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IS THE INELASTICITY DECREASING WITH ENERGY?

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ABSTRACT:

We show that when transverse momentum (pseudo) rapidity correlations are correctly taken into account the pion inelasticity remains pratically constant in the GeV-TeV energy regions. When extending our calculation to the charged particle inelasticity, we find a slow decrease with energy.

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With the increase of the available energy in high energy accelerators, now in the TeV region, increases as well the overlap between cosmic ray and accelerator experiments. It is then important to test the consistency of the physics involved and, in particular, to check some basic parameters related to particle and jet distributions. One of these parameters is the inelasticity.

By definiton, the inelasticity $K_i(s)$, where $\sqrt{s}$ is the centre of mass energy in a given hadronic collisions, is the fraction of the total energy taken away by the produced particles of type $i(i = \pi^+, \pi^-, \pi^\circ, k^+, k^-, k^\circ, \bar{k}^\circ, \ldots, \nu, \bar{\nu}, \ldots)$. Taking into account the fraction of energy X_L retained by the leading particles, one naturally expects, by energy conservation,

$$\sum_i K_i(s) + X_L(s) \equiv K(s) + X_L(s) = 1, \tag{1}$$

where $K(s)$ is the total inelasticity.

Theorists are divided with respect to the energy dependence of inelasticity[1]. In some models[2], the inelasticity decreases with energy, i.e., the incident nucleons keep an increasing fraction of their initial energy, as energy increases. In other models[3,4,5], based on the observation of growing opacity in high energy collisions and of production of larger p_T QCD jets, it is argued that the inelasticity has to increase, the leading spectrum thus becoming softer.

In the GeV region it is known that the inelasticity is of the order of 0.5: about half of the energy remains with the leading particles. Unfortunately, in the TeV region and above, not much is known. From cosmic ray experiments there is the indication that the pion inelasticity may decrease[6] and that the leading nucleon average energy fraction is not substantially changing with energy[7]. In accelerators, the leading proton spectrum was studied only up to the ISR[8], $\sqrt{s}$ = .053 TeV, and at higher energy practically no additional informations exists. Attempts to experimentally study the fraction of energy carried by produced particles at Fermilab[9] were not conclusive.

In model calculations one starts with the normalized one-particle inclusive cross-section

$$\frac{E}{\sigma}\left.\frac{d^2\sigma}{dp_L dp_T^2}\right|_i \tag{2}$$

where E is the center of mass energy, p_L the longitudinal momentum and p_T the transverse momentum. The inelasticity K_i is given by:

$$\frac{2}{\sqrt{s}}\int \frac{E}{\sigma}\left.\frac{d^2\sigma}{dp_L dp_T^2}\right|_i dp_L dp_T^2 = K_i. \tag{3}$$

Different calculations of the inelasticity[10-13] in hadron-hadron collisions reached the conclusion that there is a continuous decrease of $K(s)$ in going from the GeV to the TeV

region. According to these authors this is essentially a consequence of violation of Feynman scaling in the fragmentation region. At the same time, not much attention was given to p_T distributions. We argue here that p_T rapidity correlations have to be correctly taken into account, otherwise the results of calculations cannot be fully reliable.

Let us start with the proposal, which in fact is a generalization of a suggestion of Wdowczyk and Wolfendale[11], of treating the inclusive invariant normalized cross-section as a function of variables scaled by the mean. If the shape of the distribution does not change dramatically with energy, scaling by average values gives some control of energy dependence, as the first moment of the function is already correctly given. We thus, for instance, write

$$\frac{2}{\sqrt{s}}\frac{E}{\sigma}\frac{d^2\sigma}{dxdp_T^2} = K(s)\frac{1}{\langle x\rangle}\frac{1}{\langle p_T\rangle^2} f\left(x/\langle x\rangle, p_T/\langle p_T\rangle\right), \tag{4}$$

where x is the Feynman variable. Neglecting, for produced particles, the small contribution from the upper limit in the x integration, one can impose the energy independent constraint

$$\int\int f\left(x/\langle x\rangle, P_T/\langle p_T\rangle\right) d(x/\langle x\rangle) d(p_T^2/\langle p_T\rangle^2) = 1, \tag{5}$$

such that the equation

$$\int\int \frac{2}{\sqrt{s}}\frac{E}{\sigma}\frac{d^2\sigma}{dxdp_T^2} dxdp_T^2 = K(s) \tag{6}$$

is automatically satisfied.

