THE SPIN STRUCTURE OF THE NUCLEON

E. REYA
Institut fUr Physik, Universitat Dortmund
D-4600 Dortmund 50, FRG

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1. Introduction

During the past five years the interest in spin physics originated
mainly from the theoretically not anticipated result obtained by the EMC
experiment\(^1\) for the (longitudinally) polarized proton structure function
\(g_1(x, Q^2)\). In the following I will give a general review of the status and
the importance of the study of polarized structure functions and parton
distributions. For obvious reasons I shall concentrate mainly on the
developments during the past five years.

In the next Section I will first discuss the interesting aspects of \(g_1\),
remind you of the origin of the surprising longitudinal spin 'problem' which
initiated all the interest and excitement during the past few years, and
discuss comparatively two perturbative scenarios for its solution in terms
of either a large total sea-quark polarization or a large total gluon
polarization (or a suitable combination of both), then I shall use the
operator language to formulate these two scenarios for the spin structure
\(g_1\) of the proton which leads, via the generalized U(1) Goldberger-Treiman
relation, to non-perturbative definitions (and estimates) of the total
contributions of quark and gluon components to the proton spin. This
Section will be concluded by a brief review of the most important
experiments to be performed during the next few years which will be of
crucial importance for testing and discriminating between the various
theoretical ideas concerning the partonic spin content of nucleons; it is
finally pointed out that a recent Fermilab experiment on asymmetries in
pion production from (doubly) longitudinally polarized (anti)proton-proton
scattering is inconclusive concerning the size of the total
gluon-polarization in the proton.

In Section 3 I will consider \(g_2(x, Q^2)\), related to the transverse spin
structure function of the nucleon, which will serve as a probe of the
quark–gluon bound-state dynamics ('higher twist'), but is so far
unfortunately totally unmeasured. Finally, a new class of transverse
polarization (twist 2 and 3) chiral-odd nucleon structure functions, the
so-called 'transversity' distributions \(h_\perp(x, Q^2)\), which have received much
attention recently, will be briefly discussed in Section 4 and the
conclusions will be summarized in Section 5.

2. The Structure Function g_1 and the Spin of the Nucleon

The excitement about spin physics was generated in 1987 by
theoretically not anticipated data of the EMC collaboration\(^2\) at CERN on the
longitudinally polarized deep inelastic structure function \(g_1(x, Q^2)\) of the
proton which appears in the antisymmetric component of the hadronic
tensor

\[
W_{\mu\nu}(p,q) = \int d^4x e^{ipx} \langle p|U_\mu(x)j_\nu(0)|p,s\rangle
\]

with \(p^\mu\) being the four-momentum of the (proton) target and \(s^\mu\) its spin,
with \(p^2=0\) and \(s^2=1\), and as usual \(x=Q^2/2p\cdot q\) where \(Q^2=-q^2\).
Experimentally \(g_1(x, Q^2)\) is obtained by measuring the longitudinal
polarization asymmetry\(^3\)

\[
A_L(x, Q^2) = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}
\]

Plenary talk presented at the Workshop on 'QCD - 20 Years Later',
where the arrows refer always to a longitudinally polarized beam and target. Furthermore, within the 'naive' quark-parton model,

\[
F_1(x,Q^2) = \frac{1}{2} g_1(x,Q^2) + O(\alpha_s) \quad (3)
\]

\[
F_2(x,Q^2) = \frac{1}{2} g_2(x,Q^2) + O(\alpha_s^2) \quad (4)
\]

where \( q(x,Q^2) = q^+ + q^- \) and \( \Delta q(x,Q^2) = q^+ - q^- \), with \( q(x,Q^2) \) denoting the quark distributions of flavor \( q \) in the proton aligned (anti-aligned) with its spin. When taken together with previous data from SLAC, the more recent EMC results lead to a reasonably accurate determination of \( g_1(x,Q^2) \) for \( 0.01 \leq x \leq 0.7 \), with an average \( \langle Q^2 \rangle = 10 \) GeV\(^2\) from EMC and \( \langle Q^2 \rangle = 5 \) GeV\(^2\) from SLAC. The combined SLAC and EMC data lead to a rather small value for the first moment of \( g_1(x,Q^2) \):

\[
g_1^{[Q^2]}(x) = \int_0^1 g_1(x,Q^2) \, dx = 0.12 \pm 0.01 \pm 0.005 \quad (5)
\]

It should be emphasized that almost all of \( \int g_1^{[Q^2]} \, dx \) stems from the region \( x \leq 0.01 \), i.e.,

\[
\int_{0.01}^1 g_1^{[Q^2]}(x) \, dx = 0.12 \quad (6)
\]

This surprisingly small EMC result initiated all the interest in spin physics during the past few years (for recent reviews see, for example, Refs. 4–12), since it is almost half as large as 'naively' expected theoretically by Gourdin and Ellis and Jaffe (EJ):

\[
g_1^{[Q^2]}(x) = 0.19 \quad (7)
\]

This latter so-called Ellis-Jaffe "sum rule" has been obtained, as we shall see, by assuming that the total polarization of strange sea quarks contributes insignificantly to the spin of the proton, as one nevertheless expects intuitively due to the fact that it is easier for a gluon to create a nonstrange pair than a heavier strange pair - a situation very similar to the unpolarized broken SU(3) sea as observed by neutrino-nucleon scattering experiments. (Note that the original 1983 SLAC measurement, \( 0.17 \pm 0.05 \), was consistent with the expectation in Eq. 7.)

The surprising implications of the 'small' EMC result in Eq. 5 become immediately transparent by noting that \( \int g_1^{[Q^2]} \, dx \) measures the matrix element of the flavor-SU(3) axial vector currents \( j_i = q' \gamma_i \gamma_5 (A/2) q \) which in turn can be expressed by the total polarizations (first moments), carried by (antiquarks and gluons,

\[
\Delta q(Q^2) = \int \left[ \Delta q(x,Q^2) + \Delta q(x,Q^2) \right] \, dx, \quad \Delta g(Q^2) = \int \left[ \Delta g(Q^2) + \Delta g(Q^2) \right] \, dx, \quad (8)
\]

in the following general way:

\[
\langle p_d | j_i [p_p] | \rangle = \Delta q_i e^{\alpha} \Delta q \Delta_l \frac{\alpha_s A}{4 \pi} \frac{\Delta q_i}{2 \pi} \quad (9)
\]

where \( \Delta q \) and \( \Delta g \) denote from now on always the total polarizations defined in Eq. 8, unless otherwise stated. The detailed form of the matrix element on the r.h.s. of Eq. 9 is of course a matter of convention for choosing an appropriate factorization scheme, i.e. particular values of the gluonic Wilson coefficient \( \Delta C \) which describes the coupling of \( \Delta g(Q^2) \) to the photon induced by the \( \gamma_5 \)-triangle anomaly. Most of the (partly) controversial discussions during the past few years were and still are related to this problem and I shall discuss them in detail in the subsequent subsections. According to Eq. 4 the first moment of \( g_1^{[Q^2]} \) can be expressed in terms of the flavor non-singlet \( (a \neq 0) \) and singlet \( (a = 0) \) partonic contributions in Eq. 9 in the following way:
\[
\int g_1(x,Q^2)dx = \left[ \frac{\alpha_S}{4\pi} \ln^2(Q^2) \right] \left( 1 - \frac{\alpha_S}{\pi} \right) + \frac{1}{3} \Delta g(Q^2) \left( 1 - \frac{\alpha_S}{\pi} \right) \]
\[
= \left[ (0.123 \pm 0.001) + \frac{1}{3} \Delta g(Q^2) \right] \left( 1 - \frac{\alpha_S}{\pi} \right) \] (10)
\]

which is valid up to O(\alpha_S) and where we have included the fermionic Wilson coefficients\(^{15}\) \Delta C_q = \int \Delta C_q(x,Q^2)dx = 1 - \alpha_S/\pi with the HO \sigma_q(Q^2)/n = 0.061 for \Lambda_{\overline{MS}} = 0.2 \text{ GeV}. The flavor non-singlet contribution \Delta q_{12} + 3 \Delta q_{36} = 0.123 \pm 0.001 is fixed by the Gamov-Teller part of octet hyperon β-decays via the Bjorken sum rule\(^{16}\) (SU(2)_f symmetry)

\[
\Delta q_{12} = g - d = g + D = 1.254 \pm 0.006
\] (11)

and\(^{17}\)
\[
\Delta q_{36} = 3u - 3d - 2s = 3F - D = 0.68 \pm 0.04
\] (12)

where a SU(3)_f symmetry has been assumed for relating matrix elements of charged and neutral weak axial currents in order to extract \Delta q_{12}. Note that SU(3)_f breaking effects could be sizeable, which will be addressed later.

