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Dirac's Observable: from Particles to String and Fields

MILA
9 1992

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ABSTRACT

Starting from a class of 2-body relativistic systems with action-at-a-distance described by two 1st-class constraints, a method for the determination of the global Dirac observables for a relativistic system with 1st-class constraints is developed. It is based on a decomposition of the gauge foliation of the constraint manifold in sectors belonging to different Poincare' orbits and on the use of the Wigner theory. After the treatment of the Nambu string, it is sketched how the method can be applied to classical gauge field theory.

Talk given at the International Symposium on
"Extended Objects and Bound Systems"
Karuizawa (Nihon University), Japan, March 19-21, 1992

1. Introduction

My study of the theory of Dirac's observables started in the early seventies, when the first singular Lagrangians and/or systems of Hamiltonian constraints for two relativistic particles with action-at-a-distance become popular. In this way it was possible to undertake the systematic study of constrained systems without the complications of general relativity and gauge field theory, for whose study Dirac[1] and Bergmann[2] introduced the concept of Hamiltonian constraint in the fifties.

The following historical motivations generated these particle models:

A) Bilocal field theory. It tried to give a description of elementary particles as extended relativistic systems, in an attempt to find a solution to the divergences of local quantum field theory, which also was the main unsolved problem for the late Dirac. After an attempt by Markov[3], Yukawa[4] developed two models of bilocal field theory, which at the classical level correspond to three and one 1st-class constraints respectively. Many models were subsequently developed essentially by the Japanese school and among them particularly relevant was the simple relativistic oscillator model of Takabayasi : see the review paper of Takabayasi[5] and the talks of Hara[6] and Ishida[7].

However at the classical level most of these models were formulated in terms of center-of-mass (c.o.m.) and relative variables and the interactions were not separable. Since no intrinsic concept of individual particle positions was defined, the natural coupling to an external gravitational field shows a pole-dipole structure[8] concentrated in the c.o.m.: in Minkowski space-time one has a point-like system and an inner world of relative variables only manifests itself through a dipole (or higher multipoles) structure sitting on the c.o.m., which contributes to build the mass spectrum and the spin of the system. By natural coupling it is meant that the variables r_i^μ relative to the c.o.m. x^μ are decomposed as $r_i^\mu = \sum_a^{1..3} \Lambda_a^\mu r_{ia}$; where Λ_A^μ, $A = 0,1,.,3$, with $\eta^{\mu\nu} = \eta^{AB}\Lambda_A^\mu\Lambda_B^\nu$, is a 4-dimensional orthogonal moving frame (a set of vierbeins) attached to x^μ and the 3-vector relative coordinates r_{ia} belong to an inner Euclidean space. See Hanson-Regge[9] for the treatment of the Λ_A^μ and ref.[10] for applications to two-particles models, to the spherical rigid body and to a string model.

Analogous remarks can be done for the classical basis of the infinite component wave equations: see the talks of Barut[11] and Bohm[12].

B) Nambu-Goto string. It induced Kalb-Van Alstine[13] to investigate a model of the previous type[14] and to begin the theoretical study of singular Lagrangians.

C) The quasi-potential approach to the Bethe-Salpeter equation. Looking for an instantaneous, in the c.o.m. frame, approximation with exact relativistic kinematics, Todorov[15] introduced two 1st-class constraints with vanishing mutual Poisson bracket in the form of modified mass-shell constraints. The model was defined in terms of the particle

canonical positions and momenta and allowed separable interactions: therefore it was an extended relativistic system not of the pole-dipole type, as it was confirmed by the study of its coupling to an external gravitational field[16]. By artificially introducing individual particle positions in the previous models of A) one could obtain extended systems also in that case: however this cannot be done in a unique way and only confined systems can be described (see Appendix A of ref.[16]).

After quantization the Todorov model gives rise to two coupled integrable Klein-Gordon equations with an integro-differential interaction: their solutions, the mass spectrum, the scalar product and the definition of a Cauchy problem were discussed in ref.[17] with a methodology from which the technique for finding the Dirac observables was originated. In particular the problem of the relative time is solved without quantizing it and without fixing the gauge: this is the prototype of the way out from the problem of the spurious solutions of the Bethe-Salpeter equation, connected to the relative times, see the talks of Nakanishi[18] and Sazdjian[19], in this approach. It implies 3-dimensional normalizations of the wave functions instead of 4-dimensional ones[7]: probably one has to wait a better understanding of the functional Schroedinger representation[20] of gauge field theories and of its associated Fock space before arriving at a final clarification of this point. In this representation the Hamiltonian will be well defined and the asymptotic conditions of the interaction picture will be abandoned.

The Todorov model was subsequently extended to a pair of 1st-class constraints for spinning particles by Crater[21]: after quantization one gets coupled integrable Dirac equations and the possibility of good fits to the meson spectrum by using phenomenological potentials.

D) General relativity. Looking for toy models to investigate the properties of systems invariant under reparametrization, Komar[22] rediscovered the Todorov model.

E) Relativistic mechanics with action-at-a-distance. One aspect of this line of research was the study[23] of canonical realizations of the Poincare' algebra for systems of particles. It ended with the No-Interaction-Theorem[24], according to which the requirements: i) Hamiltonian dynamics; ii) relativistic invariance (i.e. a canonical realization of the Poincare' group); iii) covariant world-lines; are consistent only for free particles (see ref.[25] for the nonrelativistic formulation). It turns out[26] that in a description of interacting relativistic particles with 1st-class constraints, the canonical c.o.m. coordinate, after a canonical transformation adapted to the constraints (see later on), cannot be covariant under Lorentz boosts when the constraints hold (it becomes a classical analogue of objects like the Newton-Wigner position operator[27]). On the other hand every covariant c.o.m. position cannot be canonical. Instead the relative variables have Wigner covariance as we shall see. As shown in ref.[28], when one considers irreducible canonical representations of the Poincare' group, it is possible to define three different 3-vectors having the same 3-velocity: i) the Møller "center of mass"[29] $\vec{R}$ (one puts the energy density in place of the mass density in the nonrelativistic definition of c.o.m.), which is not covariant under

