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ABSTRACT

The concept of clans as intermediate parton sources independently produced has been implemented in a simplified model of single parton shower by allowing local violations of the energy-momentum conservation law, still requiring its global conservation. Analytical calculations of the average number of clans in symmetric rapidity intervals show qualitative agreement with experimental results in e^+e^- annihilation. In this framework, a new scaling law for rescaled observables is discovered.

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ABSTRACT

The concept of clans as intermediate parton sources independently produced has been implemented in a simplified model of single parton shower by allowing local violations of the energy-momentum conservation law, still requiring its global conservation. Analytical calculations of the average number of clans in symmetric rapidity intervals show qualitative agreement with experimental results in e^+e^- annihilation. In this framework, a new scaling law for rescaled observables is discovered.

1. The concept of clan

The concept of clan was originally introduced in Ref. [1] in order to interpret the wide occurrence of Negative Binomial (NB) regularity in experimental data. The NB multiplicity distribution (MD) can indeed be generated by the independent emission of N objects, called clans, which follow a Poisson distribution with average $\bar{N}$ and then decay according to a logarithmic distribution whose average $\bar{n}_c$ determines the average number of particles inside an average clan, i.e., the process is fully described by the following equation:

$$F_{\mathrm{NB}} = e^{\bar{N}(g_{\mathrm{logarithmic}}-1)} \tag{1}$$

where F_{NB} is the generating function of the NB MD and $g_{\mathrm{logarithmic}}$ the generating function of the logarithmic distribution. Notice that the proposed clan expansion is similar to cluster expansion used in statistical mechanics. At this level clan concept is indeed a purely statistical one; it describes a group of particles of common origin whose correlations are only among particles within the same clan; each clan contains at least one particle (by assumption). Therefore, multiparticle production is considered in this picture as a two steps process, which is fully characterized by the average number of clans, $\bar{N}$, and the average number of particles per clan, $\bar{n}_c$. This parametrization is known as clan structure analysis of MD's and it can be

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applied both in full phase space and in rapidity intervals Δy. The two parameters of clan structure analysis are related to the standard NB parameters by the following equations:

$$\begin{aligned} \bar{N} &= k \log\left(1 + \frac{\bar{n}}{k}\right) \\ \bar{n}_c &= \frac{\bar{n}}{\bar{N}} \end{aligned} \tag{2}$$

where $\bar{n}$ is the average number of charged particles of the distribution and k is connected to the dispersion of the MD, $D \equiv \sqrt{\overline{n^2} - \bar{n}^2}$, by the relation $D^2/\bar{n}^2 = 1/\bar{n} + 1/k$. It is remarkable that clan structure analysis of charged particle MD's in different reactions and at different energies in full phase space and in symmetric rapidity intervals revealed an extraordinary series of new regularities[2,3,4], whose interpretation is still quite puzzling:

a. $\bar{N}(\Delta y)$ grows linearly with the width of the rapidity interval Δy *for small intervals* (for given reaction, fixed energy);
b. $\bar{N}(\Delta y)$ then bends toward a constant value as the rapidity interval Δy *approaches full phase space*;
c. $\bar{N}(\Delta y)$ is approximately energy independent *in a fixed rapidity interval* in the region of linearity;
d. $\bar{N}$ *in full phase space* grows with energy;
e. $\bar{n}_c(\Delta y)$ grows, peaks and then falls as a function of the width of the rapidity interval Δy for given reaction, at fixed energy;
f. $\bar{n}_c(\Delta y)$ grows with energy at fixed rapidity interval;
g. $\bar{N}(\Delta y)$ is larger in e^+e^- than in hh or lh; $\bar{n}_c(\Delta y)$ is smaller in e^+e^- and lh than in hh.

These results led us to the conclusion that the description of MD' in terms of $\bar{N}$ and $\bar{n}_c$ is more than a new parametrization to be compared with the standard one in terms of $\bar{n}$ and k and that the new parameters have a deeper meaning than it was thought when they were introduced on purely statistical grounds.

From a theoretical point of view, three facts are worthwhile to be mentioned (see also Ref. [5] for further discussion of these points).