Instead of using the pair x, p_T variables we shall use here the pair η, p_T because η, the pseudorapidity, is a quantity experimentally more easily measured and gives no problems in preserving scaling when the limits of integration are $0 \le \eta \le \infty$.

We thus write the inclusive normalized cross-section as a function of the (scaled) variables η and p_T,

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\eta dp_T^2} = \rho(0)\frac{1}{\langle p_T\rangle^2} f\left(\frac{\sinh\eta}{\langle\sinh\eta\rangle}, \frac{p_T}{\langle p_T\rangle}\right) \tag{7}$$

where

$$\rho(\eta) \equiv \frac{1}{\sigma}\frac{d\sigma}{d\eta} \tag{8}$$

with

$$\frac{2p_T}{\sqrt{s}}\sinh\eta = x. \tag{9}$$

By integration in p_T one obtains

$$\rho(\eta) \equiv \frac{1}{\sigma}\frac{d\sigma}{d\eta} = \rho(0)F(\sinh\eta/\langle\sinh\eta\rangle). \tag{10}$$

with

$$\int \rho(\eta) d\sinh\eta \equiv \rho(0)\langle\sinh\eta\rangle. \tag{11}$$

Eq. (10) is the improved version of the scaling relation introduced in Ref. [10]. There, as the x variable was used, a p_T factor remained in the final expression, see (9), and, in an ad hoc manner, it was substituted by $\langle p_T\rangle$. In fact, no information on $\langle p_T\rangle$ is required to test pseudo-rapidity scaling, (10).

The function $F(X)$,

$$X \equiv \sinh\eta/\langle\sinh\eta\rangle \tag{12}$$

obeys the constraints

$$F(0) = 1 \tag{13}$$

and

$$\int F(X)dX = 1 \tag{14}$$

The following parametrization was used for $F(X)$:

$$F(X) = [1 + aX]/[1 + bX + cX^2]^2 \tag{15}$$

with $a = 14.44$, $b = 5.26$ and $c = 1.25$. Given a pseudo-rapidity distribution a test of the scaling (10) can be easily performed with a fit using the function $AF(X/B)$, the fitting parameters being $A \equiv \rho(0)$ and $B \equiv \langle\sinh\eta\rangle$. Tests of the scaling (10), at ISR, CERN Collider and Tevatron are shown in Figs. 1, 2 and 3.

As a consistency check of our calculations we have estimated, from (10), the average multiplicity and found, at all energies, good agreement with data. Using the step function approximation for $F(x), F(x) = 1, 0 \le x \le 1, F(x) = 0, x > 1$, we have

$$\langle n\rangle \equiv \int \rho(\eta)d\eta \simeq 2\rho(0)[\ln 2 + \ln\langle\sinh\eta\rangle]$$

a relation which is experimentally well verified.

We can next try to estimate the inelasticity. In the approximation of neglecting the mass of the produced particles - which is presumably correct for pions -, the pseudo-rapidity and the rapidity become identical, and the energy is

$$E \simeq p_T\cosh\eta. \tag{16}$$

The inelasticity is then, from (7), given by,

$$K(s) = S\rho(0)\langle p_T\rangle\langle\sinh\eta\rangle 2/\sqrt{s} \tag{17}$$

where

$$S = \int\int p_T/\langle p_T\rangle f\left(\frac{\sinh\eta}{\langle\sinh\eta\rangle}, \frac{p_T}{\langle p_T\rangle}\right) d\left(\frac{\sinh\eta}{\langle\sinh\eta\rangle}\right) d\left(\frac{p_T^2}{\langle p_T\rangle^2}\right) \tag{18}$$

One should notice that in the limit of exact Feynman scaling, $\rho(0) = const.$, $\langle p_T\rangle = const.$, and $\langle\sinh\eta\rangle \sim s^{1/2}$ such that $K = const.$. Experimentally, the width of the pseudo-rapidity distribution is expanding slower than $\sqrt{s}$, while the normalized cross-section at $\eta = 0, \rho(0)$, and the average $p_T, \langle p_T\rangle$, are growing with energy. Taking S in (18) as a constant, as it should according to the scaling (7), $K(s)$ continuously decreases with energy, as found, for instance, in Ref. [10]*.

We arrive now to the main point in this paper. There are energy dependent p_T (pseudo) rapidity correlations that make the integral S to increase with energy. The scaling (4) or (7), as assumed in [10], [11] and [13], and the x, p_T factorization in the fragmentation region of [12] do not agree with what is experimentally observed.

