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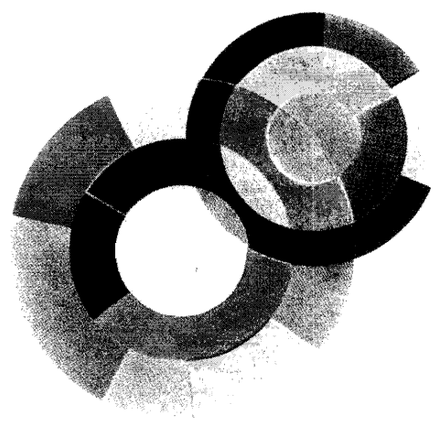

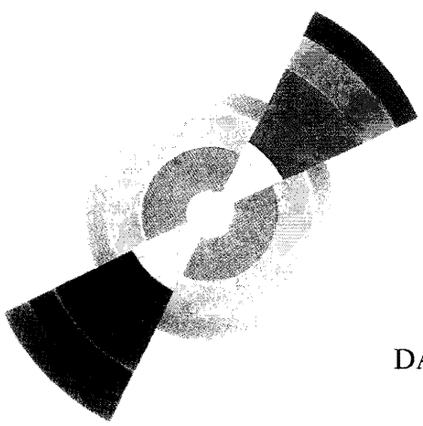

DAPNIA-04-89 04/2004

Wavelets and curvelets in denoising and pattern detection tasks crucial for homeland security.

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Talk given at the SPIE Defense of Security (Conference on Independent Component Analyses, Wavelets, Unsupervised Smart Sensors Neural Networks II), Orlando (U.S.A.), February 12-16, 2004

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DAPNIA-04-89

Couverture : Sylvie Cabrit et Marc Sauvage

Wavelets and curvelets in denoising and pattern detection tasks crucial for homeland security

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ABSTRACT

The design and successful fielding of sensors and detectors vital for homeland security can benefit greatly by the use of advanced signal and image processing techniques. The intent is to extract as much reliable information as possible despite noisy and hostile environments where the signals and images are gathered. In addition, the ability to perform fast analysis and response necessitate significant compression of the raw data so that they may be efficiently transmitted, remotely accumulated from different sources, and processed. Proper decompositions into compact representations allow fast pattern detection and pattern matching in real time, in situ or otherwise. Wavelets for signals and curvelets for images or hyperspectral data promise to be of paramount utility in the implementation of these goals. Together with statistical modeling and iterative thresholding techniques, wavelets, curvelets and multiresolution analysis can alleviate the severity of the requirements which today's hardware designs can not meet in order to measure trace levels of toxins and hazardous substances. Photonic or electro-optic sensor and detector designs of the future, for example, must take into account the end game strategies made available by advanced signal and image processing techniques. The promise is the successful operation at lower signal to noise ratios, with less data mass and with deeper statistical inferences made possible than with boxcar or running averaging techniques (low pass filtering) much too commonly used to deal with noisy data at present. SPREE diagrams (spectroscopic peak reconstruction error estimation) are introduced in this paper to facilitate the decision of which wavelet filter and which denoising scheme to use with a given noisy data set.

Keywords: wavelets, curvelets, multiresolution analysis, noise modeling, denoising, pattern detection, undecimated iterative wavelet transforms, SPREE diagrams

1. INTRODUCTION

For the better part of the past fifteen years, multiresolution analysis, which is the simultaneous study of signals on successively finer scales and on a sequence of time intervals, found one of its most fruitful realizations in discrete wavelet transforms. Following the pioneering work of Mallat, Meyer, Daubeschies, Coifman, Wickerhauser, Vetterli, Donoho and many others,[1–8] a veritable explosion of publications and applications came to the fore extending all the way from the currently used JPEG 2000 standard in image coding and compression, to the next generation video compression standard, MPEG-4, to the FBI's finger print file storage and compression, to biomedical signal and image processing, all the way to the efficient mathematical characterization of fractals, fractional Brownian motion, and Fourier integral operators and their microlocal asymptotics. An early popular exposition with references to the early history of the field can be found in the IEEE Signal Processing Magazine.[9] An interesting and peculiar feature of this field is that rigorous mathematics adopted from the fields of nonlinear approximation theory and harmonic analysis has flourished, drawn inspiration from and suggested improvements in very applied fields such as signal or image processing and digital filter design. In fact, it is possible to imagine and invent new filters, new functions, new algorithms and new phase space tiling techniques[10–16] starting from very many seemingly different directions and ending up with remarkably similar results. Neither a physicist's intuitive methods, nor a mathematician's rigorous approach, nor a practical engineer's tool bag alone seem to give anyone the edge. In fact, much has been repeatedly reinvented even in this relatively short span of time where

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the concepts of wavelets, vaguelettes, Coiflets, Duablets, ridgelets, edgelets, bandlets, contourlets, beamlets, wedgelets, chirplets, curvelets and all the rest were proposed, justified, spun, used and respun in very many directions such as statistical estimation,[17–20] denoising,[21–23] pattern detection[24–27] and data compression,[28–30] to name a few.