The surprising result now is that the non-singlet contribution in Eq. 10 alone already suffices to explain the "small" EMC measurement in Eq. 5, in contrast to the larger \(\Delta E\) estimate\(^{7}\) which implies an unexpectedly small or possibly even vanishing singlet component in Eq. 10:

\[
\int g_1(x,Q^2)dx = 0.10 \pm 0.17
\] (13)

where

\[
\Delta E(Q^2) = \Delta g(Q^2) = 0.10 \pm 0.17
\] (14)

according to Eq. 9 for \(f=3\) light quark flavors where

\[
\Delta E = \Delta q_u + \Delta q_d + \Delta q_s = \Delta q_u + 3\Delta q_s
\] (15)

with \(\Delta q_u\) being given by Eq. 12. It is now obvious that the large \(\Delta E\)-estimate in Eq. 7 follows from the large \(\Delta q_u\) contribution to \(\Delta E\) in Eq. 10 by assuming \(\Delta s = 0\) in Eq. 15 and neglecting the \(\sigma_q(Q^2)\Delta g(Q^2)\) term in Eq. 14. It should be furthermore noted that, in the 1-loop leading order (LO) of QCD, the total polarization of each quark flavor is conserved, i.e. \(Q^2\)-independent,

\[
\Delta q(Q^2) = \text{const.}
\] (16)

as a consequence of helicity conservation at the quark-gluon vertex, i.e.\(^{18}\)

\[
\Delta q_{12} = \int \Delta q_{12}(x,Q^2)dx = 0 \quad \text{and} \quad \Delta q_{36} = \int \Delta q_{36}(x,Q^2)dx = 0
\]

Furthermore\(^{18,19}\) \(\Delta q_{12} = 2\) and since \(\Delta q_{12} = \Delta q_{12}(Q^2)\), the polarized AP evolution equation\(^{18}\) for \(\Delta E(Q^2)\) simply implies that, apart from minor corrections, \(\sigma_q(Q^2)\Delta g(Q^2)\) does not renormalize in LO,

\[
\frac{d}{d \ln Q^2} \left[ \sigma_q(Q^2) \Delta g(Q^2) \right] = 0 \pm 0.0(6)
\] (17)

similarly to \(\Delta E(Q^2) = \Delta E\)-const. Therefore the combination \(\sigma_q \Delta g\) should be treated at the same level as the contribution of the conserved quark polarizations to \(g_1\) in Eq. 10 which has been originally emphasized in Refs. 20, 21.

The puzzling aspects and questions are now obvious:

(i) Why is the total polarization of the singlet combination \(\Delta E(Q^2)\) in Eq. 13 vanishingly small as compared with the "naive" expectation \(\Delta E = 0.68\) due to Eq. 15 for vanishing sea and gluonic contributions?

(ii) Why is there such a large difference in the flavor singlet sector between parton quarks and SU(6) constituent quarks

\[
\Delta E_{\text{SU(6)}} = 0.1 \ll \Delta E_{\text{partons}} = 1
\] (18)

although there is only about a 30% difference in the flavor non-singlet sector

\[
\Delta q_u = 1.25 \pm \Delta q_u^{(\text{SU(6)})} = 0.5 \quad \text{and} \quad \Delta q_s = 0.68 \pm \Delta q_s^{(\text{SU(6)})} = 1
\] (19)
due to the SU(6) values \( \Delta u = 4/3, \Delta d = -1/3, \Delta s = 0 \). One expects that, for conserved quantities, constituent and parton results should coincide; the small differences in Eq. 9 are attributed to helicity non-conservation induced at low energy scales by finite quark mass effects which break chirality and thus the symmetry in the non-perturbative region which creates a difference in the initial values for the perturbative QCD evolution.\(^5\) Thus the huge difference in Eq. 18 appears to be too large to be explained in this way and demands a different explanation.

The explanation of the smallness of the total singlet contribution in Eq. 14 has been a matter of dispute during the past years: Whether a large negative \( \Delta s \) in Eq. 15 [or possibly some SU(3) broken light sea component] or a large \( \Delta g \) in Eq. 14 or, perhaps more realistically, a combination of both is responsible for the cancellation of the \( \Delta q_8 \) term in Eq. 15, is of course entirely a matter of the convention used in defining the polarized quark distributions due to the well known factorization scheme dependence of the higher order (HO) Wilson coefficient \( \Delta C_g(x,Q) \). In other words, is it \( \Delta \Sigma \) which is to be interpreted as the total quark polarization or is it \( \Delta \Sigma \) in Eq. 15 with \( \Delta g \) as a separate independent gluon contribution. Let me now turn to a more detailed discussion of these two extreme alternative interpretations and whether physical and theoretical intuition might favor one over the other.

2.1 The Large Total Sea-Quark Polarization

Here all possible gluonic contributions in Eq. 14 are absorbed into the definition of quark polarizations, i.e. \( \Delta q = \Delta q_8 \) according to Eq. 9. To simplify the notation let me just set \( \Delta q_8 = 0 \), so that \( \Delta \Sigma = \Delta q_8 \) and thus \( \Delta \Sigma = \Delta \Sigma = 0.1 \) according to Eqs. 13 and 14 which give, using \( \Delta q_8 \) from (12) in Eq. 15,\(^{22,23}\)

\[
\Delta s = -0.19 \pm 0.06 .
\]  
(20)

This surprising result, implied by the new EMC measurement,\(^1\) of a large and negative polarization of the strange sea is in contrast to the intuitively reasonable ("canonical") expectation \( \Delta s = 0 \) leading to Eq. 7 as discussed thereafter. The finite \( \Delta s \) in Eq. 20 implies significant changes of the total polarizations carried by \( s \) and \( d \) quarks:

\[
\begin{align*}
\text{canonical} & : & \Delta u &= \frac{1}{2}(\Delta q_u + \Delta q_d) + \Delta s &= 0.97 + \Delta s = 0.78 \pm 0.06 \\
\text{new (EMC)} & : & \Delta d &= \frac{1}{2}(\Delta q_d - \Delta q_u) + \Delta s &= -0.29 + \Delta s = -0.48 \pm 0.06 .
\end{align*}
\]  
(21)

For comparison the SU(6) static quark model predictions in Eq. 19 were \( \Delta u = 4/3 \) and \( \Delta d = -1/3 \). Apart from contributing differently to the nucleon's spin, such changes of \( \Delta u \) and \( \Delta d \) may be of astrophysical relevance\(^{24}\) as well as of substantial consequences for, e.g., laboratory searches of supersymmetric dark matter (photino) candidates, and for the flux of neutrinos from dark matter (photino) annihilation in the sun which will be reduced due to the reduced photino trapping rate in the sun.

As discussed repeatedly, such a large total polarization of strange quarks is hard to swallow from a physics point of view and has therefore been repeatedly questioned.\(^4\)-\(^{12}\) It should be remembered that this result, Eq. 20, depends critically on the assumed SU(3)\(_f\) symmetry between hyperon decay matrix elements of the flavor changing charged weak axial currents and the neutral axial currents in Eq. 9 in order to estimate \( \Delta q_8 \) in Eq. 12. There are serious obvious arguments against\(^17\) a SU(3) flavor symmetry, in contrast to the fundamental SU(2)\(_f\) isospin rotation between matrix elements of charged and neutral axial currents which results in the Bjorken sum rule (11). An interesting alternative explanation of the EMC result has therefore been suggested by Lipkin\(^24\) which is not based on a SU(3)\(_f\) symmetry by assuming the sea to be only SU(2)\(_f\) symmetric, with no \( ss \) pairs, i.e. \( \Delta s = 0 \). In this case an additional assumption is necessary to enable one to extract the total helicity carried by valence quarks \( \Delta q_v = \Delta q_u + \Delta q_d \) from hyperon \( \beta \)-decay data: It is assumed\(^{25}\) that sea quarks do not participate in (flavor changing) \( \beta \)-decays which then determine \( \Delta q_v = 0.97 \) and \( \Delta q_d = -0.29 \) via Eqs. 11 and 12 for Eq. 21. The EMC measurement (13) can then be explained entirely in terms of the total polarization of the light sea quarks,\(^{24}\)
\[ \Delta_{uu} = \Delta_{dd} = -0.29 \quad \Delta_\gamma = 0 \]  
(22)

(Note that according to our definition (8), \( \Delta_{uu} = 2 \int \Delta_u(q^2) \text{d}x \).) This scenario has the welcome property that the SU(3) symmetry is strongly broken with a marginal or possibly even vanishing \( \Delta \) — a situation similar to the unpolarized broken SU(3) sea.2-4, 5

There is an alternative proposal25-27 that invokes the Skyrme model for an explanation of the small EMC result. Here one argues that the quark singlet combination \( \Delta L = Q^2 \) is suppressed by \( \Lambda \) while \( \Delta S = 0 \) for each separate flavor, and a similar suppression should hold for \( \Delta g(Q^2) \). Although it is somewhat unclear what precise relation there is between the Skyrme model and QCD, this explanation is even questionable25,27 within the Skyrme model itself.

