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ABSTRACT

By discussing recent results which are related to multiplicity distributions, it is shown how the study of the latter both in full phase space and in restricted regions of it can be very effective in order to deepen our understanding of challenging problems central to high energy physics, such as parton cascades, hadronization processes, rapidity gaps physics and final particles correlations.

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ABSTRACT

By discussing recent results which are related to multiplicity distributions, it is shown how the study of the latter both in full phase space and in restricted regions of it can be very effective in order to deepen our understanding of challenging problems central to high energy physics, such as parton cascades, hadronization processes, rapidity gaps physics and final particles correlations.

1. Introduction

The study of multiplicity distributions (MD's) is a central topic in multiparticle dynamics. It is a signature, since jet discovery in cosmic rays physics, of the intimate non-linear structure of strong interactions, which we believe today to be described by QCD. What is surprising in this respect is the crudeness of QCD predictions on MD's: the explanation lies of course in the extreme difficulty of reliable QCD calculations which drive us from the perturbative region, where partons evolve, to the non-perturbative one, where hadrons are formed. This fact emphasizes the need for QCD-inspired models which can help to understand this transition. On the experimental side, non-linearity manifests itself in the striking appearance of the negative binomial distribution (NBD) in all classes of reactions, both in full phase space and in rapidity intervals, and in the properties of consequent clan structure analysis, a fact which is known as "NB regularity".

These ideas are illustrated in this paper by discussing some recent results on MD's. From a theoretical point of view, one is confronted by a twofold problem, concerning on one hand the understanding of parton shower evolution down to low virtualities (section 2 and 3) and on the other hand the analysis of the largely unknown hadronization mechanisms (section 4). From a phenomenological point of view, the measurements of rapidity gaps probability in hard processes can benefit from the

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† Talk presented by R. Ugoccioni

acquired knowledge of MD's in order to estimate the background (section 5), while the integrated study of MD's and correlations, especially in nucleus-nucleus collisions, leads to a sharper analysis of differences and similarities with respect to hadron-hadron collisions (section 6).

2. Multiplicity distributions and parton cascades

The first analytical description of MD's in parton cascades in full phase space goes back to the Leading Log Approximation (LLA). Differential equations for the generating functions of MD's of quarks and gluons in quark- and gluon-jets were first developed in Ref. [1], in the context of jet calculus and within an inclusive approach, and in Ref. [2], taking an exclusive approach in which the same problem is formulated according to a fully Markoffian description. Let $F_G = F_G(\mathcal{Y}; z_q, z_g)$ and $F_Q = F_Q(\mathcal{Y}; z_q, z_g)$, respectively, denote the generating functions in a gluon-jet and in a quark-jet; when collinear divergences in the splitting functions are regularized with a fixed cutoff ϵ, the mentioned equations are as follows:

$$\frac{dF_G}{d\mathcal{Y}} = A(F_G^2 - F_G) + B(F_Q^2 - F_G) \tag{1}$$

$$\frac{dF_Q}{d\mathcal{Y}} = \tilde{A}F_Q(F_G - 1) \tag{2}$$

where the evolution is given in terms of jet thickness $\mathcal{Y}$

$$\mathcal{Y} = \frac{6}{11\mathcal{N}_c - 2\mathcal{N}_f} \log\left(\frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2}\right) \tag{3}$$

with Q virtuality, Q_0 virtuality cut-off of the cascade, Λ renormalization scale. The elementary splitting probabilities are

$$\begin{aligned} A &= (\mathcal{N}_c^2 - 1)/2\epsilon\mathcal{N}_c \\ \tilde{A} &= \mathcal{N}_c/\epsilon \\ B &= \mathcal{N}_f/3 \end{aligned} \tag{4}$$

These correspond (respectively) to the processes of gluon splitting into two gluons (A), of gluon emission from a quark ($\tilde{A}$) and of gluon splitting into quark-antiquark pair (B); $\mathcal{N}_c$ is the number of colours and $\mathcal{N}_f$ the number of flavours.

A simple but instructive result was found[2] by setting $B = 0$, i.e., freezing the number of quarks and letting gluons evolve:

$$F_G(\mathcal{Y}; z_q, z_g) = z_g[z_g + (1 - z_g)e^{A\mathcal{Y}}]^{-1} \tag{5}$$

$$F_Q(\mathcal{Y}; z_q, z_g) = z_q[z_g + (1 - z_g)e^{A\mathcal{Y}}]^{-\tilde{A}/A} \tag{6}$$

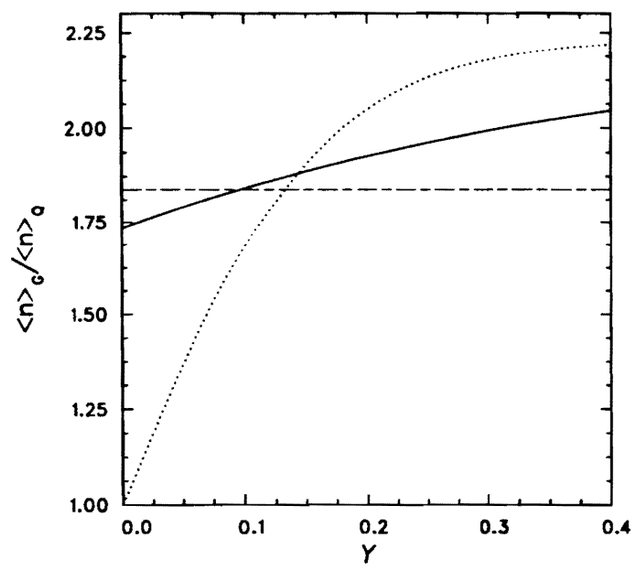

Figure 1. Ratio $\langle n\rangle_G/\langle n\rangle_Q$ in different approximations in perturbative QCD as a function of jet thickness $\mathcal{Y}$. Dotted line: LLA[6]; Solid line: MLLA[7]; Chain-dashed line: MLLA with frozen coupling constant[9].

