2\delta C_g \otimes \delta g\right] \right\} \tag{2.1}$$

with $\otimes$ denoting the usual convolutions. Needless to say that the NLO quantities δC_i and the 2-loop splitting functions $\delta P_{ij}^{(1)}$ for the Q^2-evolution have to be calculated within the same renormalization and factorization scheme so as to maintain eq. (2.1) independent of the convention chosen to the perturbative order in α_s considered ('scheme independence').

The δC_i and $\delta P_{ij}^{(1)}$ separately depend of course on the factorization scheme chosen. In other words, if one chooses a different factorization scheme according to $\delta C \to \delta C + \Delta$, in a generic but obvious notation, this change has to be compensated by an appropriate change of $\delta P^{(1)}$:

$$\delta P^{(1)} \to \delta P^{(1)} + \frac{\beta_0}{2}\Delta - \left[\Delta, \delta P^{(0)}\right] \tag{2.2}$$

in order to keep g_1 invariant, i.e. convention (scheme) independent to $\mathcal{O}(\alpha_s^2)$ [3]. This clearly requires the knowledge of all $\delta P_{ij}^{(1)}$ in one particular scheme. The completion of *all* 2-loop $\delta P_{ij}^{(1)}(x)$ in the $\overline{\mathrm{MS}}$ scheme [4,5] was the highlight in '95. Since then it became possible for the first time to perform a fully consistent NLO($\overline{\mathrm{MS}}$) analysis [6–8] for $g_1(x,Q^2)$ as in the case for unpolarized structure functions (F_i) during the past 17 years. All the required continuations of the moments $\delta P_{ij}^{(1)n} = \int_0^1 x^{n-1}\delta P_{ij}^{(1)}(x)\,dx$, required for the more convenient analysis in terms of Mellin n-moments, have been worked out in [6]. Since the unpolarized (spin averaged) parton densities $f = f_+ + f_-$ are usually extracted in the $\overline{\mathrm{MS}}$ scheme, this factorization scheme is particular convenient because the δf's have to satisfy the general positivity constraints

$$|\delta f(x,Q^2)| \le f(x,Q^2)\,. \tag{2.3}$$

It should be furthermore noted that the gluonic coefficient function in (2.1) has, in the $\overline{\mathrm{MS}}$ scheme, the property

$$\Delta C_g \equiv \int_0^1 \delta C_g(x)\,dx = 0 \qquad (\overline{\mathrm{MS}}) \tag{2.4}$$

i.e. the total gluon helicity $\Delta g(Q^2)$ decouples from the first ($n=1$) moment of g_1. Nevertheless, $\delta g(x,Q^2)$ plays an important role in analyses of $g_1(x,Q^2)$ since, for the time being, actual measurements always refer to $x \gtrsim 0.01$.

This is in contrast to the gluon 'off-shell' ($-p^2 > 0$) or 'Adler–Bardeen' scheme [9] where

$$\Delta C_g = -\frac{1}{2} \qquad (\text{'off shell' or 'AB' scheme}). \tag{2.5}$$

The appealing feature of this scheme lies mainly in the fact that here $\Delta\Sigma$ is conserved, i.e. Q^2 *independent*, and thus can remain 'constituent-like' (cf. Sect. 1) at any scale [10], with $\Delta g(Q^2)$ being then responsible for reducing the total flavor-singlet contribution to the first moment of $g_1(x,Q^2)$, as will be discussed later. For these reasons some groups prefer to work with this factorization scheme [11–13], although the transformation from the $\overline{\mathrm{MS}}$ to the AB scheme is not unique. Nevertheless it should be kept in mind that any consistent (theorist's) choice of scheme is in principle equally acceptable since the final result for any measurable quantity (g_1) has to be the same way up to $\mathcal{O}(\alpha_s^2)$ contributions as discussed above.

QCD analyses of $g_1(x,Q^2)$ data are rather ambiguous at present since the polarized input densities $\delta f(x,Q_0^2)$ are far *less* constrained than the unpolarized densities $f(x,Q_0^2)$.

This is not only due to the rather 'scarce' polarization data available, but in particular because

- *no* $\delta g(x,Q_0^2)$ sensitive *data* are available so far;
- $\delta f(x,Q^2)$ is *not* even positive definite;
- no sum rule exists like the strongly constraining $\int_0^1 x(\Sigma + g)\,dx = 1$ energy-momentum conservation sum rule for the spin-averaged case.

Therefore some groups [11–14] parametrize present g_1-data just in terms of a quark non-singlet and singlet density as well as of $\delta g(x,Q_0^2)$ which is certainly suitable in view of the limited number of independent experimental observables, g_1^p and g_1^n (or g_1^d). On the other hand, however, the knowledge of the individual densities $\delta f(x,Q^2)$ is required for calculating and estimating spin effects in different reactions ($\vec{\gamma}\vec{p} \to c\bar{c}X$, $\vec{\gamma}\vec{p} \to jX$, $\vec{p}\vec{p} \to \gamma X$, etc.) to be studied by future experiments. It is therefore more appropriate to employ theoretical prejudices and educated guesses [6–8] in order to constrain $\delta f(x,Q_0^2)$ such as [15]

$$|\delta q_v(x,Q_0^2)| \sim q_v(x,Q_0^2) \quad \text{as} \quad x \to 1 \quad (\text{'helicity retention'}) \tag{2.6}$$

$$\delta g(x,Q_0^2)\,/\,g(x,Q_0^2) \sim x \quad \text{as} \quad x \to 0 \quad (\text{'color coherence'}) \tag{2.7}$$

at some 'intrinsic' bound-state-like input scale $Q_0^2 \lesssim 1\ \mathrm{GeV}^2$. Furthermore, neutron and hyperon β-decay rates constrain the total helicities, eq. (1.2), of the following flavor non-singlet combinations [16]:

$$\Delta q_3 \equiv \Delta(u + \bar{u} - d - \bar{d}) = g_A^{n\to p} = F + D = 1.2573 \pm 0.0028 \tag{2.8}$$

$$\Delta q_8 \equiv \Delta\left(u + \bar{u} + d + \bar{d} - 2(s + \bar{s})\right) = 3F - D = 0.579 \pm 0.025\,. \tag{2.9}$$

Note that the constraint (2.8) is very solid, being nothing else than the Bjorken sum rule, since it follows from the fundamental isospin $SU(2)_{\mathrm{flavor}}$ symmetry. The 'standard' constraint (2.9), however, relies on the full $SU(3)_{\mathrm{flavor}}$ symmetry for the baryon octet (since data from $\Sigma^- \to n$ transitions are needed) and is usually employed for most QCD analyses. An alternative 'valence' constraint $\Delta(u_v + d_v) = 3F - D$ is equally acceptable, which follows from a broken $SU(3)_{\mathrm{flavor}}$ symmetry by assuming [17] that the flavor changing hyperon decay data ($\Sigma^- \to n$) fix only the total helicity of valence quarks $\Delta q_v(Q^2) \equiv \Delta q - \Delta\bar{q}$. (Here, a negative light sea component $\Delta\bar{q} \equiv \Delta\bar{u} = \Delta\bar{d}$ suffices to explain present data on the first moment of g_1, whereas the 'standard' scenario requires a negative nonvanishing $\Delta s = \Delta\bar{s}$ component as well [6].) Present polarized DIS data cannot distinguish between these two scenarios [6,8]. Finally, the flavor singlet combination remains unconstrained,

$$\Delta\Sigma = \Delta q_8 + 3\Delta(s + \bar{s}) = 3F - D + 3\Delta(s + \bar{s}), \tag{2.10}$$

using (2.9), since the total helicity of the strange sea $\Delta s = \Delta\bar{s}$ is not fixed by hyperon β-decays.

There are several ways to implement these ingredients into LO and NLO analyses [8,13] of present data (for a collection and references of all presently available g_1 data see, for example, [8]). To determine as far as possible, in particular, the individual polarized parton densities $\delta f(x,Q^2)$ it is helpful to consider some reasonable and simple ansatz for $\delta f(x,Q_0^2)$ with not too many free parameters [6,8,18,19]:

$$\begin{aligned}
\delta q_v(x,Q_0^2) &= N_{q_v} x^{a_{q_v}} q_v(x,Q_0^2), & q_v = u_v, d_v \\
\delta g(x,Q_0^2) &= N_g x^{a_g}(1-x)^{b_g} g(x,Q_0^2) \\
\delta \bar{q}(x,Q_0^2) &= N_{\bar{q}} x^{a_{\bar{q}}}(1-x)^{b_{\bar{q}}} \bar{q}(x,Q_0^2), & \bar{q} \equiv \bar{u} = \bar{d} \\
\delta s(x,Q_0^2) &= N_s \delta\bar{q}(x,Q_0^2), & (2.11)
\end{aligned}$$

with $a_{g,\bar{q}} = O(1)$, and the unpolarized "dynamical" parton densities $f(x,Q_0^2)$ are taken from [20] at the input scale $Q_0^2 \simeq 0.3$ GeV^2 which successfully predicted and explain all present (unpolarized) HERA small-x data [21] in terms of pure QCD dynamics. With this ansatz the positivity requirements (2.3) can be trivially implemented. The contribution from heavy quarks (charm, etc.) to g_1, calculated as usual via the massive $\gamma^* g \to c\bar{c}$ fusion process [22], is negligibly small in the experimentally covered x-region ($x \gtrsim 10^{-2}$) and remains small even down to $x = 10^{-4}$ (see, e.g., [8]). The resulting longitudinal asymmetries $A_1(x,Q^2) = g_1/F_1$ as well as the $g_1(x,Q^2)$ for p, n and d targets are in excellent agreement with present measurements [8,18]. The main difference with respect to the original '95-analysis [6] is related to a sizeable increase [8] of $-\delta d_v$ around $x = 0.1$ to 0.3 mainly due to the rather precise new E154 data [23] for A_1^n. Although the optimal fit resulted again in a sizeable and positive $\delta g(x,Q^2)$ [8,13,18], similar agreeable fits (χ^2) can be obtained with very different (i.e. smaller, oscillating or even negative) polarized gluon densities [6,8] — typical examples of which are depicted in fig. 1. Even the small