The characteristic feature of all these explanations of the small EMC result (13) is a small, or even vanishing, singlet quark contribution \( \Delta L = \Delta L_{\gamma} = 0.10 \pm 0.17 \). Therefore, these scenarios give rise to a "spin-surprise" (which is certainly not a "spin-crisis") by considering the helicity sum rule

\[ \frac{1}{2} \int \Delta L + \Delta g + \Delta L_L = \Delta g \]  
(23)

i.e. the quarks do not sizably contribute to the proton spin. Thus the spin of a nucleon is built up only from gluon-helicities and angular momentum \( L_L \) (which is the average value of the angular momentum of all quarks and gluons along the quantization z-axis due to the finite kT carried by partons6,7,17,28,29) — a rather unexpected and curious result. It should be emphasized that the very reason for this surprise lies in the fact that for this kind of models \( \Delta L \) is not constituent like, i.e. \( \Delta L \neq \Delta L_{\text{const}} \), as in Eq. 14. This is in contrast to the intuitively more reasonable pre-EMC Goudsmit-Ellis-Jaffe expectation13, \( \Delta S = 0 \) which implies, due to Eqs. 15 and 12, \( \Delta L = \Delta g = 0.68 \), i.e. about 70% of the proton's spin is carried by its fermionic constituents and the rest comes from \( \Delta g \). It should be mentioned that, although the \( L_L \) contribution in (23) can be formally theoretically formulated in a consistent covariant way, there appears to be no direct experimental test of the size as well as the sign of \( L_L(Q^2) \) — apart from azimuthal distributions30 which are only sensitive to some average \( L_L(Q^2) \) of rotating constituents in a polarized nucleon target.

We turn now to an alternative explanation of the EMC result where the total gluon polarization plays a dominant role and which is not plagued by the aforementioned physical drawbacks.

2.2 The Large Total Gluon Polarization

Here the smallness of the singlet component in Eq. 13 is entirely described within the QCD improved parton model and is attributed to a large gluon helicity component \( \Delta g \) in Eq. 14 using20, 31

\[ \frac{1}{2} \Delta L + \Delta g + \Delta L_L = \Delta g \]  
(24)

where \( \Delta L \) (not \( \Delta L_{\gamma} \)) is defined, via Eq. 15, as the total helicity of quarks. As an extreme assumption one takes, for definiteness, \( \Delta S = 0 \) (although a more realistic finite but small value of \( \Delta S \) cannot be ruled out experimentally) which implies a large \( \Delta L = \Delta L_{\gamma} + 3 \Delta S = \Delta L_{\gamma} = 0.68 \) and thus Eq. 24 requires

\[ \Delta g(Q^2) = 6.3 \pm 1.9 \]  
(25)

in order to explain the small EMC result. The coupling of \( \Delta g(Q^2) \) to the photon, induced by the \( \gamma \) -triangle anomaly,31 is described by the first moment of the Wilson coefficient \( \Delta C_{\gamma} = -1 \) which has been defined relative to \( \alpha/4 \pi \) for each flavor of quarks and antiquarks separately20, 32

\[ \Delta C_{\gamma}(\alpha_s) = -1 \]  
(26)

i.e. \( \Delta C_{\gamma}(\alpha_s) = \Delta C_{\gamma}(\alpha/4 \pi) \Delta C_{\gamma} \) which gives Eq. 24. Formally the gluon contribution in (24) appears as a non-leading correction at order \( a_s \) but, because of Eq. 17, \( a_s \Delta g \) is constant in \( Q^2 \) in the leading approximation and therefore should be treated at the same level as the \( Q^2 \)-independent quark contributions to \( g_1 \) in the 'naive' LO parton model. It should be furthermore emphasized that the splitting of \( \Delta E(Q^2) \) into a quark and gluon...
term depends on the definition of the quark which is formally the leading term. This partition (factorization scheme, $\Delta C_s = -1$) in Eq. 24 corresponds to $\Delta E$ defined in such a way as being conserved, also at the two-loop accuracy of massless QCD (i.e., $\Delta p^{\text{LO}}(0)$).

Since the present explanation of the low EMC measurement resorts to a large total quark polarization, $\Delta E \approx 0.7$, we have

$$\Delta E \approx 0.7 \pm \Delta E_{\text{SW}}^{(1)}$$

i.e. $\Delta E$ is constituent-like, in contrast to Eq. 18, as one expects for conserved quantities. This is now in agreement with the flavor-non-singlet sector in Eq. 19 and again the, say, 30% difference between partons and constituents in (28) represents the breaking from mass terms. Furthermore, in contrast to the previous explanation (scheme) based on a large sea polarization, we do not face a 'spin-surprise' here, since there is an (almost) "canonical" explanation of the nucleon spin

$$\frac{1}{2} = \frac{1}{2} \Delta E + \Delta g(Q^2) + L_z(Q^2)$$

where about 70% of the nucleon's spin is carried by its fermionic constituents and the rest comes from $\Delta g$, with $\Delta g(Q^2)$ being, however, sizeable according to Eq. 25.

scattering process is supposed to probe the hard pointlike parton structure of nucleons, one can, however, arrive at the definite and finite result in Eq. 26 by resorting to the following two physically intuitive assumptions: (i) the gluonic term in (24) receives only hard, perturbatively calculable contributions and (ii) $\Delta E$ remains a conserved quantity. The gluonic contribution to $g_s$ is now calculated via the polarized subprocess $\gamma g \rightarrow q\bar{q}$ depicted in Fig. 1. The collinear divergences can be regularized in various different ways by giving, for example, a small mass to gluons ($p^2(0) = m_g^2$) or to quarks ($k^2(k') = m_q^2$) or by using dimensional regularization ($n = d + 2\epsilon$, $\epsilon = 0, 2, 4$) or by assuming a lower bound $k_{\text{min}}$ for the $k_y$ of the outgoing quarks relative to the incoming gluon. A straightforward calculation of the diagram in Fig. 1 then gives

$$g_s^2 \gamma \gamma^* \rightarrow q\bar{q} = \frac{\pi}{4} \left[ 2 \Delta \rho^{(0)}(x)(\ln x - \frac{3}{2}) + 0 \right] 2 \bar{p}^2(0), m^2 = 0$$

where $\Delta \rho^{(0)}(x) = \langle 2\gamma^* - 1\rangle / 2$ and $-1/2 = -1 + \gamma - \ln(\Delta g^2/p^2)$. When convoluted with $\Delta g(x, Q^2)$ in the usual way, this gives then the gluonic contribution to $g_s^2(x, Q^2)$ provided the collinear divergence terms proportional to $\ln Q^2 / p^2$. 

Fig. 1. The matrix element squared of the polarized subprocess $\gamma g \rightarrow q\bar{q}$, being related to the $\gamma g$-triangle anomaly at the light-cone ($Q^2 = 0$), giving rise to the Wilson coefficient $\Delta C_g$, i.e. to the coupling of $\Delta g$ to $g_s$. 

