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A new method to measure the tau polarization at the Z peak

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Abstract:

A new analysis is proposed which uses the angle–dependent tau polarization to measure the weak mixing angle with smallest possible statistical error. The method is based on the reconstruction of the tau direction of flight in reactions, where the tau pair decays semileptonically.

One of the precision tests of the standard electroweak model is the measurement of the tau polarization in the reaction $e^+e^- \to \tau^+\tau^-$ at the Z resonance. This parity–violating observable is highly sensitive to the weak mixing angle. The tau polarization is equivalent to the final state polarization asymmetry

$$P_\tau = \frac{\sigma(\tau_R) - \sigma(\tau_L)}{\sigma(\tau_R) + \sigma(\tau_L)} \tag{1}$$

where $\sigma(\tau_R)$ and $\sigma(\tau_L)$ are the cross sections for the production of right-handed and left-handed τ^- leptons, respectively. Any measurement of P_τ uses the well known fact that the spin of the tau lepton is analyzed by its decays [1]. Up to now the channels $\tau \to e\bar{\nu}\nu, \mu\bar{\nu}\nu, \pi\nu, 2\pi\nu, 3\pi\nu$ have been investigated in order to extract the asymmetry P_τ from the data. Usually one measures the phase space parameters of the detectable decay products of a single tau and compares their spectrum with Monte Carlo predictions [2]. Another analysis uses the acollinearity of the decay products of both tau leptons [3]. In this paper a new set of observables is proposed which can advantageously be used to determine the weak mixing angle at the Z pole with *lowest possible statistical uncertainty* in the reactions (with unpolarized beams)

$$e^+ \; e^- \;\to\; \tau^+(k_+) \; \tau^-(k_-) \;\to\; \bar{\nu}_\tau \; A^+(p_A) \; \nu_\tau \; B^-(p_B), \tag{2}$$

where A^+ and B^- are hadronic final states emerging from τ^+ and τ^- decays, respectively. In the following we consider the reactions where each of the tau leptons decays to one pion, where each tau lepton decays to two pions, and the mixed case. These channels cover about 11 percent of all $\tau^+\tau^-$ events.

The longitudinal polarization of the tau lepton occurs due to the interference of the neutral vector and axial vector currents with the coupling constants g_V^τ and g_A^τ. Generally, in order to measure this effect one has to pick out of the differential cross section the terms which are proportional to the product $g_V^\tau g_A^\tau$ and therefore to $\sin^2\theta_W$. For this purpose we derive the observable $\mathcal{O}$ which yields the best possible sensitivity

$$R = \frac{|<\mathcal{O}>|}{\sqrt{<\mathcal{O}^2> - <\mathcal{O}>^2}}. \tag{3}$$

By varying R with respect to $\mathcal{O}$ one finds that the maximal signal–to–noise ratio is provided by observables of the form

$$\mathcal{O} \propto \frac{d\sigma_{\not{P}}}{d\sigma}. \tag{4}$$

Here $d\sigma_{\not{P}}$ are the terms in the differential cross section $d\sigma$ which are parity–violating at the stage of the tau pair production. (The authors of ref [4] give a similar expression in the context of a Monte Carlo method for the tau polarization measurement, but do not calculate the observable explicitly.) The one–dimensional function (4) contains all the desired information and makes obsolete the multi–dimensional phase space fits used in the standard

analysis. Note that the statistical error remains still optimized if arbitrary experimental phase space cuts are present.

Obviously there are contributions to $d\sigma_{\not P}$ which arise from the interference of the electron vector and axial vector couplings and are linear in $g_V^e g_A^e$. These terms do not affect the averaged tau polarization P_τ . However, if they are taken into account the sensitivity to the weak mixing angle becomes nearly twice as large. In other words, the variable (4) depends more strongly on $\sin^2\theta_W$ than the final state polarization asymmetry P_τ does.

