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Lattice models and BW type relations.

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Abstract

We show that the weights of dual partition functions of certain lattice models defined on the Temperley-Lieb algebra close relations which resembles the Birman-Wenzl algebra.At the dual point the relations collapse precisely to this algebra.As an extension of this result we find another algebra which intertwines the braid group and Hecke algebra and generalizes the BW algebra.

1. Introduction

A wide class of lattice models [1] have a partition function which in general can be expressed as

$$\mathcal{Z} = Tr\{\prod_{j=1}^{N-1}(D + A\theta_{2j})\prod_{j=1}^{N}(C + B\theta_{2j-1})\}^M \tag{1.1}$$

The letters A,B,C and D are just numbers and M and N are the numbers of rows and columns of a square lattice.The $\theta's$ form a representation of the Temperley-Lieb (TL) algebra given by generators E_i, $i = 1, 2, ..., n-1$, and relations

$$E_iE_{i\pm1}E_i = E_i$$

$$E_i^2 = qE_i$$

$$E_iE_j = E_jE_i, \quad |i-j| \geq 2, \tag{1.2}$$

or of the Hecke algebra defined by generators H_i,$i = 1, 2, ..., n-1$, and relations

$$H_iH_{i\pm1}H_i - H_i = H_{i\pm1}H_iH_{i\pm1} - H_{i\pm1}$$

$$H_i^2 = qH_i$$

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$$H_iH_j = H_jH_i, \quad |i-j| \geq 2, \tag{1.3}$$

We refer the reader to the book by P.Martin [1] for more informations about the algebras (1.2-3) and original references. Although these algebras are taken as the relevant ones for the models we show in this paper that for particular values of the anisotropies A,B,C and D the $\{\theta_i\}$ and the weights in which the argument of the trace in (1.1) factorizes together give realizations of some bigger algebras. These bigger algebras are the Birman-Wenzel (BW) algebra [3] and a new one we report about in this paper and denoted by $B\mathcal{H}$(n).These two algebras arise at the dual point as particular cases of relations satisfied by the weights coming from the partition function (1.1) and its dual to be defined in the next section.

The BW algebra is defined by the generators σ_i of the Artin braid group algebra $B(n)$ [2]

$$\begin{aligned} \sigma_i\sigma_{i+1}\sigma_i &= \sigma_{i+1}\sigma_i\sigma_{i+1} \\ \sigma_i\sigma_j &= \sigma_j\sigma_i, \quad |i-j| \geq 2, \end{aligned} \tag{1.4}$$

and E_i,$i = 1, 2, ..., n-1$ satisfying relations (1.2) and the following

$$\begin{aligned} \sigma_i + \sigma_i^{-1} &= m(1+E_i) \\ \sigma_{i\pm1}\sigma_iE_{i\pm1} &= E_i\sigma_{i\pm1}\sigma_i = E_iE_{i\pm1} \\ \sigma_{i\pm1}E_i\sigma_{i\pm1} &= \sigma_i^{-1}E_{i\pm1}\sigma_i^{-1} \\ \sigma_{i\pm1}E_iE_{i\pm1} &= \sigma_i^{-1}E_{i\pm1} \\ E_{i\pm1}E_i\sigma_{i\pm1} &= E_{i\pm1}\sigma_i^{-1} \\ \sigma_iE_i &= E_i\sigma_i = \frac{1}{l}E_i \\ E_i\sigma_{i\pm1}E_i &= lE_i \\ \sigma_i^2 &= m(\sigma_i + \frac{1}{l}E_i) - 1 \\ \sigma_i^3 &= (m+\frac{1}{l})\sigma_i^2 - (1+\frac{m}{l})\sigma_i + \frac{1}{l} \end{aligned} \tag{1.5}$$

where m and l are the parameters of the algebra and $q = m^{-1}(l+l^{-1}) - 1$. The $B\mathcal{H}(n)$ is defined by generators H_i and σ_i,i=1,...,n-1,satisfying relations (1.3) and (1.4),respectively,$q = m^{-1}(l+l^{-1}) - 1$,and the additional relations