Since the value of $\Delta g_s$ in Eq. 26 has been a matter of dispute during the past years, let me briefly comment on some details of its derivation. This is due to the fact, as has been already repeatedly emphasized, that the gluonic contribution in Eq. 24 is of course a matter of the convention used in defining the polarized quark distributions, due to the usual factorization scheme dependence of the HO Wilson coefficient $\Delta C_s(x, m^2)$ - a situation very similar to unpolarized deep inelastic scattering beyond the leading order (for a detailed discussion of various schemes as well as of the related transformations of the 2-loop anomalous dimensions, the interested reader is referred to Refs. 33,42). Since a deep inelastic
In $Q^2/m^2$ or $1/c^2$ are factorized, as usual, into $\Delta q(x,Q^2)$ and resummed by the evolution equations. The gluonic coefficient function $\Delta g(x,Q^2)$ is now determined by the remaining "finite" terms depending on the factorization scheme chosen, i.e., on how much of these "finite" terms is absorbed, together with the collinear singularities, into the definition of $\Delta q$. From Eq. 30a one might conclude

$$\Delta g(x,Q^2) = \frac{g}{4\pi} (2x-1)(\ln \frac{1}{x} - 2) \tag{31a}$$

which, when integrated gives our advocated result in Eq. 26. On the other hand, from Eq. 30b one may (naively) conclude

$$\Delta g(x,Q^2) = \frac{g}{4\pi} [2(2x-1)(\ln \frac{1}{x} - 1)+2(1-x)] \tag{31b}$$

which has a vanishing first moment

$$\int \left(\Delta g(x,Q^2)dx \right) = \frac{g}{4\pi} = 0. \tag{32}$$

This vanishing result is obviously caused by the last positive term $+2(1-x)$ in Eq. 31b. A closer inspection reveals, however, that this positive contribution $+2(1-x)$ in Eq. 30b derives from the soft non-perturbative collinear region where $k^2 > m^2(x)$ and thus should be absorbed into the definition of the light quark density $\Delta L(x,Q^2)$ 20,32-36,39. This leaves us with the hard gluonic contribution to $\Delta g$ and Eq. 31b changes to

$$\Delta g(x,Q^2) = \frac{g}{4\pi} (2x-1)(\ln \frac{1}{x} - 1) \tag{31b'}$$

which yields again the non-vanishing first moment in Eq. 26. (For heavy quarks ($Q = c, b, ...$), however, there are no contributions from the soft region and the positive contribution $+2(1-x)$ in Eq. 31b comes from the hard perturbative region 33,36,39 around $k^2 = m^2(x)$. Thus Eq. 31b is the appropriate expression for heavy quarks 34,44 with its vanishing first moment in Eq. 32, i.e., heavy quarks do not contribute to the total spin of the nucleon. It should be mentioned that different regularization procedures, for example introducing a 'brute force' lower bound for $k^2$ with $Q^2 > (m^2 + 0)^2$, result in the same hard gluon contribution to $\Delta g(x,Q^2)$ as in Eq. 31b. Furthermore it is a peculiarity of the present case that the first moment $\Delta g$ in Eq. 26 can be calculated even without introducing any regulator at all 35,36 due to the vanishing of the first moment of $\Delta g(x,Q^2)$. Having established that the finiteness of the gluonic Wilson coefficient is caused by the hard perturbatively calculable contributions of the triangle anomaly, the large total gluon polarization in Eq. 25, needed in addition for explaining the small EMC result, can of course not be calculated perturbatively and one has to resort to non-perturbative methods and/or to different hard processes where $\Delta g$ can be measured independently. Before turning to a discussion of these topics let me mention that one expects, on rather general grounds 34,45 based on the (intrinsic) bound-state dynamics of the nucleon and counting rules, the total gluon helicity to be sizeable, $\Delta g < 1$. Although the scale $\mu^2$ for this prediction is unknown, one expects an intrinsic quantity to be relevant at a low bound-state scale, probably $\mu^2 \approx 1$ GeV. Given the intrinsic polarization, one can obtain the "extrinsic" one by applying the evolution equations (radiative effects) starting at the bound-state scale $\mu^2$ which increases 45,46 this intrinsic result to a value compatible with the experimental requirement at $Q^2 = 10$ GeV. Along similar lines, the perturbatively defined $\Delta g(Q^2)$ can be dynamically estimated 46,42,47 using the renormalization group evolution equations supplemented by some non-perturbative valence or valence-like boundary conditions at a low bound-state resolution scale $Q^2 = \mu^2$. This radiative approach is very similar to the one for unpolarized nucleons 48 where the gluon and sea distributions, in particular in the unknown small-$x$ regions, can be successfully generated from an experimental valence and finite valence-like gluon and light-sea input at $\mu^2$. For the present case the relevant unpolarized input quantities are found to be 48 $g(x,\mu^2) = g(x,\mu^2)dx = 1$ and $S(x,\mu^2) = S(x,\mu^2)dx = 0$ at $\mu^2 = 0.3$ GeV. The general positivity constraints...
with $f=q,\bar{q}, g$, then constrain the first moments of the relevant polarized
distributions by

$$|\Delta g(Q^2)| \leq g(Q^2) \leq 1, \quad |\Delta s(Q^2)| \leq s(Q^2) = 0$$

where $|\Delta g(Q^2)|$ is surprisingly similar to the estimate of Ref. 45 discussed
above. The value of $\Delta g(Q^2)$ can now be easily calculated from this
input by using the relevant HO evolution equation for the combination $gE$
in Eq. 24:

$$\frac{d}{d\ln Q^2} gE(Q^2) = \left( -2f \right) gE(Q^2)$$

(35)

where $f = C_g + \bar{q} s$, and $\Delta E$ is conserved to this order according to Eq. 27.
With $\Delta E = 0$, $\Delta s = 0$, this yields 

$$-2.5 \leq \Delta g(Q^2) \leq 4$$

(36)
in agreement with experiment, Eq. 25. It should be remarked that present
EMC data, $^1$ taken at face value, allow even for a negative $\Delta g(Q^2)$
compatible with (36). This is so because experimentally $\times > 0.01$, Eq. 6, and
thus it is more appropriate to analyze the "truncated" moment $\int_0^1 x^2 \Delta g(Q^2) dx$
with its gluon contribution being given by

$$\int_0^1 x^2 \Delta g(Q^2) dx \approx 0.68$$

where $\Delta g$ and $\bar{g}$ do not factorize anymore, in contrast to considering the
(full) first moment $\int_0^1 x \Delta g(Q^2) dx \times > 0.01$, which has been done thus far (cf. Eq. 14).
In this case the ambiguous $x$-dependencies of $\Delta C$ and $\bar{g}$ become
relevant and the presently available EMC and SLAC data, $^1, ^3$ can be
explained in terms of a gluonic contribution alone ($\Delta s = 0$) even with a
$\bar{q} g$ factorization, for example in Eq. 11b, corresponding to a vanishing (1) first
moment. $^4$

2.3 Non-Perturbative Operator Current Algebra Approaches and Estimates

In the previous mechanism the effect of the $\gamma_5$-triangle anomaly is
completely described within the context of the perturbative QCD improved
parton model: $\Delta g$ can be measured in hard processes and once the total
singlet contribution $\Delta g$ is split into an anomalous gluon term and a quark
term according to Eq. 24, $\Delta E$ approaches its naive value. In this sense the
mechanism is "perturbative", although $\Delta E$ and $\Delta g$ can certainly not be
computed ab initio in perturbation theory, and the quark-like and
gluon-like terms are, by construction, individually gauge invariant and
free of any other theoretical ambiguities. This is in contrast to current
algebra (CA) definitions $^7, ^9, ^40, ^50, ^51$ which in general include non-
perturbative ($Q^2 \rightarrow 0$) contributions. $^5, ^52$ One can imagine that further
non-perturbative contributions also due to the anomaly (e.g. instanton
terms $^53$) could be present in the conserved quark term making it different
from the constituent limit even after subtraction of the anomalous gluon
component. More explicitly, the gauge non-invariant topological gluonic
current $k_\mu$ is usually employed for defining the total gluon helicity

$$\langle p, \alpha \mu, [p, \alpha] \rangle = \frac{g}{2n} \Delta g = 2\mu_T$$

(37)

with $k_\mu$ appearing in the anomalous divergence relation (neglecting the
small light quark masses $m_q$)

$$\partial \mu k_\mu = f_{\alpha\beta} [p, \alpha] [p, \beta]$$

$$= f_{\alpha\beta} [p, \alpha] [p, \beta] [p, \alpha]$$

(38)

which is due to the Adler-Bell-Jackiw $\gamma_5$-triangle anomaly. $^54$ Therefore
the singlet axial vector current $k_\mu$ is not conserved even in the chiral
limit for massless quarks. However, according to Eq. 38, a conserved (but
gauge variant) quark singlet component $\Delta L^A$ can be obtained by rewriting
$k_\mu = (\bar{q} - \gamma_5 q) + s \phi$ and identifying $\langle p, \alpha [p, \alpha] \rangle = s^A \phi$.
Furthermore,
an explicit computation by Forman of $\langle p_s l|l|p_s \rangle$ in Eq. 37 has revealed that $\Delta g_{53} \alpha(2\pi)\log (\alpha/2\pi)\log (\alpha) + R$ where $\Delta g$ corresponds to the gauge invariant hard piece of the anomaly which may be identified with our perturbative partonic definition used thus far; the remaining gauge variant Chern-Simons (instantonic) piece $\Omega$ is the soft non-perturbative contribution to the triangle-anomaly which does not correspond to a hard (large-$k_T$) subprocess (i.e. should be considered as part of the one-jet events $q^+ \rightarrow q^+$, $\gamma q^+ \rightarrow q^+$) and thus should be absorbed into the definition of the conserved total quark polarization:

$$\Delta \Delta = \Delta \Delta_{\text{kin}} - \frac{\alpha}{2\pi} \log \frac{\sqrt{\pi}}{2\pi} = \Delta \Delta_{\text{kin}} - \frac{1}{2\pi} \frac{\alpha}{\log \frac{\sqrt{\pi}}{2\pi}}$$

leaving us with our original expression (24) with both terms on the r.h.s. of (39) being separately gauge invariant. The term $-\Omega_N$ may be interpreted as an additional sea polarization induced by instantonic effects.