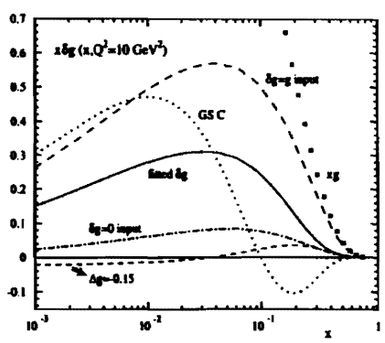

Fig. 1: Examples of experimentally allowed NLO polarized gluon densities at $Q^2 = 10$ GeV^2 [6,8]. The 'set C' gluon of [7] is denoted by 'GS C'. Also shown is the unpolarized GRV94 gluon distribution of [20] (stars).

and oscillating δg in fig. 1 (short-dashed curve), which integrates to a small total helicity $\Delta g \simeq -0.15$ (so called 'static' solution, see Sect. 3.4) cannot be ruled out by present experiments [8]. Although all these different choices yield practically indistinguishable results for $g_1^{p,n}(x,Q^2)$ in the experimentally accessible region ($x \gtrsim 10^{-2}$), they result in very different small-x extrapolations ($x < 10^{-2}$) of $g_1^{p,n}$ [8,13,14]. This δg-ambiguity is very disturbing and unfortunate, since it affects the estimates of the full first ($n = 1$) moment of $g_1^{p,n}$, $\Gamma_1(Q^2) \equiv \int_0^1 g_1(x,Q^2)\,dx$; which could shift present naive (Regge oriented) estimates of $\Gamma_1^{p,n}$ by as much as $\pm 10\%$ [8,13]!

Let me illustrate the importance of δg for the $x \to 0$ extrapolation of $g_1(x,Q^2)$ by considering the LO Q^2-evolution in the double-leading-logarithmic (DLL) approximation. The reason for this being the strong $\delta\Sigma - \delta g$ singlet mixing due to the fact that *all* $\delta P_{ij}^{(0)}(x) \sim$ const. for $x \ll 1$ (i.e. $P_{ij}^{(0)n} \sim 1/n$), in contrast to the unpolarized case. Assuming for the input densities $\delta\Sigma(x,Q_0^2)$, $\delta g(x,Q_0^2) \sim c_{\delta\Sigma,\delta g}$ as $x \to 0$, which derives from the Regge behavior $g_1(x,Q_0^2) \sim x^{-\alpha_{a_1}(0)}$, $-0.5 \lesssim \alpha_{a_1}(0) \lesssim 0$, at some hadronic scale $Q_0^2 \lesssim 1$ GeV^2 where simple Regge asymptotics might be relevant as observed by HERA experiments [21], a straightforward calculation yields in the $x \to 0$ DLL approximation [14] for $f = 3$ light active flavors

$$\begin{pmatrix} \delta\Sigma(x,Q^2) \\ \delta g(x,Q^2) \end{pmatrix} \sim \begin{pmatrix} \frac{4}{3}c_{\delta\Sigma} & - & 3c_{\delta g} \\ \frac{8}{3}c_{\delta\Sigma} & + & 12c_{\delta g} \end{pmatrix} exp\left[2\left(\frac{24}{\beta_0}\ell n\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\ell n\frac{1}{x}\right)^{1/2}\right] + \ldots \quad (2.12)$$

with $\beta_0 = 11 - 2f/3 = 9$. Thus a positive δg (i.e. $c_{\delta g} > 0$), for example, drives $\delta\Sigma(x,Q^2)$ *negative* as $x \to 0$ and Q^2 increases, due to the negative large prefactor -3 which derives from the gluon splitting into quarks ($\delta P_{qg}^{(0)}$). Therefore $g_1^p(x,Q^2)$ for example is eventually driven negative as well for $x \lesssim 10^{-3}$ and $Q^2 > 1$ GeV^2 according to an exact LO or NLO analysis [8,12,13]. (Although the DLL approximation is very useful for qualitative purposes, it does not suffice for quantitative calculations.) It should thus be emphasized that estimates of small-x contributions to $\Gamma_1(Q^2)$ using Regge extralpolations alone will be unreliable, underestimating their size, for a positive δg, and sometimes even giving them the wrong sign [8,12-14,24]!

These standard perturbative QCD renormalization group (RG) Q^2-evolution results are likely to *remain relevant* at $x \ll 1$. Potentially large small-x contributions from 'naive' LO BFKL-inspired ladder resummations of logarithmic $\alpha_s(\alpha_s \ell n^2\frac{1}{x})^k$ terms suggest [25]

$$g_1 \sim x^{-0.4\ldots-1}\left(Q^2/\mu^2_{cut-off}\right)^{0.2\ldots0.5}. \quad (2.13)$$

Such more singular terms, however, get practically cancelled [26,27], in a more consistent and complete $\overline{MS}$ calculation (including similar singular terms in coefficient functions δC_i as well as less singular terms, which have not yet been completely calculated in the singlet sector), as was emphasized in the talks by J. Kodaira and A. Vogt. Furthermore the steep small-x rise (2.13) is not indicated by the recent SMC data [24], in contrast to their old '93 measurements.

3 Sum rules (g_1–related)

To NLO($\overline{\text{MS}}$) the first moment of $g_1(x,Q^2)$ in (2.1), $\Gamma_1(Q^2) \equiv \int_0^1 g_1(x,Q^2)\,dx$, is given by

$$\Gamma_1^{p,n}(Q^2) = \left(\pm\frac{1}{12}\Delta q_3 + \frac{1}{36}\Delta q_8 + \frac{1}{9}\Delta\Sigma(Q^2)\right)\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) \tag{3.1}$$

where I have suppressed the NNLO $\mathcal{O}(\alpha_s^2)$ terms [28] since all experimental estimates of Γ_1 employ just the known NLO(2-loop) Q^2-evolution of $g_1(x,Q^2)$. The constraints (2.8) und (2.9) are usually used ('standard' scenario) to fix $\Delta q_{3,8}$, whereas the singlet $\Delta\Sigma(Q^2)$ remains undetermined due to the unknown strange sea helicity in (2.10).

3.1 Ellis–Jaffe 'sum rule'

The Ellis-Jaffe estimate [2] of $\Gamma_1^{p,n}$ is based on the assumption $\Delta s = \Delta\bar{s} = 0$ and hence eq. (2.10) fixes $\Delta\Sigma$. Thus in NLO, eq. (3.1) predicts

$$\Gamma_1^p(5)_{EJ} = 0.168 \pm 0.004\,, \qquad \Gamma_1^n(5)_{EJ} = -0.022 \pm 0.004 \tag{3.2}$$

where Q^2 is always understood in units of GeV^2. These estimates are in clear contradiction to present measurements, e.g. [24,29-32]

$$\begin{aligned} \Gamma_1^p(3)_{\text{E143}} &= 0.127 \pm 0.011, \qquad \Gamma_1^p(10)_{\text{SMC}} = 0.130 \pm 0.017 \\ \Gamma_1^n(2.5)_{\text{HERMES}} &= -0.037 \pm 0.015, \qquad \Gamma_1^n(5)_{\text{E154}} = -0.041 \pm 0.007 \\ \Gamma_1^n(10)_{\text{SMC}} &= -0.046 \pm 0.021 \end{aligned} \tag{3.3}$$

which, in the $\overline{\text{MS}}$ factorization scheme, imply via eq. (2.10) a *non*vanishing and negative Δs component in order to reduce the assumed naive $\Delta\Sigma_{EJ} = 3F - D \simeq 0.58$ value for $\Delta\Sigma$. Typical results of recent analyses ('optimal' fits) are [6-8,18]

$$\begin{aligned} \Delta\Sigma(Q^2) \simeq 0.17 - 0.22\,, \qquad \Delta\bar{q}(Q^2) \simeq \Delta s(Q^2) = \Delta\bar{s}(Q^2) \simeq -0.066 \\ \Delta g(5) \simeq 1.7 \pm 1.1 \end{aligned} \tag{3.4}$$

where $\Delta\Sigma$, $\Delta\bar{q}$ and Δs have no strong Q^2-dependence, in contrast to $\Delta g(Q^2)$ which turns out to be *sizeable* despite the fact that it decouples ($\Delta C_g = 0$, eq. (2.4)) from Γ_1 in the $\overline{\text{MS}}$ scheme. (Note that in the 'valence' scenario, discussed after eq. (2.9), $\Delta s = 0$ but $\Delta\bar{q} \simeq -0.08$ is also possible [6,8].)