On one hand, the number of gluons in a gluon jet is described by a shifted geometric distribution (shifted because the initial gluon never disappears) of average $\langle n_g\rangle_G = e^{A\mathcal{Y}}$; in the context of the negative binomial regularity, this result has led to the interpretation of clans as intermediate independent gluon jets[3, 4, 5]. On the other hand, the number of gluons in a quark jet is given by a NBD of average $\langle n_g\rangle_Q = \tilde{A}(e^{A\mathcal{Y}}-1)/A$ and parameter $k = \tilde{A}/A$. Parameter k is here independent of energy and in view of the elementary probabilities defined above its value is $k = 4/9$. a rather peculiar result for the distribution, since k is smaller than 1.

The full Eqs. 1 and 2 can in principle be solved for all moments of the distributions, although one cannot solve directly for the generating functions[6]. As an example, Fig. 1 shows (dotted line) the ratio of the average number of partons in a gluon jet to that in a quark jet as a function of jet thickness, with asymptotic value

$$\frac{\langle n\rangle_G}{\langle n\rangle_Q} = \frac{9}{4} \tag{7}$$

The same figure shows also the curve for the next to next to leading calculation[7] of the same ratio, with running (solid line) and frozen (chain-dashed line) coupling constant. Comparison of these formulae to experimental data is outside the scope of this paper (see Ref. [8]).

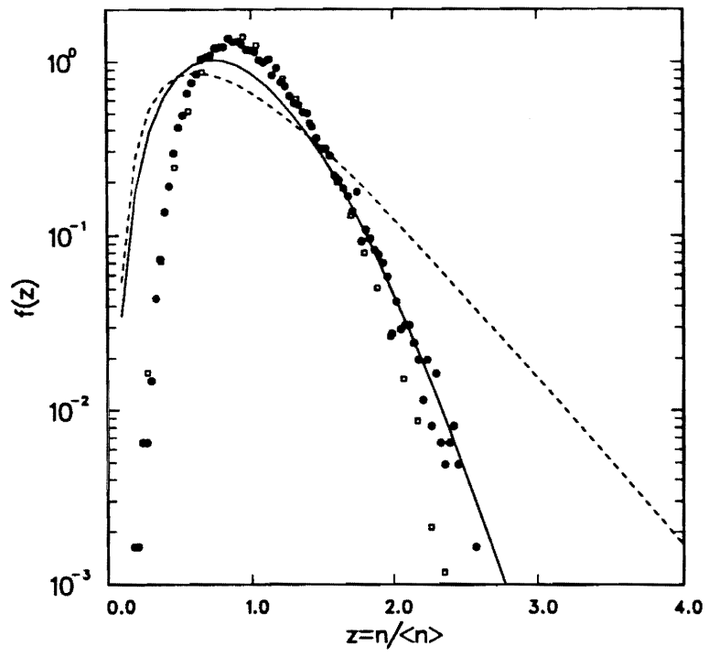

Figure 2. MD in KNO form. Open points: e^+e^- annihilation data at c.m. energy 91 GeV[10]; Solid points: Jetset 6.3 Monte Carlo at c.m. energy 2000 GeV; Dashed line: DLA prediction[11]; Solid line: MLLA prediction[12].

In general, it turned out that LLA is inadequate to describe the experimental moments of the MD. Perturbation theory was then improved by including strong angular ordering prescription to take into account destructive interference of soft gluons, resulting in the Double Leading log Approximation (DLA). Gluon emission is reduced with respect to LLA, but the kinematics of the evolution is wrong because recoil effects are not taken into account, i.e., energy is not conserved in a single emission.

A noteworthy result which is ascribed among the successes of DLA is the prediction of KNO scaling for the MD of gluons in a gluon-jet. Approximate KNO scaling in e^+e^- annihilation is observed experimentally at presently available energies, but the shape of this scaling distribution is very different from the one predicted by DLA, especially in the high multiplicity tail:

$$f(n/\langle n\rangle) \propto \exp[-(Cn/\langle n\rangle)] \tag{8}$$

where C is a calculable constant ($C \approx 2.55$). A comparison of MD's in KNO form is shown in Fig. 2: open points show data from Delphi Collaboration at $\sqrt{s} = 91$ GeV

while filled points from Jetset 6.3 Monte Carlo model at $\sqrt{s} = 2000$ GeV indicate a slow violation of scaling; points from other experiments, although not shown, fall nicely on top of the curve identified by Delphi points. The dashed line points out that DLA prediction is very far from the data, again indicating that the observed KNO scaling in e^+e^- annihilation is accidental, similarly to the case of pp collisions[13, 14]. Finally, it should be recorded that the asymptotic value of the ratio $D^2/\langle n\rangle^2$ is predicted by DLA to be 1/3.

Because the energy of the ancestor parton never decreases (there is no recoil), the production of parton is in general too prolific, as stressed by the large tail of the MD. It is then appropriate to try and take into account some proper kinematics, which implies a higher order in α_s with respect to gluon interference, i.e., include recoil terms in the evolution equations. This is the motivation behind the Modified Leading Log Approximation (MLLA), which has been treated recently, from the point of view of MD's, both with running and with frozen coupling constant; a nice approach which gives similar results is contained in the dipole model. These results will be discussed in the next section.

3. Multiplicity distributions in the Modified Leading Log Approximation

The rôle of the running coupling constant

By considering only gluon production within a gluon-initiated jet[12] (which is the approximation used also for the DLA result), pre-asymptotic terms have been added to the KNO scaling distribution, which however remains valid at extremely high energies and is violated at lower virtualities. The result for the tail is

$$f(n/\langle n\rangle) \propto \exp[-(Dn/\langle n\rangle)^{\mu}] \tag{9}$$

where $\mu = (1-\gamma)^{-1}$ and $D = C\gamma^{\gamma}(1-\gamma)^{1-\gamma}/\Gamma(1+\gamma)$ are related to the multiplicity anomalous dimension

$$\gamma = \frac{d\log\langle n(Q)\rangle}{d\log Q} \to \sqrt{\alpha_s(Q)} + \alpha_s(Q) + \ldots \tag{10}$$

responsible for the energy growth of mean particle multiplicity with virtuality Q. Because μ varies from $\simeq 1.67$ at LEP energies ($\gamma \simeq 0.40$) to the asymptotic value $\mu = 1.0$ ($\gamma = 0$) in the limit of infinite energy (recovering the DLA result), the shape of the tail widens as the energy increases. This behaviour is seen in Fig. 2 by comparing the solid line (MLLA at LEP energy) to the dashed line (DLA). It should be remarked however that even if the correction introduced by this calculation goes in the right direction, the overall description of data is still poor, especially in the head of the distribution. A comparison of DLA (dashed) and MLLA (solid) for the average number of partons as a function of c.m. energy is visible in Fig. 3: The DLA prediction grows much faster than the MLLA one: the difference is quite striking at