Lorentz boosts and not local, i.e. $\{R^i, R^j\} \neq 0$; ii) the Fokker "center of inertia"[30] $\vec{Q}$, obtained by $\vec{R}_o$ (i.e. $\vec{R}$ in the rest frame) by boosting it to the laboratory frame: $\vec{Q}$ is not local, $\{Q^i, Q^j\} \neq 0$, but covariant, i.e. it defines an invariant (that is frame independent) world-line Q^μ; iii) the canonical position $\vec{X}$ (or "center of spin"; it is the classical analogue of the Foldy-Wouthuysen mean position operator for a Dirac particle and has the same covariance problems as the Newton-Wigner operator), which is not covariant. $\vec{R}$, $\vec{Q}$ and $\vec{X}$ coincide in the rest frame but $\vec{R}$ and $\vec{X}$ define frame-dependent world-lines. If in an arbitrary given frame we consider the invariant world-line Q^μ and all the possible frame-dependent world-lines associated to $\vec{R}$ and to $\vec{X}$, then these world-lines fill a world-tube having Q^μ as a central axis and with radius $d = S/\sqrt{P^2}\,c$ (P^μ is the total timelike momentum and S is the modulus of the relative angular momentum or "classical spin"). This radius is the naive minimal one of a relativistic matter bulk with mass $M = \sqrt{P^2}$ and spin S, if the peripheral points have a velocity not exceeding the velocity of light c. Møller[29] showed that if the energy density of the matter is everywhere positive definite, the spatial extension of the matter cannot have a dimension less than d. Therefore the canonical c.o.m. position has a noncovariance induced by a spin effect and this results in a kind of "delocalization" of the c.o.m. if only frame-independent statements about the dynamics are accepted.

Another aspect of this line of research was the search of two-body relativistic models, which could be interpreted as instantaneous approximations to the Tetrode, Fokker and Feynman-Wheeler dynamics[31]. Bel[32] with his predictive mechanics (see ref.[25] for its nonrelativistic formulation) in configuration space and Droz Vincent[33] with his multi-time Hamiltonian approach emphasized the fact that, once one knows the world-lines of the two interacting particles, it is possible to rescale the affine parameters along the world-lines independently of one another and to reformulate the dynamics as a two-times problem (either with the predictive equations of motion or with two Hamiltonians with vanishing mutual Poisson bracket): it turns out[34] that one recovers the Todorov model, if the dynamics is reformulated with 1st-class constraints. As shown in ref.[27], the final canonical variables adapted to the constraints of this model, which is defined only for $P^2 > 0$, contain a noncovariant canonical c.o.m. position, a spin-1 Wigner 3-vector for the relative coordinate, a Poincare' invariant relative time, a Lorentz invariant c.o.m. time (the time in the rest frame) and the conjugated momentum variables. In this model the interaction is a 1-1 spacelike correlation between the world-lines instantaneous in the rest frame as it is confirmed by the energy-momentum tensor[16]. In the nonrelativistic analysis of ref.[25], it turns out that the No-Interaction-Theorem derives from the identification of the predictive configuration positions $q^i_a(t_a)$ with the canonical positions $x^i_a(t_1, t_2, ...)$, which is possible only in absence of interactions ($x^i_a = x^i_a(t_a) = q^i_a(t_a)$). The same holds true at the relativistic level: however let us remark that this result rests on the description of free particles as tensor products of single free particles. This allows the possibility that a free particle may be in the absolute future of another one. If the time correlations among the free particles are restricted to forbid this possibility, i.e. if the free particles are described in a way consistent with the preparation of a relativistic experiment, then the No-Interaction-Theorem will hold also for frre particles.

See ref.[34] and [35] for some bibliography to the many other variants of these models and to the description of spinning particles with Grassmann variables.

What is still lacking is a covariant Hamiltonian treatment of the Feynman-Wheeler dynamics, in which there is a 1-2 lightlike correlation between the world-lines and which is connected to field theory[34] and to an effective action for fermions in a suitable strong coupling limit of quantum field theory[36]. See however the talk of Crater[21] and ref.[37], where Dirac constraints are introduced. A final expected outcome should be that the Fock space of the Schroedinger representation should use multilocal wave functions instead of tensor products of free particle wave functions, as in the Fock space for the interaction picture and the S matrix theory. In this respect see also ref.[38].

2. Multitemporal equations and Dirac observables.

The theory of singular Lagrangians and Dirac-Bergmann theory of constraints unify the description of all relativistic systems: interacting particles, extended objects like the string, classical gauge field theories and general relativity, due essentially to the requirements of manifest covariance and of gauge invariance through the minimal coupling. These systems fall into two classes: i) Gauge theories. Here behind the gauge freedom there are Lie groups and the Dirac observables are the measurable quantities (bilinear in the Fermi fields, when fermions are present), which can be extracted from the original variables; ii) Theories invariant under diffeomorphism groups of the underlying spacetime, like general relativity, strings and reparametrization invariant systems of particles. In this case the gauge freedom is the arbitrariness for the observer in the choice of the definition of what is space and/or time, i.e. of the definitory properties of the measuring apparatus. The Dirac observables are now the independent Cauchy data for the equations of motion, which result after a convention, about what space and/or time is, has been made. Since the canonical Hamiltonian vanishes, both kinematics and dynamics are contained in the 1st-class constraints describing the system: these can be interpreted as generalized Hamilton-Jacobi equations[39,22], so that the Dirac observables turn out to be the Jacobi data.

A problem, which till now has not been cleverly investigated, is how to obtain a global and manifestly covariant description, at least at the Hamiltonian level, only in terms of a canonical basis, or at least of a Poisson algebra[40], of Dirac observables. One would expect that when this is not possible, the relativistic system is intrinsically ill defined already at the classical level: at the quantum level this should manifest itself with the presence of not curable anomalies (which can be present also for a classically well defined system). Since the mathematical theory of the anomalies relies on cohomological properties, see the descent equations[41], of the manifolds relevant to the description of the system, which have to be defined already at the classical level, one expects a classical background of these properties in the form of obstructions present in the theory of classical gauge canonical transformations.