Firstly, *clan concept is more general than NB regularity*; it characterizes the wide class of Infinitely Divisible Distributions (IDD's)[6]. The generating function $F(z)$ of any discrete IDD can indeed be written as a Compound Poisson distribution:

$$F(z) \equiv \sum_{n=0}^{\infty} P_n z^n = \exp\left\{\bar{N}[g(z) - 1]\right\} \tag{3}$$

with P_n the n-particle MD and $g(z)$ the generating function of the distribution of particles inside a clan. If $g(z)$ is the generating function of a logarithmic MD, $F(z)$ is then the generating function of a NB MD as in Eq. (1); notice that $g(z)$ might very well be in general different from the generating function of the logarithmic

MD but clan concept, which is characterized by the act of independent emission, remains valid.

Second, it is to be pointed out[5,7] that *for IDD's the average number of clans in a given rapidity interval* Δy, $\bar{N}(\Delta y)$, *can be obtained directly from the rapidity gap probability in the same interval,* $P_0(\Delta y)$, i.e., from the probability to produce no particles in the rapidity interval Δy:

$$P_0(\Delta y) = F(z=0) = e^{-\bar{N}(\Delta y)} \tag{4}$$

which determines in principle the full MD in Δy according to the following equation:

$$P_n(\Delta y) = \frac{(-\bar{n})^n}{n!} \frac{\partial^n P_0(\Delta y)}{\partial \bar{n}(\Delta y)^n} \tag{5}$$

The same relation can be also used to extract information on the rapidity gap probability from the clan structure analysis of the full MD[5,7].

A third remark concerns $\bar{n}_c$, the average number of particles per clan. It has been shown[5,8] that *hierarchical n-particle correlation structures in a rapidity interval* Δy *is controlled by the behavior of the parameter* $\bar{n}_c(\Delta y)$ *in the same interval.*

In conclusion, the act of independent emission in the production process is controlled by the parameter $\bar{N}$, which is properly defined for any discrete IDD. The successive decay of an individual clan into other particles – if characterized by one additional parameter only, like $\bar{n}_c$ – determines the hierarchical structure of n-particle correlations.

The large amount of experimental results and the above mentioned intriguing theoretical properties arise the question of the true physical meaning of the concept of clan. It should be recalled to this end the original interpretation of NB regularity within the clan model: the logarithmic distribution for particles inside an *average clan* can be interpreted[9] as the convolution of geometric *single clans* with different average multiplicities:

$$g_{\text{logarithmic}} = \int \frac{d\bar{n}_{\text{geom}}}{\bar{n}_{\text{geom}}} g_{\text{shifted geometric}} \tag{6}$$

i.e., while particles inside an *average clan* follow a logarithmic distribution, particles within each (not averaged) *single clan* follow a shifted geometric distribution. This picture is indeed remarkable and should be compared in view of Generalized Local Parton-Hadron Duality (GLPHD)[11] to what occurs at parton level, where the explicit solution of Konishi-Ukawa-Veneziano equations with cutoff regularization prescription (neglecting $q\bar{q}$-pair production) gives[10] a geometric MD for gluon MD in a single gluon-jet. Therefore, the production process in e^+e^- annihilation can be described in the following way: initial highly virtual q and $\bar{q}$ generate in a bremsstrahlung-like fashion virtual gluon sources – the *clans* – which further emit

gluons via cascading, i.e., clans at parton level can be thought of as *independent intermediate gluon sources*[11].

In order to test this point of view, in the absence of explicit satisfactory QCD calculations (see Ref. [12] for a careful discussion), the above mentioned clan concept has been implemented within the simplified model of a single partonic shower based on essentials of QCD proposed in our previous work[13]. The aim is to describe analytically clan properties in two variables, virtuality and rapidity, in order to reproduce and hopefully to explain the regularities which have been discovered by clan structure analysis. It should be pointed out that the suggested scheme is quite general and it does not lead necessarily to a NB description for final particle (parton) MD's, but it concerns the wider class of discrete IDD's as implied by the request of independent emission of clans.