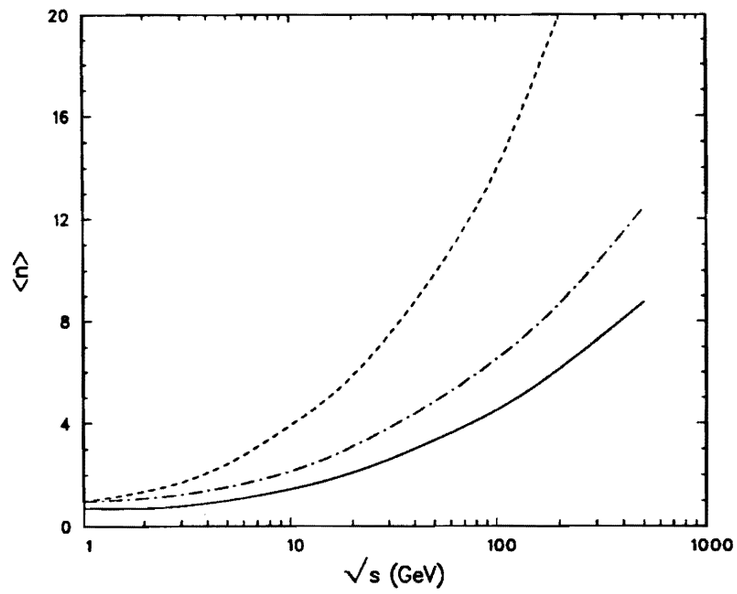

Figure 3. Average number of produced partons in different approximations as a function of c.m. energy $\sqrt{s}$. Dashed line: DLA prediction[11]; Solid line: MLLA prediction[11]; Dash-dotted line: Dipole model[15].

initial virtualities larger than 100 GeV. Notice that in the same figure is also plotted the prediction of the dipole model (dash-dotted line), to be discussed below.

The dipole model

The dipole model of cascading[18] was developed in order to perform QCD calculations in a simpler way: initial quark-antiquark pair behaves in this framework as a colour dipole, whose radiation pattern determines, via subsequent dipole splittings. gluon emission; this is possible because the QCD radiation pattern is similar to that of a dipole. Notice that in this approach only gluons are produced, the quark population being limited to the initial pair. A Monte Carlo model (Ariadne) was built on these principles, and its global results were found in agreement with experiment by using Lund string hadronization prescription in order to match parton evolution prediction with the real world.

Recently[15], the combined effects of kinematical constraints and of non-singular terms in the Altarelli-Parisi splitting kernels were taken into account as retarded terms in differential-difference equations for the MD. These terms are found to slow down the evolution of the cascade and to limit fluctuations. Different possible approxima-

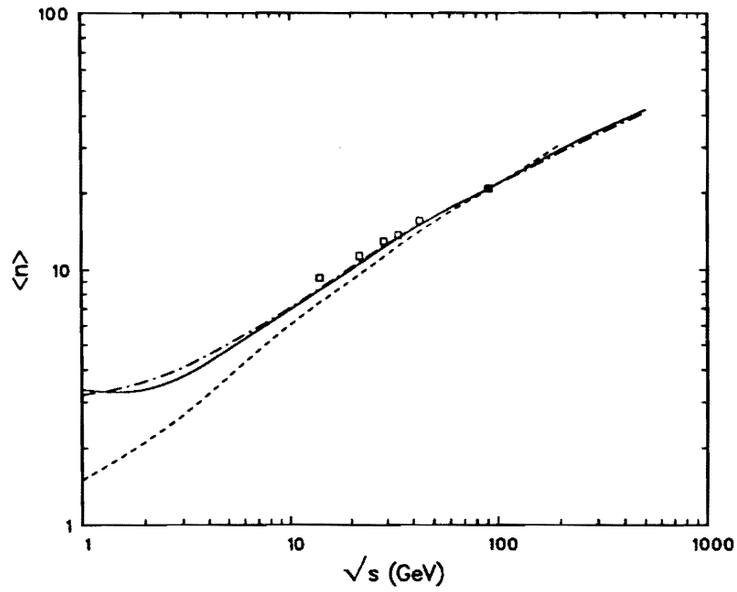

Figure 4. Average number of produced charged hadrons as a function of c.m. energy $\sqrt{s}$, obtained from figure 3 by normalizing all curves to the experimental point at 91 GeV. Points: e^+e^- annihilation data at various energies[16, 17, 10]; Dashed line: DLA prediction; Solid line: MLLA prediction; Dash-dotted line: Dipole model.

tions to these analytical equations reduce them to the DLA and MLLA forms, thus showing agreement with standard QCD calculations, and also that the equations of the dipole model actually contain more contributions than the MLLA equations alone. Unfortunately, in spite of the solid structure of the model, its analytical solution is not available, not even for the first moment of the distribution. However a numerical solution to the evolution equations for the average number of partons is shown in Fig. 3 (dash-dotted line): its behaviour turns out to be different both from DLA and MLLA solutions.

All the results in the figure are of course at parton level; the standard way to compare these results with experimental data is to use Local Parton Hadron Duality[19] as hadronization prescription and thus multiply these curves by a constant factor, of course a different one for each curve. The result is shown in Fig. 4 together with a few experimental data on e^+e^- annihilation. The multiplying factor is obtained by normalizing all curves to the experimental value at 91 GeV. All results agree in the experimentally available energy range (the DLA curve rather poorly), making a comparison of different approximations in perturbative QCD unable to distinguish among them; to carry out the comparison we need calculations of higher order moments, but

this can be done only at parton level, since we cannot use in this case LPHD (see section 4). At parton level, numerical solutions of the dipole model are found[15] to differ considerably from other approximations; results on high order moments are not available so that a comparison with the MLLA result described in the previous paragraph is not at present possible.

