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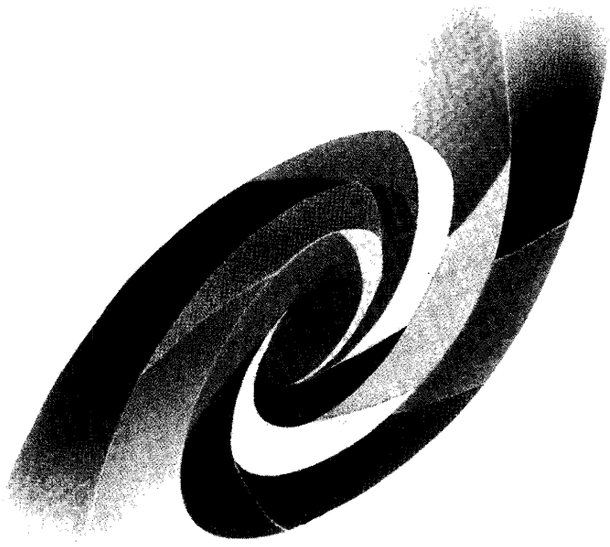

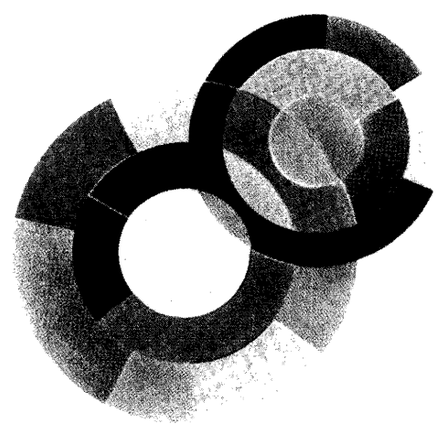

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MODELLING AND SIMULATION FOR SCATTERED GAMMA-RAY IMAGING

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ABSTRACT

Gamma-ray imaging is widely used in many fields such as medical imaging, non-destructive control and astronomy. However the image quality is considerably affected by the Compton-scatter effect. In this paper, we propose an original new imaging principle using the scattered radiation, its modelling and simulation.

In modelling the Compton diffusion [1] we have introduced the so-colled conical Radon transform (CRT), which is considered as a generalization of the classical Radon transform (integral transform defined on straight lines or plans). The invertibility of the CRT demonstrates the feasibility of reconstruction of an tri-dimensional (3D) object from a series of images indexed by the angle of scattering (or equivalently by the outgoing energy) [2].

In this work, we have simulated a medical phantom, its series of images indexed by energy (Fig. 2) and its reconstruction from this sequence of images. The latter used the a-dimensional Fourier transform and a Hankel transform for which the computation is far from being trivial. In our simulation the object under study (an organ) is a tri-dimensional cube with two nodules inside (hot and cold nodules) , the gamma source is the radiopharmaceutical ^{99m}Tc, the gamma camera is of type SPECT (Single Photon Emission Computer Tomography), the different angles of scattering θ are between 0.4^0 and 171^0 ($0.4^0 < \theta < 171^0$). The result of simulation for the series of images parameterized by θ is presented in Fig. 2.

The quality of the reconstructed object shows the relevance of the CRT in modelling the scattered radiation and the efficience of the reconstruction algorithm.

The new imaging principle leads to a great improvement of the signal-to-noise (SNR) in the images and to the possibility to perform the 3D-reconstruction without having to move the detector around the object.

KEYWORDS

Simulation, Mathematical modelling, Medical Imaging, Radon transform, Gamma-ray, Scattering, Compton scattering.

1 Nomenclature

$f(\mathbf{V})$	: object activity density function.
$\tilde{g}(\mathbf{D},\theta)$ *or* $g(\mathbf{D},t)$	: image density function.
n_e	: electron density in medium.
$p(\mathbf{D},t\|\mathbf{V})$	: Point Spread Function.
t	: $tan\theta$.
$Y(s)$	: Heaviside unit step function.
θ	: Compton scattering angle.
$\mathbf{V}$:	: a source point.
$\mathbf{M}$:	: Compton collision site.