When a good description of the system in terms of Dirac observables exists, one is going to face the problem of quantizing only the true physical degrees of freedom, which generically are nonlinear and nonlocal functions or functionals of the original variables, without fixing the gauge, becouse the gauge variables have been decoupled. When a quantization is possible, there is a high probability to get a quantum theory inequivalent to that obtained by first quantizing the original variables and then making the reduction to the physical degrees of freedom at the quantum level (see for instance the BRS method).

With regards to field theory, this method has the drawback that generically the physical Hamiltonian, and therefore also the Lagrangian, is non polynomial in the physical degrees of freedom. Power counting methods cannot be used when looking for regularizations and renormalizations of the theory, and the advantages of a global control of the dynamics of physical quantities and of the possibility to check whether a model is classically well defined are destroyed by our present inhability to solve these problems. The question, which puzzled both Dirac and Yukawa, reappears, whether it is possible to define an intrinsic ultraviolet cutoff and a regularization scheme independent from the power counting.

Motivated by all these problems, preliminarly I felt the need to study[42] the theory of singular Lagrangians and Hamiltonian constraints in the finite-dimensional case, trying to clarify all its unsolved aspects. In ref.[43] there is a review of what has been obtained till now; what is essentially still lacking is the study of the classical background of the anomalies. Here, for the case in which only 1st-class constraints are present at the Hamiltonian level, I will recall three main ingredients of the theory:

i) The Euler-Lagrange equations associated with a singular Lagrangian do not determine the gauge part of the extremals. However it cannot be totally arbitrary, but must be compatible with the algebraic properties of the Noether gauge transformations under which the action is either invariant or quasi-invariant. In the Hamiltonian formulation these properties are contained in the structure constants, or functions, of the Poisson brackets of the 1st-class constraints and the gauge arbitrariness of the trajectories is described by the Dirac multipliers appearing in the Dirac Hamiltonian. In both formulations one has to add extra equations, the either Lagrangian or Hamiltonian multitemporal equations, to have a consistent determination of the gauge part of the trajectory. When the Poisson brackets of the Hamiltonian 1st-class constraints imply a canonical realization of a Lie algebra, the extra Hamiltonian multitemporal equations have the 1st-class constraints as Hamiltonians (so that the Dirac Hamiltonian is reduced to the canonical Hamiltonian) and the time parameters (replacing the Dirac multipliers) are the coordinates of a group manifold for a Lie group whose algebra is the given Lie algebra: they enter in the multitemporal equations via a set of left invariant vector fields on the group manifold.

To understand the meaning of these equations, let us consider the constraint manifold in phase space. By itself this manifold is only an exact presymplectic manifold: it has a degenerate exact two-form and therefore no notion of Poisson brackets exists. One considers it embedded in the big unphysical phase space, to be able to use its Poisson brackets. The constraint presymplectic manifold is foliated by the gauge canonical transformations generated by the 1st-class constraints. In the ideal case in which the foliation is nice, all the leaves (or gauge orbits) are diffeomorphic and in the simplest case all of them are diffeomorphic to the group manifold of a Lie group. In this ideal case to rebuild a gauge orbit from one of its points (and therefore to determine the gauge part of the trajectories passing through that point) one needs the Lie equations associated with the given Lie group: the Hamiltonian multitemporal equations are generalized Lie equations describing all the gauge orbits simultaneously. In a generic case this description holds only locally for a set of diffeomerphic orbits, also in the case of systems invariant under diffeomorphisms.

ii) The Shanmugadhasan canonical transformation[44]. In the finite dimensional case general theorems connected with the Lie theory of function groups ensure the existence of local canonical transformations from the original canonical variables x_a, p_a, in terms of which the 1st-class constraints (assumed globally defined) have the form $\phi_\alpha(x,p) \approx 0$, to canonical bases P_α, Q_α, P_A, Q_A, such that the equations $P_\alpha \approx 0$ locally define the same original constraint manifold (the P_α are an abelianization of the 1st-class constraints); the Q_α are the adapted abelian gauge variables describing the gauge orbits (they are a realization of the times of the multitemporal equations in terms of variables pertaining to the given system); the Q_A, P_A are an adapted canonical basis of Dirac observables.

Therefore the problem of the search of the Dirac observables becomes the problem of finding Shanmugadhasan canonical transformations. The strategy is to find abelianizations P_α of the original constraints, to solve the multitemporal equations for x_a, p_a associated with the P_α, to determine the multitimes Q_α and to identify the Dirac observables P_A, Q_A from the remaining original variables, i.e. from those their combinations independent from P_α and Q_α.

iii) Global Shanmugadhasan canonical transformations. This is the main point in the search of a global description of a physical system in terms of Dirac observables. In general given an arbitrary singular Lagrangian and its associated Hamiltonian constraints, no such global canonical transformation exists. However when the system under investigation has some global symmetry group, the associated theory of the momentum map[45] is a source of globality. Now all the physical systems defined in the flat Minkowski spacetime, have the global Poincare' symmetry. This suggests to study the structure of the constraint manifold from the point of view of the orbits of the Poincare' group. If P^μ is the total momentum of the system, the constraint manifold has to be divided in four sectors (some of them may be absent for certain systems) according to whether $P^2 > 0$, $P^2 = 0$, $P^2 < 0$ or $P^\mu = 0$. Due to the different little groups of the various Poincare' orbits, the gauge orbits of different sectors will not be diffeomorphic. Therefore the gauge foliations of relativistic systems are nearly never nice, but rather one has to do with singular foliations.