We suppose that the direction of flight of the tau lepton cannot be detected directly. However, in the reactions (2) the tau momentum can be reconstructed up to a twofold ambiguity if the momenta of all final state hadrons are known and simple two–body kinematics is assumed. The correct way to integrate over the tau direction of flight would be to sum separately the nominator and the denominator of eq (4) over the two solutions for the tau momentum. Since the ambiguity (which is the component perpendicular to $\boldsymbol{p}_A$ and $\boldsymbol{p}_B$) is only of the order of m_τ , it can be neglected in the construction of the observable without loss of sensitivity. It must be emphasized that this approximation does not lead to an error in the measurement of the tau polarization.

The component of the τ^+ momentum which lies in the plane of the hadronic final states is given by

$$\boldsymbol{k} \quad = \quad \frac{c_A + \hat{\boldsymbol{p}}_A\cdot\hat{\boldsymbol{p}}_B\ c_B}{1-(\hat{\boldsymbol{p}}_A\cdot\hat{\boldsymbol{p}}_B)^2}\hat{\boldsymbol{p}}_A \quad - \quad \frac{c_B + \hat{\boldsymbol{p}}_A\cdot\hat{\boldsymbol{p}}_B\ c_A}{1-(\hat{\boldsymbol{p}}_A\cdot\hat{\boldsymbol{p}}_B)^2}\hat{\boldsymbol{p}}_B, \tag{5}$$

where hats denote unit momenta, and

$$c_i \quad = \quad \frac{m_Z E_i - m_\tau^2 - E_i^2 + \boldsymbol{p}_i^2}{2|\boldsymbol{p}_i|} \qquad (i = A, B). \tag{6}$$

In eqs (5) ,(6) all momenta refer to the c.m. frame, and the E_i are the energies of the hadronic systems A, B. Identifying eq (5) with the tau momentum, we can study — at least to a very good approximation — the tau decays in their respective rest systems: This, together with the construction of the optimized observable (4) , is the major step beyond the standard analysis.

To obtain an analytical expression for $\mathcal{O}$ we use the spin density matrix approach of refs [5 ,6] and calculate the differential cross section at tree level neglecting the width of the tau lepton. We assume lepton universality and expand to first order in the vector couplings. With these prerequisites the (approximate) optimal observable can be defined independently of coupling constants. Then the parity violating terms of the tau pair spin density matrix read (cf. eq 3.8 of ref [6])

$$\begin{aligned} M_{\not P} \quad = \quad & \hat{\boldsymbol{k}}\cdot(\boldsymbol{S}^+ + \boldsymbol{S}^-)(k_0 + (k_0 - m_\tau)(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}})^2) \ + \ m_\tau(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}})\hat{\boldsymbol{e}}\cdot(\boldsymbol{S}^+ + \boldsymbol{S}^-) \\ & + 2|\boldsymbol{k}|(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}})\hat{\boldsymbol{k}}\cdot(\boldsymbol{S}^+ + \boldsymbol{S}^-) \end{aligned} \tag{7}$$

and the leading parity conserving contribution is given by

$$
\begin{aligned}
M \quad &= \quad 1 + (\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}})^2 \;+\; \boldsymbol{S}^+\cdot\boldsymbol{S}^-(1-(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}})^2) \;-\; 2\hat{\boldsymbol{e}}\cdot\boldsymbol{S}^+\;\hat{\boldsymbol{e}}\cdot\boldsymbol{S}^- \\
&\quad + \; 2\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}(\hat{\boldsymbol{k}}\cdot\boldsymbol{S}^+\;\hat{\boldsymbol{e}}\cdot\boldsymbol{S}^- + \hat{\boldsymbol{k}}\cdot\boldsymbol{S}^-\;\hat{\boldsymbol{e}}\cdot\boldsymbol{S}^+).
\end{aligned} \tag{8}
$$

