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HIGH ENERGY QCD AND SMALL-x PHYSICS*

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HIGH ENERGY QCD AND SMALL-x PHYSICS

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Some aspects of high energy QCD and small-x physics are reviewed. The growth of νW_2 at small values of x is discussed in the context of current theories of high energy hard scattering. The relationship between higher twist contributions and the divergence of the QCD perturbation series is reviewed along with recent work on these topics. Possible 1/Q corrections are considered. Recent progress in large N_c QCD and its relationship to Skyrme models and to spin-dependent lepton-nucleon scattering is briefly summarized. Recent calculations of the energy loss of high energy quarks in a hot plasma are reviewed.

1 Small-x Physics

Small-x physics, including diffractive scattering, has become a topic of vigorous experimental activity, mainly due to recent data coming from HERA. Details of the data have been thoroughly discussed by other speakers at this conference so I shall limit discussion of data to a few highlights after which the theoretical issues concerned with small-x physics will be considered followed by a comparison of theory and experiment.

1.1 Highlights of the Data

The most spectacular aspect of the recent HERA data is the strong rise of νW_2 with decreasing values of x.[1,2] New data indicates that this rise is still quite prominent even at values of Q^2 as low as $1.5 - 2GeV$.[2] More generally, HERA has reached values of x small enough so that the total number of quarks and antiquarks in the proton, per unit rapidity, is 5-6 while the total number of gluons may be as large as 20-30, again per unit rapidity. Such large numbers of partons in the proton naturally raise questions as to where they came from and whether one is approaching a regime where unitarity and/or parton saturation effects may be important.

Diffractive events are now becoming commonplace at HERA and detailed analyses are beginning[3,4]. In some cases diffractive events at HERA and Fermilab can now be directly compared.

Analyses of final states are under way. Perhaps the most surprising result so far is the similarity between photo-absorption events and deep inelastic events[5]. It may be that one is going to have to work very hard to use final state characteristics as a discriminator between different pictures of a deep inelastic event.

1.2. *Theoretical Issues*

In order to describe the theoretical issues involved, let me in turn describe the soft pomeron, the Dokshitzer, Gribov, Lipatov, Alatarelli, Parisi (DGLAP)[6–8] equation and the Balitsky, Fadin, Kuraev, Lipatov (BFKL)[9–10] equation.

1.2.1 *The Soft Pomeron*[11,12]

The soft pomeron is the j-plane singularity which describes forward and near forward elastic and diffractive hadron-hadron scattering as well as total hadronic cross sections. The soft pomeron describes processes at high energies where there is no hard scattering or large momentum transfer. The trajectory of the soft pomeron is approximtely $\alpha_{SP} = 1.08 + 0.25t$. There is a rather elaborate and successful phenomenology of high energy soft reactions where this pomeron plays the dominant role. Unfortuantely, it has proved very difficult to relate this phenomenology, and the soft pomeron, to the fundamental theory of the strong interactions, QCD. Indeed, so far, it has not even been possible to give an algorithm for calculating α_{SP} in lattice gauge theory because of the intrinsically Minkowski nature of high energy on-shell hadron-hadron scattering. Until more progress is made in formulating high energy scattering in terms of nonperturbative QCD it is going to be difficult to gain a deeper understanding of the soft pomeron and the physics it describes. One promising direction where progress could be made is to use discrete light-cone QCD to study the soft pomeron. This should be possible when discrete light-cone QCD reaches a more mature stage of development.

1.2.2 *DGLAP Evolution*

DGLAP evolution, or the DGLAP equation, is simply the renormalization group equation used in QCD to describe parton distributions in hadrons. DGLAP evo-

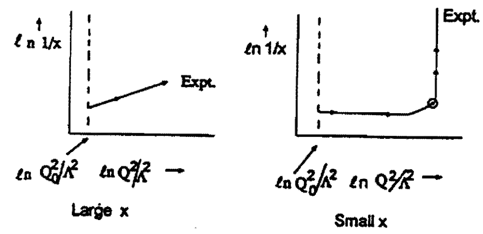

Fig.1

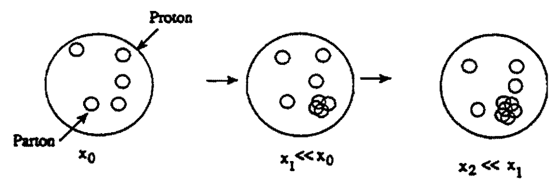

Fig.2

lution has been well tested at lower energies in deep inelastic lepton-nucleon scattering and in hard processes in QCD. The new element present in the HERA data is the very small values of the Bjorken x-variable. The situation is illustrated in Fig.1. In the large x part of the figure a typical path of evolution going from the initial distribution at Q_0^2 to the final experimental value is shown. When the measured x-values is not too large the slope of the evolution curve, in the $\ell n Q^2/\Lambda^2 - \ell n 1/x$ plane, is never large and in such a case a second order DGLAP formalism is quite adequate for an accurate description of parton evolution.

In the low-x situation illustrated in the second part of the figure there are very steep regions of evolution in the $\ell n Q^2/\Lambda^2 - \ell n 1/x$ plane. In order to adequately describe such steep regions of evolution, it may be necessary to go beyond a 2^{nd} order DGLAP formalism. (If arbitrarily high orders in the anomalous dimensions and coefficient functions were known the DGLAP equation could always be used to describe deep inelastic scattering no matter how small the x-values.) There is at this moment, a vigorous activity trying to determine whether or not terms beyond 2^{nd} order are necessary to describe deep inelastic scattering[13-16].