In fact, if we compute from the scaling relation (7) the average transverse momentum at a given (pseudo) rapidity, $\bar{p}_T(\eta)$, we obtain

$$\begin{aligned}\bar{p}_T(\eta) &= \frac{\rho(0)\langle p_T\rangle \int p_T/\langle p_T\rangle f(X, p_T/\langle p_T\rangle) d(p_T^2/\langle p_T\rangle^2)}{\rho(0)F(X)} \\ &\equiv \langle p_T\rangle\widetilde{\Phi}(X)\end{aligned} \tag{19}$$

We can further write,

$$\bar{p}_T(\eta) = \bar{p}_T(0)\Phi(X) \tag{20}$$

with $\Phi(0) = 1$. Eq. (20) tells us that $\bar{p}_T(\eta)/p_T(0)$ is an invariant, energy independent function of $X \equiv \sinh\eta/\langle\sinh\eta\rangle$. Note that in practice $\widetilde{\Phi} \simeq \Phi$ and $\langle p_T\rangle \simeq \bar{p}_T(0)$.

In Fig. (4) we show that (20) is not satisfied: the function Φ depends on the energy. In the exponential approximation,

$$\Phi(X) \to \Phi_\lambda(X) \equiv \exp(-\lambda X), \tag{21}$$

we obtain $\lambda(\sqrt{s} = 53\text{GeV})$=0.42 and $\lambda(\sqrt{s} = 630\text{GeV})$= 0.16 (see Fig.(4)).

Making now the assumption, used in [15], of a p_T distribution approximately exponential, we have

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\eta dp_T^2} = 2\rho(0)\frac{F(X)}{\Phi_\lambda^2(X)}\frac{1}{\langle p_T\rangle^2}\exp\left[-2p_T/\langle p_T\rangle\Phi_\lambda(X)\right]. \tag{22}$$

* In the notation of Ref.[10] if we put $\langle p_T\rangle \sim s^\beta$ then $\langle\sinh\eta\rangle \sim s^{-\beta-\alpha+1/2}, \rho(0) \sim s^{\alpha'}$ such that, from (17), $K(s) \sim s^{\alpha'-\alpha}, \alpha' \simeq 0.11$ and $\alpha \simeq 0.25$, decreases rapidly with energy.

We note that the well satisfied scaling (10) is still valid. The quantity S in (17) is then given by:

$$S = \int F(X)\Phi_\lambda(X)dX, \tag{23}$$

with $S(\pi^\pm, \sqrt{s} = 53\text{GeV})$=0.66, and $S(\pi^\circ, \sqrt{s} = 630\text{GeV})$=0.81.

In Table 1 we show the result of our calculation in the two cases where p_T (pseudo) rapidity correlations were studied , $\pi^\pm$ at ISR[14] and π° at the CERN Collider[15]. The conclusion is that no sizeable energy dependence of the pion inelasticity is observed. If p_T (pseudo) rapidity correlations are not included the inelasticity appears to decrease, as estimated in previous calculations.

If we apply our formula to charged particle distributions, assuming that the pion approximation (16) remains correct and that the parameter λ, see (21), in agreement with π data at 53 and 630 GeV, satisfies the relation

$$\lambda = \frac{0.494}{1 + 0.0033\sqrt{s}}$$

we obtain, for the charged inelasticity, K_{ch} the values of Table 2. A tendency for the decrease of K_{ch} with energy is observed, in particular when comparing separately CERN Collider[10] and Tevatron[16] data.

It is clear that, contrary to previous calculations, no strong decrease of the inelasticity in the GeV-TeV region is observed. For the π inelasticity we, in fact, observe that K_π is practically constant. However for K_{ch}, even includind p_T (pseudo) rapidity correlations, a slow decrease with energy remains there.

