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INCLUSIVE SEMILEPTONIC B MESON DECAYS

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ABSTRACT

We argue that the inclusive and semileptonic B-meson decays are dominated by distances close to the light-cone. We develop a model in terms of a b-quark distribution function and present several predictions. The model and its predictions are compared with the heavy quark effective theory.

The two current correlator for inclusive semileptonic decays of the B-mesons, shown in figure 1, looks very much like the diagram in deep inelastic scattering. This suggests that a similar analysis of the B decays may be possible. There are two differences, however, the initial B-meson is a heavy particle and the current, which is time-like, occurs in the final state. We studied this problem and present here the main results.

The hadronic tensor for this problem can be written as

$$W_{\mu\nu}(q^2, q\cdot p) = -\frac{1}{2\pi}\int d^4y\, e^{iq\cdot y}\left\langle B\left|\left[J_\mu(y), J_\nu^+(0)\right]\right|B\right\rangle \qquad (1)$$

It was realized some time ago[1,2] that the integral above is dominated, over large regions of phase space, by the light-cone behavior of the commutator. In figure 2 we show the physical region in the q^2vs E_e plane. We include also curves of constant hadronic mass, $s = M_x^2$. When we stay away from the physical boundaries, i.e. lines ab, bc, and select $q^2 \geq 2.0\,\mathrm{GeV}^2$, then the Fourier transform in eq.(1) receives its dominant contribution from the neighborhood of the light-cone. From the average $\langle q^2\rangle$ and $\langle q_0\rangle$ we obtained $\left\langle\sqrt{y^2}\right\rangle \approx 0.1$ fm for $b \to u$ and $\left\langle\sqrt{y^2}\right\rangle \approx 0.2$ fm for $b \to c$ decays. These are short distances where experience shows that precocious scaling holds. Thus we feel confident for applying the light-cone methods to the decay $B \to X_u + e^-\bar{\nu}$ away from the edges of the phase space and less confident for applying it to the decay $B \to X_c + e^-\bar{\nu}$. As reference scales we mention $q^2 \geq 2.0\,\mathrm{GeV}^2$ and $s \geq 0.05\,M_B^2$, so that q^2 is large for scaling to occur and there are more than ten pions for incoherence. The above values are given for

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reference and they should be established precisely from experiment as the values where the scaling phenomenon, discussed below, sets in.

The commutator is replaced by its light-cone singularity times a bilocal operator whose matrix element defines a distribution function in terms of the following Fourier transform

$$\left\langle B\left|J_\rho(x,0)\right|B\right\rangle = \frac{1}{2\pi}p_\rho \int_{-\infty}^{\infty} f(\xi)\, e^{i\xi x\cdot p} d\xi \qquad . \qquad (2)$$

The hadronic tensor depends on the distribution function as follows

$$W_{\mu\nu}(q^2, q\cdot p) = S_{\mu\nu\tau\rho}\frac{p_\rho}{2\pi}\int d\xi\, (\xi p - q)_\tau f(\xi)\, D_F(\xi p - q) + \ldots \qquad (3)$$

with $f(\xi)$ the distribution function, $S_{\mu\nu\tau\rho} = g_{\mu\nu}g_{\tau\rho} + g_{\mu\rho}g_{\tau\nu} - g_{\mu\tau}g_{\nu\rho}$ and $D_F(\xi p - q)$ the scalar propagator function in momentum space. When the argument $(\xi p - q)^2 \approx 0$, i.e. near the light-cone

$$D_F(\xi p - q) \to \delta\left(\left[\xi p - q\right]^2\right) = \delta(\xi^2 p^2 - 2\xi q\cdot p + q^2) \quad . \qquad (4)$$

The argument of the δ -function has two roots

$$x_\pm = \frac{q_0 \pm |\vec{q}|}{M_B} = \frac{q_\pm}{M_B} \quad . \qquad (5)$$

Both roots contribute to the structure functions, but the terms from x_- are so small, for heavy quark distribution functions, so that those terms will be neglected. Finally we arrive at the main results

$$W_1(q^2, q\cdot p) = 2f(x_+) \quad , \; W_2(q^2, q\cdot p) = \frac{8}{x_+ - x_-}x_+ f(x_+)$$
$$\text{and } W_3(q^2, q\cdot p) = -\frac{4}{x_+ - x_-}f(x_+) \qquad (6)$$

from which we calculate the $b \to u$ triple differential decay rate

$$\frac{d\Gamma}{dE_e dy\, dx} = \frac{G_F^2 M_B^4 |V_{ub}|^2}{8\pi^3} yxf(x)(x-y) \tag{7}$$

where $y = \frac{2E_\nu}{M_B}$, E_e and E_ν are the energies of the electron and neutrino in the restframe of the B-meson. The quantity x is a new scaling variable defined as $x = x_+ = \frac{1}{M_B^2}\left[q\cdot p + \sqrt{(q\cdot p)^2 - q^2 M_B^2}\,\right] = \frac{q_0 + q_z}{M_B}$. For more details on the variables and the kinematics the reader can consult the articles refs. 1- 3.