1.2.3 *TheBFKLPomeron*

The BFKL pomeron, or the BFKL equation, is appropriate for describing high energy scattering in a one-scale situation where that scale is large compared to the QCD Λ-parameter. For example, if one had beams of heavy quarkonium particles, with the onium radius much less than $1/\Lambda$, then high energy onium-onium scattering would naturally be given in terms of the BFKL pomeron. The BFKL equation, leading to the BFKL pomeron, naturally describes those parts of the evolution curve in the low-x part of Fig.1 which are steep. Thus, the BFKL equation describes part of the small-x data at HERA, and it is one of the main goals of the next few years to find ways of separating out these parts of the data which would then be described by the BFKL pomeron. In the remainder of this section, I shall describe properties of the BFKL pomeron and try to explain why measuring, and further understanding, this type of high energy scattering is one of the most exciting, and challenging, problems in high energy physics.

What is the BFKL pomeron? In terms of the partonic wavefunction of the proton the BFKL pomeron is a "hot spot" of partons (mainly gluons) occupying a small spatial region in the proton. The BFKL evolution leading to such a configuration in the wavefunction is illustrated in Fig.2 where the transverse spatial distribution of small-x partons of transverse dimension $\Delta x_\perp \approx 1/Q$ is shown. The BFKL equation describes the average growth of the gluon number density in a hot spot.[17]

The growth of the gluon density due to BFKL evolution is, roughly,

$$xG(x, Q^2) \sim \alpha e^{(\alpha_P - 1)\ell n 1/x} \tag{1}$$

with

$$\alpha_P - 1 = \frac{12\alpha(Q)\ell n 2}{\pi}. \tag{2}$$

When the number of gluons in a hot spot becomes as large as $1/\alpha$ the field strengths in the hot spot reach values $Q^2 F^i_{\mu\nu} \sim 1/g$, the parton picture breaks down, and one arrives at a new high field strength regime of QCD. From (1), one can expect this regime to be reached when

$$\ell n 1/x \sim \frac{1}{(\alpha_P - 1)} \ell n 1/\alpha^2. \qquad (3)$$

Long before such high energies are reached there are, in principle, large unitarity corrections to simple BFKL behavior which start at $\ell n 1/x$ values about $\frac{1}{2}$ those of the "parton saturation" value of (3).

In order to better understand the BFKL equation, and the unitarity corrections to that equation, as one follows rising parton densities with increasing energy it is convenient to study a process somewhat simpler than deep inelastic scattering. Since the BFKL equation applies to a one-scale problem, where no Q^2-evolution is allowed, a simple system to study it is in high energy heavy onium-heavy onium scattering.[18] In the large N_c limit, one can view the light-cone wavefunction of a heavy onium, consisting of a heavy quark-antiquark pair along with many soft gluons as a collection of dipoles formed from the heavy quark, the heavy antiquark, and the quark and antiquark parts of the gluons. (Recall that in the large N_c limit a gluon can be viewed as a parallel moving quark-antiquark pair.) In the center of mass of the heavy onium-heavy onium scattering, the cross section is given simply as a product of the number of dipoles in the left-moving onium times the number of dipoles in the right-moving onium times the diole-dipole scattering cross section calculated using the two-gluon exchange. Such a picture reproduces, exactly, the high energy BFKL behavior.

$$\sigma(Y) = n_L(Y/2) n_R(Y/2) \sigma_{DD} \qquad (4)$$

where $Y = \ell n s/M^2$ is the relative rapidity of the two onia, n_L is the number of dipoles in the left-moving onium and σ_{DD} is the energy independent, dipole-dipole scattering cross section. The BFKL behavior comes from the large Y growth

$$n_L(Y/2) = n_R(Y/2) \sim e^{(\alpha_P - 1)Y/2}. \qquad (5)$$

Eq.(4) is appropriate so long as $n(Y/2) << 1/\alpha$ giving a resulting cross section much less than the naive geometric cross section. (Note that $\sigma_{DD} \sim \alpha^2 R_D^2$ where R_D is the dipole radius.) When $n(Y/2) \geq 1/\alpha$ unitarity corrections corresponding to multiple dipole-dipole interactions become important and when $n(Y/2) \geq 1/\alpha^2$ parton saturation effects, in the center of mass system, become important.

While analytical calculations can be carried out completely at the level of a single dipole-dipole scattering, complete multiple dipole-dipole interactions appear beyond the limits of anayltical calculations. However, the equation governing the dipole wavefunction of the onium is a branching equation and is suitable for a Monte Carlo study. Recently, a rather complete Monte Carlo study of unitarity corrections to the total and elastic onium-onium scattering cross sections has been done[19]. For the total onium-onium cross section the BFKL formula works quite well up to rapidities of about 15 units or so. Nevertheless, there are strong unitarity corrections at small impact parameters, giving significant corrections to the BFKL formula for elastic scattering at $Y \approx 8$. The numerical study also shows that even though unitarity comes about through multiple scattering the full corrections must be done before averaging over the fluctuations in the wavefunctions of the colliding onia. The actual Glauber multiple scattering series is N! divergent due to wavefunction fluctions[18,19].

1.3 *Comparing Theory and Experiment at Small x*

1.3.1 *Structure Function(νW_2)*

The parton distributions proposed by Glück, Reya and Vogt(GRV)[20] have proved very successful in anticipating, and describing, the rise of νW_2 at small x. The GRV procedure uses a 2^{nd} order DGLAP formalism with a very low scale, $Q_0^2 \sim 0.3 - 0.35$, for the initial distribution. It is this low scale that allows νW_2 to increase with 1/x for Q^2 in the $1 - 2GeV^2$ region even without any $\alpha \ell n 1/x$ (BFKL) contributions. The original GRV idea was very attractive and supposed that at a low scale, $Q_0^2 \sim 0.3 GeV^2$, valence quark distributions should agree with the constituent quark distributions of the quark model. This proved not to work so GRV added in an initial gluon distribution with a valence-like behavior so that $xG(x, Q_0^2)$ goes to zero as $x \to 0$. The small x part of the quark and gluon distributions are then generated through 2^{nd} order DGLAP evolution. This has proved to be a successful phenomenology.