Solutions with fixed coupling constant

By keeping the coupling constant α_s frozen, it is possible to solve analytically the MLLA evolution equations including quarks as well as gluons in terms of the factorial moments of the distribution[9, 20]. The method consists in looking for solutions which satisfy KNO scaling (so that normalized factorial moments do not depend on energy), in some disagreement with the results with running α_s discussed above. The ratio of average multiplicities in different jets is found to be

$$\frac{\langle n \rangle_G}{\langle n \rangle_Q} \simeq 1.84 \tag{11}$$

as shown by the dash-dotted line in Fig. 1. Normalized factorial moments F_q are found to be well approximated by those of a NBD with parameter $k = 5$ (this corresponds to $D^2/\langle n \rangle^2 \approx 1/5$, to be compared with the asymptotic limit of MLLA $D^2/\langle n \rangle^2 \approx 1/3$ quoted earlier). Normalized factorial cumulant moments K_q, derived from F_q by the well known formula

$$K_q = F_q - \sum_{i=1}^{q-1} \binom{q-1}{i-1} K_{q-i} F_i \tag{12}$$

are found to oscillate in sign, contrary to NBD behaviour, but it is interesting that their absolute values oscillate around the values of an NBD with $k = 5$. It is therefore appropriate to study the ratio of factorial cumulant moments to factorial moments:

$$H_q \equiv \frac{K_q}{F_q} \tag{13}$$

where, being the F_q definite positive, H_q has the same sign as K_q. Remember moreover that if F_q is very sensitive to the tail of the distribution, K_q is even more; H_q can therefore be a sensitive tool to study deviations from NBD (which has always been claimed good to within 10%) in the high multiplicity tail of the distribution.

It should also be mentioned that attempts to calculate the behaviour of the ratio H_q with running α_s have appeared in the literature[21], but only the first few terms of a power expansion of H_q in q are given, and the size of the neglected coefficients was not proven to be negligible.

The problem of a truncated multiplicity distribution

Data on H_q have been presented[22] from e^+e^- and from pp and $\bar{p}p$ collisions and an oscillatory behaviour was found similar to that in Fig. 5. This figure however

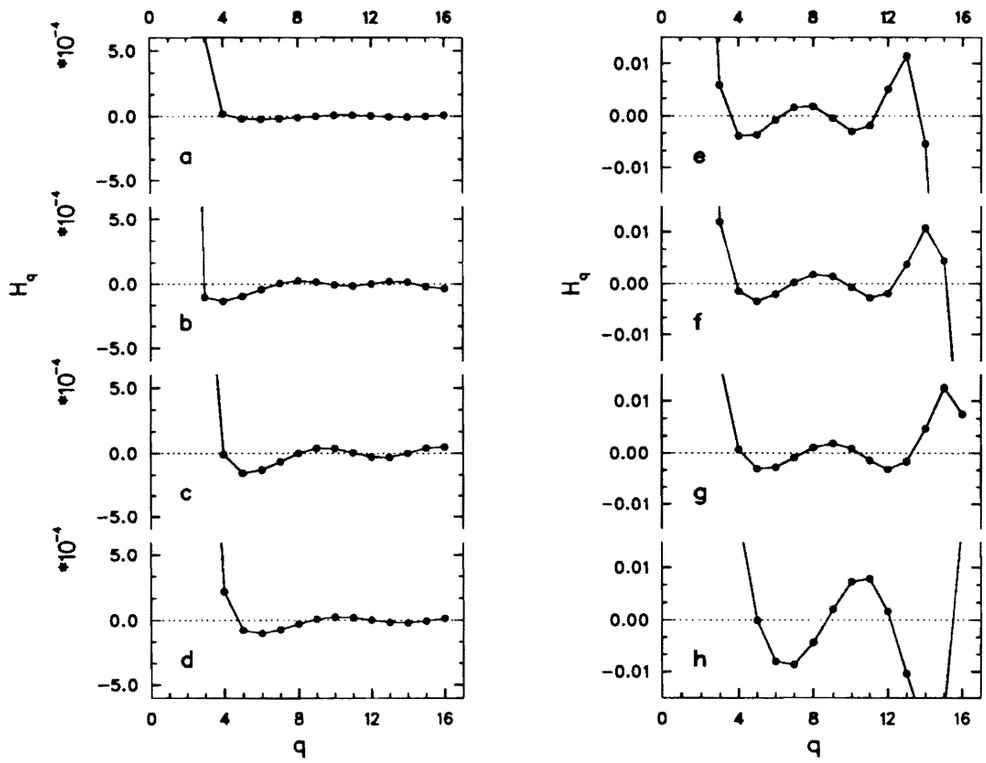

Figure 5. The ratio of factorial cumulant moments to factorial moments H_q is plotted as a function of the rank q for the distribution in Eq. 14, for different choices of parameters, corresponding to a good NBD description of experimental data in e^+e^- annihilation and $\bar{p}p$ collisions, as listed in Table 1. The lines are drawn only to guide the eye.

does not show data, but shows an exact calculation for the following *truncated* MD[23]:

$$P_n = \begin{cases} AP_n^{\mathrm{NBD}} & \text{if } (n \text{ even}) \text{ and } (n \leq n_0) \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Here n_0 is the maximum observed multiplicity and A is a normalization parameter, so that $\sum_n P_n = 1$; the "even-odd effect" (only even multiplicities are possible in full phase space because of charge conservation) was also taken into account.

We notice therefore that a MD which has monotonically decreasing moment ratios H_q, like the NBD, when truncated at a certain value n_0 shows oscillations in the sign of H_q. Of course the oscillations vanish as n_0 goes to infinity. From an experimental point of view, however, one cannot know the exact shape of the tail of the MD, because of naturally finite statistic, and we have just seen that oscillations arise when truncating a MD. It is then important that this effect of finite statistics be

Table 1. Parameters used in Figures 5. The NBD with these values of $\bar{n}$ and k reproduces well the MD of the data in different experiments and at different c.m. energies $\sqrt{s}$ as listed. n_0 is largest multiplicity in the published data.