A few comments are, at this stage required:

i) Our calculation is less reliable for charged distributions (as kaons, nucleons and behavior hadrons are treated as pions) than for π distributions.

ii) Asymptotically one expects the ratio (neutral hadrons)/(charged hadrons) to evolve from 1/2 at low energy $(\bar{n}_{\pi^\circ} = \frac{1}{2}(\bar{n}_\pi^- + \bar{n}_{\pi^+}))$ to 1 at higher energy, as a consequence of quark counting. Working with the first flavour family, (u, d), the probability of a neutral combination, $(u\bar{u})$ and $(d\bar{d})$ is the same as the probability of a charged combination, $(u\bar{d})$ or $(d\bar{d})$. A small decrease of K_{ch} with energy is not incompatible with a constant or increasing with energy K_{total}.

iii) Electroweak interactions, in particular with the production of leptons from heavy quarkonium and neutrinos from Z°, may be responsable for an additional increase with energy of the total inelasticity.

iv) The change with energy of the p_T-(pseudo) rapidity correlations is related with the development of the "seagull" effect and has been estimated in the framework of QCD[17].

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TABLE 1

	I	II	III	IV
	$(2/\sqrt{s})(\sinh\eta)$	$(p_T)\rho_0(2/\sqrt{s})(\sinh\eta)$	S	K_i
π^- $\sqrt{s} = 53$ GeV	0.703	0.168	0.657	0.111
π^+ $\sqrt{s} = 53$ GeV	0.774	0.196	0.658	0.129
π^o $\sqrt{s} = 630$ GeV	0.251	0.137	0.811	0.111

Pion inelasticity at $\sqrt{s} = 53$ and 630 GeV. The first column, $(2/\sqrt{s}) < \sinh\eta >$, shows the violation of Feynman scaling. The second column gives the calculation of inelasticity without including $p_T - \eta$ correlations. The third column shows the values for the integral S, (18). The last column gives the inelasticity, Eq.(17).

TABLE 2

	UA5 (inelastic events) ref. 10				CDF (NSD events) ref.16	
$\sqrt{s}$ (GeV)	53	200	546	900	630	1800
$\rho(0)$	1.706	2.221	2.824	3.118	3.264	4.184
$(\sinh\eta)$	21.137	54.377	72.516	87.260	108.814	137.950
K_{ch}	0.339	0.338	0.254	0.228	0.394	0.281

$\rho(0)$, $(\sinh\eta)$ and charged particle inelasticity, from ISR to CERN Collider and TEVATRON.

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Figure Captions:

Fig.1 Comparison of the data on charged particle pseudo-rapidity distributions obtained by UA5-CERN for inelastic (i.e. NSD+SD) events with model calculation. Data are from ref. [10] and the curves are calculated from Eq.10. (a) $\sqrt{s} = 53$ GeV, (b) $\sqrt{s} = 200$ GeV, (c) $\sqrt{s} = 546$ GeV and (d) $\sqrt{s} = 900$ GeV.

Fig.2 Comparison of pseudo-rapidity distribution for inclusive production of pions with model calculation. Data are from ref.14-15 and the curves are from Eq.10. (a) $\pi^{\circ} - \sqrt{s} = 630$ GeV (UA7), (b) $\pi^{-} - \sqrt{s} = 53$ GeV and (c) $\pi^{+} - \sqrt{s} = 53$ GeV.

Fig.3 Comparison of the data on charged particle pseudo-rapidity distributions obtained by CDF-FERMILAB for non-single events with model calculation. Data are from the ref.14 and the curves are calculated from Eq.10. (a) $\sqrt{s} = 630$ GeV and (b) $\sqrt{s} = 1800$ GeV.

Fig.4 This figure shows the transverse momentum (p_t) — pseudo-rapidity (η) correlation. The data are from ref.14-15and the curves are calculated from Eq.21 (the solid line is at $\sqrt{s} = 53$ Gev with $\lambda = 0.42$ and dashed line is at $\sqrt{s} = 630$ GeV with $\lambda = 0.16$).

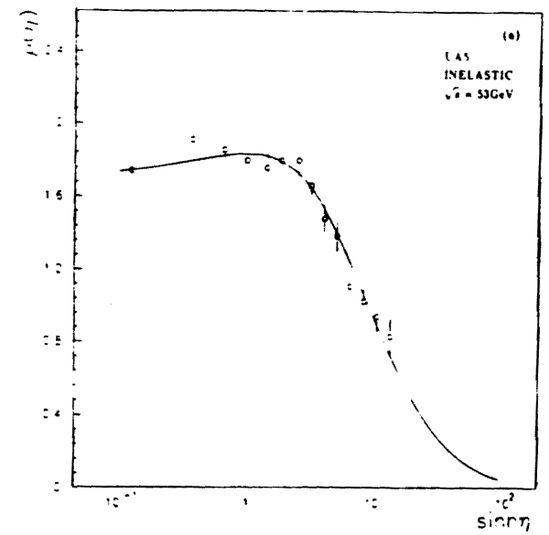


Fig.1a)

