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A SYSTEMATIC APPROACH TO HEAVY QUARK EFFECTIVE THEORY

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ABSTRACT

We derive directly from QCD lagrangian a heavy quark effective lagrangian which contains both 'particle fields' and 'antiparticle fields'. The Lorentz invariant of this effective lagrangian is trivially implied from that of full QCD theory without the need of summing over the velocity. This effective lagrangian is also reparameterisation invariant for an arbitrary velocity. Especially our form of effective lagrangian permits an expansion in powers of the heavy quark momenta characterising it's off-shellness divided by it's mass to all orders.

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The heave quark effective field theory (HQEFT) was motivated to dealing with hadrons containing a single heavy quark, where the spin and flavor symmetries become manifest in the infinite mass limit of the heavy quark with the heavy quark velocity $v_\mu = p_\mu/m_Q$ held fixed. Along this line much progress has been achieved in understanding of hadronic systems containing a single heavy quark [1-24].

Usually HQET focuses on the physical content of the heavy quark expansion (no pair creation and conservation of v_μ), and through the matching conditions it organizes all $1/m_Q$ effects.

In this note, we will show a new derivation of a $1/mQ$ expansion of the QCD lagrangian, which is found to be useful in constructing systematically a complete HQEFT directly from QCD. Firstly from the QCD lagrangian we will construct a heavy quark effective lagrangian which contains both 'particle fields' and 'antiparticle fields' and includes all orders of $1/m_Q$. We can then show that the Lorentz invarance of this effective lagrangian is trivially verified from that of the full QCD lagrangian without the need of summing over the velocity. In particular the lagrangian we constructed here is reparameterisation invariant for an arbitrary velocity. Finally we introduce a Lorentz invariant expansion parameter which characterises the off-shellness of the heavy quark. Using this parameter we show that a HQEFT can be, in principle, established by expanding and truncating the effective lagrangian in powers of $1/mQ$ and supplementing a complete α_s expansion. We will focus on in this note only the complete $1/m_Q$ expansions.

Let us begin with the QCD lagrangian

$$L_Q^{QCD} = \bar{Q}(i\not{D} - m_Q)Q + \cdots \tag{1}$$

where Q stands for a heavy quark and the covariant derivative is defined as usual by $D_\mu = \partial_\mu - igA_\mu^a T^a$ with T^a being the color SU(3) generator.

We decompose the quark field Q into two parts in the following way

$$Q \equiv \hat{Q}_v + R_v \tag{2}$$

where both $\hat{Q}_v$ and R_v contain the components of 'quark field' and 'antiquark field', i.e. $\hat{Q}_v = \hat{Q}_v^{(+)} + \hat{Q}_v^{(-)}$ and $R_v = R_v^{(+)} + R_v^{(-)}$, with

$$\hat{Q}_v^{(\pm)} \equiv \frac{1 \pm \not{v}}{2} Q^{(\pm)}, \qquad R_v^{(\pm)} \equiv \frac{1 \mp \not{v}}{2} Q^{(\pm)} \tag{3}$$

where v_μ is an arbitrary unit time-like vector, i.e. $v^2 = 1$ (intepreted as velocity), and the $Q^{(+)}$ and $Q^{(-)}$ are the components of the 'quark field' and 'antiquark field' respectively, i.e. $Q = Q^{(+)} + Q^{(-)}$, and satisfy the equations of motion

$$(i\not{D} - m_Q)Q^{(\pm)} = 0 \tag{4}$$

Our crucial point is to show that R_v corresponds to the small components relative to $\hat{Q}_v$, provided Q is restricted to its on mass shell by the equation of motion and $m_Q \gg \Lambda$. Substituting the eq.(2) into the equation of motion (eq.(4)), we obtain for R_v and $\hat{Q}_v$ a relation which makes us to be able to express the fields Q and $\bar{Q}$ in terms of the new field variables $\hat{Q}_v$ and $\bar{\hat{Q}}_v$

$$Q = [1 + (1 - \frac{i\not{v}.D + m_Q}{2m_Q})^{-1}\frac{i(\not{D} - \not{v}.D)}{2m_Q}]\hat{Q}_v \tag{5}$$

$$\bar{Q} = \bar{\hat{Q}}_v[1 + \frac{-i(\overleftarrow{\not{D}} - \not{v}.\overleftarrow{D})}{2m_Q}(1 - \frac{-i\not{v}.\overleftarrow{D} + m_Q}{2m_Q})^{-1}] \tag{6}$$