While it is clear that GRV does very well in the HERA domain it is now important to determine exactly how well it does work. I think there are two crucial issues for GRV, one theoretical and one experimental. (i) On the experimental side, it is important to compare the gluon distribution predicted by GRV with the gluon distribution obtained from 2-forward jet-production, the so-called direct real or virtual photon contribution. This will probably become the most reliable determination of the small-x gluon distribution and an important test for GRV. (ii) On the theoretical side, it is important to see whether GRV is a consistent picture of small-x QCD evolution. From the BFKL equation, it is possible to determine resummed anomalous dimensions and coefficient functions which are more appropriate for small-x evolution than a simple 2^{nd} order DGLAP formalism. For GRV to be a consistent description of small-x behavior, it should turn out that the resummation effects are small in the HERA domain. Preliminary studies do not show

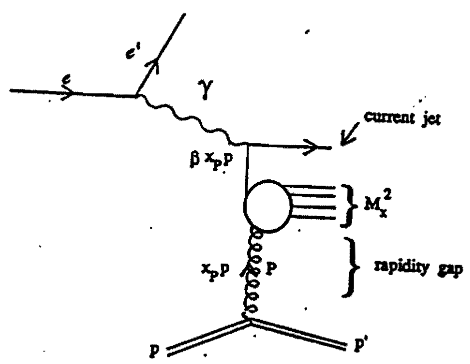

Fig.3

this to be the case[13–16], but the results depend, perhaps strongly, on the initial distributions and a detailed study of resummation effects in the context of GRV initial distribution has not yet been done.

One can imagine GRV as a successfdul phenomenology in either a strong or a weak form. In a strong form, GRV would successfully predict νW_2 at small x for Q^2 as low as $0.5GeV^2$ or so. In its weak form, the fit would break down as one goes below $Q^2 \approx 1.5GeV^2$. If GRV is successful in its strong form, it would seem to me that it calls for a vigorous theoretical response trying to realize the original GRV goal of matching perturbative and nonperturbative domains directly, without any significant transition region.

1.3.2 *Diffraction Scattering*

Let me begin with inclusive diffractive scattering at HERA. The process is illustrated in Fig.3 where it is assumed that the exchange of a soft pomeron, P in the figure, is responsible for the rapidity gap just as in the case of large rapidity gaps in soft hadron-hadron collisions[21–23]. The diffractive events give a scaling contribution to F_2 which can be written as

$$F_2^D = (1/x_P)^{2\alpha_{SP}-1} f(\beta, Q^2) \qquad (6)$$

if the soft pomeron factorizes. In (6), β is the fraction of the pomeron's momentum carried by the struck quark with $f(\beta, Q^2)$ giving the quark parton distribution of the pomeron. Of course, the whole picture holds together only if the object causing the rapidity gaps is the same object which occurs in soft hadronic physics. A consistency check is to determine $2\alpha_{SP} - 1$ from the x_P dependence of the data when β and Q^2 are fixed. This has been done by both H1 and ZEUS with somewhat confusing results. Early determinations of $2\alpha_{SP} - 1$ gave

$$2\alpha - 1 = 1.19 \pm 0.06 \pm 0.07 \quad H1^4 \qquad (7)$$

$$2\alpha - 1 = 1.30 \pm 0.08 \pm {}^{0.08}_{0.14} \quad ZEUS^3 \qquad (8)$$

while a new ZEUS determination gives

$$2\alpha - 1 = 1.47 \pm 0.03 \pm {}^{0.14}_{0.10} \quad ZEUS^{24}. \qquad (9)$$

The ZEUS analysis leading to (9) is substantially different than the analysis leading to (8). In determining (7) and (8), the contamination of rapidity gap events from non-P-exchange contributions, attributed to fluctuations of "normal events" leading to "accidental" rapidity gaps, was removed by a Monte Carlo procedure. In determining (9) the rapidity gap events are fit according to

$$\frac{dN}{d\ell n M_x^2} = D + c e^{b \ell n M_x^2} \qquad (10)$$

with D representing the true diffraction contribution and the second term the background (secondary trajectory) contributions. Fitting the x_P-dependence of the D contribution leads to (9).

While (7) and (8) are consistent with a soft pomeron picture of diffractive events in deep inelastic scattering the value of $2\alpha - 1$ obtained in (9) is not consistent with soft pomeron dominance. It seems to me that the analysis leading to (9) is the preferred analysis in that one is not relying on a poorly understood Monte Carlo analysis to determine what is the supposed pomeron contribution. If the analysis leading to (9) holds up, we are going to have to strongly modify our understanding of diffractive events in deep inelastic scattering from the prejudice that they should be governed by soft pomeron exchange[21–23]. Perhaps a few words on why one expects the soft pomeron to dominate diffractive events is in order. Refer back to Fig.3, but now suppose that the exchange, P, is some set of two or more gluon exchanges. (A single gluon is now allowed by the color neutrality of the hadronic systems on either side of the rapidity gap.) The hard reaction starts where the virtual photon hits the struck quark. If the hardness, of size Q^2, persists to the connection of the gluons making up P one obtains an amplitude which is suppressed by $1/Q^2$. Thus, one expects the hard scale to evolve down to scale of size Λ^2 at the point where the P connects to the system M^2. If that is the case then the gluons making up P are soft and P should be the soft pomeron.