We note first that there is a scaling law because the distribution function is a function of x only. Then the y-dependence is explicit. This formula is analogous to the neutrino-nucleon formula, where we learnt a lot from the y-distribution. [4] In figure 3 we show schematically the y-distribution for fixed x. A similar formula holds for $B \to X_c e^- \bar{\nu}$ decays with the same scaling function $f(x)$ and a similar factorization. The semileptonic decay rate is obtained by direct integration of the formulae and depends on moments of the distribution function

$$\frac{d\Gamma_j}{dE_e dy} = \frac{G_F^2 M_B^4}{8\pi^3} |V_{jb}|^2 \; (y < x^2 > - y^2 < x >) \quad . \tag{8}$$

In general $< x^2 >$, $< x > < 1$ and the semileptonic width is smaller than the free quark decay by 10-15%. This reduction comes from binding effects in the initial state indicated by the ellipsoidal region A in diagram 1. The free quark decay is reproduced for $f(x) = \delta(1-x)$.

Formula (7) and the analogous one for $b \to c$ decays hold over extended regions of the phase space, including the large E_e region. They should be useful in separating the $b \to u$ from the $b \to c$ decays. The formulae are analytic and smooth in all regions. They contain an unknown quark distribution function which can be either determined in experiments or be taken from theoretical analyses. In either case, it should be a function sharply peaked at large x. [5] We used the Peterson et al. [6] distribution function, as a working hypothesis, in order to obtain detailed distributions. Figure 4 shows the Argus data [7] and our prediction. The CLEO data [8] are more accurate and a similar fit is in preparation. We note that the model provides a good representation for the data. We are now working on a precise description of the $b \to c$ decays and the separation of the $b \to u$ channel. To this end we need to include radiative corrections from QED and QCD,

and perhaps the development of the distribution function in perturbative QCD. Finally, double and triple differential decay rates were also computed and are available. [2] These distributions should become useful when high-statistics data become available, because they are sensitive on the details of the decay process.

Since the model is very successful it is important to compare it with other approaches. The oldest and most popular approach for semileptonic B-meson decays is the ACCMM model. [9] This model improves the free quark decay by taking into account the Fermi motion of the spectator quark. Comparing eq. (7) with ACCMM formula I find many differences. Here there is a scaling variable and a distribution function weighted with power of x and y, as is appropriate at an infinite momentum frame. I do not know how to transform eq. (7) to the distribution function and dynamics of the spectator quark.

Another popular approach to B-meson decays is the heavy quark effective theory (HQET), [10–13] which also makes various approximations. In the HQET the momentum of the b-quark is divided into two parts, $p_b = m_b v + k$, with v the velocity of the B-meson and k a residual momentum to be identified with the covariant derivative. The decays of B-mesons are induced by a current $\bar{q}\Gamma b$, where $\bar{q}$ is a light quark. The HQET accepts that the light-quark product $q(0)\bar{q}(x)$ can be contracted into a free propagator as in figure 1. Then the Green function contains a free propagator between two Γ vertices

$$\Gamma \frac{i}{\not{p}_q - m_q} \Gamma = \Gamma \frac{i}{m_b \not{v} + \not{k} - \not{q} - m_q} \Gamma \quad . \tag{9}$$

Setting $\not{k} = i\not{D}$ one can expand the propagator in terms of $\frac{i\not{D}}{m_b \not{v} - \not{q} - m_q}$ generating an operator product expansion. It is now straight forward to identify moments of the structure functions W_1, W_2, W_3 with matrix elements of the operator occuring in the expansion. In practice the expansion is truncated by keeping the first three moments, which are determined by experiment.