Alternatively one may choose to work with a different factorization scheme, for example with the 'off-shell' or 'AB' scheme where

$$\Delta\Sigma(Q^2) = \Delta\Sigma_{AB} - 3\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta g(Q^2) \tag{3.5}$$

with $\Delta\Sigma_{AB}$ being Q^2-independent. Here one obtains [11-13,33]

$$\delta\Sigma_{AB} \simeq 0.45\,, \qquad \Delta g(5) \simeq 2.5 \pm 1 \tag{3.6}$$

i.e. (obviously) *large* changes in $\Delta\Sigma_{AB}$ but *small* changes in $\Delta g(Q^2)$ when compared with the $\overline{\text{MS}}$ results (3.4), despite the fact that Δg couples directly ($\Delta C_g = -\frac{1}{2}$, eq. (2.5)) to Γ_1 via (3.5). Since

$$\Delta(s+\bar{s})(Q^2) = \Delta(s+\bar{s})_{AB} - \frac{\alpha_s(Q^2)}{2\pi}\,\Delta g(Q^2)\,, \tag{3.7}$$

one can allow here for a vanishing strange sea helicity, $\Delta(s+\bar{s})_{AB} = 0$, and thus the experimentally required reduction of the large ('naive') $\Delta\Sigma_{AB} \simeq 3F - D$ in (3.6) is accomplished entirely by the gluon helicity Δg [9,10]. This point of view depends of course merely on the theorist's choice of scheme, whereas the directly measurable $\Delta\Sigma(Q^2)$ in eqs. (3.1), (3.4) and (3.5) has to remain independent of this choice.

3.2 Bjorken sum rule

This absolutely fundamental sum rule [34], derived from isospin invariance and thus being a pure flavor non-singlet relation, acquires significant subasymptotic corrections in perturbative QCD

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6}g_A\left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.5833\left(\frac{\alpha_s}{\pi}\right)^2 - 20.2153\left(\frac{\alpha_s}{\pi}\right)^3 - \ldots\right] + \frac{C_{HT}}{Q^2} \tag{3.8}$$

for $f = 3$ light flavors and where the NLO $\mathcal{O}(\alpha_s)$ correction to the asymptotic ('naive' parton level) Bjorken sum rule (2.8) can be directly read off eq. (3.1) and the higher order corrections have been calculated in [35]. The ellipses refer to estimates of even higher order and renormalon (Padé resummed) terms [36], and C_{HT} denotes nonperturbative higher-twist estimates [37]- [41]. The latter estimates, being mainly derived from QCD sum rules and the MIT bag model, lie in the range $C_{HT} \simeq -0.025$ to $+0.03$ GeV^2. At $Q^2 = 10$ GeV^2 the r.h.s. of (3.8) predicts in LO, NLO(α_s), etc.:

$$\Gamma_1^{p-n}(10) = \underset{\text{LO}}{0.210}, \quad \underset{\text{NLO}}{0.193, \ldots,} \quad \underset{\text{total}}{0.187 \pm 0.002} \tag{3.9}$$

and typical recent experimental (NLO) estimates of Γ_1^{p-n} are [18,19,23,24]

$$\Gamma_1^{p-n}(10)_{\text{SMC}} = 0.195 \pm 0.029\,, \quad \Gamma_1^{p-n}(5)_{\substack{\text{E143}\\ \text{E154}}} = 0.171 \pm 0.013 \tag{3.10}$$

whereas combining (all) relevant experiments results in [33,42]

$$\Gamma_1^{p-n}(5)_{\text{all}} = 0.202 \pm 0.022\,, \quad \Gamma_1^p(5)_{\text{all}} - \Gamma_1^n(5)_{\text{E154}} = 0.192 \pm 0.024\,. \tag{3.11}$$

Comparing these results with (3.9) we can safely conclude that the Bjorken sum rule is, within 1σ, *confirmed*.

An alternative general way to test the Bjorken sum rule is to *determine* g_A from an NLO analysis of $g_1^{p,n,d}(x,Q^2)$ *without* the constraint (2.8), although this might somehow overstretch presently available scarce data. Fixing $\alpha_s(M_Z^2) = 0.118 \pm 0.005$ and, very conservatively, $\Delta q_8 = 0.6 \pm 0.2$, such an analysis yields [12,13] $g_A = 1.19 \pm 0.09$ which is again consistent with the (far more accurate) result (2.8) derived from neutron β-decay.

3.3 Drell–Hearn–Gerasimov sum rule

The DHG sum rule [43] is a prediction for Γ_1 at $Q^2 = 0$,

$$I_N(0) = -\frac{1}{4}\kappa_N^2 \tag{3.12}$$

with the anomalous magnetic moments $\kappa_p = 1.79$ and $\kappa_n = -1.91$, and where [44] $I_N(Q^2) \equiv (2M_N^2/Q^2)\Gamma_1^N(Q^2)$. It can be considered as giving qualitative information on the magnitude of higher twist (resonance) effects in the region between $Q^2 = 0$ and the present DIS high energy data. As $Q^2 \to 0$ (at fixed ν), an enormous activity of $I_N(Q^2)$ is expected due to excitations of resonances (Δ, N^*) and possibly of strange resonances ($\gamma N \to K\Lambda^*, K\Sigma^*$) as well as non-resonant contributions (associated, in part, with VMD), which has been nicely reviewed and discussed by Steven Bass [45]. It should, however, be emphasized that DIS perturbative-QCD (twist-2) expectations for $I_N(Q^2)$ are fully operative down to $0.5 - 1$ GeV$^2 \lesssim Q^2$, which agree with recent measurements [46] – a situation very similar to the unpolarized HERA experiments [21]. For example, $I_p(Q^2 \gtrsim 0.5$ GeV$^2)$ continuously increases as Q^2 decreases from, say, 10 to 0.5 GeV^2, where $I_p(0.5) \simeq 0.15$ [46]. Such a behavior is not in agreement with the original expectations of a VMD inspired Q^2-extrapolation model [44,47] where the turn-over, i.e. a zero of $I_p(Q^2)$ was predicted at $Q^2 \simeq 1$ GeV^2. Recent E143 data [46], however, seem to be consistent with a zero at $Q^2 \simeq 0.2$ GeV^2, which can be reconciled with a somewhat modified Burkert–Ioffe model [46] as well as with the extrapolation model of Soffer and Teryaev [48]. Clearly, due the DHG sum rule (3.12), $\Gamma_p(Q^2 < 0.5$ GeV$^2)$ has eventually to turn negative in order to reach its strongly negative limiting value -0.80 as advocated by DHG. This is a very violent Q^2 dependence within $0 \le Q^2 < 0.5$ GeV^2 indeed! There will (hopefully) be a lot of forthcoming and interesting measurements in this at present unexplored very small Q^2 region at SLAC, DESY-Hermes, CEBAF (Jefferson, Newport News), ELSA (Bonn), MAMI (Mainz), MIT-Bates, NIKHEF, etc. [45] in the not too distant future.

3.4 Helicity sum rule: The Spin of the Proton

Since each component of the helicity sum rule (1.1) depends, in general, separately on the scale Q^2,

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta g(Q^2) + L_z^{q+g}(Q^2) \tag{3.13}$$

the question 'what partonic components of the nucleon are responsible for its total spin $\frac{1}{2}$?' will depend sensitively on the scale Q^2 where the structure of the nucleon is probed. Note that this sum rule can be rigorously formulated in terms of the QCD angular momentum operator [49] and, although the gluonic terms are not separately gauge invariant [50], eq. (3.13) is to be understood in the physical light-like gauge ($A^+ = 0$) where the expectation values of the local field operators reduce to the standard parton densities in the infinite momentum frame [51].

Let us study the separate contributions on the r.h.s. of (3.13) as obtained from recent LO and NLO($\overline{\text{MS}}$) analysis [6,8,18,19] being based on the input ansatz (2.11). Typical representative 'optimal' fit results are as follows:

	Q^2/GeV^2	$\Delta\Sigma$	Δg
LO/NLO($\overline{\text{MS}}$):	$Q_0^2 = 0.23$ / 0.34	0.23 / 0.18	0.36 / 0.051
	5	0.23 / 0.17	1.3 / 1.5
	10	0.23 / 0.166	1.57 / 1.74

It is very interesting to observe that at low input scales $Q_0^2 \simeq 0.3$ GeV^2 the nucleon's spin is dominantly carried just by the total helicities of quarks and gluons

$$\frac{1}{2}\Delta\Sigma(Q_0^2) + \Delta g(Q_0^2) \simeq \frac{1}{2}, \tag{3.14}$$

and thus the helicity sum rule (3.13) implies

$$L_z^{q+g}(Q_0^2) \simeq 0 \tag{3.15}$$

as intuitively expected for *low* (bound-state-like) scales. The spin of the proton is therefore carried solely by quarks and gluons, i.e. there is no 'spin surprise' whatsoever. It should be emphasized that this is a genuine result of LO/NLO analyses, i.e. it has *not* been implemented as an input at $Q^2 = Q_0^2$. At smaller distances, i.e. for $Q^2 \gg Q_0^2$, this picture will break down since radiative gluon and $q\bar{q}$ production off the initial partons will increase their k_T which in turn gives rise to a finite orbital component $L_z^{q+g}(Q^2)$ in (3.13). Clearly a finite (negative) $L_z^{q+g}(Q^2)$ is required to reconcile, for example, $\frac{1}{2}\Delta\Sigma(Q^2) + \Delta g(Q^2) \simeq 1.8$ at $Q^2 = 10$ GeV^2 (according to the above Table) with the sum rule (3.13). Since $\Delta g(Q^2) \sim \ell n\, Q^2$, $L_z^g(Q^2) \sim -\ell n\, Q^2$ becomes more and more negative as Q^2 increases [8]. The relevant RG Q^2-evolution equations for $L_z^{q,g}(Q^2)$ have been

written down in LO a couple of years ago [52] which predict asymptotically ($Q^2 \to \infty$)

$$J_q \equiv \frac{1}{2}\Delta\Sigma + L_z^q(Q^2) \;\to\; \frac{1}{2}\,\frac{3f}{16+3f} = 0.18$$
$$J_g \equiv \Delta g(Q^2) + L_z^g(Q^2) \;\to\; \frac{1}{2}\,\frac{16}{16+3f} = 0.32, \qquad (3.16)$$

for $f = 3$ active light flavors, which corresponds to a similar partition as the well known one of quark and gluon momenta in the unpolarized nucleon. Since $\Delta\Sigma \simeq 0.2$ is independent of Q^2 in LO (and does not change very much in NLO, see above Table) one might conlude from (3.16) that L_z^q remains small, $L_z^q(Q^2) \lesssim 0.1$, in contrast to the *large* and negative $L_z^g(Q^2) \sim -\Delta g(Q^2)$ in order to compensate the strong rise of $\Delta g(Q^2) \sim \ln Q^2$.