2 Imagerie principle

Let $\mathbf{V}$ denote an object voxel of coordinates (ξ_V, η_V, ζ_V) and $f(\mathbf{V})$ be the object activity density function, defined as $f(\mathbf{V}) = f(\xi_V, \eta_V, \zeta_V)$. This is also the number of photons emitted per unit time and per unit object (or source) volume, uniformly distributed in space at site $\mathbf{V}$. Thus, we have the number of photons receved at $\mathbf{M}$ equals to :

$$\frac{1}{4\pi\, MV^2} f(\mathbf{V}) d\mathbf{V}. \tag{1}$$

If there is a Compton collision at site $\mathbf{M}$ with the electron density in medium n_e, the number of photons reaching a unit detector surface at $\mathbf{D}$ per unit time is the flux density recorded by the detector at site $\mathbf{D}$:

$$\frac{f(\mathbf{V})d\mathbf{V}}{4\pi\, MV^2}\, n_e\, d\mathbf{M}\, \frac{r_e^2}{2}\, P(\theta)\, \frac{1}{MD^2}, \tag{2}$$

where r_e is the classical radius of the electron and $P(\theta)$ the so-called Klein-Nishina probability for deflection by an angle θ. The absorption is neglected here.

Consequently, the number of photons recorded per unit time and unit detector area at site $\mathbf{D} = (\xi_D, \eta_D)$, $\tilde{g}(\mathbf{D},\theta)$, is due to all emitting point sources $\mathbf{V}$ situated on a cone with opening angle θ, axis parallel to $O\zeta$ and with apex the scattering site on the vertical line MD:

$$\tilde{g}(\mathbf{D},\theta) = \int d\xi_M d\eta_M \frac{d\zeta_M}{\zeta_M^2} \delta(\xi_D - \xi_M)\delta(\eta_D - \eta_M) n_e$$

$$\int \frac{f(\mathbf{V})d\mathbf{V}}{4\pi} \frac{\delta(Cone)}{MV^2} \frac{r_e^2}{2} P(\theta), \tag{3}$$

where $\delta(Cone)$ restricts the integration over $\mathbf{V}$ to the circular cone. If one uses the local spherical coordinates centered at $\mathbf{M}$ in Fig. 1, we have $d\mathbf{V} = r^2 dr \sin\alpha d\alpha d\phi$ then $\delta(Cone) = \frac{1}{r}\delta(\theta - \alpha)$.

We can rewrite this transformation in a more suggestive form using $t = \tan\theta$, $\tilde{K}(\theta) = K(t)$ and $\tilde{g}(\mathbf{D},\theta) = g(\mathbf{D},t)$, as :

$$g(\mathbf{D},t) = \int d\mathbf{V}\, p(\mathbf{D},t|\mathbf{V})\, f(\mathbf{V}). \tag{4}$$

where $p(\mathbf{D},t|\mathbf{V})$ stands for the kernel of the transformation, which will be called T from now on.

Physically this kernel is also called the Point Spread Function (PSF), or image of a point source at site $\mathbf{V} = (\xi, \eta, \zeta)$. The mathematical computation of the PSF kernel [4] leads to the following expression:

$$p(\mathbf{D},t|\mathbf{V}) = Y(t)t^2 K(t) \frac{Y(t(\zeta - l) - \rho)}{\rho^2 (t\zeta - \rho)^2}$$

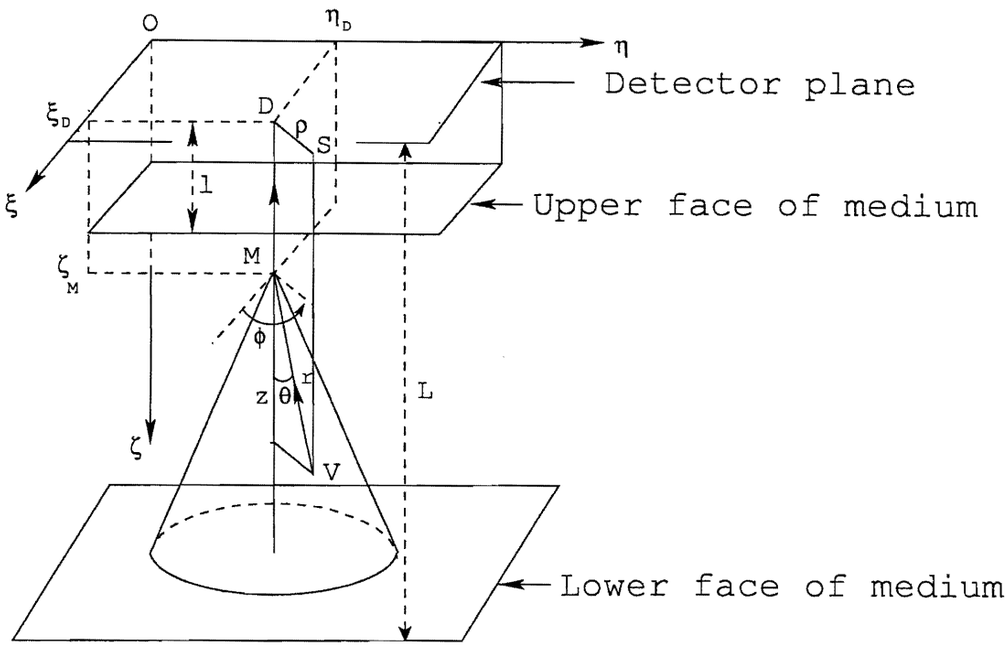

Figure 1. Coordinate system for the calculation of $\mathcal{T}$

$$+Y(-t)t^2K(t)\frac{1}{\rho^2(|t|\zeta+\rho)^2}, \tag{5}$$

valid in the whole range of t values. $Y(...)$ is the Heaviside unit step function, and