For an acceptable relativistic system the sector $P^2 < 0$ has to be absent to avoid tachions. To study the sectors $P^2 = 0$ and $P^\mu = 0$ one has to add these relations as extra

constraints. For all the sectors the next step is to do a canonical transformation from the original variables to a new set consisting of c.o.m. variables x^μ, P^μ and of variables relative to the c.o.m.. Let us now consider the sector $P^2 > 0$. By using the standard Wigner boost $L^\mu_\nu(P, P_*)$ ($P^\mu = L^\mu_\nu(P, P_*)P^\nu_*$, $P^\mu_* = \eta\sqrt{P^2}(1;\vec{0})$, $\eta = sign P^o$), one boosts the relative variables at rest. The new variables are still canonical and the base is completed by P^μ and by a new c.o.m. coordinate $\tilde{x}^\mu$, differing from x^μ for spin terms. $\tilde{x}^\mu$ has complicated covariance properties[17]; instead the new relative variables are either Poincare' scalars or Wigner spin-1 vectors, transforming under the group O(3)(P) of the Wigner rotations induced by the Lorentz transformations. A final canonical transformation[17], leaving fixed the relative variables, sends the c.o.m. coordinates $\tilde{x}^\mu$, P^μ in the new set $P\cdot\tilde{x}/\eta\sqrt{P^2} = P\cdot x/\eta\sqrt{P^2}$ (the time in the rest frame), $\eta\sqrt{P^2}$ (the total mass), $\vec{k} = \vec{P}/\eta\sqrt{P^2}$ (the spatial components of the 4-velocity $k^\mu = P^\mu/\eta\sqrt{P^2}$, $k^2 = 1$), $\vec{z} = \eta\sqrt{P^2}(\vec{\tilde{x}} - \tilde{x}^o\vec{P}/P^o)$. $\vec{z}$ is the noncovariant c.o.m. canonical coordinate of the Introduction multiplied by the total mass. Analogous considerations could be done for the other sectors.

The nature of the relative variables depends on the system. The 1st-class constraints, once rewritten in terms of the new variables, can be manipulated to find suitable global and Lorentz scalar abelianizations. Usually there is a combination of the constraints which determines $\eta\sqrt{P^2}$, i.e. the mass spectrum, so that the time in the rest frame $P\cdot x/\eta\sqrt{P^2}$ is the conjugated Lorentz scalar gauge variable. The other constraints eliminate some of the relative variables (in particular the relative time for systems of interacting relativistic particles and the string): their conjugated coordinates are the other gauge variables, which are identified with a possible set of time parameters by the multitemporal equations. The Dirac observables (apart from the c.o.m. ones $\vec{k}$ and $\vec{z}$) have to be extracted from the remaining relative variables and the construction shows that they will be either Poincare' scalars or Wigner covariant objects.

In this way in each sector preferred global Shanmugadhasan canonical transformations are identified, when no other kind of obstruction to globality is present inside the various sectors: for interacting relativistic particles and for the Nambu string globality can be achieved.

In gauge field theories the situation is more complicated, because the theorems ensuring the existence of the Shanmugadhasan canonical transformation have not been extended to the infinite-dimensional case. One of the reasons is that some of the constraints can now be interpreted as elliptic equations and they can have zero modes. Let us consider the sector $P^2 > 0$ of free Yang-Mills theory as a prototype and its 1st-class constraints, given by the Gauss laws and by the vanishing of the time components of the canonical momenta. The problem of the zero modes will appear as a singularity structure of the gauge foliation of the allowed sectors, in particular of the sector $P^2 > 0$. This phenomenon was discovered in ref.[46] by studying the space of solutions of Yang-Mills and Einstein equations, which can be mapped onto the constraint manifold of these theories in their Hamiltonian description. It turns out that the space of solutions has a "cone over cone" structure of singularities: if we have a line of solutions with a certain number of symmetries, in each

point of this line there is a cone of solutions with one less symmetry. In the Yang-Mills case the symmetries of a gauge potential are connected with the generators of its stability group, i.e. with the subgroup of those special gauge transformations which leave invariant that gauge potential. Since the Gauss laws are the generators of the gauge transformations (and depend on the chosen gauge potential through the covariant derivative), this means that for a gauge potential with non trivial stability group those combinations of the Gauss laws corresponding to the generators of the stability group cannot be any more 1st-class constraints, since they do not generate effective gauge transformations but special symmetry transformations. This problematics has still to be clarified, but it seems that in this case these components of the Gauss laws become 3rd-class constraints, which are not generators of true gauge transformations. This new kind of constraints was introduced in the second paper of ref.[42] in the finite dimensional case as a result of the study of some examples, in which the Jacobi equations (the linearization of the Euler-Lagrange equations) were singular, that is some of their solutions were not infinitesimal deviations between two neighbouring extremals of the Euler-Lagrange equations. This interpretation seems to be confirmed by the fact that the singularity structure discovered in ref.[46] follows from the existence of singularities of the linearized Yang-Mills and Einstein equations.

Other possible sources of singularities of the gauge foliation of Yang-Mills theory in the sector $P^2 > 0$ may be: i) different classes of gauge potentials identified by the field invaraints; ii) the orbit structure of the rest frame (or Thomas) spin $\vec{S}$, identified by the Pauli-Lubanski Casimir $W^2 = -P^2\vec{S}^2$ of the Poincare' group.

The final outcome of this structure of singularities is that the reduced phase-space, i.e. the space of the gauge orbits[47], is in general a stratified manifold with singularities[40]. In the sector $P^2 > 0$ of the Yang-Mills theory these singularities survive the Wick rotation to the Euclidean formulation and it is not clear how the ordinary path integral approach and the associated BRS method can take them into account. The search of a global canonical basis of Dirac observables for each stratum of the space of the gauge orbits can give a definition of the measure of the phase space path integral, but at the price of a non polynomial Hamiltonian. Moreover there are extra complications connected to the Gribov ambiguity and therefore to the existence of global Dirac observables. Much work is still needed to clarify all these problems of gauge field theories.

A final remark on the Shanmugadhasan canonical transformations. When the coordinates P_α, Q_α, P_A are globally defined and the canonical Hamiltonian vanishes, one has Liouville integrability: with the exception of the free c.o.m. motion, P_A and Q_A can be replaced with action-angle variables I_A, ϕ_A (in the sense of Jacobi data: the ϕ_A are constant of the motion), with the I_A globally defined. Every extra dynamical symmetry group of the system implies the existence of globally defined functions of the angles.