An alternative but very different scenario for the proton spin is obtained from the small and oscillating $\delta g(x, Q^2)$ in fig. 1 which results in $\Delta g(Q^2) \simeq -0.15$ as well as $\Delta\Sigma \simeq 0.3$, almost independently of Q^2. This almost 'static' (fit) solution [8], being also consistent with all present data, can be easily understood by considering the evolution equation for $\Delta g(Q^2)$ for $f = 3$ active flavors:

$$\frac{d\Delta g(Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\left(2 + \frac{\alpha_s}{2\pi}19\right)\Delta\Sigma + \frac{\alpha_s}{2\pi}\left(\frac{\beta_0}{2} + \frac{\alpha_s}{2\pi}\frac{\beta_1}{4}\right)\Delta g \qquad (3.17)$$

with $\beta_0 = 9$ and $\beta_1 = 64$. The 'static' condition $d\Delta g_{\rm static}/d\ln Q^2 = 0$ then implies

$$\Delta g_{\rm static} \simeq -0.5\,\Delta\Sigma \simeq -0.15 \qquad (3.18)$$

which depends only weakly on Q^2 since, furthermore, $d\Delta\Sigma/d\ln Q^2 = \mathcal{O}(\alpha_s^2)$. Thus $\frac{1}{2}\Delta\Sigma + \Delta g_{\rm static} \simeq 0$ 'once and forever' and from (3.13) we have $L_z^{q+g}(Q^2) \simeq \frac{1}{2}$, i.e. the proton spin is mainly of orbital origin at almost any scale. This very unusual ('funny') explanation of the proton spin is hard to swallow, but cannot be ruled out by present experiments. A similarly small and negative gluon helicity has been considered previously within the bag model [51] and we learned at this Workshop that instanton induced quark-gluon interactions give also rise to a negative total gluon polarization [53].

Although we have reached some formal theoretical understanding of the orbital contributions $L_z^{q,g}(Q^2)$ to the spin sum rule (3.13), there appears to be no immediate way to relate them to experimental observables in order to measure not only their size but also the *sign* of $L_z^{q,g}$. A possible measurement of azimuthal distributions of the current-jet has been proposed some time ago [54] but these are only sensitive to some average $\langle k_T^2 \rangle$ of rotating constituents in a polarized nucleon target. More recently, Ji [50,55] has suggested to use deeply virtual Compton scattering (DVCS) $\gamma^*(Q^2)p \to \gamma p'$ in the limit of vanishing momentum transfer $t = (p' - p)^2$ in order to get direct information about $J_{q,g}(Q^2)$ in (3.16) appearing in the spin sum rule (3.13): $\frac{1}{2} = J_q + J_g$. Although this topic has been thoroughly discussed by Anatoli Radyushkin [56] and Lech Mankiewicz [57], let me briefly sketch the main idea. Expressing the matrix element of the QCD energy-momentum tensor $T^{\mu\nu}$ at some renormalization scale Q^2 in terms of Dirac- and Pauli-like form factors

A and B, respectively,

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle \sim A_{q,g}(t, Q^2), \;\; B_{q,g}(t, Q^2), \ldots \qquad (3.19)$$

one finds that in the forward limit $t \to 0$ [50,55]

$$J_{q,g}(Q^2) = \frac{1}{2}\left[A_{q,g}(0, Q^2) + B_{q,g}(0, Q^2)\right]. \qquad (3.20)$$

The A and B form factors are in turn related to new *non*-forward twist-2 helicity non-flip and flip parton densities (amplitudes) $H(x, \xi, t, Q^2)$ and $E(x, \xi, t, Q^2)$, etc. with $\xi = Q^2/(p + p') \cdot q$, which parametrize the DVCS amplitude [50, 55, 58, 59] and which are experimentally accessible, at least in principle, at $t \neq 0$. These non-forward parton distributions have the characters of both ordinary parton densities and nucleon form factors: in the (usual) forward limit $t \to 0$, $H(x, 0, 0, Q^2) = q(x, Q^2)$, etc., whereas their first ($n = 1$) moments equal the well known Dirac and Pauli nucleon form factors $F_{1,2}(t)$. The most interesting sum rule relevant to the nucleon spin is obtained for the $n = 2$ moment [50, 56, 59]

$$\int_{-1}^{1} x\left[H_q(x, \xi, t, Q^2) + E_q(x, \xi, t, Q^2)\right] dx = A_q(t, Q^2) + B_q(t, Q^2) \qquad (3.21)$$

where (luckily) the ξ dependence, as well as contributions from other form factors of $T^{\mu\nu}$ in (3.19), drops out. Thus, once the l.h.s. of (3.21) has been measured (estimated) experimentally, one needs to extrapolate this sum rule to $t = 0$, to obtain the total quark (and hence the quark orbital L_z^q) contribution J_q to the nucleon spin according to eq. (3.20). Undoubtedly, the experimental determination of these moments in (3.21) will be very difficult and delicate, but even more so will be the required (model dependent) extrapolation to $t = 0$ [55, 56, 60, 61]. As $t \to 0$ there are, among other things, contaminations from the Bethe-Heitler process (which corresponds to the emission of a real photon by the initial and final lepton lines), since the BH amplitude is proportional to $1/t$ as compared to the DVCS amplitude $\sim 1/Q^2$. To overcome this problem one has to increase the $\gamma^*(Q^2)$ flux of the DVCS by *increasing* the beam energy in order to keep the BH process under control when extrapolating to $t = 0$ as was demonstrated by a recent model calculation [61]. Thus in this respect the CERN-COMPASS experiment ($E_\mu \simeq 100$ GeV) [62] will be in much better shape than DESY-HERMES or CEBAF (Jefferson) with their much lower beam energies.

Although the LO RG Q^2-evolution equations for the parton-helicity independent and dependent non-forward parton densities $H_{q,g}(x, \xi, t, Q^2)$ etc. are already well known [55, 56, 58, 60, 63, 64], there remain a lot of questions and unsolved problems: Is there a factorization theorem [60, 65] for DVCS? What is the ξ-dependence and the $x \to 0$ behavior of $H_{q,g}, E_{q,g}$, etc.? How to extrapolate the form factors to $t = 0$? How to measure the gluonic orbital component via $J_g(Q^2)$, i.e. the non-forward gluon densities? (perhaps via exclusive vector-meson electroproduction [57]?), etc.

Eventually such questions have to be answered, because without analyzing and *measuring* $L_z^{q,g}(Q^2)$, our understanding of the nucleon's spin is bound to remain rudimentary! The knowledge of $\Delta\Sigma$ and Δg is *not* enough! - But we don't even know experimentally $\Delta g(Q^2)$!

4 How to measure $\delta g(x,Q^2)$ [and the photonic δg^γ]

Let me briefly recapitulate the most relevant processes which are sensitive to $\delta g(x,Q^2)$, as well as to $\delta g^\gamma(x,Q^2)$ of the resolved photon, and thus should provide us with first *measurements* of the polarized gluon densities in the (hopefully) not too distant future.

• $\vec{\gamma}^{(*)}\vec{p} \to c\bar{c}X$. At presently available energies, photo- or DIS-production of charm will be the best source for δg due to the LO photon-gluon fusion subprocess $\vec{\gamma}^{(*)}\vec{g} \to c\bar{c}$. (Unfortunately, a NLO $-\alpha_s^2$ calculation is still missing.) The DESY-HERMES ($\geq$ 1998) [66] and SLAC-E156 [67] experiments will measure δg at $x \simeq 0.3$ with a statistical accuracy of $\delta(\delta g/g) \lesssim 0.3$ and ~ 0.02, respectively, with the charm asymmetry A^c being typically about 5%. (When completing this written version of my talk, I was informed that the E156 proposal was turned down by the selection committee - what unexpected sad news!) The CERN-COMPASS experiment [62] is expected to reach a statistical accuracy for $\delta g/g$ of about 0.1 at $x \simeq 0.15$. The (open) charm events will be triggered via the $D^0/\bar{D}^0 \to K^-\pi^+/K^+\pi^-$ decays. It should be noted, however, that most of the expected rates for the charm asymmetries are based on the sizeable ('optimal' fit) δg-densities in fig. 1, whereas the small oscillating δg (giving rise to a 'static' $\Delta g \simeq 0.15$) would render A^c unmeasurably small.