$$\begin{aligned} \sigma_i + \sigma_i^{-1} &= m(1+H_i) \\ \sigma_{i\pm1}\sigma_iH_{i\pm1} &= H_i\sigma_{i\pm1}\sigma_i \\ \sigma_{i\pm1}\sigma_iH_{i\pm1} - H_iH_{i\pm1} &= \sigma_i\sigma_{i\pm1}H_i - H_{i\pm1}H_i \\ \sigma_{i\pm1}H_i\sigma_{i\pm1} - \sigma_i^{-1}H_{i\pm1}\sigma_i^{-1} &= \sigma_iH_{i\pm1}\sigma_i - \sigma_{i\pm1}^{-1}H_i\sigma_{i\pm1}^{-1} \end{aligned}$$

$$\sigma_{i\pm1}H_iH_{i\pm1} - \sigma_i^{-1}H_{i\pm1} = \sigma_iH_{i\pm1}H_i - \sigma_{i\pm1}^{-1}H_i$$
$$H_{i\pm1}H_i\sigma_{i\pm1} - H_{i\pm1}\sigma_i^{-1} = H_iH_{i\pm1}\sigma_i - H_i\sigma_{i\pm1}^{-1}$$
$$\sigma_iH_i = H_i\sigma_i = \frac{1}{l}H_i$$
$$H_i\sigma_{i\pm1}H_i - lH_i = H_{i\pm1}\sigma_iH_{i\pm1} - lH_{i\pm1}$$
$$\sigma_i^2 = m(\sigma_i + \frac{1}{l}H_i) - 1$$
$$\sigma_i^3 = (m + \frac{1}{l})\sigma_i^2 - (1 + \frac{m}{l})\sigma_i + \frac{1}{l} \tag{1.6}$$

The $B\mathcal{H}$ algebra extends the BW algebra in the sense that every representation of the latter is also a representation of the former but the $B\mathcal{H}$ algebra certainly has representations which are not representations of the BW algebra. We shall give realizations of the algebras (1.5-6).

The paper is organized as follows.In section 2 we define the dual partition function and deduce a set of relations satisfied by the generators E_i of the TL algebra and the weights from the partition function and its dual that are defined on this algebra. It is shown then that at the dual point the relations found collapse to the BW algebra.Inspired by these calculations we consider in section 3 the case when the weights are defined on the Hecke algebra and deduce a set of relations involving the generators of this algebra and the weights coming from the partition function and its dual.At the dual point,these relations collapse to the $B\mathcal{H}$ algebra. Section 4 summarizes the results found and indicates the problems left opened.

2. Algebra of weights and the BW algebra

We define the dual partition function by

$$\tilde{\mathcal{Z}} = Tr\{\prod_{j=1}^{N-1}(B + C\theta_{2j})\prod_{j=1}^{N}(A + D\theta_{2j-1})\}^M \tag{2.1}$$

It is given by the same objects found in the expression for Z except for the distinct location of the constants A,B,C,and D.Lets consider first the case when $\theta_i = E_i$ of TL algebra. Denote by R and K the weights coming from Z and its dual,respectively,that is,

$$R_{2i} = D + AE_{2i} \tag{2.2}$$
$$R_{2i-1} = C + BE_{2i-1} \tag{2.3}$$
$$K_{2i} = B + CE_{2i} \tag{2.4}$$
$$K_{2i-1} = A + DE_{2i-1} \tag{2.5}$$

Using solely the TL algebra one easily shows that the following relation is closed by the above weights:

$$CR_{2i}R_{2i-1}R_{2i} - DR_{2i-1}R_{2i}R_{2i-1}$$
$$= AK_{2i}K_{2i-1}K_{2i} - BK_{2i-1}K_{2i}K_{2i-1} \tag{2.6}$$

The R's and K's,separately don't satisfy the braiding relations,though.One can show,for instance,that

$$CR_{2i}R_{2i-1}R_{2i} + D(DC + DBq + AB)R_{2i-1}$$
$$= DR_{2i-1}R_{2i}R_{2i-1} + C(DC + ACq + AB)R_{2i}$$
$$+CD(DB - AC)q \tag{2.7}$$

A simple computation gives

$$R_{2i}^{-1} = \frac{1}{D} - \frac{A}{D(D+Aq)}E_{2i} \tag{2.8}$$

$$R_{2i-1}^{-1} = \frac{1}{C} - \frac{B}{C(C+Bq)}E_{2i-1} \tag{2.9}$$

The inverses of the K_i's can be obtained by a similar calculation or just taking $D \to B, A \to C, C \to A, B \to D$ in $(2.8-9)$.