The Lagrangian in terms of the new field variables becomes

$$\begin{aligned} L^{QCD}_{Q,v} &= \bar{\hat{Q}}_v(i\not{v}.D - m_Q)\hat{Q}_v \\ &+\frac{1}{2m_Q}\bar{\hat{Q}}_v i(\not{D} - \not{v}.D)(1 - \frac{i\not{v}.D + m_Q}{2m_Q})^{-1}i(\not{D} - \not{v}.D)\hat{Q}_v \\ &+\frac{1}{2m_Q}\bar{\hat{Q}}_v(-i)(\overleftarrow{\not{D}} - \not{v}.\overleftarrow{D})(1 - \frac{-i\not{v}.\overleftarrow{D} + m_Q}{2m_Q})^{-1}(i\not{v}.D - m_Q)\hat{Q}_v \\ &+\frac{1}{4m_Q^2}\bar{\hat{Q}}_v(-i)(\overleftarrow{\not{D}} - \not{v}.\overleftarrow{D})\,(1 - \frac{-i\not{v}.\overleftarrow{D} + m_Q}{2m_Q})^{-1} \\ &\times i(\not{D} - \not{v}.D)(1 - \frac{i\not{v}.D + m_Q}{2m_Q})^{-1}i(\not{D} - \not{v}.D)\hat{Q}_v \end{aligned} \tag{7}$$

To have the flavor symmetry becomes manifest in the infinite mass limit of the quark, it is useful to eliminate the mass terms in the lagrangian. This can be achieved by making the following phase transformation

$$Q_v = e^{i\not{v}m_Q v.x}\hat{Q}_v, \qquad \bar{Q}_v = \bar{\hat{Q}}_v e^{-i\not{v}m_Q v.x} \tag{8}$$

The new fields Q_v and $\bar{Q}_v$ now carry only the momenta characterising their off-shellness. We would like to stress that in the finite quark mass case this transformation is valid only for the new field $\hat{Q}_v$, not for the field Q since only in $\hat{Q}_v$ the 'quark field' $\hat{Q}_v^{(+)}$ and 'antiquark field' $\hat{Q}_v^{(-)}$ correspond to the positive and negative eigenvalues of the operator $\not{v}$ respectively. This leads to $e^{i\not{v}m_Q v.x}\hat{Q}_v^{(\pm)} = e^{\pm i m_Q v.x}\hat{Q}_v^{(\pm)}$ the correct phase projection of momentum of the 'quark field' and 'antiquark field'. One can make such a phase change for the original field Q only when taking the infinite limit of quark mass $m_Q \to \infty$, since in this limit one has $Q \to \hat{Q}_v$ (see eqs.(5) and (6)).

As all the operators in eq.(7) commute or anticommute with $\not{v}$, it is not difficult to construct from eq.(7) a new form of the Lagrangian in terms of the new varible Q_v

$$\begin{aligned} L^{QCD}_{Q,v} &= \bar{Q}_v i\not{v}.DQ_v + \frac{1}{2m_Q}\bar{Q}_v i(\not{D} - \not{v}.D)(1 - \frac{i\not{v}.D}{2m_Q})^{-1}i(\not{D} - \not{v}.D)Q_v \\ &-\bar{Q}_v e^{2i\not{v}m_Q v.x}i(\not{D} - \not{v}.D)Q_v - \frac{1}{2m_Q}\bar{Q}_v e^{2i\not{v}m_Q v.x}(1 - \not{v}\not{D}\frac{1}{v.D}) \\ &\times i(\not{D} - \not{v}.D)(1 - \frac{i\not{v}.D}{2m_Q})^{-1}i(\not{D} - \not{v}.D)Q_v \end{aligned} \tag{9}$$

Before proceeding we would like to stress the following points: firstly this new form of lagrangian is in fact general and valid for all quarks (heavy and light) as the procedure of the derivation does not rely on whether the mass of quark is small or large. However, the practical treatment for the light quark and heavy quark becomes quite different when the quark is away from its mass shell by the order of Λ_{QCD} since for light quark $m \ll \Lambda$ and for the heavy quark $m \gg \Lambda$. The former is quite off-shell and the later is only 'slightly off-shell'. Therefore the above lagrangian is useful for heavy quark and should provide a basis of heavy quark effective field theory (see below); secondly, from the derivation It is clear that only by constructing first the effective field $\hat{Q}_v$ one is able to treat systematically both 'particle field' and 'antiparticle field' together and obtain a complete effective lagrangian; thirdly, this effective lagrangian (at the lowest order of α_s) differs from the usual one by the additional terms arising from the couplings between 'particle field' and 'antiparticle field'(the terms with factor $e^{2i\not{v}m_Q v.x}$).