Some biases do seem to persist, however, among the various constituents of this field which are mutually incompatible. For instance, it is maintained, usually by those who pursue asymptotic results in the large number of wavelet coefficients kept limit, that one wavelet family is as good as another. Or, it is held that orthogonal decompositions can do as good a job as anything else so why bother with undecimated (translationaly invariant) or overcomplete, redundant representations. It is not difficult to show that these are not correct views. One size does not fit all, and various requirements depending on the goals of the particular exercise (denoising, data compression, pattern detection, etc.) are often mutually incompatible and thus best served by different filters and different procedures or algorithms, altogether. A counter position to this has been to advocate that ever expanding libraries of filters and functions should be amassed with which to obtain multiple redundant decompositions and, adopting a criterion such as best basis or matching pursuit or basis pursuit or minimum entropy or total variation diminishing,[5,7] to isolate and choose the optimum (hopefully small) set of coefficients from within the elements of this library of libraries in order to adapt ideally to a given signal. This may well be prone to inefficiencies in implementation or stalled convergence or be unstable and have unpredictable behavior. The truth may lie somewhere in between these two extreme positions and require solid statistical theoretical backing[31–37] before it filters down and becomes convincing to the enthusiastic yet unsuspecting users of the bewildering variety of tools and methods that are in circulation today.

The events since 9-11 have caused a sea change in the efforts and concentration that are being devoted to the detection, interception and neutralization of harmful substances in public settings. Whether it is the needs of the military in hot zones or large crowds of civilians potentially targeted by terrorists, the objective of *quickly* identifying chemical, biological or radiological compounds in *trace* amounts has become of vital importance.[38–42] The needle in a haystack quality of these tasks encourages the pursuit of technologies that can enhance the signal to noise ratio of a given measurement, can detect patterns with high probability of success with controlled incidences of false positive readings,[36,37] can be a platform for efficient multiple sensor information fusion, and where automatic target recognition and rapid data transmission can be implemented. Sensors and detectors that have such advanced front end dedicated signal processing elements are necessary for the success of the massive endeavors being undertaken to protect ports, large urban congregation centers, sensitive public installations and the military. Especially useful in this regard are remote sensing techniques which rely on electromagnetic radiation (from gamma rays to radio waves). Coherent laser based techniques seem very promising in this arena as well.[43–45] The need then is to denoise typical spectra that arise from measurements such as the absorption spectra of air borne chemicals in the presence of much larger concentration levels of water vapor, nitrogen, methane and other common gases. Denoising spectra that arise from analytical chemistry or astrophysical data using wavelets has been tried in the past.[46,47] The systematic study of denoising schemes involving iterative algorithms which preserve all large peaks, the seamless extension to GC (gas chromatography) and GCMS or LCMS (gas or liquid chromatography mass spectrometry) data[48–50] and the incorporation of a set of statistical methods and mulitresolution tools deploying wavelets and curvelets[51–54] which can treat signals, images and hyperspectral data for specific *sensor adapted performance optimization* is an endeavor which we hope will bear fruit.

In this paper we show the relative merits of denoising synthetic infrared spectroscopic data made up of Toluene, methyl ethyl keytone (MEK), isoprepylacetate (IPA), formaldehyde (H2CO) and water (H2O), in the wavelength range 2910 nm to 3050nm, to which Gaussian noise with a mean of zero and standard deviations corresponding to signal to noise ratios of 9 and 6. We chose these σ values because they correspond to the rule $3\sigma = N\% \times A_{peak}$, where $A_{peak} = 0.4$ below and N = 33 and 50. We compared four different wavelet filters (Haar, Daubechies 4,[2,7,8] Antonini 7/9 biorthogonal[28] and the translationally invariant or the undecimated, highly redundant version of same (see the article by Donoho and Coifman[17] for details on cycle spinning and the à trous algorithm[8,23] which is a fast implementation of the same idea). We also compared the effects of different noise estimates assuming that the signal and noise are separated at the 3σ to the 5σ level in wavelet coefficient space. When we added the Van Cittert iteration scheme,[8] in order to preserve the energy in the largest features, we found very good fidelity in the reconstruction of the spectroscopic peak heights.

We will also show the relative merits of these same biorthogonal wavelet filters and iterative denoising schemes in 2D and compare their performance with that of curvelets with proper noise modeling (comparing 3σ to 5σ) including two cases where intentional incorrect assumptions are made concerning the *type* of noise. This image is *not* synthetic, however, and the noise is not Gaussian, nor of any other well known variety. It comes from x-ray imaging on film of laser driven cylindrical hydrodynamical mixing experiments conducted by Los Alamos.[55,56] This particular image was chosen to compare denoising techniques since it has very many edges in various stages of deformation and merging embedded in noisy inner and outer disks. It is the determination of the actual sizes and shapes of these "petals" that is of vital interest in order to assess the hydrodynamic mixing processes.

