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M_w WITHOUT Δr

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Abstract

The problem of predicting the W mass in the standard model is considered. Renormalization is performed by fitting the bare parameters of the Lagrangian in terms of three data points, namely α, G_F and M_Z. The $\rho-$parameter is a computable quantity and no special definition of $\sin^2\theta_w$ is introduced.

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Electroweak interactions are now entering in a new era of precision experiments. In a systematic investigation of the validity of the standard model we must therefore consider the radiative corrections as an indispensable part of the verification of the theory. The model contains three free parameters, namely g, s_θ and M. The relation of these parameters to measurable quantities is the content of a renormalization scheme.

A first example of the extension from QED to the standard model of a renormalization scheme is perhaps given by the calculation of Ross and Taylor [1]. However the first complete calculation of $e^+e^- \to \mu^+\mu^-$ in the standard electroweak theory was performed in ref.[2], where renormalization was implemented in a way that has become known as *on shell* scheme. At that time this procedure was understood as a sort of compromise because admittedly some very interesting radiative corrections to vector boson masses were missing. Renormalization from low energy data was thereafter explained in full details by Green and Veltman [3], leading to the vector boson mass shift discovered independently by Veltman [4] and Antonelli, Consoli and Corbò [4]. Since then the on shell scheme has become very popular and therefore the definition of $\sin^2\theta_w = 1 - M_W^2/M_Z^2$, proposed by Sirlin [5], is more or less commonly accepted. Another fundamental ingredient of the standard model is the $\rho-$parameter, introduced by Ross and Veltman [6]. The previous definition of $\sin^2\theta_w$ makes ρ equal to one by construction, while the importance of the $\rho-$parameter for confrontation with experimental data has been pointed out and repeatedly reviewed by Veltman [7].

Here we want to emphasize once more the importance of a renormalization scheme were, given the data, we subtract radiative corrections and determine g, θ_W and M. Once three measurements are obtained the rest must follow because of gauge invariance. As soon as a fourth measurement will be available, for example the W mass, then we can compute the $\rho-$parameter. Our scheme [8,9] is therefore nothing more than a set of fitting equations as introduced by Veltman in a prophetic paper [10] on mass differences in the standard model.

One quantity we can predict from the knowledge of α, G_F and M_Z is the W mass. This is of course done in the on shell scheme by introducing the factor Δr. Here we give our solution for M_W in a scheme where ρ is a computable quantity.

An unwanted feature of the standard model is that the $Z^0-\gamma$ transition at $q^2 = 0$ is not zero. A solution has been given in different approaches by Kennedy and Lynn [11], Hollik [12], and one of the authors [9]. Following ref.[9] we start with a triplet of vector bosons B^i_μ and a singlet B^0_μ. Before diagonalizing the lagrangian we introduce a new coupling constant $\bar{g}$

$$\bar{g} = g\,(1 + g^2\,\Gamma) = g\left[1 + \frac{g^2}{8\pi^2}\,B_0\,(0, M, M)\right]$$

where B_0 is the scalar two-point function [2]. Defining A_μ and Z^0_μ fields in terms of $\bar{g}$ gives for the quadratic part

$$\mathcal{L}_2 = -M^2\,W^+_\mu W^-_\mu - \tfrac{1}{2}\frac{M^2}{c_\theta^2}\,Z^0_\mu Z^0_\mu + \bar{g}^2 M^2 \Gamma\left(Z^0_\mu Z^0_\mu - \frac{s_\theta}{c_\theta}\,Z^0_\mu A_\mu + 2\,W^+_\mu W^-_\mu\right) + O\left(\bar{g}^4\right)$$

Charged and neutral currents receive extra contributions

$$\mathcal{L}_{CC}^{extra} = -\frac{i\,\bar{g}^3}{2\sqrt{2}}\Gamma\,\left[W_\mu^+\,\bar{\nu}\gamma^\mu\left(1+\gamma^5\right)e + W_\mu^-\,\bar{e}\gamma^\mu\left(1+\gamma^5\right)\nu\right]$$

$$\mathcal{L}_{NC}^{extra} = \frac{i}{4}\bar{g}^3\Gamma\,\left(s_\theta A_\mu + c_\theta Z_\mu^0\right)\left[\bar{e}\gamma^\mu\left(1+\gamma^5\right)e - \bar{\nu}\gamma^\mu\left(1+\gamma^5\right)\nu\right]$$