2. The Generalized Simplified Parton Shower (GSPS) model

The general scheme of bi–dimensional parton shower evolution in virtuality and rapidity for a parton a splitting into two partons b and c is based on LLA QCD Altarelli-Parisi-type evolution equations[14]

$$\frac{dP_{a\to bc}}{dt_a} = \frac{\alpha_s(t_a)}{2\pi} \int_{z_{\min}}^{z_{\max}} P_{a\to bc}(z_b)dz_b \tag{7}$$

where

$$\alpha_s(t_a) = \frac{12\pi}{(33-2\mathcal{N}_f)t_a} \quad , \qquad t_a = \log\frac{Q_a^2}{\Lambda^2} \quad ,$$

$P_{a\to bc}(z)$ is the Altarelli-Parisi splitting kernel and z_b is the energy fraction carried out by the parton b. In this scheme, each splitting determines the virtuality at which the parent parton splits and the energy fraction carried out by the daughter partons (with $z_b + z_c = 1$). The problem in this approach is how to take care of the energy-momentum conservation law, since the kinematical bounds on z_b ($z_{\min}$ and $z_{\max}$) depend on all three virtualities Q_a,Q_b, Q_c, whereas the virtualities of the daughter partons are fixed only after the subsequent splitting. In our model, we decided to act differently and to describe the branching process in terms of finite splitting functions according to the suggestions in [15]; here, one requires that the virtuality of the parent parton is known from a previous splitting and the splitting function determines the virtualities of produced partons; in this case the joint probability density $\mathcal{P}(Q_bQ_c|Q_a)$ for a parton of virtuality Q_a to split into two partons of virtuality Q_b and Q_c is given by

$$\begin{aligned}\mathcal{P}(Q_b,Q_c|Q_a)dQ_bdQ_c =& N(Q_a,Q_b,Q_c)p(Q_b|Q_a)p(Q_c|Q_a)dQ_bdQ_c \\ &\text{with } Q_b+Q_c \le Q_a \ ,\end{aligned} \tag{8}$$

where $N(Q_a, Q_b, Q_c)$ is a normalization factor and $p(Q_i|Q_a)$ represents the probability for a parton of virtuality Q_a to split at Q_i. In this way, the kinematical limits on the energy fraction z_b are fixed by standard two-body kinematics[13]:

$$B - \sqrt{B^2 - \frac{Q_b^2}{Q_a^2}} < z_b < B - \sqrt{B^2 - \frac{Q_b^2}{Q_a^2}} \qquad B = \frac{1}{2}\left[1 + \left(\frac{Q_b}{Q_a}\right)^2 - \left(\frac{Q_c}{Q_a}\right)^2\right] \quad (9)$$

For the virtuality dependence of the splitting function, a factorized form inspired to QCD and suitably normalized by a Sudakov form factor term has been chosen:

$$p(Q_b|Q_a)dQ_b = C(Q_a)\frac{d}{dQ_b}\left(\frac{1}{C(Q_b)}\right)dQ_b = d\left(\frac{\log Q_b}{\log Q_a}\right)^A \quad (10)$$

where A is the only free parameter of the model and controls the length of the cascade. For the rapidity dependence of the splitting function, the singular part of the Altarelli-Parisi gluon splitting kernel has been selected:

$$P(z_b)dz_b \propto \frac{dz_b}{z_b(1-z_b)} \; . \quad (11)$$

The Simplified Parton Shower (SPS) model proposed in Ref. [13] is based on the above mentioned assumptions. In order to check its predictive power, the model has been implemented in a Monte Carlo programme; it has been found that the resulting MD's are well described by a NB MD both in full phase space and in rapidity intervals; in addition, clan structure parameters, $\bar{N}$ and $\bar{n}_c$, obtained from NB fits, show trends similar to those observed experimentally. This consideration notwithstanding, we judged the model still unsatisfactory; it is very hard in fact to do analytical calculations in its framework and clans are here still a statistical tool only.

The real improvement came when we decided to include explicitly clans as *independently produced intermediate parton sources* and to modify accordingly the kinematics of the process; the new version of the model is called[16] Generalized Simplified Parton Shower (GSPS) model. The new idea is to allow *local violations of the energy-momentum conservation law, still requiring its global conservation.* The mentioned local violations have been obtained by allowing local fluctuations in the virtualities of the produced partons, Q_i:

$$Q_b + Q_c \not\leq Q_a \quad , \quad Q_i \leq Q_a \quad \text{[i=b,c]}$$

With this assumption, the joint probability density $\mathcal{P}(Q_b Q_c|Q_a)$ becomes factorized:

$$\mathcal{P}(Q_b, Q_c|Q_a)dQ_b dQ_c = p(Q_b|Q_a)dQ_b p(Q_c|Q_a)dQ_c \quad (12)$$