3. Particles, strings and fields.

In this final section I will sketch what has till now been done in the search of global Dirac observables:

i) Particles. The Todorov-Komar-Droz Vincent model, with only $P^2 > 0$, has been completely understood. In Appendix D of ref.[17] it is given a final canonical transformation, which connects the original canonical coordinates x_i^μ, p_i^μ, $i = 1, 2$, to a set composed by $T = P \cdot x/\eta\sqrt{P^2}$, $\epsilon = \eta\sqrt{P^2}$, $T_R = P \cdot r/\eta\sqrt{P^2}$, $\epsilon_R = P \cdot q/\eta\sqrt{P^2}$, $\vec{z}$, $\vec{k}$, $\vec{r}$, $\vec{q}$ ($P^\mu = p_1^\mu + p_2^\mu$, $x^\mu = (x_1^\mu + x_2^\mu)/2$, $r^\mu = x_1^\mu - x_2^\mu$, $q^\mu = (p_1^\mu - p_2^\mu)/2$). $\vec{r}$, $\vec{q}$ are Wigner spin-1 vectors. This transformation is the closest one to a Shanmugadhasan canonical transformation, becouse in the particle case the sum of the 1st-class constraints becomes an algebraic equation for the mass spectrum ϵ in terms of $\vec{r}$, $\vec{q}$. The diference of the two constraints is $\epsilon_R - (m_1^2 - m_2^2)/2\epsilon \approx 0$, implying that the relative time T_R is a gauge variable. A true Shanmugadhasan transformation could be defined for each branch of the mass spectrum.

ii) Nambu string. The study of the abelianization of its constraints by using light-cone coordinates was done in ref.[48]. Due to the total loss of manifest covariance and to the existence of two overlapping charts with different abelianizations and different local Shanmugadhasan transformations, a study of the sector $P^2 > 0$ (also the sectors $P^2 = 0$ and $P^\mu = 0$ are present at the classical level) with the Wigner technique used for the Todorov model was done in ref.[49], for both the open and closed string.

Let us consider the open string: the closed one is technically more involved, but the steps are the same. After a canonical transformation from the original variables $x^\mu(\tau,\sigma)$, $p^\mu(\tau,\sigma)$ to c.o.m. and relative variables x^μ, P^μ, $y^\mu(\tau,\sigma)$, $\mathcal{P}^\mu(\tau,\sigma)$, the relative variables have been boosted at rest and the new basis $\tilde{x}^\mu$, P^μ, $\tilde{y}(\tau,\sigma)$, $\tilde{\mathcal{P}}(\tau,\sigma)$, $\vec{\tilde{y}}(\tau,\sigma)$, $\vec{\tilde{\mathcal{P}}}(\tau,\sigma)$ was obtained. $\vec{\tilde{y}}$, $\vec{\tilde{\mathcal{P}}}$ are Wigner spin-1 vectors, while $\tilde{y}$, $\tilde{\mathcal{P}}$ are Poincare' scalars ($\tilde{y}(\tau,\sigma)$ are the infinite relative times). After rewriting the original constraints $A_\pm^2(\tau,\sigma) = (p(\tau,\sigma) \mp N x'(\tau,\sigma))^2 \approx 0$, $x'^\mu(\tau,\sigma) = \frac{\partial x^\mu(\tau,\sigma)}{\partial\sigma}$, in the new variables ($A_\pm^\mu \to \tilde{A}_\pm, \vec{\tilde{A}}_\pm$), the following abelianization was discovered: there are two sets of global Poincare' invariant Abelian constraints $\chi_\pm^\eta(\tau,\sigma)$ corresponding to the two disjoint sheets, $\eta = signP^o$, of the mass hyperboliod $P^2 > 0$. The conjugated global Poincare' invariant Abelian gauge variables $T_\pm^\eta(\tau,\sigma)$ (the multitimes of the multitemporal approach) were found and it was shown that the Dirac observables had to be extracted from the Wigner unit vectors $\vec{\tilde{n}}_\pm^\eta(\tau,\sigma) = \vec{\tilde{A}}_\pm(\tau,\sigma)/\eta\sqrt{\vec{\tilde{A}}_\pm^2(\tau,\sigma)}$. The solutions of the multitemporal equations for the unit vectors allowed the identification of nontrivial functions $\tilde{B}_\pm^{\eta\parallel}(\tau,\sigma)$ of the gauge variables, needed to perform a generalized harmonic analysis of the unit vectors. The generalized Fourier coefficients $\vec{\alpha}_n$ of the unit vectors are Wigner spin-1 Dirac observables, satisfying σ-model like constraints implied by $\vec{\tilde{n}}_\pm^{\eta 2}(\tau,\sigma) \stackrel{?}{=} 1$. Therefore the $\vec{\alpha}_n$ are not independent oscillators,

as it is confirmed by their Poisson brackets.

One of these constraints, together with a combination of the original constraints, gives the mass spectrum $\eta\sqrt{P^2}$ in terms of the $\vec{\alpha}_n$. This is equivalent to the $L_o \approx 0$ Virasoro constraint; all the other Virasoro constraints are decoupled and live in the gauge world spanned by $\chi_\pm^\eta$, $T_\pm^\eta$, so that they are not going to be quantized: this avoids the Virasoro cohomology mechanism for producing the critical dimension in the covariant approach. Moreover there are the c.o.m. observables $\vec{z}$, $\vec{k}$. Both for the open and closed string the unit vectors $\vec{\tilde{n}}_\pm^\eta(\tau,\sigma)$ are tangent to closed curves with the functions $\tilde{B}_\pm^{\eta\parallel}(\tau,\sigma)$ as natural parameters.

To get a true Shanmugadhasan canonical transformation, one has to extract a canonical basis of Dirac observables from the $\vec{\alpha}_n$. To avoid the other mechanism for the critical dimension, i.e. the nonlinearity of the Poincare' realization of the noncovariant approach, a reduction like in σ-models has to be discarded. Instead one has to try to extend the Pauri-Prosperi[49] theorems on the canonical realizations of the Lie algebras to this infinite-dimensional case, in which the relevant algebra is o(3)(P). One has to look for variables adapted to the Thomas spin $\vec{\tilde{S}}$, which is an observable: the new canonical basis will be formed by two pairs $\{\tilde{S}_3, arctg\tilde{S}_2/\tilde{S}_1\}$, $\{\tilde{S} = \sqrt{\vec{\tilde{S}}^2}, \phi\}$ (ϕ is an angle) and by Poincare' scalar action-angle variables I_n, ϕ_n. The Poincare' generators are realized only by using the first two pairs and $\vec{z}$, $\vec{k}$, so that they can be consistently quantized in 4 dimensions. The results of Pron'ko[50], who applied the inverse scattering method for periodic potentials to the Nambu string, seems to show that the I_n exist and are globally defined, because they are the integral invariants of the closed curves having $\vec{\tilde{n}}_\pm^\eta(\tau,\sigma)$ as unit tangent vectors: if this is true one obtains a possible definition of Liouville integrability for the Nambu string. If the infinite-dimensional generalized algebra of Pohlmeyer-Rehren[51] conserved charges (it contains the Poincare' algebra and, for this extended object, it avoids the Coleman-Mandula no-go theorem, origin of supersymmetry for pointlike systems, because it has structure functions depending on the total momentum instead of structure constants) implies that also the angles ϕ_n are globally defined, then one can quantize the Nambu string in 4 dimensions and due to the nature of these variables one would also obtain the decomposition of the spin levels over each mass level into irreducible components. Otherwise the Virasoro cohomological problem, notwithstanding all the nonlocal operations needed to build these new variables, could reappear in connection with the transition functions on the overlap of the charts of the symplectic atlas needed to describe I_n and ϕ_n and a critical dimension will show up like in the other approaches. In any case it would be worthwhile to suceed in building this canonical transformation explicitely.