Now, turn to exclusive and almost exclusive diffractive production of vector mesons[25–28] or of a two-jet system as illustrated in Fig.4. When a diffractive event consists only of a single vector meson or of two jets then the simplest connection into the hard quark-antiquark system is that of two gluons. If the final state proton is

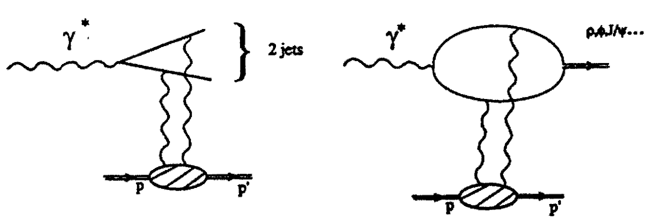

Fig.4

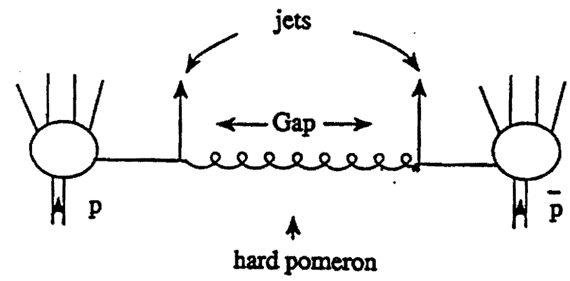

Fig. 5

scattered at almost zero momentum transfer, then the amplitudes for the two processes illustrated in Fig.4 are proportional to $xG(x, Q^2)$ where $x = Q^2/s$. $xG(x; Q^2)$ appears in the leading double logarithmic approximation and so the reliability of the predicted cross sections is far from perfect. Nevertheless one expects diffractive vector meson production to grow rapidly with decreasing x and such reactions should give an independent, rough, determination of xG. As the calculations and experiments become more refined and precise, one can expect to get much new information on the wavefunctions of vector mesons and on high energy QCD.

Finally, let me turn to rapidity gap events where one has jets bordering the rapidity gaps on both sides. Such events were first seen and studied by the DO collaboration and now there are analyses coming from DO,[29] CDF[30] and ZEUS[31]. The process for hadronic collisions is illustrated in Fig.5[32,33]. The transverse momenta of the two jets bordering the rapidity gap are of equal magnitude but opposite in direction. This hard transverse momentum from one jet to the other flows through the hard pomeron connecting the two jets. It would seem that we have the perfect process to measure the hard (BFKL) pomeron! However, the picture as presented in Fig.5 is a bit too simple. It would appear there that the cross section should be given in terms of a quark (gluon) distribution in the proton times an antiquark (gluon) distribution in the antiproton times a hard scattering cross section with the high energy and large momentum transfer hard part of the process being given by the BFKL pomeron. That is

$$\sigma_{2-jet} = x_1 q(x_1, Q^2)\hat{\sigma}(Y, Q^2)x_2\bar{q}(x_2, Q^2). \qquad (11)$$

However, quark and gluon distribution can be expected to appear only in inclusive process, and the requirement that there be a large rapidity gap means that we are not dealing with a purely exclusive process. Technically, the difficulty is the following: In a proton-antiproton collision involving a hard subprocess, soft interactions between spectators and remnants of the proton and antiproton cancel between real and virtual terms. (This is the cancellation of soft gluon exchange and soft gluon production which is necessary for QCD factorization to hold.) However, when one requires a rapidity gap some of the soft production is lost. This has been parametrized[33,34] by putting in a factor $< S^2 >$, a survival probabililty, on the right-hand side of (11). Unfortunately, the energy dependence of this factor is not calculable and so it is, so far, difficult to use these rapidity gap processes to determine the trajectory of the hard pomeron.

As far as data are concerned about 1% of Fermilab 2-jet events are accompanied by a rapidity gap while about 5-6% of HERA 2-jet events have a rapidity gap. This corresponds to an estimated survival probability of about 0.1 at Fermilab and 0.5 at HERA. It would seem that this field is ripe for some good theory or some good phenomenology to go along with the striking data in order to determine the BFKL intercept.

1.3.3 *Final States*

Detailed studies of final states in deep inelastic events at HERA are now beginning. The hope is that from such studies one may learn about the dynamics governing the events. There are some clear signs that this is not going to be easy. In particular, the distribution of transverse energy[35] seems to vary little between $Q^2 = 0$ and $Q^2 \approx 20 GeV^2$ although we believe the physics at $Q^2 = 0$

is purely soft physics while the physics at $Q^2 \approx 20GeV^2$ involves DGLAP, and maybe BFKL, in an essential way. One can imagine two simple possibilities as to why this is the case. (i) Perhaps the final states are determined mainly from soft dynamics which is only loosely connected with the presence of DGLAP evolution or which dominates the contributions of DGLAP evolution to final states. (ii) It may be that the variables that one is using to measure final state characteristics, for example, the transverse energy per unit rapidity, is dominated by a small number of the events which are not really characteristic of either photoabsorption or deep inelastic scattering. In such a case, one must search for other final state properties which better characterize a "typical" event in real or virtual photon proton inelastic collisions.

As I have emphasized earlier, BFKL dynamics is far more interesting than DGLAP dynamics. However, it may well be the case that νW_2 is not a very good observable for measuring BFKL dynamics. After all BFKL dynamics governs high energy scattering for processes where there is only one scale, and where that scale is in the perturbative regime. In νW_2 there are two distinct scales, Λ corresponding to the inverse radius of the proton and Q corresponding to the hardness of the virtual photon. Exactly the same situation holds true for jet measurements at Fermilab. Nevertheless, it is in principle, possible to trigger on a specific class of events, either at HERA or at Fermilab, where DGLAP evolution is suppressed while BFKL evolution is enhanced. Referring back to the small-x part of Fig.1, if one could trigger on a path of evolution like the one shown there the x-dependence would be determined without Q^2-evolution. Indeed, the circle with a dot in the center in that figure is meant to be a trigger which requires that the evolution path pass through a high transverse momentum while x-values are still large. The trigger is, in principle, simple. If a jet, associated with a deep inelastic event, is measured to have transverse momentum $k_{1\perp}$ and a longitudinal momentum fraction x_1, then the evolution curve will pass through the point $(\ell n k_{1\perp}^2/\Lambda^2, \ell n 1/x_1)$ in Fig.1. If $k_{1\perp}^2 \approx Q^2$ then, the evolution between $\ell n 1/x_1$ and $\ell n 1/x$ will be purely a BFKL evolution. The predicted associated jet structure function is[36–38]