Finally, if there are N_f flavors of quarks, each moving with a velocity v_μ the Lagrangian density becomes $L^{QCD}_v = \sum_Q L^{QCD}_{Q,v}$. If they move in different velocity it generalizes to $L^{QCD}_v = \sum_Q L^{QCD}_{Q,v_Q}$

Summarizing above procedure of constructing an effective lagrangian, one sees that

$$L^{QCD}_Q = \bar{Q}(i\not{D} - m_Q)Q$$

$$= \bar{Q}_v i\not{v}.DQ_v + L^{QCD}_{Q,v}(\frac{1}{m_Q}) \equiv L^{QCD}_{Q,v} \tag{10}$$
$$= \bar{Q}_{v'} i\not{v}'v'.DQ_{v'} + L^{QCD}_{Q,v'}(\frac{1}{m_Q}) \equiv L^{QCD}_{Q,v'}$$

which shows that the Lagrangians in terms of the new variable Q_v with different velocity v_μ are equivalent in spite of the fact that $Q_v \neq Q_{v'}$. This proves that the effective lagrangian is reparameterisation invariant for an arbitrary velocity v_μ. Note that we should distinguish the reparameterisation invariance in the present case from the one discussed in the recent papers [25, 26]. The reparameterisation invariance discussed in [25,26] is valid when summing over the velocity of the usual effective lagrangian.

As the QCD lagrangian L^{QCD}_Q is Lorentz invariant eq.(10) trivially implies that the effective lagrangian $L^{QCD}_{Q,v}$ is also Lorentz invariant without the need of summing over the velocity which is different from the usual one.

We are now in the position to discuss how a complete $1/m_Q$ expansion of the HQEFT can be, in principle, established by expanding and truncating above effective lagrangian [1]

The effective lagrangian given above becomes useful only when the heavy quark is near its mass shell so that the operators $i(\not{D} - \not{v}v.D)$ and $(i\not{v}v.D - m_Q)$ act on $\hat{Q}_v$ (or $i(\not{D} - \not{v}v.D)$ and $(i\not{v}v.D)$ act on Q_v) to give a 'slightly off-shell' scale of order $\Lambda(\ll m_Q)$. To be more explicit we write in the momentum representation a heavy quark with velocity v_μ as

$$p_\mu = m_Q v_\mu + k_\mu \tag{11}$$

Thus the 'slightly off-shell' means $(p^2 - m_Q^2)/m_Q \sim O(\Lambda)$, which is satisfied by choosing the residual momentum k_μ and velocity v_μ in such a way that

$$k \cdot v \sim O(\Lambda), \qquad k^2 \sim O(\Lambda^2) \tag{12}$$

Such conditions allow us to make an expansion in terms of the powers of $1/m_Q$ for the non-local operators, i.e.

$$(1 - \frac{i\not{v}v.D}{2m_Q})^{-1} = \sum_{n=1}^{\infty}(\frac{i\not{v}v.D}{2m_Q})^{n-1} \tag{13}$$

It is convenient and useful to write the effective lagrangian in the following form

$$L_{Q,v} = L^{(0)}_{Q,v} + L^{(m_Q)}_{Q,v} \tag{14}$$

[1] I wish to thank Benjamin Grinstein for drawing my attention to this point

with

$$L^{(0)}_{Q,v} = \bar{Q}_v i\not{v}v.DQ_v, \qquad L^{(m_Q)}_{Q,v} = \sum_{n=1}^{\infty} L^{(n)}_{Q,v} \tag{15}$$

where

$$\begin{aligned} L^{(n)}_{Q,v} \;=\; & \frac{1}{(2m_Q)^n}\{\bar{Q}_v i(\not{D} - \not{v}v.D)(i\not{v}v.D)^{n-1} i(\not{D} - \not{v}v.D)Q_v \\ & +\bar{Q}_v(-i\not{v}v.\overleftarrow{D})e^{2i\not{v}m_Q v.x}(i\not{v}v.D)^{n-1} i(\not{D} - \not{v}v.D)Q_v \\ & +\bar{Q}_v e^{2i\not{v}m_Q v.x}(1 - \not{v}\not{D}\frac{1}{v.D})i(\not{D} - \not{v}v.D)(i\not{v}v.D)^{n-1} i(\not{D} - \not{v}v.D)Q_v\} \end{aligned} \tag{16}$$

One may also take the effective lagrangian eq.(7) to make an expansion in powers of $1/m_Q$ with the Lorentz-invariant 'slightly off-shell' condition $v.p - m_Q \sim O(\Lambda)$.