Fig.	$\bar{n}$	k	n_0	Experiment	Reaction	$\sqrt{s}$	Ref.
a)	12.9	212.0	28	HRS	e^+e^-	29 GeV	17
b)	13.6	54.0	36	Tasso	e^+e^-	34 GeV	16
c)	15.5	30.8	38	Tasso	e^+e^-	43 GeV	16
d)	21.4	24.3	52	Delphi	e^+e^-	91 GeV	10
e)	10.7	11.0	26	ISR	pp	30 GeV	24
f)	12.2	9.4	32	ISR	pp	44 GeV	24
g)	13.6	8.2	38	ISR	pp	62 GeV	24
h)	28.3	3.7	100	UA5	$\bar{p}p$	540 GeV	25

taken away when presenting experimental data, in order to expose the true physical picture.

It is worthwhile to notice a consequence of the existence of the mentioned dynamical sign oscillations in the ratio H_q, namely that many distributions which are used to describe the MD in QCD cascades would be ruled out because they do not show oscillations. One can prove, in fact, that every infinitely divisible distribution (IDD) must have positive cumulant moments, as follows. A discrete IDD can always be cast in the compound Poisson form:

$$F(z) = \exp\{\bar{N}[g(z) - 1]\} \tag{15}$$

where $\bar{N}$ is the average number of Poissonianly produced objects (we call them "clans") and where $g(z)$ is the generating function of the MD inside an average clan. By their definitions, the factorial cumulant moments of the distribution generated by $F(z)$:

$$\begin{aligned} K_q &\equiv \frac{d^q \log F(1+z)}{dz^q}\Big|_{z=0} \\ &= \bar{N}\frac{d^q g(1+z)}{dz^q}\Big|_{z=0} \equiv \bar{N} F_q^{(g)} \end{aligned} \tag{16}$$

are proportional to the factorial moments of the inside-clan distribution $g(z)$, which by definition cannot be negative: therefore $K_q < 0$ for some order q implies that $F(z)$ is not IDD.

In conclusion, no definite answer can be given on this topic, neither from the theoretical point of view (hadronization corrections are not known, as will be explained further below) nor from the experimental point of view (because of finite resolution).

A final comment should be made about the search for deviations from the negative binomial form. In addition to investigating the tail of the MD, also the head, i.e., the low multiplicity part, should be considered, especially in view of the studies on the rapidity gap probability (discussed in section 5). In order to do this one can use the *combinants*[26], some properties of which it is worthwhile to mention here.

Combinants are defined in terms of the MD generating function $F(z)$ as

$$C_q \equiv \frac{1}{q!}\frac{d^q \log F(z)}{dz^q}\Big|_{z=0} \qquad q = 1, 2 \ldots \tag{17}$$

provided $F(0) > 0$, and the combinant of order q is given by a finite combination of the probabilities $P_0, P_1 \ldots P_q$ as

$$C_q = \frac{P_q}{P_0} - \frac{1}{q}\sum_{n=1}^{q-1} nC_n \frac{P_{q-n}}{P_0} \tag{18}$$

It should be noticed that for the Poisson distribution

$$C_1 = \langle n \rangle \qquad C_q = 0 \quad (q = 2, 3 \ldots) \tag{19}$$

so that a value different from zero immediately identifies deviations from the Poissonian shape. Moreover, it can be shown[27, 28] that if, and only if, all combinants are positive and their sum is finite, then the resulting MD is a compound Poisson distribution, and the MD inside a (generalized) clan, p_q, is directly given by the combinants of the full distribution:

$$p_q \equiv \frac{C_q}{\bar{N}}; \qquad \bar{N} \equiv \sum_{q=1}^{\infty} C_q \tag{20}$$

Finally, we note the interesting consequence of Eq. 18 that combinants are not affected by the truncation of the multiplicity distribution, and thus are not affected by the problem of finite statistics.

Open problems

Multiplicity distributions are a valuable tool to analyze our understanding of parton showers, but even though noteworthy efforts have been made in order to improve our knowledge of MD's in terms of QCD cascading, the result, while encouraging, is still not satisfactory. It is not surprising that the improvement has come by including conservation laws and more kinematics in the evolution equations. We know that although these are formally of higher order in perturbation theory, their rôle is not negligible at present energies; after all, all successful Monte Carlo models of QCD cascades are based on LLA with angular ordering and correct kinematics and they give very good descriptions of experimental data on MD's, and not only in full phase space.

Indeed one of the major limitations of the analytical QCD approach thus far is its applicability to full phase space distributions only, while the striking fact about NB regularity and consequent clan structure analysis is the behaviour of the average number of clans, $\bar{N}$, and of the average number of particles per clan, $\bar{n}_c$, in symmetric rapidity intervals. None of the analytical calculations mentioned in the previous sections addresses this problem. Lacking the possibility of a QCD calculation, a favourable approach is that of an approximate model which is inspired to QCD parton evolution but is analytically calculable.

An example of such a programme is the Generalized Simplified Parton Shower (GSPS) model, whose novel idea is that of inserting clans a priori as a physical intermediate step in the parton cascade, instead of analyzing their properties as a statistical tool a posteriori. This task is carried out by starting from a kinematically correct simplified model of parton evolution[29] and relaxing locally the energy-momentum conservation law (but maintaining the global conservation) according to the hypothesis of the physical existence of clans. The model has indeed been successful in calculating the properties of the average number of clans[30, 5], and will be extended in order to estimate also the average number of partons per clan.

It should be pointed out the anti-parallelism of this approach to that of QCD calculations: in going from DLA to MLLA, conservation laws which were completely neglected are now beginning to be taken into account; in going from the SPS to the GSPS, strict conservation laws which were imposed on the parton shower are now relaxed. In both cases an improvement is seen over the previous situation, which is an encouraging result. In both cases, however, results are obtained at parton level, and they must be interpreted at hadron level: at present only guesswork is available to describe this process, but the study of MD's can play a primary rôle in solving this difficult problem.

4. Multiplicity distributions and hadronization mechanisms

Single-particle inclusive distributions are well described by QCD cascades plus Local Parton Hadron Duality (LPHD). In the original formulation[19], LPHD was invoked on the basis of preconfinement and was explicitly described as a proportionality between the partonic *single-particle inclusive distribution* and the hadronic one. Physically, this seems a very sound hypothesis, and it is also very convenient for analytical calculations, as witnessed by the success of the prediction of the so-called humpbacked plateau[11]. Other hadronization mechanisms are used in Monte Carlo models, where it is possible to follow in detail the partonic evolution, but in the context of analytical calculations these methods are not applicable. As far as MD's are concerned however, the simple LPHD is not enough, linking only the single-particle distributions, i.e., the first moment of the MD (to this end, it was invoked in building Fig. 4 from Fig. 3).