With this choice the vector boson self-energies are modified. Dropping the bar we get

$$S_{\gamma\gamma} = \frac{g^2\,s_\theta^2}{16\,\pi^2}\,p^2\,\Pi_{\gamma\gamma}$$

$$S_{ZZ} = \frac{g^2}{16\,\pi^2\,c_\theta^2}\,\left[\left(S_{ZZ}^0 - 32\,\pi^2\,M^2\,\Gamma\right) - 2\,s_\theta^2\,\left(S_{Z\gamma}^0 + 16\,\pi^2\,M^2\,\Gamma\right) + s_\theta^4\,p^2\,\Pi_{\gamma\gamma}\right]$$

$$S_{Z\gamma} = \frac{g^2}{16\,\pi^2}\,\frac{s_\theta}{c_\theta}\,\left[\left(S_{Z\gamma}^0 + 16\,\pi^2\,M^2\,\Gamma\right) - s_\theta^2\,p^2\,\Pi_{\gamma\gamma}\right]$$

$$S_{WW} = \frac{g^2}{16\,\pi^2}\,\left[\left(\Pi_{WW}^0 + s_\theta^2\,\Pi_{WW}^1\right)p^2 + \left(\Sigma_{WW}^0 - 32\,\pi^2\,M^2\,\Gamma\right) + s_\theta^2\Sigma_{WW}^1\right]$$

with

$$\begin{aligned}\Pi_{\gamma\gamma} = &\tfrac{2}{3} - 12\,B_{21}(M,M) + 7\,B_0(M,M)\\ &+ 4\sum_f\,\left[B_f(m_e,m_e) + \tfrac{4}{3}\,B_f(m_u,m_u) + \tfrac{1}{3}\,B_f(m_d,m_d)\right]\end{aligned}$$

$$\begin{aligned}S_{Z\gamma}^0 = &\,p^2\,\{\tfrac{2}{3} - 10\,B_{21}(M,M) + \tfrac{13}{2}\,B_0(M,M)\\ &+ \sum_f\,[B_f(m_e,m_2) + 2\,B_f(m_u,m_u) + B_f(m_d,m_d)]\} - 2\,M^2 B_0(M,M)\end{aligned}$$

$$S_{ZZ}^0 = p^2\,\Pi_{ZZ}^0 + \Sigma_{ZZ}^0$$

$$\begin{aligned}\Pi_{ZZ}^0 = &\tfrac{2}{3} - 9\,B_{21}(M,M) + \tfrac{25}{4}\,B_0(M,M) - B_{21}(M_0,m_H)\\ &- B_1(M_0,m_H) - \tfrac{1}{4}\,B_0(M_0,m_H)\\ &+ \tfrac{1}{2}\sum_f\,[B_f(m_e,m_e) + B_f(m_\nu,m_\nu) + 3\,B_f(m_u,m_u) + 3\,B_f(m_d,m_d)]\end{aligned}$$

$$\begin{aligned}\Sigma_{ZZ}^0 = &-2\,M^2 B_0(M,M) + \tfrac{1}{2}\,M_0^2 B_1(M_0,m_H) + \tfrac{5}{4}\,M_0^2 B_0(M_0,m_H)\\ &- \tfrac{1}{2}\,m_H^2\,B_1(M_0,m_H) - \tfrac{1}{4}\,m_H^2 B_0(M_0,m_H)\\ &- \tfrac{1}{2}\sum_f\,[m_\nu^2 B_0(m_\nu,m_\nu) + m_e^2 B_0(m_e,m_e)\\ &+3\,m_u^2 B_0(m_u,m_u) + 3\,m_d^2 B_0(m_d,m_d)]\end{aligned}$$

$$\begin{aligned}\Sigma_{WW}^0 = &\tfrac{9}{2}\,(M_0^2 - M^2)B_1(M_0,M) + \tfrac{1}{4}\,(13M_0^2 - 21M^2)B_0(M_0,M)\\ &+ \tfrac{1}{2}\,(M^2 - m_H^2)B_1(M,m_H) + \tfrac{1}{4}\,(5M^2 - m_H^2)B_0(M,m_H)\\ &+ \sum_f\,[(m_e^2 - m_\nu^2)B_1(m_\nu,m_e) - m_\nu^2 B_0(m_\nu,m_e) + 3\,(m_d^2 - m_u^2)B_1(m_u,m_d)\\ &-3\,m_u^2 B_0(m_u,m_d)]\end{aligned}$$