It was claimed that, by evaluating the matrix element $\langle p_s l|l|p_s \rangle$ on the lattice, one obtains the bound

$$\langle p_s l|l|p_s \rangle < 0.05$$

This result is surprisingly small and, moreover, can obviously not be used to extract some information about $\Delta g$. It remains to be seen whether this result can be trusted in view of the approximations involved (quenched quarks, small $8^4\times 10$ lattices and how to accommodate instantons, ...) and in view of the delicate problem of how to define the topological charge on a lattice.

The splitting of $\Delta g^2(Q^2)$ into a quark-like and a gluon-like term, both of them gauge invariant and in particular with the correct behavior under the renormalization group (RG), has been formulated and discussed in very general terms by Shore and Veneziano. For this reason I will briefly outline their main results, although there is no general consensus on their interpretation.

Generalizing the classic results of current algebra and PCAC, the Goldberger-Treiman (GT) relation for the flavor non-singlet isoriplet channel ($F_{\text{emc}} = 2M_{\gamma \pi}^2(0) = M_{\gamma \pi}$ with $F_{\text{emc}} = 23.3$ MeV), to the flavor singlet channel one obtains $G_{1}(0) \leq 2M_{\gamma \pi}(0)$, the so called "$U(1) \text{ GT-relation}"

$$F_{\text{emc}}(0) G_{1}(0) = 2M_{\gamma \pi}(0)$$

The singlet axial-vector form factor $G_{1}(0)$ is defined from $\langle p_s l|l|p_s \rangle$ which is related, via the OPE, to the large $Q^2$ limit of the matrix element $\langle p_s l|l|p_s \rangle$ in Eq. 1 measured indirectly by the polarized EMC/SLAC experiments: $1-3$ $G_{1}(0) = \Delta \Delta_{\text{kin}} = \frac{1}{2\pi} \frac{\alpha}{\log \frac{\sqrt{\pi}}{2\pi}} = 0.10 \pm 0.17$ according to Eq. 13. It is important to note that $n_0$ appearing on the l.h.s. of the singlet GT-relation does not correspond to the physical $SU(3)$-singlet $n'$ but instead to the unphysical would-be Nambu-Goldstone boson in the OZI limit of QCD where the triangle anomaly is absent. This is so because $G_{1}(0)$ on the r.h.s. of (41) is not RG invariant, due to the non-conservation of the singlet axial-vector current in Eq. 38; therefore the l.h.s. of the singlet GT-relation must be scale dependent as well in order to be consistent with the RG, and so cannot simply correspond to the substitution of the $n'$ for the $n$ in the classical flavor non-singlet GT-relation where the anomaly is absent. This RG non-invariance resides in $F_{\text{emc}}(0) = 2M_{\gamma \pi}(0)$, where $X'0(0) = \delta x^{O(3)}(0) + X'0(0)$ with the topological charge density $b \cdot \kappa$ defined in (38), being related to the topological susceptibility of massless QCD. It is interesting to note in passing that $X(0)$ has been estimated from QCD sum rules and lattice calculations to be $X(0) = - (1.3 \pm 0.7)$. This, together with the OZI approximation, yields $F_{\text{emc}} = 2M_{\gamma \pi} = 0.13$ which can be even considered as a prediction of the small EMC result $G_{1}(0) = 0.1$ according to the singlet GT-relation (41) in order to gain more insight into the mechanism responsible for the smallness of the l.h.s. of the $U(1) \text{ GT-relation} (41)$, it is useful to rewrite it in terms of the physical $n'$ :
Here $F$ is now a RG invariant process independent decay constant, related to the zero-momentum limit of the propagator for the composite operator $\bar{q}q$, which is not the $n'$ decay constant $F_{n'}$ defined analogously to $F$. Furthermore, $g_{\text{NN}}$ is the scale dependent coupling of the nucleon to the gluonic operator $a_k$ in Eq. 38. From the requirement that the total quark polarization $\Delta E$ should be RG invariant, one may identify $^{10,51}$ in the $U(1)$ GT-relation (42)

$$2 M \Delta E = F g_{n'\text{NN}}, \quad 2 M - f_{n'}^2 2 M_{n'} g_{n'\text{NN}}$$

as non-perturbative (gauge- and isospin-invariant) definitions of the "quark" and "gluon" components.

To go beyond these formal definitions one needs some guiding principle which allows to estimate the actual size of each individual term in the singlet GT-relation (42): One expects $^{51,58}$, the $O(\alpha_s)$ approximation to apply to those quantities which are RG invariant (e.g. $F g_{n'\text{NN}}$), i.e. conserved, and thus have the same RG behavior in QCD itself and in the $O(\alpha_s)$ limit, for those quantities which are not RG invariant in QCD (e.g. $g_{G\text{NN}}$), but which are RG invariant in the $O(\alpha_s)$ limit, one expects the $O(\alpha_s)$ approximation to be strongly violated and therefore not applicable. Thus

$$\frac{\Delta M}{2M} = \frac{1}{2M} F g_{n'\text{NN}} = \frac{1}{2M} \sqrt{\frac{6}{\pi} g_{n'\text{NN}}} \approx 0.7$$

(44a)

$$\Delta g / \Delta q_{\text{NN}} = 0$$

(44b)

$$g_{n'} / g_{\text{NN}} = \Delta g / \Delta q = 0.68$$

(44c)

which implies that the quark component $\Delta E$ of the proton spin is not small, and, moreover, the observed small EMC value of $g_{n'} = 0.1$ would then be due to an approximate cancellation in (42) between $\Delta E$ and a large gluon component $\Delta g$, defined in (43), which must therefore display large OZI violations according to (44b).

This interpretation is suggested (but certainly not required) by the success of the OZI approximation for the decay $n' \rightarrow \gamma \gamma$ where the analogous relation to (42) is given by $^{58}$

$$F_{n'\gamma\gamma}(0) \approx \frac{1}{2f_{n'}} \frac{1}{4\pi} \frac{g_{n'\gamma\gamma}(0)}{2f_{n'}} = \frac{4N_c}{3\pi} F_{n'\gamma\gamma}(0)$$

(45)

Here each term is separately RG invariant and thus the OZI approximation should apply reliably which gives simply

$$\sqrt{\frac{6}{\pi} g_{n'\gamma\gamma}} = \frac{4N_c}{3\pi} F_{n'\gamma\gamma}(0)$$

(45')

since $g_{n'\gamma\gamma} = 0$ in the OZI limit. This latter relation is quite accurately satisfied experimentally $^{58}$ which supports the reliability of the above arguments.

It is certainly interesting to have rather general non-perturbative arguments in favor of a large, conserved and thus constituent-like quark component $\Delta E$ which, on account of the small EMC measurement, $^{1}$ implies a large gluon polarization $\Delta g$. Nevertheless it is clear that only future (dedicated) experiments can ultimately decide about the physical reality of the various theoretical ideas and scenarios discussed so far. In particular, the polarized gluon distribution $\Delta g(x, Q^2)$, about which no detailed experimental information whatsoever exists for the time being, is here of crucial importance. It is thus clear that we need much MORE information from polarized experiments in the (hopefully near) future.