$$\tilde{K}(\theta) = \frac{n_e}{4\pi}\frac{r_e^2}{2}P(\theta)\sin\theta \tag{6}$$

We have shown that the transformation $\mathcal{T}$ is *invertible* [3]. This is the foundation of the new tree demensional object reconstruction.

$$F(u,v,w) = \frac{1}{\mathcal{J}_l^L(w)}\int dz\, e^{2\imath\pi zw}[-|z|\sqrt{u^2+v^2}]\, e^{2\imath\pi lw}$$

$$\int_0^\infty t\, dt J_1(2\pi|z|t\sqrt{u^2+v^2})\{Y(z)\frac{\partial}{\partial t}\frac{G(u,v,t)}{K(t)} + Y(-z)\frac{\partial}{\partial t}\frac{G(u,v,-t)}{K(-t)}\} \tag{7}$$

The tree-dimensional Fourier transform of f : F is expressed as a transform of the bidimensional Fourier transform of g : G, where $J_1(.)$ is the first order Bessel function and

$$\mathcal{J}_l^L(w) = \frac{e^{2\imath\pi lw}}{l} - \frac{e^{2\imath\pi Lw}}{L} + 2\imath\pi w\,[ExpIntegral(2\imath\pi Lw) - ExpIntegral(2\imath\pi lw)] \tag{8}$$

Finally the object activity density $f(V)$ may be recovered by inverse 3D Fourier transform from (7).

3 Numerical simulations

Our problem is to carry out the computation of (7), which is continuous and defined on an infinite support. Always this same problem appears and specially three times in the computation of (7).

The image pixels from our computation show the value of gamma ray on their middle. A real camera gives the average of gamma-rays on its pixels. So the particular discrete nature of our virtual camera leads to strong discontinuities in the calculation of $\frac{\partial}{\partial t}\frac{G(u,v,t)}{K(t)}$. To limit this effect, 16 points are used in the computation of one pixel. But the problem does not appear only in space domain. To determine an image for a given value of gamma-ray energy, we use in fact four different values of energy.

The second difficulty comes from the Hankel transform (with the bessel function as the kernel). There is no universal way for this calculation. We choose the fast Hankel tranform (FHT) method of A.E. Siegman [6]. This algorithm has many attractive advantages : the precision and quickness of calculation and its simplicity of carrying out. Unfortunately data have to be sampled with an exponential step, witch is inconsistant with the energy scale of the detector. To satisfy this two conditions, we need to interpolate data of camera to perform the FHT. The result shows also an exponential step and a second interpolation is required to go ahead the computation. Those stages occurs for each sampled values of u and v in (7), that means we need a great number of images from our virtual detector at various energy (about 32 times more than a real one).

The computation ends up with a Fourier transform (on z) and a filtering ($\frac{1}{\mathcal{J}_l^L(w)}$). In certain cases the filter could bring some aberrant value because of the finite dimension of data. A simple and efficient solution is to periodize data.

Each of the three stage before has its own numerical requirement mutually exclusive with the other. The essential is to found for each variables in (7) the sampling method to obtain the result having physical meaning.

4 Results

As an illustration of simulating and reconstruction of 3D objects from scattered gamma-rays, we present the numerical computations of equation (7) with an inhomogeneous cubes as example. The working conditions are the following:

- The gamma detector is a SPECT-camera. It is a discretized square of dimensions $N \times N$ length units with $N = 128$.
- The pixel size is 1 mm^2.
- The scattering medium is represented by a cube of dimensions $N \times N \times N$. To avoid boundary effects as much as possible, emission sites with non zero activity density are assumed to be localized inside a cube of size $64 \times 64 \times 64$ of same center as the larger cube with size $N = 128$.
- The electron density in biological medium is $n_e = 3.510 \times 10^{23}$ electrons/cm^3.
- The radio pharmaceutical employed is Technecium 99 with an activity density 2.210×10^{-2} Ci/cm^3.
- The acquisition time per image is about 0.1 sec.
- The 3D original object (cubes with 8 pixels per side) is placed at the center of the cubical scattering medium. In Fig. 2 is represented the original object: an inhomogeneous cube with two "hot" nodules and one "cold" nodule.
- The distance from camera to the upper face of the scattering medium cube is $l = 80$ units.