From (2.2-3) and (2.8-9) we get

$$R_{2i} + R_{2i}^{-1} = D + \frac{1}{D} + (A - \frac{A}{D(D+Aq)})E_{2i} \tag{2.10}$$

$$R_{2i-1} + R_{2i-1}^{-1} = C + \frac{1}{C} + (B - \frac{B}{C(C+Bq)})E_{2i-1} \tag{2.11}$$

The following group of relations intertwine the R and K operators. It is tedious but straightforward to achieve them.One uses solely relations (2.2-5) and the TL algebra. They are:

$$R_{2i}R_{2i-1}E_{2i} = (DC + qAC + AB)E_{2i} + DBE_{2i-1}E_{2i} \tag{2.12}$$

$$R_{2i-1}R_{2i}E_{2i-1} = (CD + qBD + BA)E_{2i-1} + CAE_{2i}E_{2i-1} \tag{2.13}$$

$$E_{2i}R_{2i-1}R_{2i} = (CD + qCA + BA)E_{2i} + BDE_{2i}E_{2i-1} \tag{2.14}$$

$$E_{2i-1}R_{2i}R_{2i-1} = ACE_{2i-1}E_{2i} + (DC + DBq + AB)E_{2i-1} \tag{2.15}$$

$$R_{2i-1}E_{2i}R_{2i-1} = K_{2i}E_{2i-1}K_{2i} \tag{2.16}$$

$$R_{2i}E_{2i-1}R_{2i} = K_{2i-1}E_{2i}K_{2i-1} \tag{2.17}$$

$$R_{2i}E_{2i-1}E_{2i} = K_{2i-1}E_{2i} \tag{2.18}$$

$$R_{2i-1}E_{2i}E_{2i-1} = K_{2i}E_{2i-1} \tag{2.19}$$

$$E_{2i}E_{2i-1}R_{2i} = E_{2i}K_{2i-1} \tag{2.20}$$

$$E_{2i-1}E_{2i}R_{2i-1} = E_{2i-1}K_{2i} \tag{2.21}$$

$$R_{2i}E_{2i} = E_{2i}R_{2i} = (D + Aq)E_{2i} \tag{2.22}$$

$$E_{2i-1}R_{2i-1} = R_{2i-1}E_{2i-1} = (C + Bq)E_{2i-1} \tag{2.23}$$

$$E_{2i-1}R_{2i}E_{2i-1} = (Dq + A)E_{2i-1} \tag{2.24}$$

$$E_{2i}R_{2i-1}E_{2i} = (Cq + B)E_{2i} \tag{2.25}$$

Relations (2.12-15) close with similar ones for the K's.Other relations where R and K are interchanged and mixing the inverse operators are,of course ,possible but they become redundant in the special case of interest here as we will explain soon.The full set of relations closed by these weights will be considered in a future publication.

We also give the following relations:

$$R_{2i}^2 = (D + \frac{1}{D})R_{2i} + [(D + qA)A - \frac{A}{D}]E_{2i} - 1 \tag{2.26}$$

$$R_{2i-1}^2 = (C + \frac{1}{C})R_{2i-1} + [(C + Bq)B - \frac{B}{C}]E_{2i-1} - 1 \tag{2.27}$$

$$R_{2i}^3 = [(D + \frac{1}{D}) + (D + qA)]R_{2i}^2 - [1 + (D + \frac{1}{D})(D + Aq)]R_{2i} + (D + Aq) \tag{2.28}$$

$$R_{2i-1}^3 = [(C + \frac{1}{C}) + (C + Bq)]R_{2i-1}^2 - [1 + (C + \frac{1}{C})(C + Bq)]R_{2i-1} + (C + Bq) \tag{2.29}$$