### 2.4 Phenomenological Signatures of $\Delta g$ and $\Delta Q^2$

There have been many suggestions for measuring polarized parton distributions with hard processes $^{4,5,7,8}$ but it appears to be particularly difficult to extract information about $\Delta g(x, Q^2)$. Since it is beyond the scope of this talk to discuss all polarized processes suggested so far, I will concentrate on those which I believe are the most important ones for delineating $\Delta g$. (All realistic measurements are of course sensitive to $\Delta E(x, Q^2)$ and not just to their first moments. The interested reader is referred to the reviews in Refs. 5,7,8,35,52, and the references therein, for a discussion of the detailed $x$- and $Q^2$-dependence of polarized parton distributions.) As in the unpolarized case, only processes where $\Delta g$ occurs directly already in the LO (with no $\Delta q$ and $\Delta Q$ contributions present)
appear to be the most promising source for measuring $\Delta g(x,Q^2)$. This is the case for

1. deep inelastic (or photon) production of heavy quarks $Q=c,b$ via the fusion process $g^*(\nu) g\rightarrow Q\bar{Q}$ responsible for open charm $c$ or $b$ production.\(^{59,60}\) Although this production mechanism for open charm production is theoretically clean with a sizeable polarization asymmetry\(^{44}\) $P_{c\bar{c}}=20-30\%$, it will be hard for the presently ongoing polarized fixed target DIS experiments\(^{61,62}\) to observe a sufficient amount of events since $a^2=400$ nb for $E_{\nu}=100$ GeV. Here, I believe, because of the high-intensity $e^-$-beam, the SLAC experiment\(^{62}\) is in a much better shape than the SMC experiment at CERN. The situation is similar for $J/\psi$ production\(^{59,60}\), but here, however, one faces the additional model dependence (duality\(^{63}\) or color singlet\(^{64}\) model) for bound-state production. Anyway, the hope is\(^{60}\) that SMC could measure $\Delta g(x,Q^2)$ in the region $0.03\leq x \leq 0.25$.

2. hadronic heavy quark production\(^{65,66}\) via the LO subprocesses $gg\rightarrow Q\bar{Q}$, $Q\bar{Q}\rightarrow Q\bar{Q}$ which depends "quadratically" on $\Delta g$ and where the $\Delta g\Delta Q$ contribution is small,\(^{66}\) appears to be a very sensitive and presumably the best (most realistic?) test of $\Delta g$. Here the polarized $p\bar{p}$ RHIC collider ($\sqrt{s}$=50-500 GeV) with high luminosity ($L \sim 10^{32}$ cm$^{-2}$sec$^{-1}$) will play a decisive role, provided, of course, our experimental colleagues succeed in polarizing longitudinally both proton beams;

3. more recently it has been emphasized that singly polarized hadron-hadron collisions,\(^{67-70}\) i.e. where only one incoming (possibly fixed target) hadron is polarized, provide access to $\Delta g(x,Q^2)$ as well which appears to be experimentally easier than working with both initial hadrons polarized. However, the prize one has to pay here is that for Drell-Yan dilepton production\(^{67,68}\) either the polarization of one of the final leptons has to be measured,\(^{67}\)

$$p^2 + \gamma X = \mu \mu' X$$

or the angular distribution of the produced lepton pair\(^{68}\) as a polarimeter for the virtual photon. Alternatively one could also consider the production of a polarized direct-photon\(^{69,70}\) via $p\bar{p}\rightarrow T$. It should be emphasized that one could use here the energetic unpolarized proton beams of the TEVATRON $pp$ or HERA $ep$ colliders ($E_p=1$ TeV) to be scattered off a polarized fixed $p$-target\(^{69}\) still giving rise to a sizeable $\sqrt{s}=43$ GeV.

There have been many further suggestions and studies to measure $\Delta g(x,Q^2)$ and/or $\Delta Q^2(x,Q^2)$ in other suitable hard processes initiated by doubly polarized hadron-hadron collisions such as the production of large-$p_T$ photons,\(^{70-74}\) Drell-Yan dimuons,\(^{34,70,73}\) large-$p_T$ jet asymmetries of final $jj$, $j\bar{p}$, $jj\bar{p}$ events,\(^{75-79}\) etc. Again, the future high luminosity RHIC and UNK colliders will be essential for performing such experiments. Recently, polarization asymmetries for semi-inclusive pion production have been measured from doubly longitudinally polarized (anti)proton-proton collisions at the Fermilab $pp$\(^{80}\) resulting in $\Delta g^0(0)p^0=1.4$ GeV at $E_{p\bar{p}}=200$ GeV, i.e. $\sqrt{s}=20$ GeV. It should be pointed out that this result does not necessarily imply a vanishing\(^{80,77}\) gluon polarization $\Delta g$, but it is equally consistent with a large $\Delta g \sim 3-6$.\(^{81}\)

A clean distinction between a large and a small $\Delta g$ scenario could be achieved, if it were possible to perform such a semi-inclusive experiment at, say, $\sqrt{s}=100$ GeV with $p_T^{ \gamma \pm} \geq 5$ GeV.\(^{81}\)

Furthermore, measurements of $g(x,Q^2)$ at various fixed values of $Q^2$ would be very desirable in order to extract from measured scaling violations $\Delta g(x,Q^2)$ and $\Delta Q^2(x,Q^2)$ using, as usual, the polarized evolution equations.\(^{15,55,72,73,79,82,83}\) Strictly speaking, this would be possible only in the LO at present.\(^{55,52,73,79,82,83}\) Since $\Delta g$ enters Eq. 14 formally in $\gamma_2$, a theoretically consistent analysis in Bjorken-$x$ space requires, however, the polarized 2-loop splitting functions $\Delta \alpha_{(2)}(x)$, which are not yet available, in order to allow for a renormalization/factorization scheme invariant analysis of $g(x,Q^2)$ in Eq. 10:

$$\frac{g(x,Q^2)}{\Delta g(x,Q^2)} = \frac{g(x,Q^2)}{\Delta Q^2(x,Q^2)} \left[ \Delta C^a \bar{C}^a + 3 \Delta C^a \bar{C}^a + \frac{1}{2} \left[ \Delta C^a \bar{C}^a + 3 \Delta C^a \bar{C}^a \right] \right]$$

where $\Delta C^a \bar{C}^a$ is the fermionic Wilson
coefficient is given by

\[ \Delta C_q(x, Q^2) = \frac{4}{m} \int \frac{\ln(1-x)}{1-x} + \frac{1}{2} \ln(1-x) \frac{1-x^4}{1-x} \ln x \]

\[ + 2 + x - \frac{1}{2} \frac{1-x^2}{1-x} \ln(1-x) \]  

(48)

and with the gluonic Wilson coefficient \( \Delta C_g(x, Q^2) \) given by Eq. 31b, although this latter choice is a matter of convention.

Finally, I remind you that the small EMC result (5) implies further dramatic consequences for the polarized neutron structure function \( g_1^N(x, Q^2) \) by making use of the "sacred" (isospin invariance) Bjorken sum rule in Eq. 11,

\[ \int_0^1 [g_1^N(x, Q^2) - g_1^N(x, Q^2)] \, dx = \frac{1}{2} \frac{1}{Q^2} \ln(1-x) \]  

(49)

where I have included the small \( a_s/\pi \) correction. This gives, using Eq. 5,

\[ \int_0^1 g_1^N(x, Q^2) \, dx = -0.07 \pm 0.02 \]  

(50)

which is about ten (!) times larger than the pre-EMC Ellis-Jaffe expectation \( g_1^N(0, Q^2) \) = -0.006 based on Eq. 7. These predictions for \( g_1^N \) can be straightforwardly translated into Bjorken-\( x \) space,

\[ g_1^N(x, Q^2) = g_f^N(x, Q^2) - \frac{1}{6} \Delta u_f(x, Q^2) - \Delta d_f(x, Q^2) \]  

(51)

where the insignificant HO contribution due to \( \Delta C_q \) in Eq. 49 has been suppressed, which are shown in Fig. 2 for two different polarized valence densities \( g_f^N(x, Q^2) \) taken from EMC/SLAC measurements.\(^{1,3}\) Due to the large negative area of \( g_1^N \) in Eq. 50, the predictions in the small-x region (\( x \leq 0.02 \)) differ of course significantly from the original pre-EMC Gourdin-Ellis-Jaffe\(^{13}\) estimate based on assuming \( \Delta C = 0 \).