iii) Gauge fields. In the case of the electromagnetic field most of the work for the search of the Dirac observables was done by Dirac[52] himself. The potentials $A_\mu(t,\vec{x})$ have conjugated momenta $\pi^o(t,\vec{x}) = 0$, $\pi^i(t,\vec{x}) = -E^i(t,\vec{x}) = F^{oi}(t,\vec{x})$. The Dirac Hamiltonian is $H_D = H_c + \int d^3x[A_o(t,\vec{x})\Gamma(t,\vec{x}) + \lambda(t,\vec{x})\pi^o(t,\vec{x})]$ with $H_c = \int d^3x[\vec{\pi}^2(t,\vec{x}) + \vec{B}^2(t,\vec{x})]$, $B^i(t,\vec{x}) = \epsilon^{ijk}\partial^j A^k(t,\vec{x}) = 1/2\epsilon^{ijk}F^{jk}(t,\vec{x})$, and the Gauss law $\Gamma(t,\vec{x}) = \vec{\partial} \cdot \vec{\pi}(t,\vec{x}) \approx$

0 is the only secondary constraint. We assume boundary conditions such that we can integrate by parts and such that the Laplacian Δ has no zero modes. From now on we shall omit the time t inside the fields. By means of the Hodge decomposition we obtain $\vec{A}(\vec{x}) = \vec{\partial}\eta(\vec{x}) + \vec{D}(\vec{x})$, $\vec{\pi}(\vec{x}) = (\vec{\partial}/\Delta)\Gamma(\vec{x}) + \vec{\mathcal{P}}(\vec{x})$ with $\vec{\partial}\cdot\vec{D}(\vec{x}) = \vec{\partial}\cdot\vec{\mathcal{P}}(\vec{x}) \equiv 0$. Here $\eta(\vec{x}) = (\vec{\partial}/\Delta)\cdot\vec{A}(\vec{x}) = \int d^3y\vec{c}(\vec{x},\vec{y})\cdot\vec{A}(\vec{y})$, where $\vec{c}(\vec{x},\vec{y})$ is the distribution defined in ref.[52] , which satisfies $\vec{\partial}_x\cdot\vec{c}(\vec{x},\vec{y}) = \delta^3(\vec{x}-\vec{y})$. The gauge world is composed by the canonical pairs $(A_o(\vec{x}),\pi^o(\vec{x}))$, $(\eta(\vec{x}),\Gamma(\vec{x}))$. The Dirac observables are the transverse fields $\vec{D}(\vec{x})$, $\vec{\mathcal{P}}(\vec{x})$ $(B^i(\vec{x}) = \epsilon^{ijk}\partial^j D^k(\vec{x}))$ satisfying $\{D^i(\vec{x}),\mathcal{P}^j(\vec{y})\} = (\delta^{ij} - \partial^i\partial^j/\Delta)\delta^3(\vec{x}-\vec{y})$ and whose dynamics is governed by the physical Hamiltonian $H_P = 1/2\int d^3x[\vec{\mathcal{P}}^2(\vec{x}) + \vec{B}^2(\vec{x})]$, so that $\dot{\vec{\mathcal{P}}}(\vec{x}) = \vec{D}(\vec{x})$. To obtain a Shanmugadhasan transformation one should extract the transverse degrees of freedom from $\vec{D}(\vec{x})$, $\vec{\mathcal{P}}(\vec{x})$, possibly in an Euclidean covariant way. The Coulomb gauge $\eta(\vec{x}) \approx 0$ is compatible[53] with the temporal gauge $A_o(\vec{x}) \approx 0$, when the Gauss law holds, since $\dot{\eta}(\vec{x}) = \{\eta(\vec{x}),H_D\} = A_o(\vec{x}) - (1/\Delta)\Gamma(\vec{x}) \approx A_o(\vec{x}) \approx 0$, $\dot{A}_o(\vec{x}) = \lambda(\vec{x}) \approx 0$, and we get a special case of the Lorentz gauge $\partial_\mu A^\mu(x) = 0$.

When massive Dirac fermions, described by Grassmann spinors, are present, the Gauss law becomes $\Gamma(\vec{x}) = \vec{\partial}\cdot\vec{\pi}(\vec{x}) - e\psi^\dagger(\vec{x})\psi(\vec{x})$ and the Dirac Hamiltonian is $H_D = \int d^3x[i\psi^\dagger(\vec{x})\vec{\alpha}\cdot\vec{\partial}\psi(\vec{x}) + e\psi^\dagger(\vec{x})\vec{\alpha}\cdot\vec{A}(\vec{x})\psi(\vec{x}) - m\psi^\dagger(\vec{x})\beta\psi(\vec{x}) + 1/2(\vec{\pi}^2(\vec{x}) + \vec{B}^2(\vec{x})) + A_o(\vec{x})\Gamma(\vec{x}) + \lambda(\vec{x})\pi^o(\vec{x})]$, with $\{\psi_\alpha(\vec{x}),\psi^\dagger_\beta(\vec{y})\} = -i\delta^3(\vec{x}-\vec{y})$. Dirac[52] introduced the gauge invariant fermion fields $\psi^*(\vec{x}) = \psi(\vec{x})exp(i\eta(\vec{x}))$, satisfying the same Poisson brackets of the original ones: the bilinears in them are even Dirac observables, but are not local objects not having compact support. They describe the electron with its Coulomb cloud. The Coulomb gauge is now not compatible with the temporal one, becouse $\dot{\eta}(\vec{x}) \approx A_o(\vec{x}) + e(1/\Delta)\psi^\dagger(\vec{x})\psi(\vec{x}) \approx 0$. $\vec{D}(\vec{x})$, $\vec{\mathcal{P}}(\vec{x})$, $\psi^*(\vec{x})$, $\psi^{*\dagger}(\vec{x})$ are the Dirac observables, with a dynamics governed by the physical Hamiltonian