$$\frac{k_{1\perp}^2 x_1 d(\nu W_2(x,Q^2))}{dk_{1\perp}^2 dx_1} = x_1[G(x_1,Q^2)+4/9(q+\bar{q})(x_1,Q^2)] \cdot \sum_f e_f^2 \frac{33\pi\alpha}{128}\sqrt{\frac{Q^2}{k_{1\perp}^2}} \frac{e^{(\alpha_P-1)\ell n x_1/x}}{\sqrt{14\alpha N_c\zeta(3)\ell n x_1/x}} \quad (12)$$

If x is varied for fixed x_1 and Q^2, $\alpha_P - 1$ can be extracted from the data and compared with the (lowest order) theoretical expectation of $\alpha_P - 1 = \frac{12\alpha(Q)\ell n 2}{\pi}$. An early analysis of H1 has been done[39] which is encouraging even though one is not jet able to extract $\alpha_P - 1$.

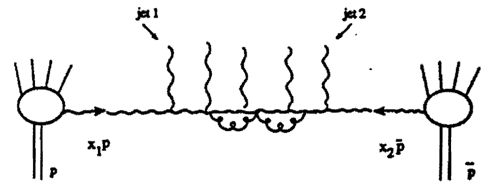

FIG. 6

A cross section analogous to that given in (12) can be measured, perhaps more easily, at Fermilab.[40]. What needs to be measured is simply the 2-jet *inclusive* cross section, say, where the jets have $k_\perp > Q$ and are separated by a rapidity Y. Then the two-jet cross section is given by

$$\sigma_2 = x_1[G(x_1,Q^2) + 4/9(q+\bar{q})(x_1,Q^2)] \, x_2[G(x_2,Q^2) + 4/9(q+\bar{q})(x_2,Q^2)] \cdot (\frac{3\alpha}{\pi})^2 \frac{\pi^3}{2Q^2} \frac{e^{(\alpha_P-1)Y}}{\sqrt{\frac{7}{2}\zeta(3)\alpha N_c Y}} \quad (13)$$

where the situation is illustrated in Fig.6. To measure $\alpha_P - 1$ it is necessary to fix x_1 and x_2 along with Q^2 while varying Y. This can only be done when different beam energies are available. The DØcollaboration expects to make such a measurement during the next year.

2 Power Corrections to Hard Processes

Theorists have long organized hard processes in terms of leading and higher twist contributions. At sufficiently large Q^2, the hard scattering scale, one can neglect the higher twist contributions. There have been two recent developments which relate to this. (i) There may be $1/Q$ terms in certain processes which are not visible in any operator product expansion[42–47]. (ii) There are difficulties, in principle, with determining the higher twist terms in an unambiguous way[48–54]. This latter point is another way of saying that the original separation of hard processes into a leading twist part and a higher twist part has essential ambiguities.

2.1 Some Observables May Have $1/Q$ Corrections

Recently, it has been observed, that some measurable quantities have corrections to a leading twist perturbation series which are of size $1/Q$ rather than the more usual $1/Q^2$ terms. It is perhaps useful to think of observables as being of two varieties, those that are directly related to a matrix element of some operators formed out of quark and gluon fields and those that are defined in addition by some kinematic weighting or cuts. These latter type I will call "synthetic" observbables in what follows. For example, jets are invariably of a synthetic character because it is always necessary to impose some kinematic cut; in Sterman-Weinberg jets, one defines a somewhat artificial cone and declares particles to be in the jet or outside of the jet depending on whether they fall in the cone or outside ofthe cone. Energy-energy correlations in $e^+e^- \to$ hadrons defined by

$$EEC(\theta, Q) = \frac{1}{\sigma}\int_0^1 \frac{d\sigma}{dx_1 dx_2}\, x_1 x_2 dx_1 dx_2, \qquad (14)$$

with $\frac{d\sigma}{dx_1 dx_2}$ the two-particle inclusive distribution with the two particles at a relative angle θ is another example because of the $x_1 x_2$ weighting in (14). Of course, the $x_1 x_2$ weighting is what suppresses the logarithmic infrared divergences in (14) allowing EEC to be calculable in perturbation theory. However, there are higher twist corrections in (14) coming from particles having x_1 or x_2 or order Λ/Q. Clearly, such soft particles cannot be calculable perturbatively. Since these regions of small x are suppressed only linearly in (14) there must be nonperturbative terms of size $1/Q$ in (14). Similar corrections occur in thrust and in Sterman-Weinberg jet cross sections.

It may well be that $1/Q$ terms only occur in synthetic obervables and that observables which can be directly written in terms of matrix elements of QCD operators will have higher twist corrections depending on the possible dimensions of higher local operators. It is an important open problem to understand if this is the case or whether $1/Q$ corrections can also be present in more natural observables.

It has also been suggested that the $1/Q$ corrections occuring in synthetic observables may be universal[42–46] in that the $1/Q$ correction always seem to come from soft gluons. If such is really the case, and a good proof remains to be found, it would mean that there would be only a single unknown parameter characterizing all $1/Q$ corrections. Since there are many such observables, one would have a very powerful phenomenology characterizing the leading nonperturbative corrections to these synthetic observables. This has become a very active field. It is likely that we are now witnessing the birth of a new type of phenomenology.