$$\Sigma_{WW}^1 = 2\,(M^2 - M_0^2)\,[2B_1(M_0,M) + B_0(M_0,M)] - 2\,M^2\,[2B_1(\lambda,M) + B_0(\lambda,M)]$$

$$\begin{aligned}\Pi_{WW}^0 = &\tfrac{2}{3} - 9B_{21}(M_0,M) - 9B_1(M_0,M) + \tfrac{7}{4}\,B_0(M_0,M)\\ &- B_{21}(M,m_H) - B_1(M,m_H) - \tfrac{1}{4}\,B_0(M,m_H)\\ &+ 2\sum_f\,[B_{21}(m_\nu,m_e) + B_1(m_\nu,m_e) + 3\,B_{21}(m_u,m_d) + 3\,B_1(m_u,m_d)]\end{aligned}$$

$$\begin{aligned}\Pi_{WW}^1 = &\,8\,B_{21}(M_0,M) - 2\,B_0(M_0,M) + 8\,B_1(M_0,M) - 8\,B_{21}(\lambda,M)\\ &- 8\,B_1(\lambda,M) + 2\,B_0(\lambda,M)\end{aligned}$$

where $B_f = 2\,B_{21} - B_0$ and λ is the photon mass. In a one loop calculation for the electric charge we find a replacement [3]

$$4\,\pi\,\alpha = g^2\,s_\theta^2(1+\delta\alpha), \qquad \delta\alpha = \delta^W + \delta^V + \delta^\gamma + \delta^{Z\gamma}$$

where δ^W represents wave-function factors given in ref.[3] in terms of B form factors. Two vertex diagrams give δ^V which can be cast into the form

$$\delta^V = \frac{g^2}{32\,\pi^2}\sum_{f=e,\mu}\left[C_f(0) - \frac{v_\theta + 1}{4\,c_\theta^2}\,C_{ff}(0)\right]$$

where $v_\theta = 4\,s_\theta^2 - 1$ and using the C form factors [6]

$$C_{ff}(s) = -2\,C_{24} + (C_{11} + C_{23})\,s + 1, \quad C_f(s) = 6\,C_{24} - (C_0 + C_{11} + C_{23})\,s - 1$$

The non e.m. contributions add up to a non zero result

$$\delta^W + \delta^V = \frac{g^2}{16\,\pi^2}\,(\Delta - \ln\,M^2)$$

where $\Delta = -2/(n-4) + \gamma - \ln\,\pi$. Also vector bosons give rise to a novel feature

$$\delta^{Z\gamma} = -v_\theta\,\frac{c_\theta}{s_\theta}\,\frac{S_{Z\gamma}(0)}{M^2} \neq 0$$

Indeed the $Z^0 - \gamma$ transition at zero momentum is

$$S_{Z\gamma}(0) = -\frac{g^2}{8\pi^2}\frac{s_\theta}{c_\theta} M^2 \left(\Delta - \ln M^2\right)$$

In our approach the following happens

$$\delta^{Z\gamma} = 0, \qquad \delta^W + \delta^V + \delta^{extra} = 0, \qquad \delta^{extra} = -\frac{g^2}{16\pi^2}\left(\Delta - \ln M^2\right)$$

where δ^{extra} is the contribution from the extra $ee\gamma$ vertex. At the same time the infinite parts in the eeZ^0 vertex cancel against the extra eeZ^0 vertex. With the outlined procedure we only need to worry about the photon self-energy diagrams. The combination $gs_\theta = e$ is easily fixed in terms of α from $e\mu$ scattering

$$\frac{1}{g^2 s_\theta^2} = \frac{1}{4\pi\alpha} + \frac{1}{16\pi^2}\Pi_{\gamma\gamma}(0)$$

The second experimental quantity we need is given by G_F or rather by the muon lifetime

$$\frac{1}{\tau_\mu} = \frac{m_\mu^5}{192\pi^3}\frac{g^4}{32M^4}\left(1 + \delta_\mu^{e.m.} + \delta_\mu^{weak}\right)$$

where

$$\delta_\mu^{e.m.} = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)$$

and the contribution to δ_μ^{weak} from wave-function factors, four vertex diagrams, five box diagrams and the extra vertex from $\mathcal{L}_{CC}^{extra}$, which we call $2\,\delta_G$, is infrared and ultraviolet finite

$$\delta_G = \frac{g^2}{8\pi^2}\left[3 + \frac{7 - 4s_\theta^2}{4s_\theta^2}\ln c_\theta^2\right] + \tfrac{1}{2}\delta_\mu^{e.m.}$$