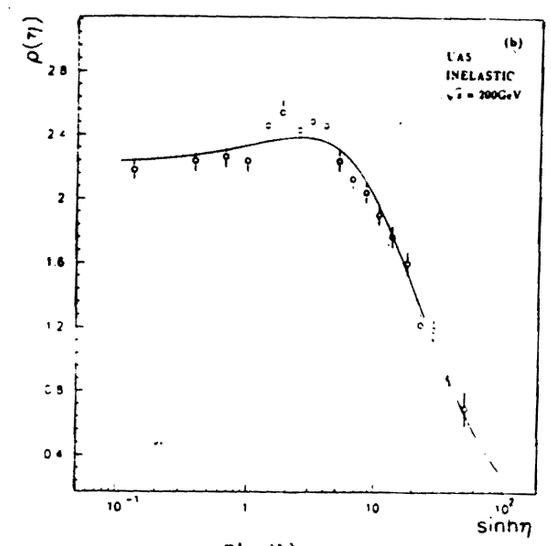


Fig.1b)

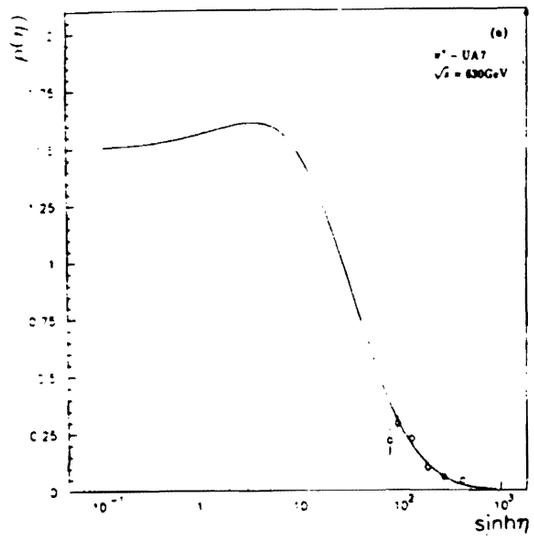


Fig.2a)

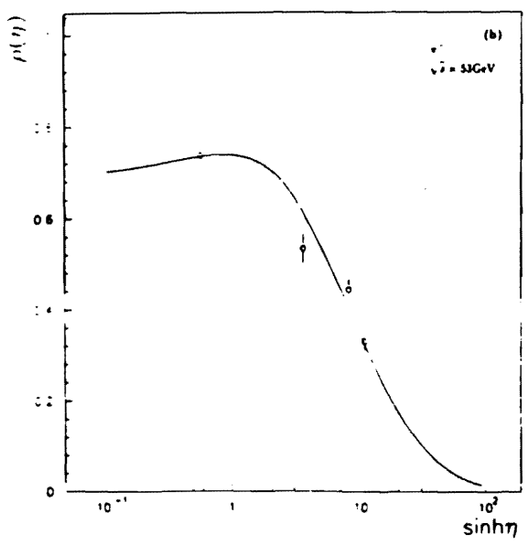


Fig.2b)

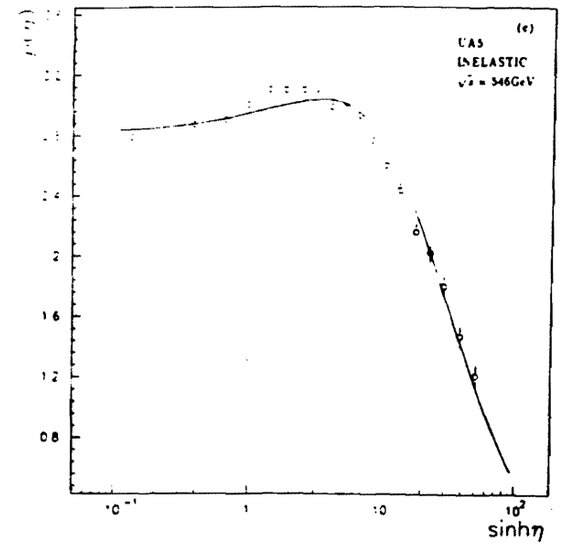
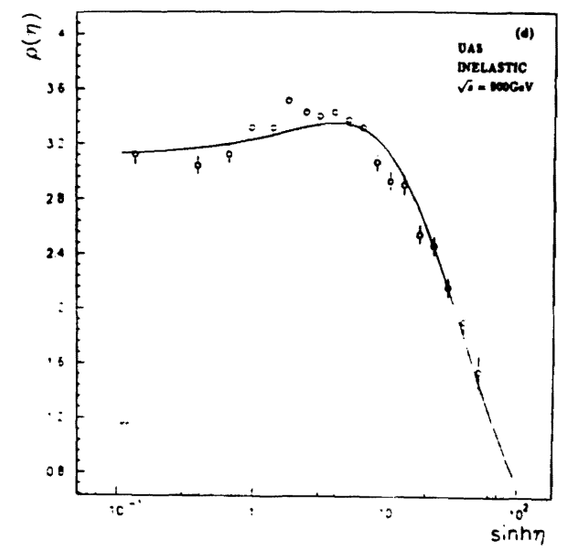