Each parton emission is therefore independent in virtuality. Since exact kinematics is locally violated, constraints on the energy fraction are also no longer valid (here $z_b + z_c \neq 1$); therefore kinematical bounds in rapidity are loosened:

$$|y_i - y_a| \leq \log \frac{Q_a}{Q_i} \qquad [\text{i=b,c}] \tag{13}$$

Accordingly, the splitting kernel in z must be modified and becomes decoupled:

$$P(z_b, z_c) dz_b dz_c \propto \frac{dz_b}{z_b} \frac{dz_c}{z_c} \tag{14}$$

By combining Eqs. (12) and (14), one sees that the total bi–dimensional splitting function is also decoupled and corresponds to two independent emissions of a single parton; accordingly, in order to describe the production process, is enough to follow just one branch of the shower evolution. By identifying now in each splitting one of the produced partons as the ancestor parton of a clan, one obtains that clans' production process is Markoffian and therefore clans are independently produced. The explicit calculation of clans' MD in a parton shower originated by an ancestor parton of virtuality W gives in fact a shifted poissonian distribution with average number

$$\bar{N}(W) = 1 + A \log \left(\frac{\log W}{\log 2} \right) \tag{15}$$

Going to symmetric rapidity intervals via an ordinary binomial convolution, one finds that the clans distribution in the smallest rapidity intervals is a Poissonian distribution (in larger intervals, the contribution of the ancestor parton must be taken properly into account).

We thus succeeded to implement explicitly in the model the concept of *clans* as *independent intermediate parton sources*, simply by allowing local fluctuations in the virtualities of produced partons along the shower. Some further comments on this result are needed: first, notice that conservation laws play an important role in full phase space and their role should weaken in restricted domain of phase space, the domain which one is interested in. Second, strong fluctuations in virtuality with subsequent large violation of energy-momentum conservation law are not allowed and kinematical constraints in the evolution of the parton shower are not neglected. Furthermore, both the kinematical phase space domain allowed for parton emission and the idea of decoupling in the evolution are parallel to that implemented in the dipole model developed in Lund[17]. In fact, Eq. (13) is nothing else than the kinematical triangle of dipole model, whereas decoupling is highly connected with the idea of dipole emission, where two subsequent dipole emissions are needed in order to describe a full branching; the GSPS model and dipole model substantially differ, however, in their way of describing the shower evolution, which is based on finite splitting functions in the GSPS model (see Eqs. (12) and (14)) and on elementary splitting kernels of the type shown in Eq. (7) in the dipole model.

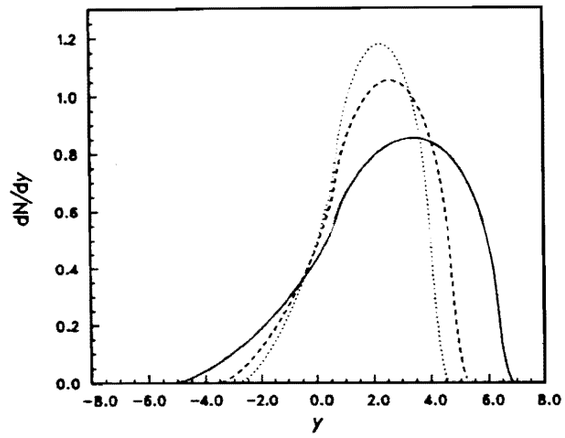

Figure 1. Clan density in rapidity for one parton shower at $W = 50$ GeV (dotted line), 100 GeV (dashed line) and 500 GeV (solid line). Normalization is to the average number of clans in full phase space.

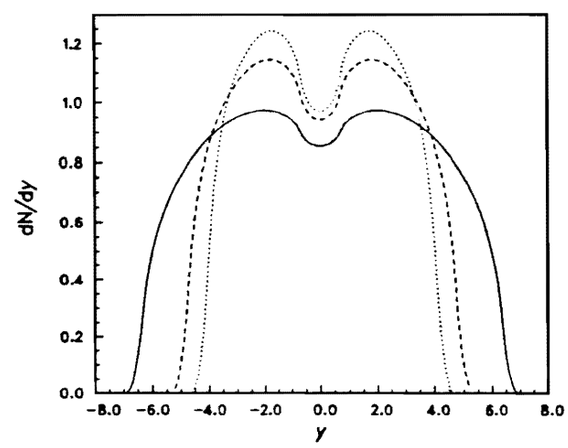

Figure 2. Clan density in rapidity resulting by the addition of two back-to-back showers at $W = 50$ GeV (dotted line), 100 GeV (dashed line) and 500 GeV (solid line). Normalization is to the average number of clans in full phase space.