The combination M^2/g^2 is fixed by our second fitting equation, obtained from μ-decay when we include W self-energy diagrams

$$8\frac{M^2}{g^2} = \frac{1+\delta_G}{G_F} + \frac{1}{2\pi^2}\overline{\Sigma}_{WW}(0), \qquad \overline{\Sigma}_{WW} = \overline{\Sigma}^0_{WW} + s_\theta^2\,\Sigma^1_{WW}$$

A bar implies that the corresponding function has been opportunely subtracted. The third fitting equation, namely the position of the Z^0 peak, gives

$$\begin{aligned}\frac{M^2}{c_\theta^2} &= M_Z^2 + \frac{g^2}{16\pi^2 c_\theta^2} Re\left[\overline{S}^0_{ZZ}\left(-M_Z^2\right) - 2s_\theta^2\overline{S}^0_{Z\gamma}\left(-M_Z^2\right) - s_\theta^4 M_Z^2\Pi_{\gamma\gamma}\left(-M_Z^2\right)\right]\\ &\equiv M_Z^2 + \frac{g^2}{16\pi^2 c_\theta^2} Re\, f_Z\left(-M_Z^2\right)\end{aligned}$$

The previous equation defines $f_Z(p^2)$. Inside the function f_Z we used the zero'th order value for s_θ^2, which follows from the fitting equations

$$\bar{s}_\theta^2 = \tfrac{1}{2}\left[1 - \sqrt{1 - \frac{2\pi\alpha}{G_F M_Z^2}}\right]$$

Up to first order the solution for s_θ^2 is found to be

$$\begin{aligned}s_\theta^2 &= \bar{s}_\theta^2\left(1 + \frac{\bar{c}_\theta^2}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\kappa\right)\\ \kappa &= \delta_G - \frac{\alpha}{4\pi}\Pi_{\gamma\gamma}(0) + \frac{G_F}{2\pi^2}\left[\overline{\Sigma}_{WW}(0) - Re\, f_Z\left(-M_Z^2\right)\right]\end{aligned}$$

Once more the corrections to s_θ^2 are not finite. To determine M_W, for instance from the total cross section of $e^+e^- \to W^+W^-$, what we need is

$$g^2\Delta_W = \frac{g^2}{p^2 + M^2 - S_{WW}(p^2)}$$

The pole of the propagator, $p^2 = -M_W^2$, gives M_W. First we introduce

$$f_W(p^2) = \left(\Pi^0_{WW} + \bar{s}_\theta^2\,\Pi^1_{WW}\right)p^2 + \overline{\Sigma}^0_{WW} + \bar{s}_\theta^2\,\Sigma^1_{WW}$$

Since

$$g^2\Delta_W = \left[\frac{p^2}{g^2} + \frac{M^2}{g^2} - \frac{1}{16\pi^2}f_W(p^2)\right]^{-1}$$

the second fitting equation gives

$$g^2\Delta_W = 8G_F\left[\frac{8G_F}{g^2}p^2 + 1 + \delta_G + \frac{G_F}{2\pi^2}\overline{\Sigma}_{WW}(0) - \frac{G_F}{2\pi^2}f_W(p^2)\right]^{-1}$$

Finally we need G_F/g^2, which can be derived from the remaining equations

$$8\frac{G_F}{g^2} = \frac{1}{\bar{c}_\theta^2 M_Z^2}\left(1 + \frac{\bar{c}_\theta^2}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\kappa\right) + \frac{G_F}{2\pi^2}\bar{s}_\theta^2 \Pi_{\gamma\gamma}(0)$$