In order to deal effectively with high order moments, it is necessary to *extend*

LPHD to n-particle inclusive distributions. Such an extension, called Generalized Local Parton Hadron Duality (GLPHD), was proposed[3] on the basis of the approximate occurrence of NBD at final parton level (final here means: at a virtuality at which perturbation theory is no more applicable) in Monte Carlo simulations as well as at final hadron level (final here means: after resonances have decayed) in the data, with parameters at hadron (h) and at parton (p) level connected by the following formulae:

$$\langle n\rangle_h \simeq 2\langle n\rangle_p \qquad k_h \simeq k_p \tag{21}$$

The hypothesis of GLPHD links n-particle to n-parton inclusive distributions

$$Q_n^{(h)}(y_1,\ldots,y_n) = \rho^n Q_n^{(p)}(y_1,\ldots,y_n) \tag{22}$$

with ρ constant (≈ 2). Its application to MD's can be carried out as follows, in order to derive explicit relations between the MD at parton level and that at hadron level.

For any rapidity domain Δy, the generating functions of the MD $F^{(a)}(z)$, with $a = h, p$, are obtained by constructing the functionals

$$\Phi_a[u(y)] = 1 + \sum_{n=1}^{\infty}(n!)^{-1}\int_{\Delta y} Q_n^{(a)}(y_1,\ldots,y_n)\prod_{i=1}^{n} u(y_i)dy_i \tag{23}$$

and making use of the function

$$u(y) = \begin{cases} z-1 & \text{if } y \in \Delta y \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

Then we find the following rule for transforming the partonic (hadronic) generating function into the hadronic (partonic) one:

$$F^{(h)}(z) = F^{(p)}(z') \quad \text{with} \quad z'-1 = \rho(z-1) \tag{25}$$

As an example, assume now that the partonic distribution is of NBD type:

$$F^{(p)}(z') = [1-(\langle n\rangle_p/k_p)(z'-1)]^{-k_p} \tag{26}$$

then by applying the above Eq. 25 we obtain again a NBD

$$F^{(h)}(z) = [1-(\langle n\rangle_h/k_h)(z-1)]^{-k_h} \tag{27}$$

for the hadronic level, with parameters

$$\langle n\rangle_h = \rho\langle n\rangle_p \qquad k_h = k_p \tag{28}$$

thus explaining relation 21.

The validity of Eq. 22 is more general than its application to the NBD. In fact it is seen that Eq. 25 leaves the distribution invariant in form, with the average value multiplied by ρ and all other parameters unchanged, whenever the generating

function depends on z and $\langle n \rangle$ through the product $\langle n \rangle (z-1)$ only. This property is shared by a wide class of MD's, e.g., Poisson, NB, Pólya-Aeppli, Thomas and in general all those expressible as Poisson transforms

$$P_n = \int_0^\infty p(x) \frac{(x\langle n \rangle)^n}{n!} e^{-x\langle n \rangle} dx \tag{29}$$

These are the distributions for which

$$(n+1)P_{n+1} - nP_n = -\langle n \rangle \frac{\partial P_n}{\partial \langle n \rangle} \tag{30}$$

As a further application of GLPHD to multiplicity distributions, let us look at factorial moments, F_q, and factorial cumulant moments, K_q. Their definitions in terms of inclusive distributions and correlation functions imply for the normalized quantities the following result:

$$F_q^{(h)} = \frac{1}{\langle n \rangle^q} \int Q_n^{(h)}(y_1, \ldots, y_n) dy_1 \ldots dy_n = F_q^{(p)} \tag{31}$$

$$K_q^{(h)} = \frac{1}{\langle n \rangle^q} \int C_n^{(h)}(y_1, \ldots, y_n) dy_1 \ldots dy_n = K_q^{(p)} \tag{32}$$

and therefore also

$$H_q^{(h)} = H_q^{(p)} \tag{33}$$

that is, if the GLPHD hadronization prescription is used, the parton level result on these quantities can be directly applied to the hadrons. Notice that this is true in limited domains of phase space as well as in full phase space.

Notwithstanding the fact that short range correlations appear at the hadronic level (due to resonances) which are not present at the partonic one, thus weakening the validity of Eq. 22, one sees that the comparison of experimental data on high order moments to partonic calculations necessarily makes use of GLPHD. In particular, the validity of Eq. 22 for all n at once is put to test by the study of multiplicity distributions, which has therefore still much to say on the understanding of the hadronization process.

5. Multiplicity distributions and rapidity gaps

The study of hard processes which present jets separated in rapidity by a large interval in which no particles appear ("rapidity gaps") starts with cosmic ray showers[31]; its relevance in high energy physics was pointed out more recently[32, 33], in particular with reference to hadron-hadron collisions. If the hard scattering proceeds via colour-singlet exchange, one expects to observe two jets well separated in rapidity; since the colour-singlet does not radiate, no particles are seen in the region between

the jets. If the hard scattering proceeds via colour-octet exchange, and two jets are produced, the region between them is expected to be populated due to soft gluon radiation; however also in this case can rapidity gaps be observed as the result of fluctuations in the number of emitted gluons. A naive theoretical estimate of rapidity gap probability shows that it does not depend on the size of the gap for colour-singlet exchange, and that it falls exponentially for colour-octet exchange (this corresponds to Poissonian statistics in the number of particles).

The DØ and CDF Collaborations[34, 35, 36] have performed a direct measurement of the probability P_0 of a rapidity gap between two high-p_T jets: they found that the probability of an empty interval of width Δy decreases at first roughly exponentially and then levels off with the widening of the size of the interval. Even though the observed behaviour is qualitatively consistent with the naive theoretical expectations, a quantitative result is not yet achievable. Proton remnants can in fact fill the region between jets, so that the originally produced gap does not "survive": experiments can therefore only set an upper limit on the gap probability.