3. Results

The MD of clans in the GSPS model has been calculated to be a shifted Poisson in full phase space and a Poisson in the smallest rapidity intervals. A full treatment of the MD in symmetric rapidity intervals Δy is in progress. The analytical expressions for the clan density and, after integration, for the average number of clans in symmetric rapidity intervals have been obtained.

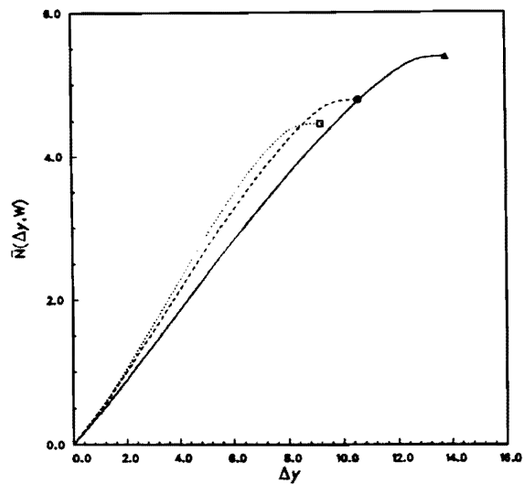

Figure 3. Average number of clans $\bar{N}(\Delta y, W)$ as a function of rapidity interval Δy for a single shower at $W = 50$ GeV (dotted line), 100 GeV (dashed line) and 500 GeV (solid line). The symbols represent the values of the average number of clans in full phase space.

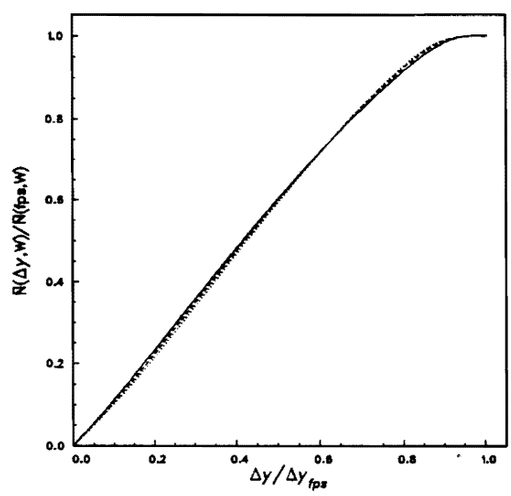

Figure 4. Normalized average number of clan $\bar{N}(\Delta y, W)/\bar{N}(\text{fps}, W)$ as a function of rescaled rapidity $\Delta y/\Delta y_{\text{fps}}$ for a single shower at $W = 50$ GeV (dotted line), 100 GeV (dashed line) and 500 GeV (solid line).

In Figure 1, clan density normalized to the average number of clans in full phase space is shown for a single parton shower generated by an ancestor parton with different initial virtualities $W = 50, 100, 500$ GeV. The value of the parameter of the model has been fixed, A=2, but results do not depend qualitatively on this choice. In Figure 2 clan density is shown for two back-to-back parton showers at the same initial virtualities indicated in Figure 1. Notice that the width of the

distribution is logarithmically increasing with the initial virtuality W according to the widening of available phase space, $\Delta y_{fps} = 2 \log W$; the height of the distribution is decreasing with the initial virtuality simply because the curve in the Figure is normalized to the average number of clans in full phase space, which increases with W only through a double logarithm (see Eq. (15)). In addition, notice that the central dip at $y \simeq 0$ is slowly removed by increasing the virtuality of the ancestor parton; it should be kept in mind that the structure of Figures 1 and 2 refers to clan production; the different behavior found for particle production is not in contradiction with that in Figures 1 and 2 since parton production within a single clan has still to be included. In Figure 3 the average number of clans $\bar{N}(y_c, W)$ obtained by integrating clan density for a single shower in a symmetric rapidity interval Δy is shown as a function of the width Δy at different virtualities of the ancestor parton. The dependence of $\bar{N}(y_c, W)$ on the rapidity interval Δy is studied up to full phase space, whose extension is fixed by the virtuality of the ancestor parton.