$$H_P = \int d^3x[i\psi^{*\dagger}(\vec{x})\vec{\alpha}\cdot\vec{\partial}\psi^*(\vec{x}) - m\psi^{*\dagger}(\vec{x})\beta\psi^*(\vec{x}) + 1/2(\vec{\mathcal{P}}^2(\vec{x}) + \vec{B}^2(\vec{x})) + e\psi^{*\dagger}(\vec{x})\vec{\alpha}\cdot\vec{D}(\vec{x})\psi^*(\vec{x})] + e^2\int d^3xd^3y[\psi^{*\dagger}(\vec{x})\psi^*(\vec{x})]c(\vec{x},\vec{y})[\psi^{*\dagger}(\vec{y})\psi^*(\vec{y})]$$

with $\Delta_x c(\vec{x},\vec{y}) = \delta^3(\vec{x}-\vec{y})$. The last term is the nonlocal Coulomb interaction and the preceding term is the coupling to the external radiation field. The difficult problem is how to quantize the Coulomb term.

Instead the problem of obtaining a manifestly covariant formulation can be solved[54] by reformulating the theory in parametrized form on arbitrary spacelike hypersurfaces, following Dirac[1]. If $z^\mu(\tau,\vec{\sigma})$ are the coordinates of a point on the hypersurface, then $z^\mu_A(\tau,\vec{\sigma}) = \frac{\partial z^\mu(\tau,\vec{\sigma})}{\partial\sigma^A}$, $\sigma^o = \tau$, are a set of vierbeins and the metric tensor on the hypersurface is $g_{AB}(\tau,\vec{\sigma}) = z^\mu_A(\tau,\vec{\sigma})\eta_{\mu\nu}z^\nu_B(\tau,\vec{\sigma})$. The independent fields are $A_A(\tau,\vec{\sigma}) = z^\mu_A(\tau,\vec{\sigma})A_\mu(\tau,\vec{\sigma})$, so that $F_{AB} = z^\mu_A z^\nu_B F_{\mu\nu}$ and the action is $S = -1/4\int d\tau d^3\sigma[\sqrt{g}g^{AC}g^{BD}F_{AB}F_{CD}](\tau,\vec{\sigma})$, where g^{AB} is the inverse metric. The canonical Hamiltonian vanishes and one has the 1st-class constraints $H_\mu(\tau,\vec{\sigma}) = \rho_\mu(\tau,\vec{\sigma}) + T_{\tau\tau}(\tau,\vec{\sigma})l_\mu(\tau,\vec{\sigma}) - T_{\tau r}(\tau,\vec{\sigma})z_{r\mu}(\tau,\vec{\sigma}) \approx 0$, $\pi^\tau(\tau,\vec{\sigma}) \approx 0$, $\chi(\tau,\vec{\sigma}) = \partial^r\pi^r(\tau,\vec{\sigma}) \approx 0$. Here $\rho_\mu(\tau,\vec{\sigma})$ is the momentum conjugated to $z^\mu(\tau,\vec{\sigma})$, $l^\mu(\tau,\vec{\sigma})$ the unit normal to the hypersurface and $T_{\tau\tau}(\tau,\vec{\sigma})$, $T_{\tau r}(\tau,\vec{\sigma})$ are components

of the electromagnetic energy momentum tensor. By eliminating the electromagnetic constraints and gauge variables as before, one remains with the 4 Abelian constraints $H_{P\mu}(\tau,\vec{\sigma}) = \rho_\mu(\tau,\vec{\sigma}) + T_{P\tau\tau}(\tau,\vec{\sigma})l_\mu(\tau,\vec{\sigma}) - T_{P\tau r}(\tau,\vec{\sigma})z_{r\mu}(\tau,\vec{\sigma}) \approx 0$, which depend on the transverse observables $D_r(\tau,\vec{\sigma})$, $\mathcal{P}_r(\tau,\vec{\sigma})$. Then, since we are working in special relativity, we restrict the hypersurfaces to spacelike hyperplanes $z^\mu(\tau,\vec{\sigma}) = x^\mu(\tau) + b^\mu_r(\tau)\sigma^r$, where $b^\mu_A(\tau)$ with $b^\mu_o(\tau) = l^\mu(\tau)$ are orthonormal vierbeins. Only 10 constraints are left

$$H^\mu_P = P^\mu - 1/2l^\mu(\tau)\int d^3\sigma(\vec{\mathcal{P}}^2 + \vec{B}^2)(\tau,\vec{\sigma}) - b^\mu_s(\tau)\int d^3\sigma(\vec{\mathcal{P}}\times\vec{B})^s(\tau,\vec{\sigma}) \approx 0$$

$$H^{\mu\nu}_P = S^{\mu\nu} - b^{[\mu}_r(\tau)l^{\nu]}(\tau)1/2\int d^3\sigma\sigma^s(\vec{\mathcal{P}}^2 + \vec{B}^2)(\tau,\vec{\sigma}) - b^{[\mu}_r(\tau)b^{\nu]}_s(\tau)\int d^3\sigma\sigma^s(\vec{\mathcal{P}}\times\vec{B})^r(\tau,\vec{\sigma}) \approx 0$$

where $P^\mu = \int d^3\sigma\rho^\mu(\tau,\vec{\sigma})$ and $S^{\mu\nu}$ are respectively the total momentum and the spin tensor conjugated to the vierbeins b^μ_A (see ref.[9] for their treatment). Therefore manifest covariance is obtained at the price of introducing 10 extra degrees of freedom, associated with the Poincare' group. If we restrict ourselves to the sector of field configurations with $P^2 > 0$, we can select the special foliation of the Minkowski spacetime given by the family of hyperplanes normal to P^μ: this can be done by imposing $b^\mu_A = L^\mu_{\nu=A}(P,P_*)$. Then the indices $A = r$ become spin-1 Wigner indices.. Only the 4 1st-class constraints H^μ_P are left and by using the c.o.m. variables defined in section 2 they can be written in the form

$$H_\tau = \eta\sqrt{P^2} - V_\tau = \eta\sqrt{P^2} - 1/2\int d^3\sigma(\vec{\mathcal{P}}^2 + \vec{B}^2)(\tau,\vec{\sigma}) \approx 0$$

$$V^r = \int d^3\sigma(\vec{\mathcal{P}}\times\vec{B})^r(\tau,\vec{\sigma}) \approx 0$$