2. DENOISING SPECTROSCOPIC DATA

We can demonstrate the relative merits and pitfalls of various wavelet filters and hard thresholding as a means to denoise a signal by looking at theoretical or clean spectra of a number of hazardous chemicals to which significant levels (33% and 50%) of noise are added. This is shown in Fig. 1 where the clean and noisy signals are plotted as well as the denoising that is achieved using the undecimated wavelet transform using the Antonini 7/9 biorthogonal filter with a 4σ noise level estimator and a Van Cittert largest peak preserving iterative scheme.[8] This is necessary when using redundant transforms which would not conserve energy otherwise. Figure 2 shows the same with the decimated version of the previous figure as well as by the use of the orthogonal transforms Haar and Daub4. Clearly, the undecimated case is superior. This is explored further by showing 9 successive spectral band decompositions in Figures 3 and 4. These figures show blow ups of our best denoised vs clean signal plots for 33% and 50% Gaussian additive white noise. Almost all the peaks are very well reproduced with some small ones missed and mistaken for noise. To better gauge the performance of various filters and compare various options, we propose a new diagnostic for spectroscopic data called Spectroscopic Peak Reconstruction Error Evaluation (SPREE) diagrams, which are plotted in the next two figures. SPREE diagrams allow one to ascertain quickly to what extent peak heights in a spiky signal (IR, UV, X ray, GC, GCMS, etc.) are faithfully reproduced by some denoising scheme, how many false positives are generated and how many false negatives. It is a way to compare two different reconstructions, or in the case of synthetic data, the original clean signal to reconstructions of its noisy versions. The advantage of SPREE is that it shows just how well peaks are being reproduced (by plotting the relative amplitude of the reconstruction of the original) without any regard to location in wavelength. Instead the issue of reconstruction fidelity is mapped onto the question of proximity to the 45 degree line in a SPREE diagram. Also, false positives of a certain size are countable on the right edge of the plot while false negatives, again distinguished by the sizes of peaks missed, are displayed on the top of the plot. The former correspond to zeros in the signal (plotted from 1 to 0) and the latter correspond to zeros in the reconstructed signal (also plotted from 1 to 0). The descending order in the axes makes sure that the false detections and misses are not plotted on the x and y axes but on the opposite edges of the plots for clarity. Clearly, the analysis of SPREE diagrams can be automated and criteria imposed such as the adoption of the most successful candidate in a search through noise modeling algorithms (which have adjustable parameters) and different filter families.

3. DENOISING IMAGES WITH PRONOUNCED CURVED EDGES

The cylmix x ray image was denoised using decimated and undecimated versions of the 7/9 filter with iterative hard thresholding with two ways of estimating the noise level as being unspecified but stationary. These can be seen in the first two rows of Fig. 7. The next row contains curvelet analysis with the proper model for the noise while the last row shows results from intentional misguided assumptions for the noise to show just how sensitive the mechanism of denoising is to having a good noise model. This simply allows us to restate that the proper noise model and good filters can do very impressive denoising together as the third row in Fig. 7 shows clearly. The iterative technique which preserves the large peaks in the image is necessary for all redundant representations, or else overall energy conservation will be lost.

Our next step will be to incorporate stronger false positive detection rejection methods by incorporating adaptive thresholding on various levels of the multiresolution analysis.[34–37] These false detection rate (FDR) reduction techniques can further enhance the rather impressive performance of 7/9 Antonini undecimated iterative multiresolution based denoising we deployed in this paper on a noisy spectroscopic signal and an image of

hydrodynamic mixing. We will also look at more systematic scans of synthetic as well as laboratory data and marry these tools with supervised machine learning algorithms such as neural networks.[26,57]

ACKNOWLEDGMENTS

We thank Matt Balkey of LANL for providing the cylindrical mix x-ray film image in order to allow us to test our denoising tools. We also thank David Donoho, Emmanuel Candes and Bruno Torrésani for insightful comments, suggestions and encouragment. This work was partially supported by a DOE SBIR Phase II grant DE-FG03-01ER83294 as well as by Sandia National Laboratories.