It has been found that:

a. $\bar{N}(\Delta y)$ grows in rapidity in a way very close to linear for intervals $\Delta y < 2$, then grows again linearly but with a somewhat different slope;
b. $\bar{N}(\Delta y)$ bends toward a constant value as Δy approaches full phase space.
c. $\bar{N}(\Delta y)$ shows an approximate (5%) energy independence at ancestor energies below 100 GeV. This is actually a very slow decrease which is reminiscent of the results obtained in Monte Carlo simulations for single gluon jets[18,19].
d. $\bar{N}$ in full phase space grows with energy.

All the above mentioned results are qualitatively consistent with experimental results obtained in e^+e^- annihilation at hadron level.

In addition to these results, a new regularity for rescaled quantities is discovered: the ratio of $\bar{N}(\Delta y, W)$ to the value in full phase space $\bar{N}(W)$ is approximately energy independent (within a much higher degree of approximation than in point c.), when it is plotted as a function of rapidity interval rescaled to full phase space (see Figure 4).

4. Conclusions

Clan multiplicity and rapidity distributions have been calculated within a simple model of a single partonic jet based on essentials of QCD, the GSPS model; by locally weakening energy-momentum conservation law, still requiring its global conservation, clans have been implemented in the model as physical objects, namely, independent intermediate parton sources. The analytically obtained dependence of the average number of clans with rapidity interval and energy was found to reproduce qualitatively the behavior of the corresponding experimental quantities at hadron level. These results support therefore the physical relevance of the identification of clan as independent intermediate gluon sources and go far beyond the

original statistical interpretation of clans as introduced in the past in order to interpret NB regularity.

5. References

1. A. Giovannini and L. Van Hove, *Z. Phys. C - Particles and Fields* **30** (1986) 391.
2. P. Abreu et al., DELPHI Collaboration, *Z. Phys. C - Particles and Fields* **56** (1992) 63.
3. M. Arneodo et al., EMC Collaboration, *Z. Phys. C - Particles and Fields* **35** (1987) 335.
4. R.E. Ansorge et al., UA5 Collaboration, *Z. Phys. C - Particles and Fields* **43** (1989) 357.
5. A. Giovannini, S. Lupia and R. Ugoccioni, "What can we learn from clan structure analysis on rapidity gap probability", these proceedings.
6. W. Feller, *An introduction on probability theory and its applications* Vol. I, 3rd ed. (J. Wiley & Sons, New York, 1968).
7. S. Lupia, A. Giovannini and R. Ugoccioni, "Clan Structure Analysis and Rapidity Gap Probability", DFTT 28/94, to appear in *Z. Phys. C - Particles and Fields.*
8. S. Lupia, A. Giovannini and R. Ugoccioni, *Z. Phys. C - Particles and Fields* **59** (1993) 427.
9. A. Giovannini and L. Van Hove, *Acta Phys. Pol.* **B19** (1988) 495. APPOA
10. A. Giovannini, *Nucl. Phys.* **B161** (1979) 429.
11. L. Van Hove and A. Giovannini, *Acta Phys. Pol.* **B19** (1988) 917.
12. R. Ugoccioni, A. Giovannini and S. Lupia, "Multiplicity Distributions", these proceedings.
13. R. Ugoccioni and A. Giovannini, *Z. Phys. C - Particles and Fields* **53** (1992) 239.
14. G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298.
15. K. Kajantie and E. Pietarinen, *Phys. Lett.* **B93** (1980) 269.
16. R. Ugoccioni, A. Giovannini and S. Lupia, "Clan Properties in Parton Showers", DFTT 7/94, to appear in *Z. Phys. C - Particles and Fields.*
17. B. Andersson, *Nucl. Phys.* **B360** (1991) 109; G. Gustafson, *Nucl. Phys.* **B392** (1993) 251; G. Gustafson and M. Olsson, *Nucl. Phys.* **B406** (1993) 293.
18. F. Bianchi, A. Giovannini, S. Lupia and R. Ugoccioni, *Z. Phys. C - Particles and Fields* **58** (1993) 71.
19. R. Ugoccioni, A. Giovannini and S. Lupia, in *Proceedings of the XXIII International Symposium on Multiparticle Dynamics* (Aspen, CO, USA, 1993), Eds. M. Block and A. White (World Scientific, Singapore, 1994) p. 297.