From above expressions one easily reads off terms of any order in $1/m_Q$. To the lowest order of α_s, the advantages of this derivation can easily be seen when compared to an alternative method in which one first writes down all possible allowed operators in HQEFT at each order in $1/m_Q$ with arbitrary coefficients. These are determined by comparing term by term with the full QCD theory. This is the so-called 'matching approach'. Another alternative derivation [27] from a sequence of Foldy-Wouthuysen transformations also provides a way to construct a new form of lagrangian. But it does not exhibit the essential steps for the construction of an effective theory as already commented in [22].

As the 'slightly off-shell' conditions in eq.(12) are Lorentz-invariant, they hold in any boosted frame and so does the expansion in powers of Λ/m_Q. The term $L^{(m_Q)}_v$ should be small in the context of effective theory and will be therefore treated as a perturbative term in perturbation theory. In this way, the corrections of the order of powers of $1/m_Q$ to the Green functions and therefore to physical observables are computed by making an insertion of the powers of $L^{(m_Q)}_v$ followed by a truncation. In this note, we have only presented the complete $1/m_Q$ expansion of the effective lagrangian, which should be supplemented by a complete α_s expansion for a complete HQEFT. The latter expansion is, of course, also an important subject which is under investigation.

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References

[1] S. Nussinov and W. Wetzel, Phys. Rev. D36 (1987) 130.

[2] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. 45 (1987) 292; 47 (1988) 199.

[3] G.P. Lepage and B.A. Thacher, Nucl. Phys. B4 (Proc. Suppl.) (1988) 199.

[4] E. Eichten, Nucl. Phys. B4 (Proc. Suppl.) (1988) 170.

[5] H.D. Politzer and M.B. Wise, Phys. Lett. B206 (1988)681; B208 (1988) 504.

[6] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989) 113; Phys. Lett. B237 (1990) 527.

[7] E. Eichten and B. Hill, Phys. Lett. B234 (1990) 511.

[8] H. Georgi, Phys. Lett. B240 (1990) 447.

[9] B. Grinstein, Nucl. Phys. B339 (1990) 253.

[10] J.D. Bjorken, talk presented at Les Recontre de Physique de la Vallee d'Acoste, La Thuile, Italy, March 18-24, 1990, SLAC-PUB-5278 (1990) unpublished.

[11] F. Hussain, J.G. Körner, K. Schilcher, G. Thompson and Y.L. Wu, Phys. Lett. B249 (1990) 295.

[12] A. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B343 (1990) 1.

[13] M.E. Luke, Phys. Lett. B252 (1990) 447.

[14] T. Mannel and Z. Ryzak, Phys. Lett. B247 (1990) 2388.

[15] Y.L. Wu, Mod. Phys. Lett. A6 (1991) 1277.

[16] M. Neubert, SLAC report, SLAC-PUB-5770, March 1992.

[17] Y.L. Wu, 'Accuracy of spin-flavor symmetry and decay (coupling) constants of heavy mesons', Mainz report, MZ-TH/92-13, March 1992, to be published.

[18] M.B. Wise, 'New Symmetries of the Strong Interaction' Lectures given at Lake Louise Winter Inst., Lake Louise, Canada, Feb., 1991. CALT-68 -1721, and references therein.

[19] H. Georgi, 'Heavy Quark Effective Field Theory', HUTP-91-A039, 1991, and references therein.

[20] B. Grinstein, 'Lectures on Heavy Quark Effective Theory' SSCL-Preprint-17, 1991, and references therein.

[21] A. Falk, B. Grinstein and M. Luke, Nucl. Phys. B357 (1991) 185.

[22] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B368 (1992) 204.

[23] C.L.Y. Lee, Caltech Preprint, CALT-68-1663 (1991).

[24] K. Schilcher, Mainz report, MZ-TH/92-15, 1992.

[25] M.J. Dugan, M.Golden and B. Grinstein, Phys. Lett. B282 (1992) 142.

[26] M. Luke and A.V. Manohar, UCSD/PTH 92-15, 1992.

[27] J.G. Körner and G. Thompson, Phys. Lett. B264 (1991) 185.