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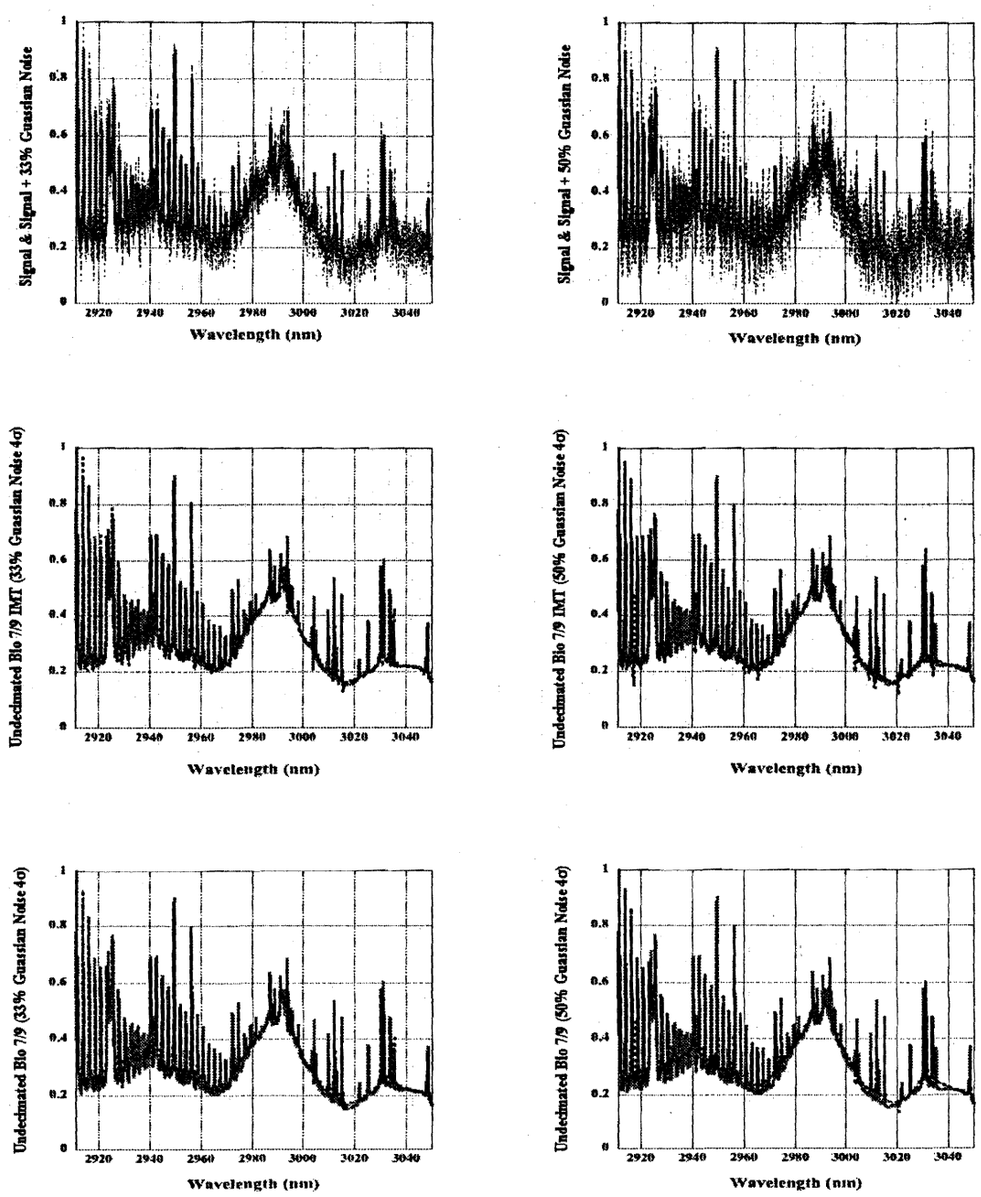

Figure 1. The spectroscopic signal and the signal plus 33% and 50% white Gaussian noise are shown in the first row. The second row shows our best denoising results using undecimated wavelet transforms implemented with the à trous algorithm and a Van Cittert large peak preserving iterative scheme and 4σ hard threshold. The third row shows denoising without the iterative scheme.