In addition, a well-founded evaluation of the contribution due to colour-octet exchange is still lacking, but further knowledge on this point can be gathered by studying the MD of particles in the region between jets. In the first place, in addition to a direct measurement of P_0, one can analyze the global shape of the MD in a given interval and then use it to extrapolate the rapidity gap probability. In this way, experimental uncertainties should be reduced. In the second place, one should remember a study of the UA1 Collaboration[37], where MD's in minimum bias events have been compared to MD's between jets in hard scattering events: the two were found very similar. A detailed analysis of rapidity gap probability and MD in minimum bias events could then provide useful insights on the behaviour expected for colour-octet exchange in hard processes. Actually, analyses of MD's in minimum bias events have been performed at lower energies in hadron-hadron collisions and in other reactions; their discussion in terms of rapidity gap probability[27, 38] shows that the latter is approximately energy independent and, more important, that it is consistent with the naive Poissonian expectation in small intervals only, a leveling being visible for larger intervals. A more careful analysis of colour-octet exchange thus suggests itself at this point, since data of this type are already available at Tevatron[39].

The idea of using the MD in connection with rapidity gaps probability has been followed by DØ and CDF Collaborations: they study the MD of events obtained by colour-octect exchange, both in a Monte Carlo model (Pythia), containing no singlet exchange, and in 3-jet events, by excluding the particles assigned to the central (in rapidity) jets: they find that the MD is well described by a NBD, thus confirming the similarity with minimum bias data pointed out above. When studying the MD in the gap for the inclusive two-jet sample, they fit the bulk of it by a NBD and find that while both the maximum of the distribution and the tail are well described, the data are in excess of the fit at very low multiplicity (see Fig. 6). This excess is shown to be a factor 4-5, which makes one qualitatively sure of the presence of a singlet

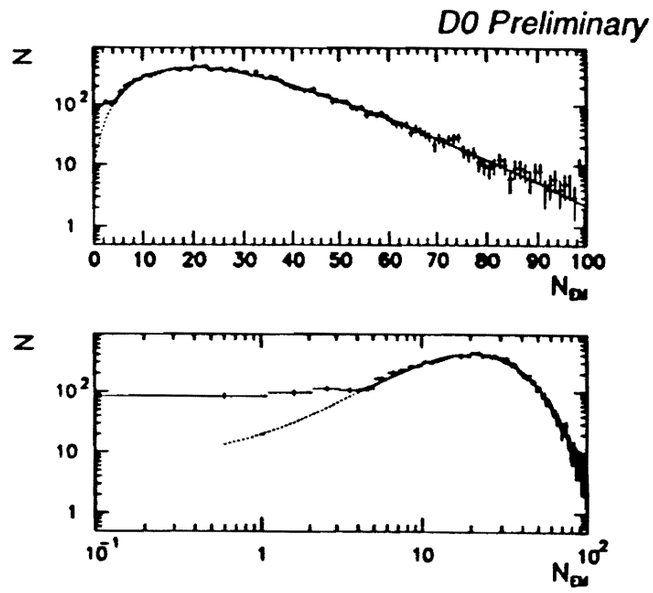

Figure 6. The NB fit performed by DØ Collaboration[35] to the MD in the rapidity region between jets for the inclusive two-jet events sample (above), together with an expanded view, in logarithmic scale, of the low multiplicity tail (below).

component. On the quantitative side, however, one should be aware of difficulties that may arise. In a recent study[40] of Monte Carlo events generated by the Herwig program at Tevatron energy with two high p_T jets with a rapidity gap of 2.5 units where the predominant mechanism is $q\bar{q}$ scattering with gluon exchange, the MD of particles between these jets (with fixed gap) were fitted with one NBD, with rather poor results, and with the sum of two NBD's, with better results. The relevant fact here is that it was found in both cases that the fits underestimate P_0 by a factor 2, a fact which indicates that caution is needed in quantitative data interpretation.

In conclusion to this section, it must be said that many considerations which were applied to hadron-hadron collisions are also valid in deep inelastic scattering, where a large fraction of events with large rapidity gaps has been observed[41], but no detailed analysis of the background has been performed. In addition in the near future it is expected that the rapidity domain accessible to experiment will be increased at Hera and at Tevatron, with the possibility to measure larger rapidity gaps. In view of this, it is clear that the development of a complementary approach to the study of rapidity gaps, inspired to multiparticle techniques, and to multiplicity distributions in particular, as outlined in this section, is of great importance.

6. Multiplicity distributions and correlations

It is well known that there is a strong connection between the study of MD's and the study of correlations, as the moments of the former are the integrated version of the latter (as was already hinted at in Eq. 32). When the integration domain becomes very small, it is seen that indeed the integral study becomes a differential study, so the analysis of MD's in very small domains is even more closely connected to correlations.

It should be recalled that the wide occurrence of NB regularity provides already a strong link with correlations; the relationship between the parameter k and the two-particle correlation function $C_2(y_1, y_2)$ is well known[42]

$$\frac{1}{k(\Delta y)} = \frac{1}{[\bar{n}(\Delta y)]^2} \int_{\Delta y} C_2(y_1, y_2) dy_1 dy_2 \tag{34}$$

Notice that this equation follows from the definition of k in term of the dispersion D as

$$\frac{1}{k} = \frac{D^2}{\bar{n}^2} - \frac{1}{\bar{n}} \tag{35}$$

Whether the distribution is actually of NB type depends not on C_2 but on higher order correlations. Data show indeed that the MD is of NB type in all classes of reactions. This implies a hierarchical structure of correlations, so that higher order correlations are linked to $C_2(y_1, y_2)$ in a simple way. Interesting results have been found recently in relativistic heavy ions collisions in emulsion[43], confirming that in central collisions, in central rapidity intervals NB parametrization is very good. This is a an important result because the initial situation is much more complex than in e^+e^- or $p\bar{p}$ collisions and can lead in principle to a different behaviour for the parameters.

It is to be noticed, from the purely theoretical point of view, that also for nucleus-nucleus collisions, evolution equations corresponding to the LLA ones of parton dynamics are beginning to appear and to be solved in the *time* variable[44]. In fact, in the case of a parton evolving in vacuum, where the virtuality can only decrease by successive branchings, time is a variable completely determined by virtuality and rapidity. On the contrary, a parton inside dense nuclear matter can undergo scattering and fusion processes and consequently the steady decrease of average virtuality with time is disrupted, and time becomes the evolution variable of the system. So far it was shown in this framework that evolution is dilated and accompanied by an enhancement of particle production; these effects become stronger with increasing nuclear density. Notice that these results correspond to the LLA of QCD and have not yet reached the level of quantitative comparison with data.