Comparing the two approaches, we note that both approaches accept that the two currents are close to the light-cone. In our approach the light-cone algebra is used and in HQET a free propagator is introduced for the light quark. The rest of the expansion, however, seems to be different. Our approach relates the bilocal operator to an unknown

distribution function and uses the kinematics of an infinite momentum frame. The distribution function includes non-perturbative effects, coming from the binding of the b-quark and indicated with ellipsoidal regions A and B in figure 1. The final state interactions in region C are taken by closure to add up to one. In the HQET, on the other hand, the b-quark is considered to be very heavy so that it can be treated in the equal velocity approximation. It seems to me that this formalism includes a part but not all binding effects from the ellipsoidal regions A and B. The HQET pays greater attention to the region C in figure 1. The improved propagator in eq. (9) is a light-quark propagator in the gluonic background field of the B-meson whose expansion in powers of the covariant derivative was desribed above. A consequence is that several physical predictions are different. For instance, the treatment of the electron spectrum at large electron energies in the two approaches is drastically different. We obtain a smooth spectrum whose shape depends on f(x). HQET analyses begin, to zeroth order, with sharply peaked distributions and then correct them in order to obtain realistic spectra.[14]

In closing, I point out several properties which must be present in the data :

(1) The short distance behavior, which is present in both cases, should become evident as the scaling of the structure functions. Scaling should set in at lower values of q^2 for $b \to u$ than for $b \to c$ decays.

(2) The $y = \frac{2E_\nu}{M_B}$ dependence, in figure 3, is a property of the spin-½ constituents.

(3) Comparisons of $B \to e\nu X$ with $B \to \tau\nu X$ decays should show quantitative changes in the distributions brought about by the smaller phase space.[15,16]

(4) Particle spectra in two or three variables are very important, because they probe the decay dynamics in a deeper way.

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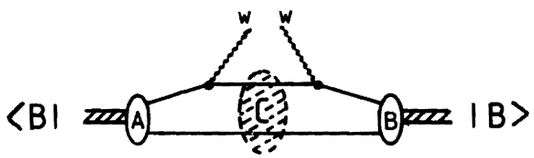

Fig. 1 *The two current correlator*

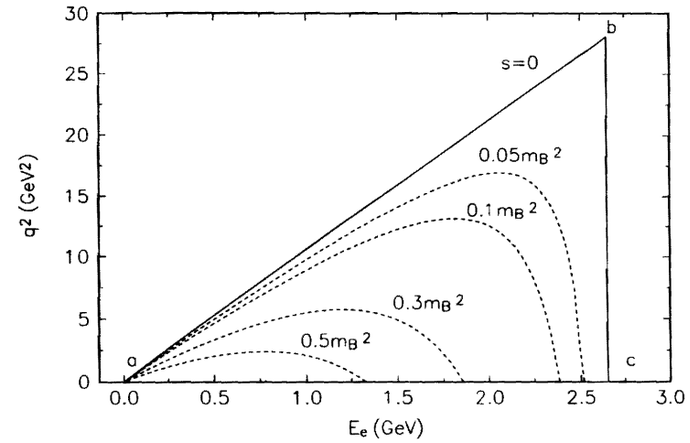

Fig. 2 *The physical region in the q^2 vs E_e plane for fixed values of s*

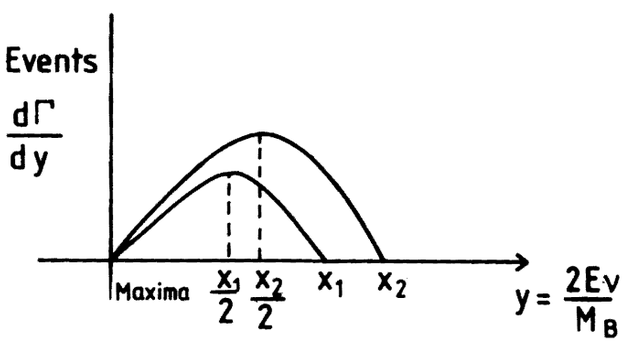

Fig.3 *The y -distribution for fixed x*

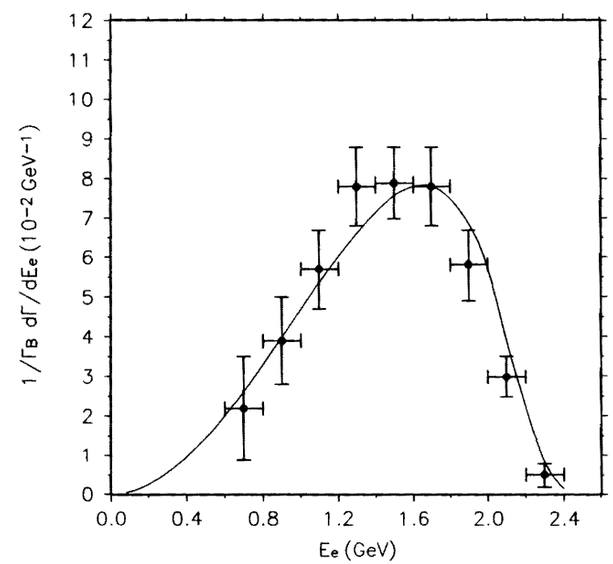

Fig. 4 *The electron spectrum and its comparison with experimental data*