The relations listed above have a striking resemblance with those defining the BW-algebra (1.5).If we require that

$$C = D \tag{2.30}$$

$$A = B \tag{2.31}$$

and,for $C = D = x, A = B = y$,

$$x^2 + qxy + y^2 = 0 \tag{2.32}$$

then,

$$R_i = x + yE_i \tag{2.33}$$

and

$$K_i = y + xE_i = xyR_i^{-1} \tag{2.34}$$

will satisfy the braid relations with the relations (2.10-29) collapsing to the following

$$R_i + R_i^{-1} = (x + \frac{1}{x}) + (y + \frac{1}{y})E_i \tag{2.35}$$

$$R_{i\pm1}R_iE_{i\pm1} = E_iR_{i\pm1}R_i = xyE_iE_{i\pm1} \tag{2.36}$$

$$R_{i\pm1}E_iR_{i\pm1} = (xy)^2R_i^{-1}E_{i\pm1}R_i^{-1} \tag{2.37}$$

$$R_{i\pm1}E_iE_{i\pm1} = (xy)R_i^{-1}E_{i\pm1} \tag{2.38}$$

$$E_{i\pm1}E_iR_{i\pm1} = (xy)E_{i\pm1}R_i^{-1} \tag{2.39}$$

$$R_iE_i = E_iR_i = (x + qy)E_i \tag{2.40}$$

$$E_iR_{i\pm1}E_i = (y + xq)E_i \tag{2.41}$$

$$R_i^2 = (x + \frac{1}{x})R_i + (x + qy)(y + \frac{1}{y})E_i - 1 \tag{2.42}$$

$$R_i^3 = [(x + \frac{1}{x}) + (x + qy)]R_i^2 - [1 + (x + \frac{1}{x})(x + yq)]R_i + (x + qy) \tag{2.43}$$

Together,conditions (2.32) and the additional one

$$(x + yq)(xq + y) = 1 \tag{2.44}$$

imply

$$xy = 1 \tag{2.45}$$

We call this last relation the duality condition since (2.30-31) imply that

$$Z(x, y) = (xy)^{NM}\tilde{Z}(1/y, 1/x) \tag{2.46}$$

and at the dual point (2.45) we then have

$$Z(x, 1/x) = \tilde{Z}(1/x, x) \tag{2.46}$$

A name for relation (2.31) is the Serre condition. Relations (2.35-43) then become those for the BW algebra (1.5) with $m = x + x^{-1}$,$l = -x^3$ and $q = -x^2 - x^{-2}$. Result (2.34) is the basic reason why we excluded certain relations, in particular,those intertwinning the inverse operators R^{-1} and K^{-1} from the list above.We may conclude that in (1.1) when $C = D = x$ and $B = A = 1/x$ the sets of E_i's and weights $R_i = x^{\pm} + x^{\mp}E_i$ form a representation of the BW algebra for the above value of the parameters.In general the value of q associated to the lattice models considered here is a positive number but we can take $x = it$ to get that.The case $t = 1$ is interesting for $m = 0$.This case corresponds to the Brower algebra.

3. The BH algebra

In section 2 we defined local transfer operators on the TL algebra which were then used to generate a set of relations that has the BW algebra relations as a particular case.As is well known and simply checked the TL algebra is a quotient of the Hecke algebra.Just compare relations (1.2) and (1.3).It is therefore natural to think that if we define the operators R_i's and K_i's in the Hecke algebra, the relations they will generate are more general and have as a particular case an algebra the BW algebra is a particular case of.This is exactly what we find in this section.

We define new weights R_i and K_i by relations

$$R_{2i} = D + AH_{2i} \tag{3.1}$$

$$R_{2i-1} = C + BH_{2i-1} \tag{3.2}$$

$$K_{2i} = B + CH_{2i} \tag{3.3}$$

$$K_{2i-1} = A + DH_{2i-1} \tag{3.4}$$

Again,A,B,C and D are constants but now the H operators are Hecke algebra generators satisfying (1.3). The inverse operators are given by

$$R_{2i}^{-1} = \frac{1}{D} - \frac{A}{D(D+Aq)}H_{2i} \tag{3.5}$$

$$R_{2i-1}^{-1} = \frac{1}{C} - \frac{B}{C(C+Bq)}H_{2i-1} \tag{3.6}$$

From (3.1-2) and (3.5-6) we get

$$R_{2i} + R_{2i}^{-1} = D + \frac{1}{D} + (A - \frac{A}{D(D+Aq)})H_{2i} \tag{3.7}$$

$$R_{2i-1} + R_{2i-1}^{-1} = C + \frac{1}{C} + (B - \frac{C}{C(C+qB)})H_{2i-1} \tag{3.8}$$

Using solely relations (1.3) and (3.1) -(3.4) one can easily check the following relations.