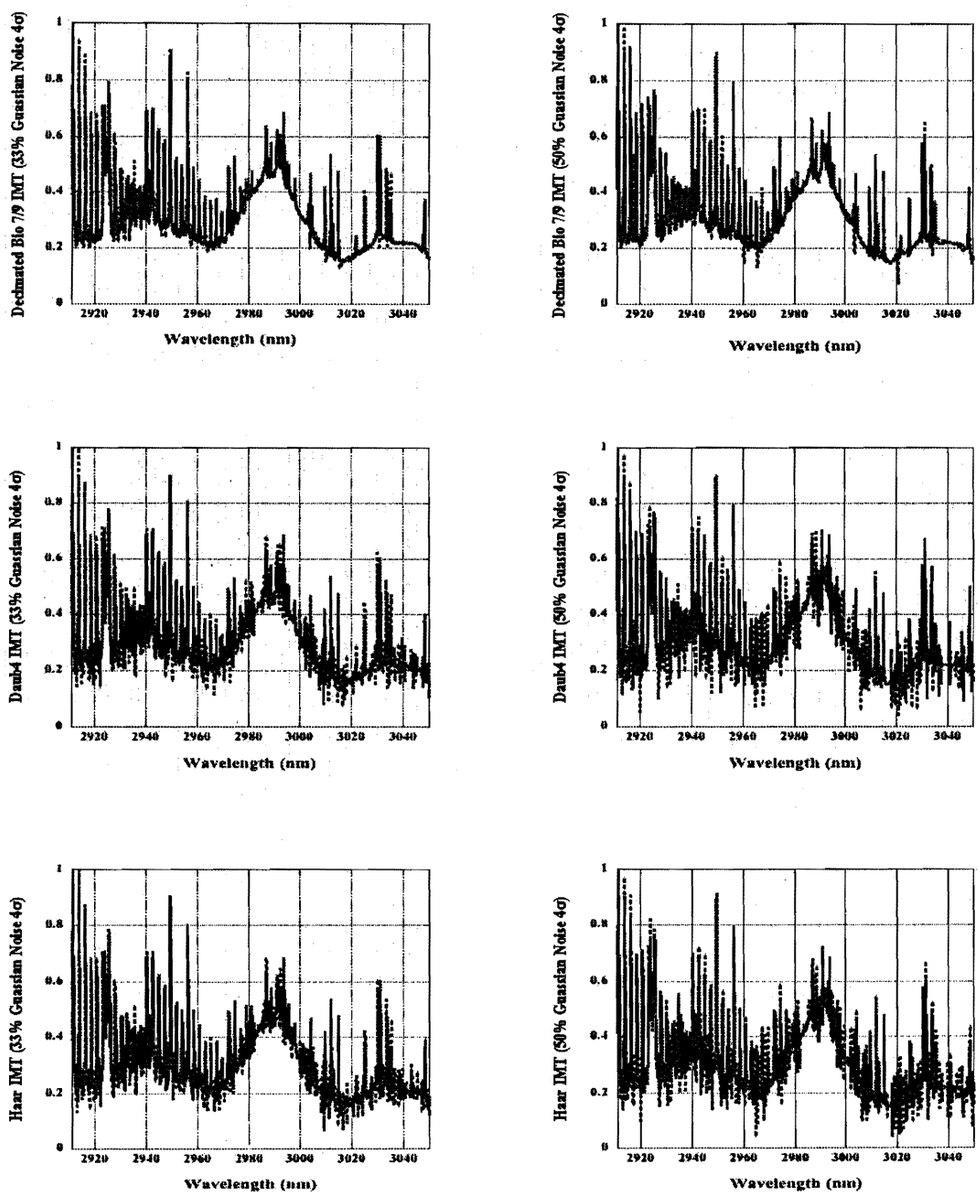

Figure 2. Three filters far less potent than the method adopted in the previous figure give the following results. Haar, Daub4 and the decimated Antonini 7/9 biorthogonal systems are used to denoise the 33% and the 50% noise added data used previously. The results are full of false positives and false negatives almost everywhere. These approaches are not recommended.