2.2 Higher Twist Terms in the Operator Product Expansion. How to Use Them?

As an example of the questions arising when one tries to use higher twist terms in an operator product expansion consider the Bjorken sum rule for spin-dependent lepton-nucleon scattering[55]

$$\int_0^1 dx[g_1^P(x,Q^2) - g_1^N(x,Q^2)]$$
$$= \frac{G_A}{6}[1 - \frac{\alpha}{\pi} - 3.58(\frac{\alpha}{\pi})^2 - 20.21(\frac{\alpha}{\pi})^3 - c_4(\frac{\alpha}{\pi})^4] + HT \qquad (15)$$

in $\overline{MS}$ scheme with 3 massless quarks. c_4 is estimated to be about 120 and HT stands for the Λ^2/Q^2 higher twist term. Eq.(15) comes from

$$i\int d^4x e^{iqx} j_\mu(x) j_\nu(0) \underset{q^2\to\infty}{\longrightarrow} \epsilon_{\mu\nu\rho\sigma}\frac{q_\rho}{q^2}[E_1 j_{5\sigma} + \frac{1}{Q^2}E_2 O_\sigma] \qquad (16)$$

where j_μ and j_ν are electromagnetic currents, $j_{5\sigma}$ is an axial vector current and $O_\sigma = \bar{q}\gamma_\lambda \tilde{F}_{\lambda\sigma} q$. We have suppressed flavor indices for simplicity. E_1 has the perturbative expansion given on the right-hand side of (15). It would be interesting to have a rough evaluation of the HT term which is determined by the proton matrix element of O_σ. One might hope that a lattice evaluation could give this. However, a moment's thought shows that this is not possible. Write (15) schematically as

$$\int_0^1 dx[g_1^P - g_1^N] = PT + HT \qquad (17)$$

with PT standing for the perturation series on the right-hand side of (15). Now the left-hand side of (17) is a well defined measurable quantity. If the higher twist contribution, HT, were well defined it would mean that the perturbation series, PT, was also well defined. But we know that the perturbation series is divergent. Thus, the higher twist term must not be well defined. Indeed, the higher twist contributions are well defined only after one gives a rule for dealing with the (Borel nonsummable) divergent perturbation series.

This is all quite fine and it is perhaps not too surprising that one cannot really separate high order terms in the perturbation series from higher twist terms. What is needed is a procedure for defining the perturbation series which can be matched to nonperturbative methods of calculating the higher twist contributions. Novikov, Shifman, Vainshtein and Zakharov[54] suggested doing calculation with an infrared cutoff with the cutoff separating perturbative and nonperturbtive contributions. However, higher order perturbative corrections are surely going to be difficult here[53]. If one tries to calculate the matrix element O_μ above nonperturbatively, say in QCD lattice theory, the difficulty is that O_μ mixes with $j_{5\mu}$ giving terms of size $1/a^2$, with a the lattice spacing, rather than size Λ^2. Martinelli and Sachrajda[52] have given a nonperturbative prescription for defining operators like O_μ which stop the mixing. The challenge, now, is to find a procedure which can be used with the existing order α^3 terms in perturbation theory. It is clear that if (15) is to be useful, we need a nonperturbatives definition of the higher twist term which does not double count perturbative contributions at α^3, with the perturbative terms being given in some particular renormalization scheme.

It is possible to subtract out the divergences of the perturbation series, due to infrared renormalons, in a well defined, if scheme-dependent, way[49,51]. One can then take $HT = C/Q^2$ with c to be determined phenomenologically. In such a procedure, one adds a new nonperturbative parameter as one goes to accuracy greater than Λ^2/Q^2. This parameter is universal so long as care is taken in removing the perturbative divergences in a systematic way in going from process to process.

There is a very vigorous theoretical activity concerning the role and definition of higher twist terms in a short distance expansion. The problems encountered here touch some of the most profound parts of a quantum field theory which exists in a nonperturbative sense and not just as a perturbative series.

2.3 *Regularities in the Perturbation Series*

Brodsky and Lu[56] recently introduced the idea of commensurate scale relations. The idea is the following. For each observable assign an effective charge[57]. For example for the ratio, R, of cross sections for e^+e^- going to $\mu^+\mu^-$ define α_R by

$$R = 3\sum_f e_f^2(1+\alpha_R(Q)). \tag{18}$$

Then the predictions of QCD can be written as one effective charge in terms of another. If $\alpha_1(Q)$ and $\alpha_2(Q)$ are the effective charges of observables 1 and 2, then

$$\frac{\alpha_1(Q)}{\pi} = \frac{\alpha_2(Q^*)}{\pi} + r_2\left(\frac{\alpha_2(Q^{**})}{\pi}\right)^2 + r_3\left(\frac{\alpha_2(Q^{***})}{\pi}\right)^3 + \cdots. \tag{19}$$

The $Q^*, Q^{**}\cdots$ are determined by requiring that all running coupling effects be included in the $Q^{*\prime}s$ and not in the $r_i's$[58]. When this is done one finds that the $r_i's$ are not large, for relations where 3^{rd} order calculations exist, and in some cases $r_i = 1$ for all i. The explanation of this simplicity seems to be that after running coupling effects are put into the $Q^{*\prime}s$ the r_i are the expansion parameters of a conformally invariant QCD[59,60]. In some cases, $r_i = 1$ results from the Crewther relation.

The idea of trying to put all running coupling dependence into the scale of α is a nice idea.[58] It seems to me that there are two major unresolved issues here. (i) Can one give a prescription for separating running coupling corrections from other corrections in higher orders? If so, then one can define a conformally invariant QCD, at least perturbatively. In QED, one knows how to make this separation, but there is no obvious procedure in QCD. (ii) If one is able to put running coupling corrections into the $Q^{*\prime}s$ is it generally true that the resulting $r_i's$ are small in the first 3-4 orders of perturbation theory? If so, it means that the dominant terms in the perturbation series come from running coupling effects and thus are likely governed by the first infrared renormalon. It is not at all clear that this is generally the case. One needs a larger variety of example where order α^3 corrections to the leading term have been calculated.