- A series of images corresponding to the object at various scattering angles θ ($0.4^0 < \theta < 171^0$) have been simulated as example. They are presented in figures 2 the object.

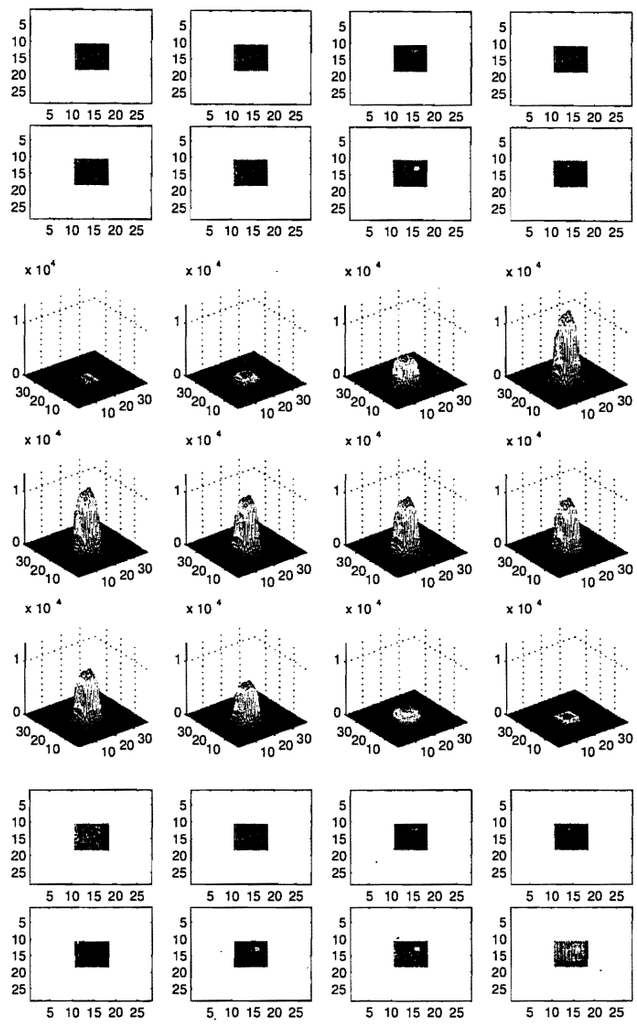

Figure 2. The original object (a cube with two nodules inside) is presented (plane by plane) in the first two lines, the series of images labelled by θ ($0.4^0 < \theta < 171^0$) is on three next lines and the reconstructed objet on the last two lines.

The reconstructed object is in Fig. 2. In the absence of noise, one can observe a good agreement with the original object, in particular the small structures (nodules) are well recovered. The root mean square error (RMSE) is for the reconstructed inhomogeneous object equal to 13%. Thus we observe a good performance of the Compound Conical Radon Transformation for modelling the new imaging process based on scattered radiation.

Since our main objective is to show how to exploit advantageously Compton scattered radiation to generate a new imaging principle we present only results on image formation as well as object reconstruction from scattered rays.

In real situations, of course, one must take into account other factors such as absorption by the medium, Poisson emission noise and the imperfections of the detector (collimator and measuring electronics).

Concerning emission noise there are well known methods to deal with it such as the method of Maximum Likelihood or the methods of wavelets. They may be used for "denoising" the measured data beforehand or jointly with the inversion process.

As for the imperfections of the detector, the standard way for treating this problem is to make use of a response function of the Gaussian type operating with spatial coordinates as well as with energy coordinate. These issues are discussed in detail in reference [5].

5 Conclusion

In this work the feasibility of 3D object reconstruction using Compton-scattered rays in gamma emission imaging is derived from a new linear integral transform directly from a Compton-scattering analysis of the image formation [3]. The proposed new imaging principle takes advantage of properties of scattered rays instead of rejecting them as usually done in most actual scatter correction methods. It improves the signal-to-noise ratio, and consequently the contrast. Moreover on one hand, in this procedure the angle of scattering appears as a free parameter and serves to label the collected data, on the other hand the multi-views of the object are obtained without the need of moving the detector. This possibility is particularly interesting in applications where the number of measurements is very limited, e.g. in non-destructive evaluation. Several perspectives of the proposed approach are possible: for example extension to transmission imaging and to prospective higher order Compton scattering imaging processes.

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