$$\begin{aligned} & R_{2i}R_{2i-1}H_{2i} - (DC + ACq + AB)H_{2i} - DBH_{2i-1}H_{2i} \\ = & H_{2i-1}R_{2i}R_{2i-1} - (DC + DBq + AB)H_{2i-1} - ACH_{2i-1}H_{2i} \end{aligned} \tag{3.9}$$

$$\begin{aligned} & R_{2i-1}R_{2i}H_{2i-1} - ACH_{2i}H_{2i-1} + (DC + ACq + AB)H_{2i} \\ = & R_{2i}R_{2i-1}H_{2i} - DBH_{2i-1}H_{2i} + (DC + BDq + AB)H_{2i-1} \end{aligned} \tag{3.10}$$

$$\begin{aligned} & (B^2 - C^2)[R_{2i}H_{2i-1}R_{2i} - K_{2i-1}H_{2i}K_{2i-1}] \\ = & (A^2 - D^2)[R_{2i-1}H_{2i}R_{2i-1} - K_{2i}H_{2i-1}K_{2i}] \end{aligned} \tag{3.11}$$

$$R_{2i}H_{2i-1}H_{2i} - K_{2i-1}H_{2i} = H_{2i}H_{2i-1}R_{2i} - H_{2i}K_{2i-1} \tag{3.12}$$

$$R_{2i-1}H_{2i}H_{2i-1} - K_{2i}H_{2i-1} = H_{2i-1}H_{2i}R_{2i-1} - H_{2i-1}K_{2i} \tag{3.13}$$

$$H_{2i}R_{2i} = R_{2i}H_{2i} = (D + Aq)H_{2i} \tag{3.14}$$

$$H_{2i-1}R_{2i-1} = R_{2i-1}H_{2i-1} = (C + Bq)H_{2i-1} \tag{3.15}$$

$$\begin{aligned} &BH_{2i-1}R_{2i}H_{2i-1} - B(A + Dq)H_{2i-1} \\ &= AH_{2i}R_{2i-1}H_{2i} - A(B + Cq)H_{2i} \end{aligned} \tag{3.16}$$

We also give the following relations:

$$R_{2i}^2 = (D + \frac{1}{D})R_{2i} + [(D + qA)A - \frac{A}{D}]H_{2i} - 1 \tag{3.17}$$

$$R_{2i-1}^2 = (C + \frac{1}{C})R_{2i-1} + [(C + Bq)B - \frac{B}{C}]H_{2i-1} - 1 \tag{3.18}$$

The cubic relations for R_{2i}^3 and R_{2i-1}^3 are exactly the same ones given by (2.28-29). One more relation is

$$\begin{aligned} BR_{2i}R_{2i-1}R_{2i} - AR_{2i-1}R_{2i}R_{2i-1} &= (BD - AC)(R_{2i}R_{2i-1} + R_{2i-1}R_{2i}) \\ +AR_{2i}(C^2 + B^2 + BCq) &- BR_{2i-1}(ADq + A^2 + D^2) \\ +A^2BC &- AB^2D \end{aligned} \tag{3.19}$$

From (3.19) we can read the conditions allowing the R operators to satisfy the braiding relations.They are exactly the same ones given by (2.30-32). In this case we get

$$R_i = x + yH_i \tag{3.20}$$

and

$$K_i = xyR_i^{-1} \tag{3.21}$$

with

$$R_i^{-1} = \frac{1}{x} + \frac{1}{y}H_i \tag{3.22}$$

where x and y satisfy the condition $x^2 + qxy + y^2 = 0$. The relations above then collapses to the special case given by relations:

$$R_i + R_i^{-1} = (x + \frac{1}{x}) + (y + \frac{1}{y})H_i \tag{3.23}$$

$$R_{i\pm1}R_iH_{i\pm1} = H_iR_{i\pm1}R_i \tag{3.24}$$

$$R_{i\pm1}R_iH_{i\pm1} - (xy)H_iH_{i\pm1} = R_iR_{i\pm1}H_i - (xy)H_{i\pm1}H_i \tag{3.25}$$

$$R_{i\pm1}H_iR_{i\pm1} - (xy)^2R_i^{-1}H_{i\pm1}R_i^{-1} = R_iH_{i\pm1}R_i - (xy)^2R_{i\pm1}^{-1}H_iR_{i\pm1}^{-1} \tag{3.26}$$