Besides the transverse electromagnetic degrees of freedom we have the remaining 8 c.o.m. variables associated to the hyperplanes. V^τ, V^r is the total momentum of the field configuration. The previous construction clearly points toward the existence of a canonical transformation from the transverse variables $\vec{D}(\vec{x})$, $\vec{\mathcal{P}}(\vec{x})$ to a basis containing the field c.o.m. variables V^τ, V^r, T^τ, Z^r and an infinite number of relative variables. T^τ and $\vec{Z}$ are the rest frame time and the noncovariant canonical c.o.m. position of the field configuration, which naturally have to be restricted so that these variables and the 10 Poincare' generators exist and are finite. It would be important to find this canonical basis, since it would allow to show explicitely the unification of the Wigner description of particles, strings and fields. Once this basis is known, one can eliminate the last degrees of freedom of the hyperplanes in the sector $P^2 > 0$ by imposing the gauge-fixings $\vec{z} - \vec{Z} \approx 0$, $P\cdot x/\eta\sqrt{P^2} - T^\tau \approx 0$: only the field degrees of freedom $\vec{Z}$, T^τ, $\vec{V}$, V^τ and all the transverse field variables relative to the c.o.m. (transforming like Wigner spin-1 vectors) would be left to be quantized.

I have found[54] the generalization of the procedure for finding the transverse Dirac observables of the free Yang-Mills theory in the case of a trivial principal bundle[47] and for a gauge potential without stability group, to avoid the singularities of ref.[46]. In this case the Gauss laws do not have zero modes and a Green function, which solves them,

has been found. By solving the multitemporal equations and by introducing a generalized Hodge decomposition, what could be called a generalized Coulomb gauge has been found and the transverse Dirac observables and the physical Hamiltonian have been constructed. The Gribov ambiguity becomes a problem of coordinate singularities, but I have still to understand the global problems connected with the center of the gauge group, which is isomorphic to the center of the holonomy group of the given connection. To pass from invariance under the small gauge transformations to gauge invariance in large one has to identify some global topological information, for instance which are those special loops on the base whose holonomy identifies the center of the holonomy group.

Since the physical Yang-Mills Hamiltonian is nonpolynomial and since also without fermions the Coulomb gauge and the temporal one are not compatible[53] due to the fact that the bosonic field is charged, we do not know how to quantize the theory before or after its covariantization. However the emergence of a canonical noncovariant c.o.m. coordinate $\vec{Z}$ (or better $\vec{Z}' = \vec{Z}/\eta\sqrt{P^2}$) also for field configurations, shows that also in the case of fields one can define the noncovariance world-tube of radius $d = S/\sqrt{P^2}c$. The same radius can be expressed in terms of the Poincare' Casimirs in the sector $P^2 > 0$, $W^2 \neq 0$, i.e. $d = \sqrt{-W^2}/P^2c$. To $\vec{Z}'$ one can apply the results found by Hegerfeldt[55] studying the Newton-Wigner position operator. He found that if this position is quantized as a self-adjoint operator, one can define good localization properties, but then one has violation of Einstein causality (i.e. a spreading of a wave packet with a velocity higher than the light velocity) in the following cases: i) initial localization on a compact set; ii) Gaussian and exponential wave packets. Only with wave packets with power tails there is the possibility of no violation of causality: but to accept only such a class of wave packets is equivalent to abandon the self-adjointness of the position operator and to have bad localization properties. Now the noncovariance world-tube points in the direction of having a "delocalization", i.e. of excluding the possibility of localize the c.o.m. inside the world-tube to avoid frame-dependent statements. This problematics, together with the fact that for a field configuration belonging to a Poincare' irreducible representation with $P^2 > 0$, $W^2 \neq 0$ (like for the Wigner definition of elementary spinning particles), one has an intrinsic unit of mass, $M(P) = \eta\sqrt{P^2}$, and an intrinsic unit of lenght, the radius d, points toward the existence of an intrinsic ultraviolet cutoff for these field configurations. By restricting the Heisemberg relations to forbid localization inside the noncovariance world-tube, i.e. below a radius proportional to the Compton wavelenght of the field configuration: $\Delta Z'^k \geq \hbar/\Delta P^k \geq \hbar s/\sqrt{P^2}c$ (with $S \rightarrow \hbar s$), one obtains $\Delta P^k \leq (1/s)\sqrt{P^2}c$. This can be reinterpreted as an intrinsic cutoff $P^k_{max} = \sqrt{P^2}c/s$. There would be a smearing of short distances to avoid frame-dependent statements induced by spin effects.

As suggested in ref.[57], a different alternative possibility to be explored is noncommutative geometry: according to me particularly relevant could be the framework of ref.[58], emphasizing the analogies with the classical Lorentz and Poincare' group of their quantum analogues; however see also ref.[59]. The radius r could play the role of the classical basis of a deformation parameter independent form $\hbar$, which describes the deformation of the Poisson brackets to the quantum commutators.

We are back to the original problems, which pushed Dirac and Yukawa to begin the line of research here considered. Whether all these qualitative fragments can be used to define a regularization procedure for nonpolynomial Hamiltonians is a completely open problem. I think that there is the urgent need to make an effort in this direction, in parallel with the attempt to understand better the functional Schroedinger representation.

Finally one has to extend this multitemporal approach to classical general relativity, trying in particular to solve the constraints of the Ashtekar complex tetrad gravity[60].

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