A somewhat related program of work has been progressing in the past few years where one tries to estimate, and control, higher order terms in the QCD perturbation series using Padé approximants[62,63]. For example, for the Bjorken sum rule one can use the Padé approximants to estimate the order α^4 correction. One finds agreement with earlier estimates[64]. More importantly, however, is the fact that Padé approximants can be used to evaluate the strength of the leading infrared renormalon singularity in the Borel plane. This strength gives the essential uncertainty in perturbation theory evaluations of the Bjorken sum rule. This uncertainty is found to be of the same order as the higher twist contributions estimated using QCD sum rules. Indeed, the higher twist term should be at least this size in order to compensate the perturbative uncertainty. The use of Padsé approximants may prove a powerful method for estimating perturbation theory contribution beyond the level at which explicit higher order calculations can be done.

3 The Quark Model, Skyrme Model, Large N_c QCD and Spin-Dependent Structure Functions

3.1 *Baryons in Large N_c QCD*

Recently, a number of very interesting papers[65−67] have appeared which study baryons and their couplings to mesons and currents in a large N_c expansion. One result of this study is the apparent equivalence of quark model and Skyrme model realizations of large N_c QCD and also an equivalence in higher order $1/N_c$ corrections. This means that the quark model and Skyrme model cannot really disagree in any predictions they may give, although results that are simple in a Skyrme model may appear complicated when expressed in the quark model and vice versa. While large N_c QCD gives many reults, in leading orders of $1/N_c$, it does not lead to the full SU(6) constituent quark model. For example, the SU(6) non-relativistic quark model result that $G_A = 5/3$, a number which is lowered to about 5/4 in relativistic quark models, does not emerge from large N_c QCD.

3.2 Spin-Dependent Deep Inelastic Scattering

Experimental data on spin-dependent deep inelastic lepton-nucleon scattering continue to grow and become more precise.[69–72] The primary issue continues to be the value of the first moment of g_1 and its implications for the consitituent quark model of nucleons. The Bjorken sum rule (15) has been tested at the 10% level and is now being used to determine the QCD α-parameter. The quantity of interest is

$$\Gamma_1^T = \int_0^1 dx g_1^T(x, Q^2) = \frac{1}{2}\sum_f e_f^2 (p\lambda = \frac{1}{2}|j_{50}^f|p\lambda = \frac{1}{2}) \tag{20}$$

for a target T, a proton or neutron, where $j_{5\mu}^f = \bar{q}_f \gamma_\mu \gamma_5 q_f$ is the axial vector current for quarks of flavor f. In the Q^2 range of the spin-dependent scattering experiments u, d, s are the active quarks. In a naive quark parton picture

$$\Gamma_1^P(Q^2) = \frac{1}{2}\Sigma e_f^2 \Delta q_f(Q^2) \tag{21}$$

with Δq_f being the fraction of the proton's spin carried by quarks of flavor f.

I shall not review the data here but just note that a new SMC analysis gives[73,74]

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.19 \pm 0.07 \tag{22}$$

where the low-x region has been given considerable attention in this analysis. I is important to note, however, that the 0.07 error in (22) does not really reflect all the possible sources of errors in the data and especially in the analysis of the data. In particular, the assumption of a Q^2-independent asymmetry has been made. With the large amount of high quality data now at hand, we can expect more complete and reliable anaylses to come in the next few years.

The basic dilemma is still the fact that the constituent quark model suggests that

$$\Delta u \approx 1, \Delta d \approx -\frac{1}{4}, \Delta s \approx 0$$

$$\Delta\Sigma \approx \frac{3}{4} \tag{23}$$

while the experimental numbers, especially for $\Delta\Sigma$ are far away. Backing away from the quark model for the moment, what does large N_c QCD have to say about these quantities? It is straightforward to see that

$$\Delta u + \Delta d = O(N_c^0) \tag{24a}$$

$$\Delta u + \Delta d - 2\Delta s = \Delta u + \Delta d + \Delta s + O(1/N_c) \tag{24b}$$

in large N_c QCD. The left-hand side of (24b) is 0.59 in the real world while the right-hand side is very likely less than 1/3. Specific Skyrme models[75] suggest that $\Delta\Sigma = O$ but this cannot be a general result of Skyrme models since they are equivalent to large N_c QCD and (24) does not require $\Delta\Sigma = O$. We can also expect to have interesting new lattice QCD results on Δq_f in the next year or so.

3.2.1 *The Importance of the Sea*

It seems very likely that the sea of quark-antiquark pairs are playing an essential role in the divergence of experimental values from consitituent quark value for $\Delta u, \Delta d$ and Δs. Roughly

$$\Delta u \approx 0.80$$

$$\Delta d \approx -0.45$$

$$\Delta s \approx -0.12. \tag{25}$$

If one were to add about 0.15 to $\Delta u, \Delta d$ and Δs these numbers would be very compatible with the quark model numbers. This is suggestive that there is a flavor singlet (sea) contribution which is causing the problem. This raises a number of interesting issues. (i) In most lattice calculations of hadronic properties a quenched approximation is used. It is generally believed that the sea contribution makes a small effect on static properties. When lattice calculations for $\Delta\Sigma$ become more accurate it will be very interesting to see whether or not the sea contribution is essential for agreement with experiment. It may well be that $\Delta\Sigma$ is an "unusual" quantity which is much more sensitive to sea contributions than other static quantities. ($\Delta\Sigma$ is a static quantity because it is equal to the *forward* proton matrix element of the flavor singlet axial vector current.) (ii) Is the quark model smart enough to know about the sea? Perhaps the success of the quark model is in large part due to the fact that the sea has been an unimportant part of the observables considered. The failure of the quark model would then be due to its inability to properly handle the sea. On the other hand, is not part of the idea of a constituent quark to include the chiral condensate and the sea in terms of its static properties? (iii) Could we have expected large N_c to do better than indicated in (24b) or are sea corrections generally large in large N_c QCD?