• $\vec{\gamma}\vec{p} \to c\bar{c}$, j, jjX. Photoproduction would be one of the *most* interesting processes to be studied at a future (fully) polarized HERA($\vec{e}\vec{p}$) collider [68,69] as has been discussed in great detail by Werner Vogelsang [70]. In particular single-jet and dijet production will give realistic experimental access to the *resolved* photon structure as well, i.e. to measure the gluon and quark content of photons, δg^γ and δq^γ, with $\delta g^\gamma(x,Q^2)$ dominating the 1-j and 2-j event rates, which are *new*, experimentally entirely unknown quantities. The usual 'direct' photon (pointlike $\gamma - q$ coupling) contributes via the subprocesses $\vec{\gamma}\delta q \to q$ and $\vec{\gamma}\delta g \to q\bar{q}$ etc. to 1-j and 2-j rates, whereas the 'resolved' photon contributes in addition via purely hadronic subprocesses $\delta g^\gamma \delta q \to q$, $\delta g^\gamma \delta g \to q\bar{q}$ etc. In other words, the experimental signatures for δg are 'contaminated' by the (experimentally) unknown δg^γ. A careful analysis in specific kinematic regions where the 'direct' contributions are suppressed [68,70] will then allow for an experimental extraction of $\delta g^\gamma(\delta q^\gamma)$. In order to calculate the expected size of the 'resolved' contributions, we need some theoretical estimates for $\delta f^\gamma(x,Q^2) = f_+^\gamma - f_-^\gamma$. Since these parton densities of a longitudinally (more precisely, circularly) polarized photon are not as well established within the spin-physics community as are the parton densities of an unpolarized spin-averaged photon

(see, e.g. [71] for a recent review), let me briefly comment on them. Being formally very similar to the spin-averaged counterparts $f^\gamma = f_+^\gamma + f_-^\gamma$, the $\delta f^\gamma(x,Q^2)$ are theoretically 'under control': the LO Q^2-evolution equations have been known for quite some time [72] and have been extended to NLO recently [73]. The essential difference to the purely hadronic parton densities $\delta f(x,Q^2)$ is that the $\delta f^\gamma(x,Q^2)$ satisfy *inhomogeneous* RG evolution equations. For example in LO

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\delta\Sigma^\gamma\\ \delta g^\gamma\end{pmatrix} = \frac{\alpha}{2\pi}\begin{pmatrix}\delta P_{q\gamma}^{(0)}\\ 0\end{pmatrix} + \frac{\alpha_s}{2\pi}\delta\hat{P}^{(0)} \otimes \begin{pmatrix}\delta\Sigma^\gamma\\ \delta g^\gamma\end{pmatrix} \tag{4.1}$$

where the inhomogeneous term derives from the 'pointlike' photon splitting $\gamma \to q$ which can be calculated from first principles. The general solution decomposes into $\delta f^\gamma(x,Q^2) = \delta f_{PL}^\gamma + \delta f_{had}^\gamma$ with the 'pointlike' component being driven by the inhomogeneous photon splitting $\delta P_{q\gamma}^{(0)}$ which uniquely determines $\delta f_{PL}^\gamma(x,Q^2) = [\alpha/\alpha_s(Q^2)]\delta P_{q\gamma}^{(0)}(x)\ldots$ once an appropriate hadronic input scale Q_0^2 has been specified. This in in contrast to the hadronic components δf_{had}^γ which are derived from the usual homogeneous part of the evolution equation (4.1) and thus require, as usual, also the specification of the (VMD oriented) input densities $\delta f^\gamma(x,Q_0^2)$. Being guided by the successful dynamical predictions for unpolarized parton densities with an input at $Q_0^2 \simeq 0.3$ GeV^2 [20,71,74], somewhat realistic estimates for $\delta f^\gamma(x,Q^2)$ have been suggested in LO and NLO [73,75] which are based on a 'minimal' input $\delta f_{had}^\gamma(x,Q_0^2) = 0$ and on a 'maximal' (c.f. (2.3)) input $\delta f_{had}^\gamma(x,Q_0^2) = f_{had}^\gamma(x,Q_0^2)$. Typical quantitative predictions for δq^γ and δg^γ can be found in [73], for example. These results have been used to calculate the expected rates for $c\bar{c}$, 1-j and 2-j production [68,70]. For example, the dominant δg ('direct' photon) and δg^γ ('resolved' photon) contributions to $\vec{\gamma}\vec{p} \to jX$ can be experimentally separated and measured at pseudorapidities $\eta_{lab} < 0$ and $\eta_{lab} > 0$, respectively. Similarly, a measurement of the pseudorapidity distributions of dijet production in $\vec{\gamma}\vec{p} \to jjX$ would be a sensitive probe of δg and could even discriminate between the various δg scenarios shown in fig. 1 in the 'direct' photon regime (where, by definition, $x_\gamma^{obs} > 0.75$), whereas the 'resolved' regime ($x_\gamma^{obs} < 0.75$) will allow for a measurement of δg^γ. Even a measurement of just a non-vanishing asymmetry in the latter case would prove already the existence of the polarized 'resolved' contribution, i.e. the existence of δg^γ! For both cases, an integrated luminosity of about 100 pb^{-1} would be sufficient at a future polarized HERA collider.

It should be emphasized that a polarized HERA($\vec{e}\vec{p}$) collider will be a *unique* facility worldwide for measuring $\delta f^\gamma(x,Q^2)$! It should be kept in mind that, in contrast to previous intentions, there will be no polarized LEP($\vec{e}^+\vec{e}^-$) collider and therefore no measurements of the polarized photon structure function $g_1^\gamma(x,Q^2)$ in the foreseeable future.

• $\vec{p}\vec{p} \to c\bar{c}$, $b\bar{b}X$. Hadronic heavy quark production proceeds via the (so far available) LO subprocesses $\delta g\delta g \to Q\bar{Q}$ and $\delta q\delta\bar{q} \to Q\bar{Q}$. It appears to be a very sensitive and presumably the most realistic test of $\delta g(x,Q^2 \simeq 4m_Q^2)$ since δg enters 'quadratically', with the $\delta q\delta\bar{q}$ contribution being small [76]. (Again, the NLO-α_s^3 contributions are entirely unknown.) Realistic measurements of δg will be possible preferably at medium RHIC [77]

energies, $\sqrt{s} \lesssim 100$ GeV, or at future HERA-$\vec{N}$(phase II) energies [78], $\sqrt{s} \simeq 40$ GeV, where the charm and bottom asymmetries A^c_{pp} and A^b_{pp} are about a few percent. Since $A^{c,b}_{pp}$ decrease for increasing energies, they will become unmeasurably small (less than 1%) for $\sqrt{s} \gtrsim 200$ GeV. The situation is similar for J/ψ production [78,79].

• $\vec{p}\vec{p} \to \gamma, \gamma j, \ldots X$. Prompt photon production will serve as a clean and discriminative probe of $\delta g(x, Q^2 \simeq p_T^{\gamma 2})$ since it is dominated already in LO by the gluonic Compton subprocess $\delta g \delta q \to \gamma q$, with the annihilation subprocess $\delta \bar{q} \delta q \to \gamma g$ being small. Here the NLO α_s^2 contributions have been fully calculated as well [80] (and references therein). The situation is very similar to unpolarized prompt photon production, $pp \to \gamma X$ dominated by $gq \to \gamma q$, which serves as the cleanest direct probe of $g(x, p_T^{\gamma 2})$ in the medium to small-x region (see, e.g. [81]). Indeed the longitudinal double-spin asymmetry A^γ_{LL} at HERA-$\vec{N}$(II), $\sqrt{s} \simeq 40$ GeV, will be a very sensitive probe of δg and is capable of discriminating between almost all present δg scenarios (cf. fig. 1) for $p_T^\gamma < 10$ GeV and $-1 \lesssim \eta \lesssim +1$ [80]. Similarly at RHIC, $\sqrt{s} \simeq 500$ GeV, a measurement of $A^\gamma_{LL}(\eta = 0)$ will provide us with a very sensitive and discriminative probe of $\delta g(x, p_T^{\gamma 2})$ as long as $p_T^\gamma \lesssim 60$ GeV [82]. Here the expected statistical accuracy is $\delta[\delta g/g(0.05 \lesssim x \lesssim 0.3)] \simeq 0.01 - 0.3$ for 800 pb^{-1}. The strong sensitivity to δg can be easily understood from the approximate relation $A^\gamma_{LL}(\eta = 0) \simeq \frac{3}{8}[\delta g(x_T, Q^2)/g(x_T, Q^2)] A_1^p(x_T, Q^2)$ where $x_T = 2p_T^\gamma/\sqrt{s}$, $Q^2 \simeq p_T^{\gamma 2}$ and with the DIS asymmetry $A_1^p = g_1^p/F_1^p$ [82]. Analogous favorable results have been obtained for $\gamma + j$ and $J/\psi + j$ production [78].

5 Semi-inclusive asymmetries

The important aim of semi-inclusive measurements is, apart from measuring fragmentation functions [83], flavor separation, i.e. to disentangle the individual distributions δu_v, δd_v, $\delta \bar{u}$, $\delta \bar{d}$ and $\delta s = \delta \bar{s}$. Let me start with the DIS semi-inclusive reaction

• $\vec{e}\vec{p} \to ehX$ with $h = \pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0$, etc. which is presently being measured and analyzed by SMC [84] and HERMES [85]. Particularly interesting and useful are asymmetries for the difference of π^+ and π^- production where, in LO, the fragmentation functions $D^h_{\bar{q},q}$ cancel (see, e.g. [86]):

$$A_p^{\pi^+ - \pi^-}(x, Q^2) = \frac{4\delta u_v - \delta d_v}{4u_v - d_v}, \qquad A_d^{\pi^+ - \pi^-}(x, Q^2) = \frac{\delta u_v + \delta d_v}{u_v + d_v}. \tag{5.1}$$

Likewise, for kaon production

$$A_p^{K^+ - K^-}(x, Q^2) = \delta u_v / u_v, \qquad A_p^{K^0 - \bar{K}^0}(x, Q^2) = \delta d_v / d_v \tag{5.2}$$

which will allow to measure *directly* the polarized valence densities and to test their large-x behavior. We also learned from HERMES [85] that they have measured the $\pi^0 = \frac{1}{2}(\pi^+ + \pi^-)$ asymmetry $A_n^{\pi^0}(x, Q^2)$ and we are eagerly awaiting its analysis since it

can be directly used to extract $\delta s(x, Q^2)$ due to [86] $A_n^{\pi^0} \sim \delta s/s(A_1^n, D_q^{\pi^0})$, i.e. provided one uses $s(x, Q^2)$, A_1^n and the π^0 fragmentation function as input from other experiments. The NLO corrections to these asymmetries have been analyzed too [87].