Here $\hat{\boldsymbol{e}}$ is the unit direction of the positron beam, and the $\boldsymbol{S}^{\pm}$ are the spin operators of the $\tau^{\pm}$ leptons in their respective rest systems. The tau spins are traced by the momenta of the decay products: We replace the $\boldsymbol{S}^{\pm}$ by the (appropriately normalized) spin–dependent parts of the respective tau decay density matrices taken in the laboratory frame. Now the observable (4) can be defined as the ratio

$$
\mathcal{O} = \frac{M_{\boldsymbol{P}}}{M}, \tag{9}
$$

which is a function of the measurable phase space parameters and depends on the tau decay channels. The spin dependence of the decay mode $\tau \to \nu\pi$ is given by

$$
[\boldsymbol{S}^{\pm}]_{\pi^{\pm}} \quad = \quad \frac{2}{m_\tau^2 - m_\pi^2}\left(\mp m_\tau \boldsymbol{p}_{\pi^{\pm}} + \boldsymbol{k}\frac{m_\tau^2 + m_\pi^2 + 2m_\tau E_{\pi^{\pm}}}{2(k_0 + m_\tau)}\right). \tag{10}
$$

For the channel $\tau \to \nu\pi^0\pi$ we have

$$
[\boldsymbol{S}^{\pm}]_{\pi^0\pi^{\pm}} \quad = \quad \mp\,\frac{\mp H_0^{\pm}\boldsymbol{k} + m_\tau \boldsymbol{H}^{\pm} + \boldsymbol{k}\;(\boldsymbol{k}\cdot\boldsymbol{H}^{\pm})/(k_0 + m_\tau)}{k_{\pm}H_{\pm} - m_\tau^2(p_{\pi^{\pm}} - p_{\pi^0})^2}, \tag{11}
$$

where the four-vectors $H^{\pm}$ are defined as

$$
(H^{\pm})^{\nu} \quad = \quad 2(p_{\pi^{\pm}} - p_{\pi^0})^{\nu}(p_{\pi^{\pm}} - p_{\pi^0})k_{\pm} + (p_{\pi^{\pm}} + p_{\pi^0})^{\nu}(p_{\pi^{\pm}} - p_{\pi^0})^2. \tag{12}
$$

We use the standard metric: $p^2 > 0$ when p is time–like. Fortunately the line shape of the rho resonance does not appear in the ratio eq (9) . This is not the case for observables which are designed for the decay channel $\tau \to \nu 3\pi$ that is not considered here.

To get an idea about the sensitivity of the optimized observables $\mathcal{O}$ we calculate their expectation values and widths, neglecting the effects of radiative corrections: Assuming 10^7 Z events, $\sin^2\theta$ could be measured with a relative statistical error below the percent level (Tab. 1). At present the LEP experiments [7] reach this accuracy using the standard analysis. There the best sensitivity is obtained with the single–tau method where only one charged particle is measured. Complementary to these events the reactions of type (2) can be exploited for a high precision measurement of the weak mixing angle. For this purpose the analysis proposed in this paper should be a valuable tool.

We remark that observables of the form (4) can be used in various reactions to analyze effects which appear linearly in the differential cross sections. An interesting application is the search for CP–violating effects in the tau pair production at electron–positron–colliders. This will be discussed in a future publication [8].

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$\tau^+\tau^- \rightarrow$	$\frac{\Delta \sin^2\theta}{\sin^2\theta}$	number of events
$\bar{\nu}\pi^+ \ \nu\pi^-$	0.81 %	5000
$\bar{\nu}\pi^0\pi^+ \ \nu\pi^- + c.c.$	0.55 %	15000
$\bar{\nu}\pi^0\pi^+ \ \nu\pi^0\pi^-$	0.56 %	15000

Tab. 1 Relative statistical error (1 s.d.) with which $sin^2\theta$ can be determined in $\tau^+\tau^-$ decays to one or two pions for typical numbers of events, corresponding to 10^7 Z decays.

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