3.2.2 *The AxialAnomaly*

The axial anomaly causes a misidentification of spin with the axial vector current, the proper identification being[76–79]

$$\Delta q_f = (p\lambda = 1/2|j_{50}^f|p\lambda = 1/2) + \frac{\alpha}{2\pi}\Delta G \qquad (26)$$

with ΔG the amount of spin carried by gluons in the proton. For example, $\Delta G \approx 2$ could give a 0.24 contribution to $\Delta\Sigma$ for $\alpha \approx 1/4$. Clearly, it is important to measure ΔG, an interesting quantity in its own right. I think it very unlikely that ΔG will be larger than 2. Thus, it is also very important to firm up the errors on $\Delta\Sigma$. If $\Delta\Sigma$ turns out to be, say, 0.15 or less, it is very unlikely that the axial anomaly is playing an important role in saving the consitiutent quark model. One would naturally look for reasons why $\Delta\Sigma$ is so small, and the Skyrme model[75] (a particular skyrme model) might be the answer. On the other hand, *if* $\Delta\Sigma$ turns out to be as large as 0.35 the anomaly might indeed play an important role in preserving the consitituent quark model.

4 Energy Loss of Quarks and Gluons in Nuclear Matter

There has been striking new prgress in understanding the energy loss of high energy quarks and gluons as they pass through matter. The corresponding QED atomic physics problem is the energy loss of a high energy electron as it passes through atomic matter. The QED problem has an illustrious history especially concerning the Landau-Pomeranchuk-Migdal (LPM) effect,[80,81] which effect suppresses induced radiation from that which one might expect using the Bethe-Heitler formula incoherently in the medium. There has been a recent elegant rederivation of the LPM result in QED[82]. The corresponding QCD problem is much more subtle and difficult to deal with from a technical point of view. There are many potential applications at hand ranging from corrections to hard scattering formulas taking place in cold nuclear matter, to jet quenching in hot nuclear matter, to calculations of the energy available for thermalization, and the rate of thermalization, in the early stages of high energy heavy ion collisions.

The QCD problem for hot matter, the quark-gluon plasma, was set up some time ago by Gyulassy and Wang[83] and recently solved by Baier, Dokshitzer Peigné and Schiff(BDPS)[84]. The BDPS result is at first sight unintuitive and was certainly not anticipated. Suppose a quark is produced by a hard collision, for example, by a highly virtual photon, in a hot plasma. Then the energy loss of that quark, coming from gluon radiation induced by scatterings in the plasma, after it travels a distance z from its production point is

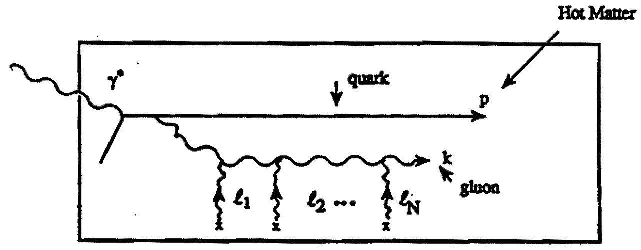

FIG.7

$$-\Delta E = c\alpha z^2 \qquad (27)$$

where α is the QCD coupling and c is a slowly varying (logarithmic) function of z. This means that

$$-\frac{dE}{dz} = -\frac{\Delta E}{z} = c\alpha z. \qquad (28)$$

Thus, the rate of energy loss of the quark depends on how long it has travelled in the medium. Until the calculation by BDPS, it was generally expected that $\frac{dE}{dz}$ would vary at most logarithmically with z so (27) and (28) have come as a great shock.

Indeed, there is a very simple way of getting (27) which I shall now describe. The picture of what is happening is illustrated in Fig.7 where a quark in the plasma absorbs a highly virtual photon and begins moving through the medium. A gluon, k, is radiated due to multiple scattering in the plasma, the exchanges $\ell_1, \ell_2 \cdots \ell_N$. The energy loss in a distance z will be dominated by the maximum energy gluon which can be emitted in that distance. Such a gluon must be freed over a time t=z. Now, the lifetime of the gluon k is $\frac{2k}{k_\perp^2}$. One takes k as large as possible while still satisfying the LPM requirement that the lifetime be not greater than z. Thus,

$$z = \frac{2k}{k_\perp^2}. \qquad (29)$$

What is $k_\perp^2$? To free the gluon $k_\perp^2$ must not be larger than the momentum transfer squared which k takes from the medium. Assuming that each scattering in the medium has $\ell_\perp^2 \approx \mu^2$ it is clear that

$$k_\perp^2 = N\mu^2 \qquad (30)$$

with N the number of collisions. The number of collsions is determined by the mean free path of the gluon so

$$N = z/\lambda_g \tag{31}$$

with λ_g the gluon's meson free path. Combining (29), (30) and (31) gives

$$k = \frac{1}{2}\mu^2/\lambda_g z^2 \tag{32}$$

as an estimate of the energy lost by the quark over a distance z in the plasma. Of course, the probability that the quark emit the (virtual) gluon which is to be freed carries a factor of α so one get finally

$$-\Delta E = c\alpha z^2. \tag{33}$$

Although the BDPS calculation only applies directly to hot matter it seems pretty clear that (27) and (28) must also hold for cold matter. It is an exciting challenge to develop a usable formalism for cold matter and to check experimentally the rather surprising result contained in (27) and (28).

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