• $p\vec{p} \to W^\pm X$. This $W^\pm$ production process, with only the target being longitudinally polarized, will give rise to non-vanishing parity-violating *single* spin asymmetries $A_L^{W^\pm}$ due to the combination of the γ_5 from the polarized particle and the γ_5 from the axial-vector coupling [88]. Clearly RHIC ($\sqrt{s} \simeq 500$ GeV) will play a decisive role for $W^\pm$ and Z^0 production where $A_L^{W^\pm}(y)$ can be direct measures of $\delta q(x, M_W^2)$, $x \simeq M_W/\sqrt{s}$, in specific kinematic regions like

$$\begin{aligned} A_L^{W^+} &\sim \delta u/u, \quad & A_L^{W^-} &\sim \delta d/d \quad & \text{for} \quad y \simeq +1 \\ A_L^{W^+} &\sim -\delta \bar{d}/\bar{d}, \quad & A_L^{W^-} &\sim -\delta \bar{u}/\bar{u} \quad & \text{for} \quad y \simeq -1 \end{aligned} \tag{5.3}$$

which amount to about 5 - 50% [82,88]. Alternatively, of course, the doubly polarized Drell-Yan process (being in the unpolarized case the classical probe for sea densities), $\vec{p}\vec{p} \to \mu^+\mu^- X$, will also provide us with direct access to $\delta \bar{q}$ at RHIC as well as HERA-$\vec{N}$(II) [89].

• $\vec{e}\vec{p} \to \nu D X$. Semi-inclusive DIS D-meson production via charged current reactions at a future polarized HERA($\vec{e}\vec{p}$) collider will be a direct probe of the strange sea densities: due to the main subprocesses $W^+ s \to D$ and $W^- \bar{s} \to \bar{D}$, the single asymmetries (a polarized proton beam would suffice) are

$$A^D_{W^+} \sim \delta s/s, \qquad A^{\bar{D}}_{W^-} \sim -\delta \bar{s}/\bar{s} \tag{5.4}$$

which turn out to be sizeable, about $\pm$ (10 to 40)% [90].

• $\vec{e}p(\text{or } e\vec{p}) \to e\vec{\Lambda}X$. This reaction will serve as a sensitive test of the so called 'polarization-(spin-)transfer' to the final state Λ. There are two possible scenarios for the spin-transfer to the $\Lambda \sim (uds)$. In the naive approach [91], which relies on the naive quark model where the spin of the Λ is carried by the strange quark's spin, it is expected that the negative strange sea polarization in the polarized proton will be transferred to the longitudinal Λ polarization in the current fragmentation region. The longitudinal polarization of the Λ is thus given by

$$\mathcal{P}_\Lambda(x, z, Q^2) = \delta s(x, Q^2) \delta D^\Lambda_s(z, Q^2) / s(x, Q^2) D^\Lambda_s(z, Q^2), \tag{5.5}$$

i.e. one assumes 'naively' $\delta D^\Lambda_{f \neq s} = 0$. Beyond the nonrelativistic quark model, the Λ spin is shared, in general, among the u, d, s quarks in the Λ which will contribute to $\mathcal{P}_\Lambda$. This is the second scenario [92] where $\mathcal{P}_\Lambda \sim \Sigma_q e_q^2 \delta q \delta D^\Lambda_q \ldots$. It has been shown in [93] to LO and NLO that recent LEP data (i.e., a negative large polarization-transfer at large z) are insufficient to discriminate between the two scenarios, whereas a clear distinction would be possible in semi-inclusive DIS with the ongoing and planned HERMES [94] and COMPASS [62] experiments!

• $\vec{e}\vec{p} \to \overset{(-)}{\nu} X$. High energy DIS at large Q^2 allows to probe spin effects in weak charged (as well as neutral) current interactions, giving rise to polarized weak structure functions such as $g_1^{W^+} = \delta d + \delta s + \delta\bar{u}$, $g_5^{W^+} = -(\delta d + \delta s - \delta\bar{u})$, etc., which will provide us with clean tests of the (flavor non-singlet) valence densities δq_v [90,95]. (Note that there is, unfortunately, no uniform notation for these structure functions in the literature: g_5 has been originally defined as g_3, and sometimes is also denoted by $-g_5$, etc.). Furthermore, there is a wealth of new (integral) relations and sum rules between them [96] (for twist-2 and twist-3 contributions). The polarized HERA($\vec{e}\vec{p}$) collider will be again a *unique* facility worldwide for such measurements and involved tests, although it may take some time before they can be realized.

6 Cosmological spin–offs

Let me just briefly remind you that polarization experiments may also be relevant to astrophysics and cosmology [36,97]. For example, one of the favored (supersymmetric) candidates for Cold Dark Matter is the lightest neutralino χ, which has spin-dependent couplings with nucleons that would be responsible for its capture by the sun (which could be detected by high-energy solar neutrinos produced in $\chi\chi \to f\bar{f}$ with $f = \tau, c, b$), and would contribute to elastic χ scattering off nuclei in laboratory searches of supersymmetric CDM candidates. Since the spin-dependent matrix elements contributing to $\sigma(\chi p \to \chi p)$ is related to axial-current matrix elements, the neutralino capture rate and laboratory CDM searches will strongly depend on the total (anti)quark helicities, $\sigma(\chi p \to \chi p) \sim (\Delta q + \Delta\bar{q})^2$. Thus, changes of

$$\begin{aligned} \Delta(u+\bar{u}) &= \tfrac{1}{2}(\Delta q_8 + \Delta q_3) + 2\Delta s \simeq 0.79 \\ \Delta(d+\bar{d}) &= \tfrac{1}{2}(\Delta q_8 - \Delta q_3) + 2\Delta s \simeq -0.47 \end{aligned} \tag{6.1}$$

from their canonical values (2.8) and (2.9), $\Delta(u+\bar{u}) = \frac{1}{2}(\Delta q_8 + \Delta q_3) = 2F \simeq 0.92$ and $\Delta(d+\bar{d}) = \frac{1}{2}(\Delta q_8 - \Delta q_3) = F - D \simeq -0.34$, due to the finite (negative) Δs in (3.4) can reduce the neutrino flux from the sun and can greatly enhance elastic χ scattering off neutron-rich nuclei, $\sigma(\chi n \to \chi n)$, in laboratory CDM searches [97].

7 Non–perturbative approaches

Lattice calculations provide us with total helicities Δq (or more generally with the $n-th$ moments of parton densities) in the continuum limit $am_q \to 0$. The most recent improvement in this field is the implementation of an improved action by the DESY/HLRZ collaboration [98], i.e. a systematic procedure for the removal of all terms linear in the lattice spacing a from the lattice observables [99,100] which reduces the cut-off errors order by order in a, yielding a better extrapolation towards the continuum limit [98]. This $\mathcal{O}(a)$

improved lattice theory yields, for example, for the valence helicities $\Delta u_v = 0.841(52)$ and $\Delta d_v = -0.245(15)$, in reasonable agreement with DIS experiments (cf. Sect. 3 and [18]). Previous lattice calculations [101] also attempted to calculate sea quark contributions (which might be questionable in a quenched approximation) with the total result

$$\Delta(u+\bar{u}) \simeq 0.64, \quad \Delta(d+\bar{d}) \simeq -0.35, \quad \Delta(s+\bar{s}) \simeq -0.11 \; : \; \Delta\Sigma \simeq 0.2 \tag{7.1}$$

which compares favorably with the experimental results (3.4) [18]. While furthermore the lattice determination of the $F/D \simeq 0.63$ ratio is consistent with data from hyperon β-decays in eqs. (2.8) and (2.9), i.e. $F/D \simeq 0.58$, a discrepancy of about 10% persists [98,101] between the lattice results for $g_A \simeq 1.1$ and the neutron β-decay value in (2.8), $g_A \simeq 1.26$.

Very promising appears to be the chiral soliton approach towards the structure of the nucleon within the effective chiral theory which allows for a calculation of the *full* x-dependence of structure functions and parton densities from first principles [102]. Polarized, as well as unpolarized, valence and sea quark input densities have been calculated, taking into account the influence of the instanton vacuum, at a typical scale set by the inverse average instanton size $\bar{\rho}$, i.e. $Q_0^2 \simeq (\bar{\rho})^{-2} \simeq (0.6\,\text{GeV})^2$. What makes this approach so very promising is the fact that it predicts [102], besides the valence densities, a *valence*-like input (unpolarized) sea density in the small-x region at $Q_0^2 = 0.3 - 0.4\,\text{GeV}^2$, which forms the basic ingredient for understanding and predicting all small-x unpolarized DIS HERA data [21] from pure (parameter-free) QCD dynamics [20]. So far, the polarized sea and the (un)polarized gluon input densities have not been calculated. It is in particular the valence-like *gluon* densities [6,8,18,20] which, being $1/N_c$ 'suppressed', have to come out *sizeable* at $Q_0^2 = 0.3 - 0.4\,\text{GeV}^2$! Otherwise the chiral soliton approach does not refer to a perturbative (twist-2) input scale reachable by perturbative RG Q^2-evolutions, but instead would refer to some nonperturbative bound-state-like constituent-quark scale. Nevertheless, for the time being, this approach seems to be a the most important forefront of nonperturbative QCD and might eventually link, from first principles, the confining world to the perturbative sector.