Fig.1c)



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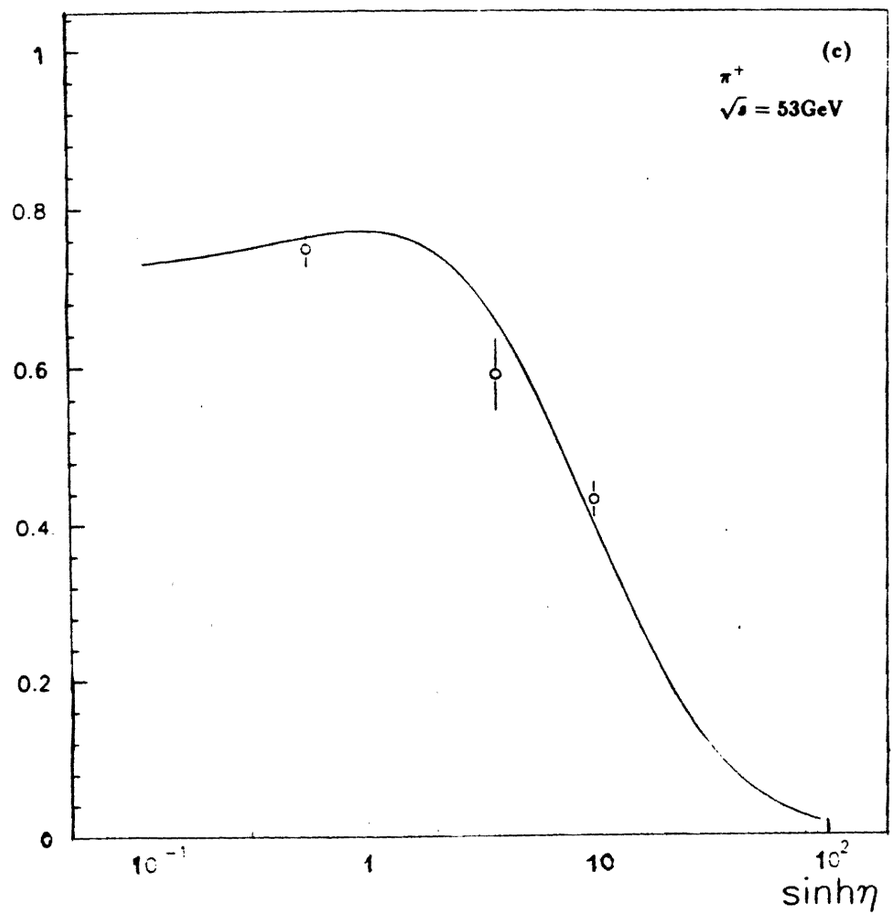

Fig.2c

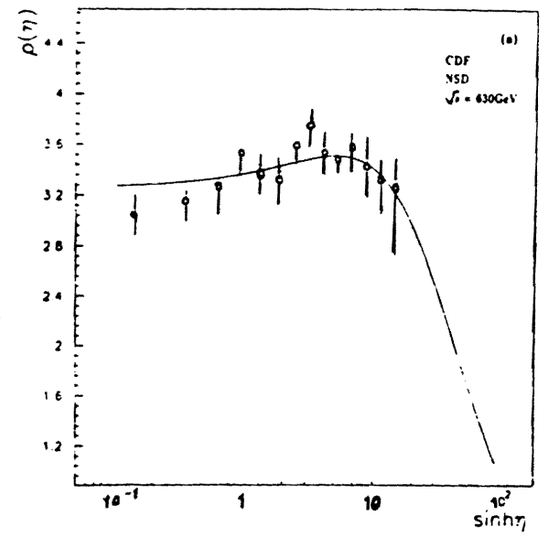

Fig.3a)