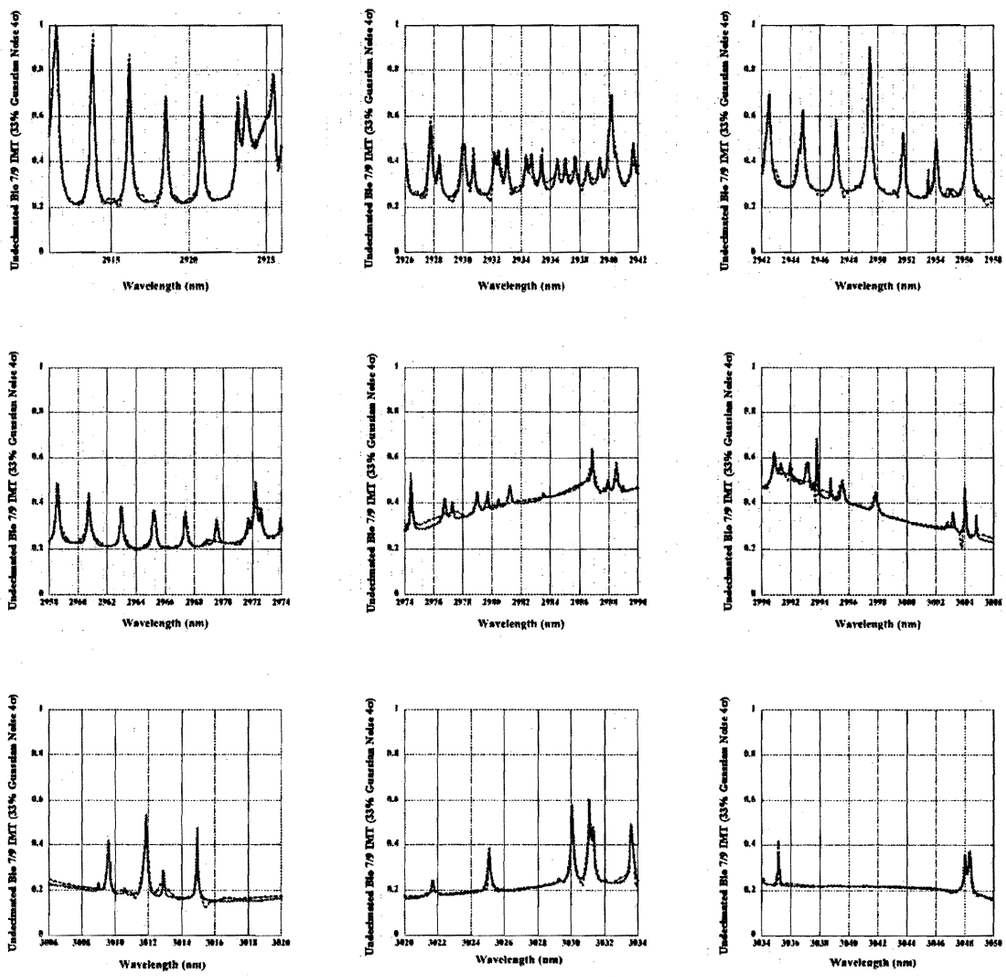

Figure 3. The performance of the undecimated Antonini wavelet system with Van Cittert iterative denoising on the spectroscopic data plus 33% Gaussian white noise is shown in 9 successive wavelength partitions. The (extremely rare) false positives, (small and annoying) false negatives and detected peak height under and over shoots, all very mild, are apparent.

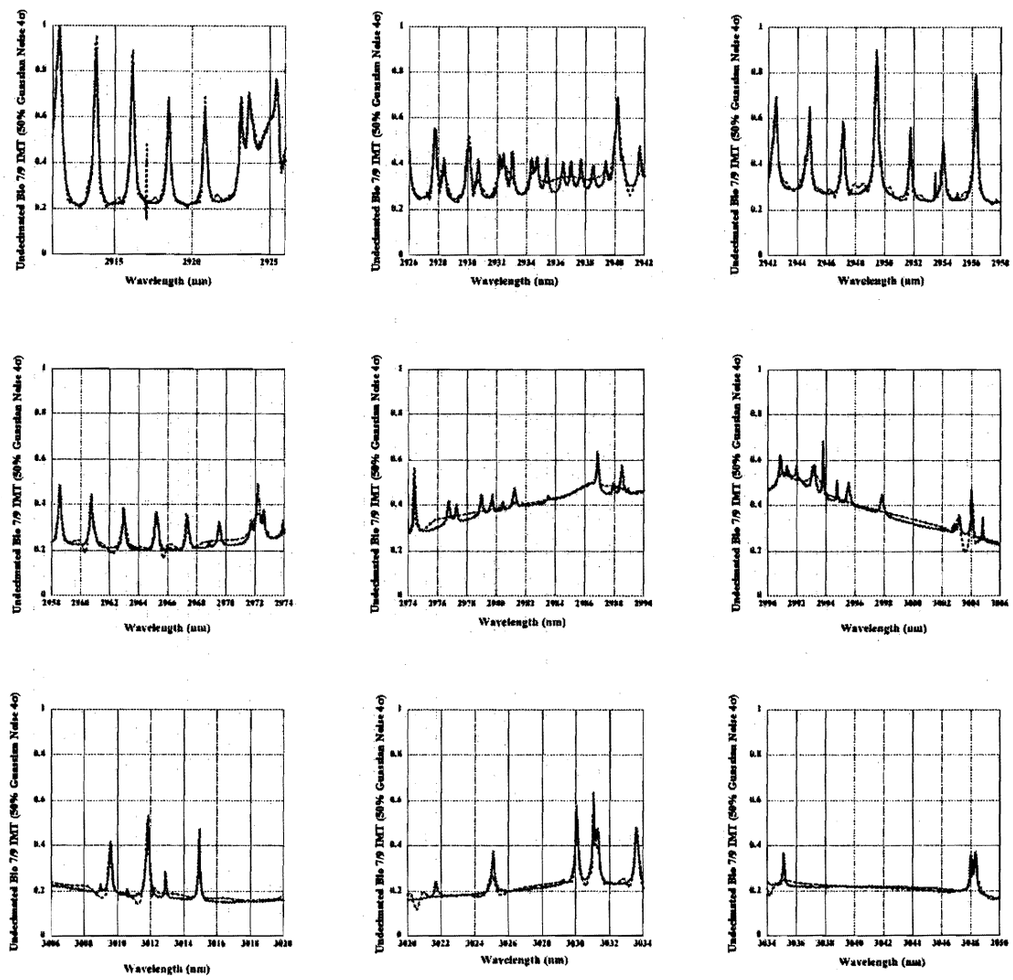

Figure 4. The performance of the undecimated Antonini wavelet system with Van Cittert iterative denoising on the spectroscopic data plus 50% Gaussian white noise is shown in 9 successive wavelength partitions. The (rare) false positives, (numerous) false negatives and detected peak height (slight) under and over shoots are apparent.