8 The transverse spin structure function $g_2(x, Q^2)$

For DIS off a transversely polarized nucleon target, $\vec{e}p^{\uparrow} \to eX$, the resulting transverse asymmetry A_2 is proportional to the combination $g_T \equiv g_1 + g_2$ which is really the 'transverse spin structure function' although, for obvious reasons, one usually just refers to g_2 [103]. Since g_2 is related to a transverse polarization, no simple parton interpretation can be made for it [103,104]. (Recall that the quark-spin operator projected along the transverse nucleon spin, $\Sigma_T = \gamma_0\gamma_5\not{s}_T$ with $\not{s}_T \sim \gamma^1$, does *not* commute with the free Hamiltonian $H_0 = \alpha_x p_x$ and thus there is *no* energy eigenstate $|p_x\rangle$ such that $\Sigma_T|p_x\rangle = \lambda_T|p_x\rangle$.) Nevertheless, a transverse-spin *average* for quarks can still be defined in the nucleon and it is just g_T which is sensitive to the quark-gluon interactions. Furthermore, g_2 vanishes [104]

for a free (massless or massive) quark, i.e. for a pointlike nucleon, and thus g_2 cannot be expressed as an incoherent sum over free on-shell partons. The partons must be interacting (quark-gluon correlations) and/or virtual in order to contribute to g_2. Therefore g_2 will serve as a unique test ground for '*higher* twists' (> 2), in particular for twist-3 (not suppressed by inverse powers of Q^2).

Therefore one makes the general decomposition

$$g_2 = g_2^{WW} + \bar{g}_2 \tag{8.1}$$

with the unknown twist-3 contribution $\bar{g}_2$. The known twist-2 'Wandzura-Wilczek' piece g_2^{WW} receives contributions from the same class of twist-2 light-cone operators which determine g_1, giving rise to the so-called Wandzura-Wilczek relation [105]

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y}\, g_1(y,Q^2). \tag{8.2}$$

This relation immediately follows from the light-cone OPE which gives for the n-th moments

$$\int_0^1 x^{n-1} g_1\, dx = \frac{1}{2} a_n, \qquad n \geq 1 \tag{8.3}$$

$$\int_0^1 x^{n-1} g_2\, dx = \frac{1}{2}\frac{n-1}{n}(d_n - a_n), \qquad n > 1 \tag{8.4}$$

where a_n and d_n denote the nucleon matrix elements of the twist-2 and twist-3 operators, respectively. If the latter are negligible, i.e. $d_n = 0$, then (8.3) and (8.4) gives the WW relation (8.2) [103]. It has become customary to extract the pure twist-3 matrix element from (8.3) and (8.4),

$$d_n(Q^2) = 2\int_0^1 x^{n-1}\left[g_1(x,Q^2) + \frac{n}{n-1} g_2(x,Q^2)\right] dx = 2\frac{n}{n-1}\int_0^1 x^{n-1}\, \bar{g}_2(x,Q^2)\, dx \tag{8.5}$$

which is a direct probe of *non*-partonic (twist-3) effects or, in other words, a direct measure of deviations from the (twist-2) WW relation (8.2). Having g_1 measured, it can be experimentally determined by measuring g_2. Theoretically one expects d_n to be small for ultra-relativistic on-shell quarks ($s^\mu \sim p^\mu$) and because of small non-relativistic m_q/M_N corrections, but considerable twist-3 contributions might be due to the off-shellness ($k^2 \neq 0$) of the interacting quarks [103, 104, 106] and bag model estimates even yield $d_n \sim a_n$, i.e. $\bar{g}_2 \sim -g_2^{WW}$ at some low bound state scale $Q^2 \sim \Lambda^2$.

Several recent experimental attempts [107, 108] to determine d_n in (8.5) by measuring $g_2^{p,n,d}(x,Q^2)$, in addition to g_1, did *not* result in any statistically relevant twist-3 contribution ($\bar{g}_2$) to g_2, i.e. present data are in agreement with the twist-2 WW prediction (8.2), $g_2(x,Q^2) \simeq g_2^{WW}(x,Q^2)$, at presently attainable values of Q^2. For example, at $Q^2 \simeq 5$ GeV^2, $d_3^p(5) = (5.4 \pm 5.0) \times 10^{-3}$ and $d_3^n = (-10 \pm 15) \times 10^{-3}$ which cannot even discriminate between the vastly different expectations derived from the bag model and QCD sum rules [108].

A further important issue to be tested experimentally is the validity of the Burkhardt-Cottingham sum rule [109]

$$\int_0^1 g_2(x,Q^2)\, dx = 0 \tag{8.6}$$

which can be derived as a superconvergence relation based on Regge asymptotics (see [103] for a review). It appears to be very robust and is most probably true, but it would be surprising if it were violated due to long-range effects [104, 110, 111]. Note that the light-cone OPE is *non*-committal about the BC sum rule since for the first ($n = 1$) moment of g_2 there is no twist-3 operator within the OPE (and thus $n > 1$ in (8.4)), which might be a hint though no proof of the BC sum rule since we do not know the behavior of d_n as $n \to 1$. Furthermore, g_2^{WW} in (8.2) obeys automatically the BC sum rule (8.6). Present measurements at an average Q^2 of 3 to 5 GeV^2 imply [108]

$$\int_{0.03}^1 g_2^p(x,Q^2)\, dx = -0.013 \pm 0.028, \quad \int_{0.014}^1 g_2^n(x,Q^2)\, dx = 0.06 \pm 0.15 \tag{8.7}$$

which are consistent with (8.6) but certainly not conclusive.

Another interesting, but less problematic ($n > 1$), sum rule concerns the valence content of g_1 and g_2, originally derived by Efremov, Teryaev and Leader [96],

$$\int_0^1 x\left[g_1(x,Q^2) + 2g_2(x,Q^2)\right]^{\text{valence}} dx = 0 \tag{8.8}$$

which amounts to the vanishing of all twist-3 contributions to the second ($n = 2$) moment of g_2. There is also a wealth of similar sum rules and relations between structure functions in the electroweak sector [96] which are unfortunately beyond experimental reach for the time being and will require dedicated experiments at a future polarized HERA($\vec{e}\vec{p}$) collider.

9 Single spin asymmetries

Let me consider for definiteness semi-inclusive single transverse spin asymmetries A_N as being measured in purely hadronic singly polarized reactions, $pp^\uparrow \to \pi, \gamma X$, where $A_N = \left[d\sigma(pp^\uparrow) - d\sigma(pp^\downarrow)\right] / [\ldots + \ldots]$. (Other processes, such as the singly polarized Drell-Yan process $pp^\uparrow \to \ell^+\ell^- X$ have been thoroughly discussed in [112].) Within perturbative QCD at the leading twist-2 level, $A_N = 0$. On the other hand, Fermilab experiments [113] exhibit surprisingly large single spin asymmetries at 200 GeV: $A_N^{\pi^\pm}(x_F \gtrsim 0.3)$ rises considerably from about ± 0.2 to ± 0.4, while $A_N^{\gamma}(x_F \lesssim 0.1) \simeq 0$ within large errors. This latter process, $pp^\uparrow \to \gamma X$, is particularly interesting because direct-photon production proceeds (dominantly) without fragmentation, i.e. the photon carries directly all the information from the hard scattering process. At present there are basically three sources for a *non*vanishing A_N:

(i) dynamical contributions, i.e. 'hard' partonic scattering twist-3 effects, which result from a short distance part calculable in perturbative QCD combined with a long distance part related to quark-gluon correlations. The latter correlations can correspond to fermionic (quark) pole [114] and/or gluonic pole [112,115] dominance in the calculation of the twist-3 partonic scattering cross sections;

(ii) intrinsic k_T effects in parton distribution functions which, being non-perturbative universal nucleon properties, give rise to twist-3 contributions when convoluted with hard partonic scattering cross sections. Such contributions are usually referred to as 'Sivers effect' [116,117] where $A_N \sim \langle k_T \rangle / p_T$ or the possible nonvanishing 'quark distribution analyzing power' proportional to $f_{q/p^\uparrow}(x,\vec{k}_T) - f_{q/p^\downarrow}(x,\vec{k}_T)$ can give rise to $A_N \neq 0$ [118,119];

(iii) intrinsic k_T effects in parton fragmentation functions which is known as 'Collins' or 'sheared jet' effect [119,120]. Here one expects the 'quark fragmentation analyzing power' proportional to $D^{\pi,\gamma}_{q^\uparrow}(z,\vec{k}_T) - D^{\pi,\gamma}_{q^\downarrow}(z,\vec{k}_T)$ to be different from zero which gives rise to a nonvanishing A^π_N, for example [121].

Finally it should be remembered that large spin effects in proton-proton elastic scattering, $pp^\uparrow \to pp$, have been discovered many years ago [122] with proton beams of 24 and 28 GeV. The single spin asymmetry was found significantly different from zero for $p_T \gtrsim 2$ GeV of the outgoing protons. Although we do not have any detailed quantitative theoretical understanding of elastic single spin asymmetries for the time being, it has been emphasized by Dennis Sivers [117] that, apart from possible helicity *non*conserving effects occurring in the hadronic wave function, one expects in general (i.e. at least qualitatively) a *non*vanishing elastic single spin asymmetry due to *degenerate multiple* Regge exchanges which give rise to different phases in different helicity amplitudes.

Detailed studies and calculations have been performed [78,123,124] to examine to what accuracy these single spin asymmetries could be measured at RHIC and HERA-$\vec{\text{N}}$(I) – I refer here to the 'phase I' program of HERA-$\vec{\text{N}}$, i.e. to a polarized fixed target experiment with an unpolarized beam. They have shown that measurements with a sufficient statistical accuracy ($\delta A_N \lesssim 0.05$) can be performed up to $p_T \lesssim 8$ GeV. Indeed, the HERA-$\vec{\text{N}}$(I) facility would be particularly suited for such studies.