Since this latter estimate disagrees with the more recent EMC data\(^{1}\) in Eq. 5, a confirmation of the large and negative predictions for \( g_1^N(x, Q^2) \) in Fig. 2 could also serve as a sensitive cross-check of the EMC data on

\[ g_1^N(x, Q^2) \] in the small-x region. Such neutron measurements will be available soon\(^{61,62}\) which, furthermore, will serve as a fundamental test of the crucial Bjorken sum rule and of the QCD-improved parton model.\(^{1}\) Any significant deviation from these predictions would indicate the importance of non-perturbative higher-twist contributions invalidating Eq. 51.\(^{1,3,13}\) Alternatively, there are non-QCD models that contemplate a violation\(^{85}\) of the Bjorken sum rule, where the r.h.s. of (49) is expected to be about half as large (\( \approx 0.01 \)) due to physically motivated final states ("fire-strings") as opposed to quarks and gluons; this violation, being of non-partonic origin, is thus not related to a breakdown of isospin symmetry.
3. The Transverse Spin Structure Function $g_2$

The second spin-dependent deep inelastic structure function $g_2$, which is kinematically suppressed, appears in the antisymmetric component of the hadronic tensor in Eq. 1 in the following way (for recent reviews, see Refs. 86,87,4):

$$\mathcal{W}_{\mu
u} = \text{...} + \epsilon_{\mu
u\rho\sigma} \frac{M}{p \cdot q} \left[ s \eta_1 + \frac{1}{p \cdot q} \left( p \cdot q + s \cdot q \right) \right] \text{...} \tag{52}$$

For the longitudinal polarization considered so far, i.e. $s$ (anti)parallel to the beam direction, one gets

$$\mathcal{W}_L = \left[ g_1 - (2xM/Q)^2 g_2 \right] \tag{53}$$

which gives access dominantly to $Aq(x,Q^2)$ and $A\eta(x,Q^2)$ via, e.g., Eq. 47. For a transversely polarized nucleon, i.e. $s$ transverse to the beam direction, one has

$$\mathcal{W}_L = \left( (2xM/Q) g_1 + g_2 \right) \tag{54}$$

so that $g_1$ and $g_2$ enter with equal coefficients, which allows for a measurement of $g_2(x,Q^2)$ although the rates will be down by a factor $xM/Q$ with respect to (53). Strictly speaking, it is therefore the combination $g_1^* g_2$ which is the 'transverse spin structure function' although, for obvious reasons, one usually refers just to $g_2$. Since $g_2$ is related to a transverse polarization, it may not be easy to find a partonic interpretation because a massless parton can only have a longitudinal polarization. Furthermore, $g_2$ vanishes for a free (massless or massive) quark, i.e. for a pointlike nucleon, and thus $g_2$ cannot be expressed as an incoherent sum over free on-shell partons. The partons must be interacting (quark-gluon interactions) and/or virtual in order to contribute to $g_2$. Therefore $g_2$ will serve as a unique probe of 'higher twist' (twist $\neq \dim.-\spin$) as well.

Regardless of the difficulties with a partonic interpretation, one usually writes

$$g_2 = g_2^{WW} + g_2^{\text{HM}} \tag{55}$$

where the twist-2 'Wandzura-Wilczek' piece $g_2^{WW}$ receives contributions from the same class of twist-2 light-cone operators which determine $g_1$, giving rise to the so-called Wandzura-Wilczek relation

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \frac{1}{2} \int \frac{dz}{z} g_1(z,Q^2), \tag{56}$$

and where $\mathcal{E}_2(x,Q^2)$ refers to the twist-3 contribution which can be non-vanishing due to quark-gluon correlations in the target nucleon and due to quark masses. In other words, $g_2$ is determined by the partons' transverse momenta and their off-shellness ($\sim A^3$) which are unknown in the parton model. The origin of the WW-relation (56) can be easily understood in terms of a light-cone operator expansion of (52) where two classes of operators occur. One class is represented by totally symmetric operators of twist-2 (spin = n + U),

$$\mathcal{O}^{WW}_{\text{sym}}[T a, T b ... T n q, etc.] \tag{57}$$

where $n = 0$ and the appropriate subtraction of trace-terms is always implied. To the second class belong twist-3 operators (spin = n) with mixed symmetry that need at least two (antisymmetrized) indices (n > 0),

$$\mathcal{O}^{WW}_{\text{asym}}[T a, T b ... T n q, etc.] \tag{58}$$

The (Mellin) moments of $g_1$ and $g_2$ are then given by

$$g_1^{WW} = \int x g_1(x,Q^2) dx = a_n, \quad g_2^{WW} = \int x g_2(x,Q^2) dx = a_0 - \frac{1}{n+1} (a_n - a_0) \tag{59}$$

where $a_n$ and $a_0$ denote the nucleon matrix elements of $\mathcal{O}^{WW}_{\text{sym}}$ and $\mathcal{O}^{WW}_{\text{asym}}$, respectively. If the twist-3 operators are negligible, i.e. all $a_n > 0$, then Eq. 58 immediately gives the WW relation (86,86,90) (56) which completely determines the twist-2 contribution $g_2^{WW}$ to $g_2$ in Eq. 55 in terms of the measured $g_1(x,Q^2)$. The expected shape of $g_2^{WW}(x,Q^2)$ is shown in Fig. 3 derived from the existing EMC data for $g_1(x,Q^2) = 10 \text{ GeV}^2$. Note that an
experimental confirmation of the prediction for $g_2(x, Q^2)$ in Fig. 3 (dashed curve) would imply the absence of any twist-3 $g_2$ contribution to $g_2$ in Eq. 55. The expectation that $g_2$ should be small is based on the fact that it vanishes for ultra-relativistic on-shell quarks where $s^0 \sim p^0$ (i.e., in this case there are not enough four-vectors to form an antisymmetric combination in Eq. 52) and that non-relativistic corrections, being of the order $m_F M$ or $m_F^2 \Lambda$, are small for light quarks. In general, however, this twist-3 piece $g_2'$, being the contribution from the $g_2$ sequence in Eq. 59, is not expected to vanish due to the off-shellness of interacting quarks $^6, ^{86, 89-91}$ with virtuality $k^2$ where $k^2 A^2$ is not small. Indeed, the first of the twist-3 operators can be shown $^9$ to be an operator of order $g_s$ involving the gluon field strength tensor, $g_s^2 F^{\mu\nu} F_{\mu\nu} + \ldots \in \mathcal{G}$. (Note that even for a free quark, where $g_2$ vanishes, $g_2'$ has to be exactly cancelled by $g_2$ in Eq. 55; in this case the contribution to $g_2'$ is entirely given by the mass operator in Eq. 58.) In the chiral bag model, for example, the quarks inside the bag are much off-shell $^91, ^92$ and consequently the departures from the $\mathcal{WW}$ prediction of $g_2$, Eq. 56, are sizable, as can be seen from Fig. 4 where the predictions $^{93}$ for $g_2$ of an improved and extended (one-gluon exchange) version $^{92}$ of the MIT bag model $^9$ are shown. It should, however, be emphasized that the results of such bound-state models are expected to hold at some non-perturbative bound-state scale, typically $Q^2 \approx \Lambda^2$). Strictly speaking it would therefore not even be possible to use these predictions as an input for an evolution to a larger, experimentally relevant scale $Q^2 \gtrsim 1 \text{ GeV}^2$, unless one arbitrarily chooses the bag bound-state scale $Q^2 \approx \Lambda^2$ from where a perturbative RG evolution could be started. In order to demonstrate the importance of different scale effects, the $\mathcal{EMC}$ prediction for $g_2'$ at $Q^2 = 10 \text{ GeV}^2$ is shown in Fig. 4 as well (this dashed curve is the same as the one in Fig. 3). The difference between the bag and $\mathcal{EMC}$ prediction is indeed very large, indicating that expectations from bound-state models are not very relevant for actual
deep inelastic measurements. This is not very surprising since the bag predictions\(^\text{91,92}\) for \(g_1\) (as well as for unpolarized structure functions) disagree with actual measurements. Therefore the large bag-model prediction for \(g_2\) in Fig. 4 appears to be also not too relevant for future experiments. This is here particularly disturbing because the \(Q^2\)-evolution of \(g_2(x, Q^2)\) is theoretically unknown and in general an unsolved intricate problem, because the number of independent twist-3 operators \(O^{(0)}_3\) in (58) increase with \(n\).\(^94\) Thus there exist no Altarelli–Parisi type evolution equations and any extrapolation to the interesting small-\(x\) region is entirely unknown. There have been attempts\(^95\) to construct practical approximations to the \(Q^2\)-evolution in the limit \(x \to 1\) (large \(n\)) but the very large-\(x\) region is experimentally (and theoretically) less relevant.