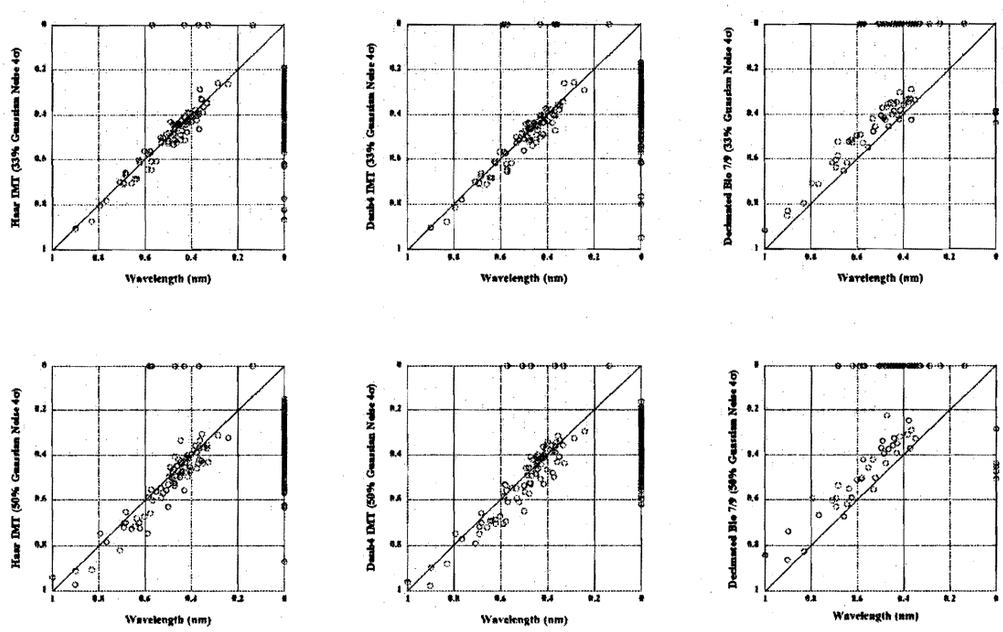

Figure 5. The degree to which the spectroscopic peaks are reproduced by Haar, Daub4 or decimated Antonini biorthogonal 7/9 wavelet filters with a 4σ noise threshold is captured by these SPREE diagrams. False positives are vertically against the right edge while false negatives are horizontal along the top edge of the figure. The first row shows denoising using 3 filters with 33% noise added to the signal. The second row shows SPREE plots for additive 50% noise data. Perfect reconstruction would have placed all the points along the 45^0 diagonal with no clustering of points along the top and right edges of the plot area.

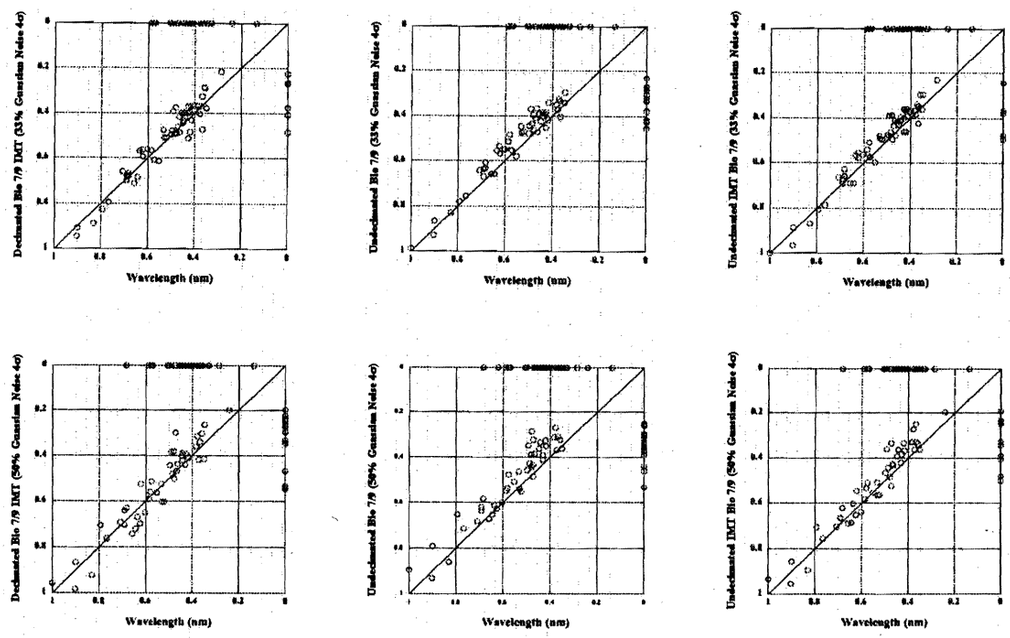

Figure 6. The degree to which the spectroscopic peaks are reproduced by decimated and then undecimated biorthogonal filtering with or without Van Cittert iteration to preserve significant peak heights is shown in these SPREE diagrams for 33% and 50% Gaussian white additive noise cases. Our best results were achieved by the use of the iterative scheme and undecimated wavelets (last column) where there are very few false positives, and the adherence to the 45^0 fidelity diagonal is remarkable. The false negatives are largely at small values which are not a detriment to the usefulness of the most effective denoising technique proposed in this paper.