10 Transverse chiral–odd ('transversity') structure functions

In complete analogy to the unpolarized F_1 and polarized (spin) g_1 structure functions, the 'transversity' structure function [125–128] is, in LO, given by (cf. eq. (2.1))

$$h_1(x,Q^2) = \frac{1}{2}\Sigma_q e_q^2 \left[\delta_T q(x,Q^2) + \delta_T \bar{q}(x,Q^2)\right] \tag{10.1}$$

where $\delta_T q \equiv q^\uparrow - q^\downarrow$ describes the quark 'transversity' distribution with $q^\uparrow(q^\downarrow)$ being the probability of finding a quark in a *transversely* polarized proton with spin parallel

(antiparallel) to the proton spin. The $\delta_T \overset{(-)}{q}$ are related to the tensor current $\bar{q} i\sigma^{\mu\nu}\gamma_5 q$ which is chiral (and charge conjugation) *odd*, i.e. measure correlations between left-and right-handed quarks, $q_L \leftrightarrow q_R$ with $q_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5)q$, induced for example by non-perturbative condensates $\langle \bar{q}_L q_R \rangle$ in the nucleon. Thus, unlike in the case of the q and δq, there is no gluonic transversity density at leading twist. The $\delta_T \overset{(-)}{q}(x,Q^2)$ are leading twist-2 densities and *complete* the twist-2 sector of nucleonic parton distributions, and are therefore as interesting in principle as the familiar $f(x,Q^2)$ and $\delta f(x,Q^2)$! (Note that the q and δq densities are related to the matrix elements of the vector and axial-vector currents, $\bar{q}\gamma_\mu q$ and $\bar{q}\gamma_\mu\gamma_5 q = \bar{q}_L\gamma_\mu\gamma_5 q_L + \bar{q}_R\gamma_\mu\gamma_5 q_R$, respectively, which preserve chirality, i.e. are chiral-even ($q_L \to q_L$, $q_R \to q_R$). Thus $g_T = g_1 + g_2$ of Sect. 8, which preserves quark chirality, must not be confused with h_1 which flips chirality.)

Transversity structure functions are experimentally entirely unknown so far. It is obviously not possible to measure them in usual ep inclusive DIS where, apart from possible quark mass corrections, chirality is conserved ($\gamma^* q_L \to q_L$, etc.). One would need, for example, to measure a single transverse asymmetry $A_T^{h^\uparrow}$ in single transversely polarized *semi*-inclusive DIS or proton-proton scattering, $ep^{\uparrow\downarrow} \to eh^\uparrow X$ or $pp^{\uparrow\downarrow} \to h^\uparrow X$ with h = jet, Λ, ..., which requires a (difficult) measurement of the transverse polarization of the final state $h^\uparrow$ [127]. Bob Jaffe has put forward a novel idea for extracting $\delta_T q$ from DIS two-meson production, e.g. $ep^{\uparrow\downarrow} \to e\pi^+\pi^- X$ via a Collins-angle-like ϕ distribution [120] by measuring the observable $\vec{\pi}^+ \times \vec{\pi}^- \cdot \vec{s}_T$, i.e. the correlation of the normal to the plane formed by the three-momenta $\vec{\pi}^\pm$ with the nucleon's transverse spin [127]. This requires the cross section to be held fully differential to avoid averaging the meson-meson final state interaction phase to zero. It is conceivable that HERMES and (in the future) COMPASS can perform such measurements. Moreover HERA-$\vec{\text{N}}$(I) and RHIC could equally probe transversity densities via singly transversely polarized proton-proton scattering since the single transverse asymmetries appear to be sizeable, for example $A_T^{j^\uparrow} \sim 10\%$.

Originally there have been many suggestions [127,128] to measure the quark transverse polarization in *doubly* transversely polarized hadron-hadron initiated reactions, like $p^\uparrow p^{\uparrow\downarrow} \to \mu^+\mu^-, \gamma, jj, c\bar{c}, \ldots X$. The expected double transverse asymmetries A_{TT} turn out, however, to be prohibitively small [127,128], $A_{TT} \ll A_{LL}$, i.e. much smaller than the doubly longitudinally polarized asymmetries (including the DIS A_1 and A_2) considered thus far. Nevertheless it will be a challenge for RHIC and HERA-$\vec{\text{N}}$(II) to study A_{TT} as well.

All these expectations for transverse asymmetries are based on theoretical model estimates [128]. In the non-relativistic quark model, $\delta_T q(x,Q^2) = \delta q(x,Q^2)$ due to rotational invariance, whereas in the bag model [126] $\delta_T q(x,Q^2) \gtrsim \delta q(x,Q^2)$ – both expectations hold presumably at some non-perturbative bound-state-like scale $Q^2 = \mathcal{O}(\Lambda^2)$. Somewhat smaller results [128], $\delta_T q(x,Q^2) \lesssim \delta q(x,Q^2)$, are obtained by the chiral soliton approach [129] at $Q^2 \simeq 0.3$ GeV^2 and with QCD sum rules [130] at $Q^2 \gtrsim 1$ GeV^2. It

seems that transversity densities are sizeable and not too different from the (longitudinal) helicity distributions. Anyway, $\delta_T q \neq \delta q$ ($h_1 \neq g_1$) directly probes relativistic effects in the wave function.

Furthermore, the $\delta_T \overset{(-)}{q}(x, Q^2)$ obey simple Altarelli-Parisi type of Q^2-evolution equations with the LO splitting function $\delta_T P_{qq}^{(0)}(x)$ having been calculated some time ago [131]. Recently the calculation of the NLO 2-loop splitting functions $\delta_T P_{qq\pm}^{(1)}(x)$ has been completed in the $\overline{\text{MS}}$ factorization scheme [132, 133] for the simple non-singlet type evolution equations (since no gluons are involved) for the flavor combinations $\delta_T q_\pm \equiv \delta_T q \pm \delta_T \bar{q}$. One of the interesting applications of these LO and NLO Q^2-evolutions [133, 134] is the validity of the Soffer inequality [135] at any $Q^2 > Q_0^2$ which reads

$$|\delta_T\, q(x, Q^2)| \leq \frac{1}{2}\left[q(x, Q^2) + \delta q(x, Q^2)\right] = q_+(x, Q^2), \tag{10.2}$$

valid for each quark flavor, and likewise for antiquarks. It turns out that this inequality is *preserved* at any $Q^2 > Q_0^2$ provided it is valid at the input scale $Q^2 = Q_0^2$, although $q(x, Q^2)$, $\delta q(x, Q^2)$ and $\delta_T q(x, Q^2)$ have entirely different evolution kernels!

There have also been attempts [126, 128] to estimate the nucleon's 'tensor charge', i.e. the total transversity $\Delta_T q(Q^2)$ carried by quarks,

$$\int_0^1 \left[\delta_T\, q(x, Q^2) - \delta_T\, \bar{q}(x, Q^2)\right] dx \equiv \Delta_T\, q(Q^2) \tag{10.3}$$

which is a flavor non-singlet valence quantity (quarks *minus* antiquarks, since the tensor current is C-odd). This is in contrast to the nucleon's 'axial charge', i.e. the total helicity of each quark flavor $\Delta(q + \bar{q}) \equiv \int_0^1 [\delta q(x, Q^2) + \delta \bar{q}(x, Q^2)]\, dx$, discussed at the beginning, since the C-even spin operator $\bar{q}\gamma_\mu\gamma_5 q$ measures quarks *plus* antiquarks. One of the outstanding puzzles is how to obtain an independent measure of $\Delta_T q(Q^2)$ and thereby formulate a 'transversity sum rule' analogous to those that have been so helpful in the study of $\Delta(q + \bar{q})$ in connection with the spin of the proton.

We face again the curious situation of having reached a remarkably advanced theoretical sophistication, without having any experimental measurement whatsoever!

11 'Shopping List'

It would be somewhat redundant to end a 'Summary and Outlook' talk with a summary, so let me just present you a shopping list with items of top priorities, consisting mainly of required *experiments* for the years to come:

- $\delta g(x, Q^2) = ?$
  to be *measured*, and eventually extract $\Delta g(Q^2)$;

- $L_z^{q,g}(Q^2) = ?$
  to relate the *orbital* contribution to the proton spin to a measurable observable is still a theoretical challenge and will eventually become an experimental one;
- $\delta_T \overset{(-)}{q}(x, Q^2) = ?$
  a measurement of the 'transversity' densities will *complete* the leading twist-2 sector of the nucleon's parton densities (f, δf, $\delta_T \overset{(-)}{q}$);
- $\delta q^\gamma(x, Q^2)$, $\delta g^\gamma(x, Q^2) = ?$
  the polarized HERA($\vec{e}\vec{p}$) collider would be a unique facility worldwide for measuring the polarized photonic parton densities in the foreseeable future.

  ⋮

- Theorists should eventually succeed to calculate all polarized input parton densities, in particular $\delta g(x, Q_0^2)$ and $\delta \bar{q}(x, Q_0^2)$, from first principles; the chiral soliton approach seems to be on the most promising track to link the confirming world to the perturbative sector.

Needless to say, we need *all* the *experimental* help we can get!

Acknowledgements

Being the last speaker I would like to express my warmest thanks to Johannes Blümlein and Wolf-Dieter Nowak for having organized this excellent, very topical and stimulating workshop. Thanks are also due to the DESY directorate, in particular to Paul Söding, for the generous financial support of this meeting.

This work has been supported in part by the 'Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie', Bonn.

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