To establish an example of the connected study of MD's and correlations let us look at the data on AA collisions from E802[45]. It is an analysis of central O+Cu collisions; analyzing MD's in symmetric rapidity intervals centered at rapidity 1.7, with widths down to 0.1, it was found that NBD fits nicely the data and that

its parameter k rises linearly and steeply with the width of rapidity interval. This behaviour can be realized if adjacent bins are statistically independent and share the same NBD with the same parameters. It is then concluded that the correlation length is of the order of the smaller available bin size (0.1 units of rapidity), and much smaller than in hh collisions. On the contrary, NA35[46] Collaboration found that the parameter k is constant with the width of the rapidity interval but independent of the position of the interval. The analysis of MD's in asymmetric rapidity intervals (given width with center moving along the rapidity axis) will be very helpful in clarifying the situation, if we recall that this kind of analysis in hadronic reactions has revealed that NBD fits are barely acceptable and k is not constant. Because of the possibly much smaller correlation length, results in the case of AA collisions could be different.

A further test of this interpretation was proposed some time ago by Léon Van Hove[47] in the study of the class of distributions of points in a continuous domain D characterized by the property that the MD in D and in every subdomain of D is a NBD. For non-overlapping domains D_i such that $\bigcup D_i = D$ the following additivity property was proven to occur:

$$\sum_i \frac{\langle n \rangle_{D_i}}{k_{D_i}} = \frac{\langle n \rangle_D}{k_D} \tag{36}$$

This rule is known to be violated in muon-hadron[3] and in hadron-hadron collisions[48], thus implying that the MD cannot be of NB type in *every* subdomain. A test of this rule would be very interesting as it could point out the peculiar characteristics of AA collisions. In this case one could make the hypothesis that the rôle of stimulated emission in producing the final NBD may be important, because the complex initial state could very well spawn more than one source of cascading. Remember that stimulated emission was ruled out in hadron-hadron experiments in favour of cascading[48], by comparing the value of k for the whole sample to that for negative particles only: $1/k$ would be in fact the fraction of particles which stimulate a further emission. Because of Bose-Einstein interference, stimulation would be stronger for like-charge particles, i.e., one would expect $1/k|_{\text{same charge}}$ to be larger than $1/k|_{\text{all charges}}$, contrary to the results of the analysis in Ref.[48]. It would then be interesting to measure $1/k$ for like-charge particles also in nucleus-nucleus collisions in order to verify whether also in this case the mechanism of stimulated emission can be excluded.

7. Conclusions

Through the discussion of results recently appeared in the literature, it has been shown how pervasive the topic of multiplicity distribution is in the realm of high energy physics.

One point which makes the subject particularly stimulating is that QCD calculations on parton showers can provide so far only qualitative predictions on final

parton MD's and related observables in full phase space. In order to be more quantitative, QCD has to take into account more accurately the energy-momentum conservation law in parton shower evolution, and this is very often quite a difficult task: attempts in this direction have started but satisfactory results have not yet been obtained. In addition, the huge amount of high quality experimental data which is available on MD's demands to be explained by a theory of strong interactions not only in full phase space, as QCD calculations usually attempt to do, but also in restricted regions of it; in more general terms, rapidity and transverse momentum dependence of the observables should be quantitatively justified. One way to overcome the just mentioned limitations is therefore to build QCD-inspired models of parton showers, within a correct kinematical framework. It turns out that the inclusion of recoil effects in QCD calculations as well as the weakening of strict conservation laws in a QCD-inspired model (e.g., the GSPS model) provide at final parton level the results that one would expect on purely phenomenological grounds. Although this fact is encouraging, an analytical description of how an initial parton of high virtuality evolves to final partons in rapidity and transverse momentum is still lacking.

Of course the problem of how final partons hadronize is still open and requires further studies, far beyond pure guesswork hadronization prescriptions currently used in QCD Monte Carlo numerical calculations. It is remarkable that generalized local-parton hadron duality, assuming NB regularity to be valid, provides a nice consistent scheme of parton evolution and hadronization. Indeed the wide occurrence of NB regularity in high energy collisions is experimentally observed. Accordingly, it is not hazardous to consider NB regularity as the "Born term" of a sound physical description of MD's. What makes it even sounder is its occurrence with a high degree of accuracy also in events of different jet topologies and at single jet level (as shown by our studies with Monte Carlo in e^+e^- annihilation at LEP energy). All these findings suggest to extend and improve the experimental search and analysis of MD's in these directions. Would these results be confirmed, they would strengthen and widen the idea of jet universality, i.e. of the striking fact that single (quark and gluon) jet structures are the building blocks of multiparticle dynamics in all reactions and that NBD is their approximate description. In fact, deviations at most into 10% from NB behaviour should be expected; they should be checked of course not only in the standard direct measurement of MD's, but also in the careful study of a special class of observables sensitive to particular properties of MD's, which have been discussed in this paper and are summarized in the following:

a) The quite unconventional analysis of MD's by means of combinants, C_q, and of the ratio of factorial cumulant moments to factorial moments, H_q, which are expected to show in greater detail how deviations affect the head and the tail of the NB distribution respectively; remember that attention should be paid for the tail to effects, like MD's truncation, which might simulate dynamical violations.

b) The study of the "void scaling function" in rapidity in order to test hierarchical structure of final particles correlations.

c) The study of the probability to detect no particle in restricted rapidity regions, $P_0(\Delta y)$, in order to deepen our understanding of rapidity gaps physics.

The studies proposed in points *b)* and *c)* find of course their natural formulation in the framework of the clan structure analysis of the MD, being the void scaling function the inverse of the average number of particles per clan and $-\log P_0(\Delta y)$ the average number of clans of the MD.

d) The study in different reactions and at different c.m. energies, and in particular in nucleus-nucleus collisions, of the additivity rule in rapidity intervals for its intriguing connection with second order correlation functions.

Notice that all the above mentioned possible research lines on the deviations from NB behaviour point in the same direction: the need of an integrated theoretical and experimental study of final particles MD's and n-particle correlation functions, which might very well be the new frontier in the field.

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