$$R_{i\pm1}H_iH_{i\pm1} - (xy)R_i^{-1}H_{i\pm1} = R_iH_{i\pm1}H_i - (xy)R_{i\pm1}^{-1}H_i \tag{3.27}$$

$$H_{i\pm1}H_iR_{i\pm1} - (xy)H_{i\pm1}R_i^{-1} = H_iH_{i\pm1}R_i - (xy)H_iR_{i\pm1}^{-1} \tag{3.28}$$

$$R_iH_i = H_iR_i = (x+qy)H_i \tag{3.29}$$

$$H_iR_{i\pm1}H_i - (y+xq)H_i = H_{i\pm1}R_iH_{i\pm1} - (y+xq)H_{i\pm1} \tag{3.30}$$

$$R_i^2 = (x+\frac{1}{x})R_i + (x+qy)(y+\frac{1}{y})H_i - 1 \tag{3.31}$$

$$R_i^3 = [(x+\frac{1}{x}) + (x+qy)]R_i^2 - [1+(x+\frac{1}{x})(x+yq)]R_i + (x+qy) \tag{3.32}$$

Using now the Serre and duality conditions we get the relations (1.6) taking $x + x^{-1} \to m, -x^3 \to l$ and $-x^2 - x^{-2} \to q = m^{-1}(l+l^{-1}) - 1$.

The algebra $B\mathcal{H}(n)$ above could have been obtained by observing the simple way (1.2) can be generalized to produce (1.3) and then repeating the procedure to find relations (1.6) from (1.5).Those relations involving the same index should be kept unchanged.This procedure gives a more general second relation than the one appearing in (1.6).In any case,it is clear that every representation of the BW algebra is also a representation of $B\mathcal{H}$ but the space of representations of the latter is certainly bigger.Those representations of the Hecke algebra that are not of TL give via (3.20-22) representations of the algebra $B\mathcal{H}$ that are not of the BW algebra.

4. Discussion

In this paper we have shown that the weights coming from the dual partition functions (1.1) and (2.1) together close certain very interesting relations.In particular,relation (2.6) is very intriguing but it remains to be seen whether any deep implications will follow from it and from the other relations.Those obtained with the weights defined on the TL algebra have a striking resemblance with the BW algebra relations and they actually coincide at the dual point where the set $\{E_i, weights\}$ then gives a realization of the BW algebra.Another class of models have weights defined on the Hecke algebra.Repeting similar calculations to this case we were able to find a new algebra that gives the structure obeyed by the set $\{H_i, weights\}$ at the dual point.This algebra generalizes the BW algebra since its representations are also representations of the $B\mathcal{H}$ algebra but the latter's space of representation should be bigger as explained in the previous section.One should of course ask for the mathematical structure of this algebra.Most important,perhaps,what are its other realizations?This is very relevant not only from the mathematical point of view but from the physics point of view as well since a large number of exactly solved lattice models were built from representations of the algebras (1.2-4).Solutions of Yang-Baxter equation can in general be obtained from Baxterization of representations of braid group representations.Jones [4] did that for representations coming from the the BW

algebra and found a nice connexion with the Fateev-Zamolodchikov model.It might be that a similar connexion might exist between the $B\mathcal{H}$ algebra and the Chiral Potts model but this is a possibility that we should discuss elsewhere.

Each of the algebras (1.2-3,5) have a Markov trace defined on them by means of which invariant of knots are defined [5-7].Remains to be seen whether a Markov trace may be defined on the $B\mathcal{H}$ algebra and whether a topological invariant can be obtained from it.

In [3] Birman and Wenzel showed how to obtain the Brower algebra as a limiting case of their algebra.Applied to the $B\mathcal{H}$ relations their calculation leads to a generalization of that algebra.

The relation we called the Serre condition involves scalars only whereas the well known Serre relations associated to certain quantum group algebras involve only noncommuting quantities but the resemblance is striking and makes one think since they are both related to solutions of braiding type relations and besides it is known that a connexion exists between the the TL and Hecke algebras with certain quantum group algebras [8].

To conclude I would like to add that an algebra mixing the Temperley-Lieb algebra and the Hecke algebra can be found from the algebra of weights but we will consider this elsewhere.

We shall adress all the problems mentioned above in future publications.

Note.I recently learned from B.Westbury that the relations (1.6) also appears in his work.La Roche pointed out to me that the same algebra appears in work by Wadati et al but I was not able to find the references.

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