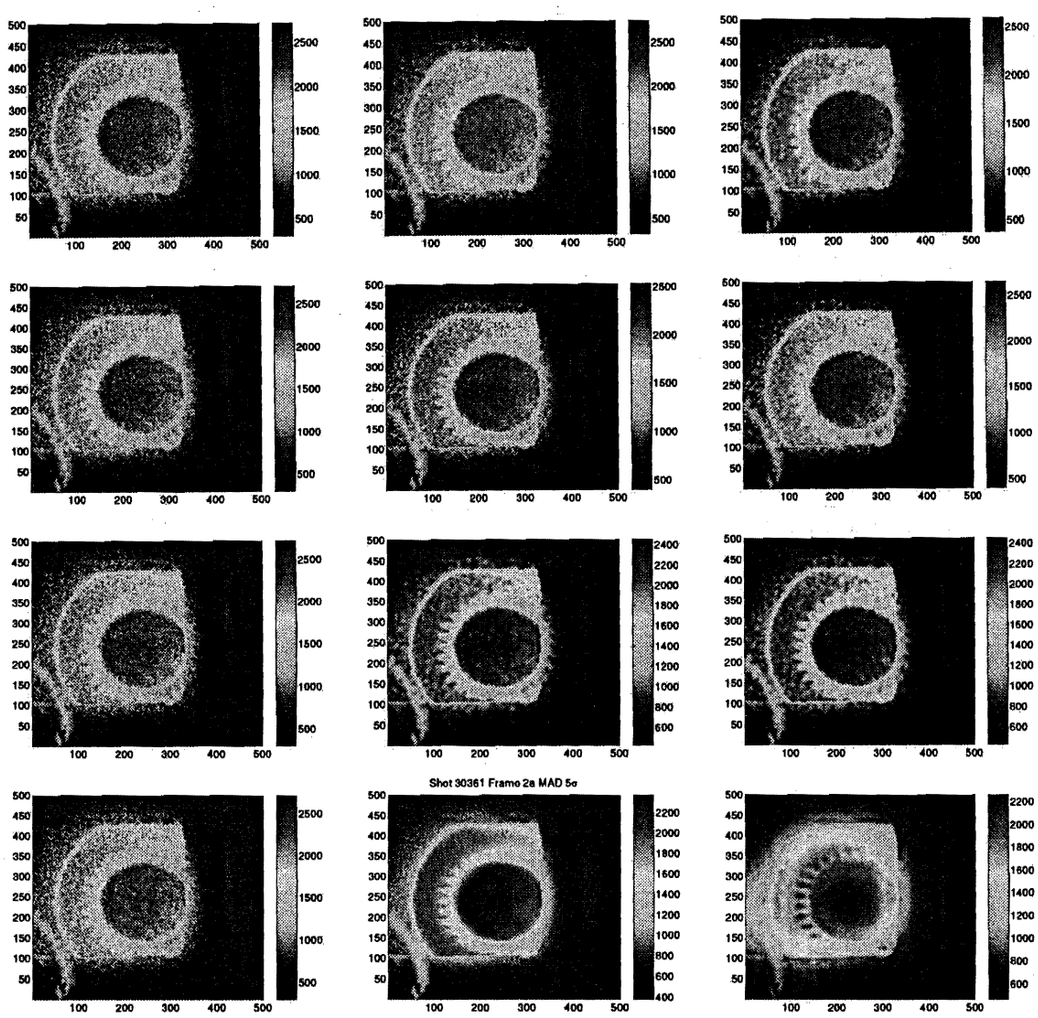

Figure 7. Denoising a cylindrical mix x-ray image from LANL. Decimated (first row) and undecimated (second row) Antonini 7/9 biorthogonal wavelets as well as curvelets (third and fourth rows) are used. The noise is estimated starting at 3σ and 5σ respectively in the second and third columns of the first three rows and assumed to be of an unspecified stationary variety. In the last row MAD (median absolute deviation) and white Gaussian noise models (respectively) are used to show the effects of wrong noise model adoption with an otherwise good choice of filter. The first column contains the raw noisy image (four times). The hard thresholding has been done with an iterative approach which attempts to preserve the energy in the image for the redundant transforms (in the second, third and fourth rows). Our goal here is to identify the extent to which the fingers or petals are distorted, merging and deforming in a series of time lapse images of which this is one. Curvelets allow this assessment far better than all wavelet based denoising we have tried. But with a wrong noise model (row four), even hard thresholding with curvelets produces artifacts and distortions which are not desirable.