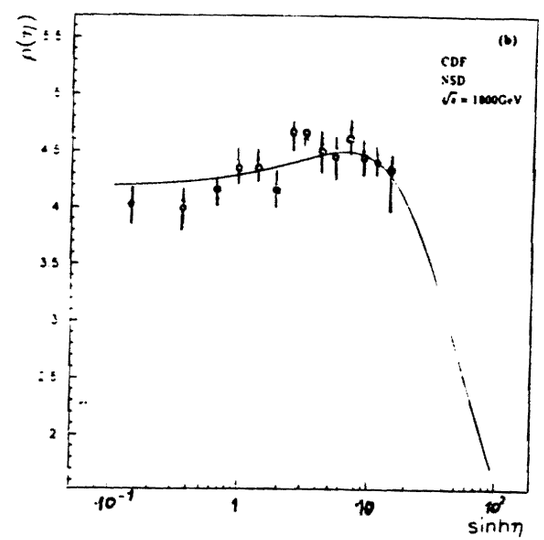

Fig.3b)

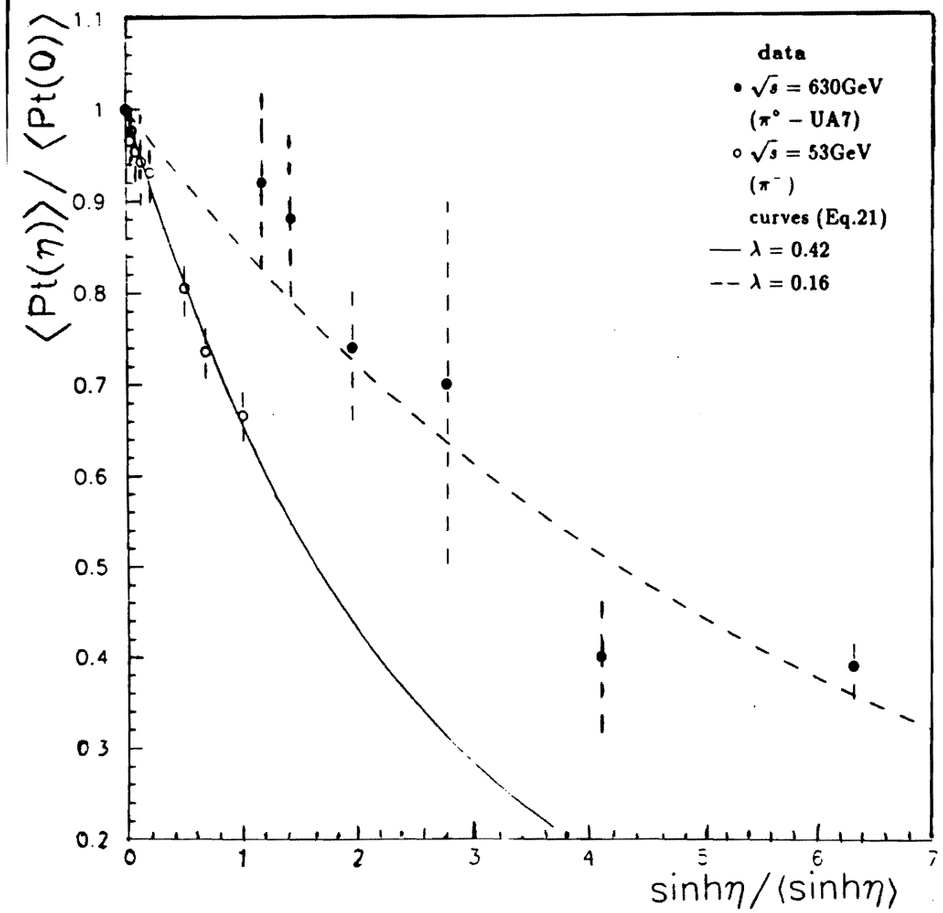
data
• √s = 630GeV
(π° – UA7)
∘ √s = 53GeV
(π⁻)
curves (Eq.21)
— λ = 0.42
-- λ = 0.16
⟨Pt(η)⟩ / ⟨Pt(0)⟩
sinhη / ⟨sinhη⟩
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
1
2
3
4
5
6
7

Fig.4