The W propagator is

$$g^2\Delta_W = 8G_F\bar{c}_\theta^2\left\{-x + \bar{c}_\theta^2 + \frac{\alpha}{4\pi}\frac{\bar{s}_\theta^2}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\Pi_{\gamma\gamma}(0)\,x + \left(1 - \frac{x}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\right)\bar{c}_\theta^2\delta_G \right.$$
$$\left. + \frac{G_F\bar{c}_\theta^2}{2\pi^2}\left[\overline{\Sigma}_{WW}(0) - f_W(p^2)\right] - \frac{G_F}{2\pi^2}\frac{\bar{c}_\theta^2}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\left[\overline{\Sigma}_{WW}(0) - Re\, f_Z\left(-M_Z^2\right)\right] x\right\}^{-1}$$

with $x = -p^2/M_Z^2$. As a first step we use the explicit expressions for the self-energies and prove that all ultraviolet divergent factors cancel out in the above formula. In this way the W propagator is finite and scale independent. The first term in braket, $-x + \bar{c}_\theta^2$, is nothing but the lowest order pole. The one loop corrected result is given by the zero of

$$F_W(x) = -x + \bar{c}_\theta^2 + \frac{\alpha}{4\pi}\frac{\bar{s}_\theta^2}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\Pi_{\gamma\gamma}(0)\,x + \left(1 - \frac{x}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\right)\bar{c}_\theta^2\delta_G$$
$$+ \frac{G_F\bar{c}_\theta^2}{2\pi^2}\left[\overline{\Sigma}_{WW}(0) - Re\, f_W(-xM_Z^2)\right]$$
$$- \frac{G_F}{2\pi^2}\frac{\bar{c}_\theta^2}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\left[\overline{\Sigma}_{WW}(0) - Re\, f_Z\left(-M_Z^2\right)\right] x$$

For the numerical evaluation of M_W we split $\Pi_{\gamma\gamma}$ into hadronic part plus rest [12]

$$\Pi_{\gamma\gamma}(0) = \Pi^h_{\gamma\gamma}(0) + \Pi^{rest}_{\gamma\gamma}(0), \qquad \Pi^h_{\gamma\gamma}(0) = Re\,\Pi^h_{\gamma\gamma}\left(-M_Z^2\right) - Re\,\Pi^{h,sub}_{\gamma\gamma}\left(-M_Z^2\right)$$

where $\Pi^{sub}(p^2) = \Pi(p^2) - \Pi(0)$. The first term is computed perturbatively with $m_u = 4.5\,MeV, m_d = 7.9\,MeV, m_c = 1.35\,GeV, m_s = 155\,MeV$ and $m_b = 5.3\,GeV$. For the second one we use effective quark masses, derived from a fit to a dispersion integral [12], $m_u = m_d = 0.041\,GeV, m_c = 1.5\,GeV, m_s = 0.15\,GeV$ and $m_b = 4.5\,GeV$. The top quark mass is left as a free parameter. For large m_t we use the asymptotic expressions for the B form factors [9] to obtain

$$F_W(x) \sim -x + \bar{c}_\theta^2 + \tfrac{3}{8}\frac{G_F m_t^2}{\pi^2}\frac{\bar{c}_\theta^2}{\bar{c}_\theta^2 - \bar{s}_\theta^2}\,x$$

Thus M_W clearly increases with m_t, a situation which has an obvious correspondence with what we get from low energy data. There M_Z is not input parameter and $\delta M_W^2 \sim const$, $\delta M_Z^2 \sim -m_t^2/M_Z^2$ as $m_t \to \infty$.

Data points are now $\alpha = 1/137.0359895$ and $\sqrt{2}G_F = 1.16637 \times 10^{-5}\,GeV^{-2}$. As a result for $M_Z = 92GeV$ we find M_W to be

W mass (all masses in GeV)			
m_t/m_H	10	100	1000
50	80.87	80.80	80.66
100	81.14	81.06	80.92
150	81.42	81.35	81.21
200	81.77	81.70	81.55

We have made several checks for our numerical calculation. The B form factors are computed using the FORTRAN code QFORMF [13], written in REAL*16 precision for the VAX/8600. Cancellation of ultraviolet factors is guarantee since the results are invariant against variations in Δ, the quantity substituted for $1/n - 4$ in the various divergent functions. Finally we have used α, G_F, M_W and M_Z as input in order to compute ρ according to the results of ref.[9]. By requiring $\rho = 1$ we obtain a value for m_t for each pair M_W, M_Z, for example $M_Z = 91.9\,GeV$ and $M_W = 80.9\,GeV$ correspond to $m_t = 94.11\,GeV$ (when $m_H = 100\,GeV$). Reversing the procedure we use the same value of M_Z and the derived m_t to compute M_W. We found agreement, within few MeV, between the two values of M_W.

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