It is therefore very important to check experimentally first the \(WW\) relation (56) (i.e. is \(g_2=\Delta g^\perp\) ) and then to extract \(g_Z=g^\perp-W\). A measurement of \(g_Z(x, Q^2)\) which appears to be easily accessible to experiment, will provide, for the first time, important information about the nucleon matrix elements of specific interaction-dependent quark-gluon operators of higher-twist and thus further insight into the bound-state dynamics involved.

A further important issue to be tested experimentally is the validity of the Burkhardt–Cottingham (BC) sum rule \(^\text{1}\)
\[
\int_0^1 g_2(x, Q^2) dx = 0 .
\]
(Note that \(g_2^{(0)}\) in (56) obeys, rather general grounds, this sum rule, \(\int_0^1 g_2^{(0)}(x, Q^2) dx = 0\)). This sum rule is a sufficiently general result of a superconvergence relation based on Regge asymptotics \((\alpha(0)=0)\). It turns out to be very robust and is most probably true\(^96,91,97\) (unless there are Regge cuts with branch points at \(\alpha(0)=0\)) or specific non-polynomial \(J=0\) fixed poles in Compton amplitudes that could invalidate the BC sum rule only by terms of order at most\(^98\) \(1/Q^2\). It is pointed out that the light-cone approach does not predict the first moment in Eq. 60, since the \(n\)-moment of \(g_2\) in Eq. 59 is only valid for \(n>0\). Although the presence of the factor \(n\) in Eq. 59 indicates the validity of the BC sum rule, we would need the \(n=0\) continuation of the twist-3 matrix element \(d_\perp\); if \(d_\perp^{(1/2)}\) as a function of \(n>0\), the BC sum rule (60) would hold; it would not hold if \(d_\perp^{(1/2)}\) for \(n=0\) (but we know that the twist-2 matrix element \(a_0\) is not singular because the first moment of \(g_1\) is finite). It is important to check the validity of the BC sum rule by experiment as far as possible. It would be very interesting if it were not true, because this would imply a non-conventional behavior of twist-3 operators \(d_\perp^{(1/2)}\) or the importance of long-range effects\(^91,99\).

4. Transverse Chiral–Odd Structure Functions

For completeness let me finally briefly mention the chiral-odd "transversity" distributions\(^\text{100,101,107,15,94}\) which received some interest recently. Analogously to \(g_1\), the leading twist-2 "transversity" distribution \(h_1(x, Q^2)\) is defined as measuring the probability in a transversely polarized nucleon to find a quark aligned with the nucleon polarization minus the probability to find it oppositely polarized. Thus this distribution can be directly measured, for example, in a doubly transversely polarized hadron-hadron scattering process where it appears in a transverse spin asymmetry \(A_{TT}\) defined as a straightforward generalization\(^\text{101}\) of the longitudinal one in Eq. 2. A transversity distribution is chiral-odd in the sense that it measures the correlation between left- and right-handed quarks \((L \sim R)\). This is in contrast to the deep inelastic structure functions \(g_1, g_2\) discussed thus far which, for obvious reasons, are always chiral-even \((L \sim L, R \sim R)\) apart from small quark mass corrections. The chirality of a quark can be flipped (without suppression) via the hard subprocess part, for example, in a Drell–Yan process with two initial hadrons, in contrast to a fully inclusive deep inelastic ep scattering process where the chirality is always conserved. Therefore, \(h_1\) and \(h_2\) (which is the chiral-odd analogon to \(g_1\) and receives twist-3 contributions as well) cannot be measured in fully inclusive deep inelastic lepton-hadron scattering and the transverse structure function \(g_1^{**} g_2\) in Eq. 54 are of course not the same as the ones bemeasured by the pure twist-2 quantity \(h_1(x, Q^2)\). Furthermore this latter twist-2 distribution \(h_1\), where only fermions contribute, obeys a simple simple Altarelli–Parisi type of evolution equation\(^102\).
There have been various suggestions\textsuperscript{101-105} to measure the quark transverse polarization in doubly transversely polarized hadron-hadron initiated reactions using the azimuthal dependence of the produced particles, like Drell-Yan dileptons, direct photons, heavy quarks and quark/gluon jets. For example, for the case of the dilepton production the spin asymmetry $A_{TT}$ for transverse-transverse collisions measures directly $h_T$, whereas for longitudinal-transverse collisions $A_{LT}$ gives access to $h_L$ provided $g_1$ and $g_1^* g_2$ are known; longitudinal-longitudinal collisions obviously give $g_1$ via $A_{LL}$. The experimental feasibility of these suggestions seems to be limited both by the low rates and by the difficulty to produce intense high energy polarized colliding hadron (in particular antiproton) beams. For this reason it has been suggested more recently to use single transverse polarization experiments where only one initial hadron is polarized, with the other one being unpolarized, and the polarization of the final state particle is observed instead\textsuperscript{106-108}. Alternatively, transversity distributions can also be measured by a semi-inclusive deep inelastic $e^p$ scattering experiment, with an unpolarized electron and a transversely polarized proton target, producing a polarized quark jet, for example\textsuperscript{107,109,110}. It will not be easy to measure the transverse handedness of the quark in the final state and old ideas\textsuperscript{111} have been recently revived\textsuperscript{107,110,112} for determining the polarization of an outgoing quark (or gluon) via the hadron distribution in the jet. Since transverse and "transversity" distributions are experimentally entirely unexplored, we hopefully can look forward to many new results in the years to come.

5. Summary and Conclusions

The explanation of the spin content of the nucleon in terms of a large quark-sea or gluon component (or a combination of both) is, strictly speaking, a matter of the (theoretical factorization) convention used in defining polarized quark distributions. If the gluon component is entirely absorbed into the definition of total quark polarizations, a large negative strange sea polarization is required for explaining the "small" EMC measurement of $g_1(x,Q^2)$ at $Q^2 = 10\text{ GeV}^2$, or a SU(3) broken sea polarization with $\Delta S = \Delta S > \Delta S = 0$ can do as well. However, a sizeable gluon contribution $\Delta g$ appears to be intuitively more plausible since its coupling to the photon, induced by the triangle anomaly, is non-vanishing if only hard perturbative contributions, appropriate for a deep inelastic process, are taken into account; the soft non-perturbative pieces are absorbed into the definition of quark polarizations which corresponds to a large and conserved total polarization of the flavor singlet quark component $\Delta q = \Delta u + \Delta d + \Delta s$. Thus $\Delta q$ is constituent-like, as are the conserved flavor non-singlet quark polarizations fixed by hyperon $\beta$-decays. The large total gluon polarization $\Delta g$ needed in addition for explaining the "small" EMC measurement, cannot be calculated perturbatively and one has to resort to non-perturbative methods and/or to different hard processes where $\Delta g$ can be measured independently.

Various theoretical arguments have been given in favor of a sizeable $\Delta g$ and in particular recent non-perturbative (current algebra) arguments result in an estimate of a large value for $\Delta g$. It is clear that only future dedicated polarization experiments can ultimately decide about the size of $\Delta g$; there have been many suggestions, but I believe that polarized photoproduction of heavy quarks and, in particular, purely (longitudinally polarized) hadronic heavy quark production, which depends "quadratically" on $\Delta g$, will be the most sensitive and important experiments for delineating $\Delta g(x,Q^2)$. Perhaps a singly longitudinally polarized proton-proton scattering experiment, using the 1 TeV unpolarized proton beams of TEBRAF or HERA to be scattered off a polarized fixed target, could do the job as well provided the experimentalists manage to measure the polarization of one of the final leptons of a produced Drell-Yan lepton pair, say. In addition, of course, the EMC result has to be checked by re-measuring $g_1^p$ and a measurement of the polarized neutron structure function $g_1^n(x,Q^2)$, hopefully available soon, will serve as a fundamental test of the crucial (isospin symmetry) Bjorken sum rule and of the QCD-improved parton model.

A measurement of the transverse spin structure function $g_2(x,Q^2)$, or more precisely $g_2^u(x,Q^2)$, will provide important information about specific quark-gluon operators of higher twist and thus further insight into the
bound-state dynamics of nucleons. This will be achieved by looking for deviations of $g_3$ from its twist-2 Wandzura-Wilczek contribution, uniquely determined by $g_1$. A further important issue to be tested experimentally is the validity of the Burkhardt-Cottingham sum rule for $g_3(x,Q^2)$ which is expected to hold on rather general grounds.

Finally, the so far entirely unknown "transversity" distributions, although not related to the spin structure of nucleons, should be experimentally explored either in doubly transversely polarized hadron-hadron scattering experiments or in semi-inclusive deep inelastic reactions with just the proton target being transversely polarized and by measuring the polarization of the produced outgoing quark.

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7. References


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