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A Correlation between Quantised-Alignment and Identical-Band Mechanisms from the Perspective of the Many-Body Solutions

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Abstract

The problem of nuclear rotation treated within the framework of the two-dimensional cranking and particles-rotor hamiltonians is solved in the many-body space for a deformed single-j shell configuration with pairing. The properties of "identical" (twinned) bands in neighboring, in particular, odd- and even-systems are studied. It is shown that the twinned band mechanism is accompanied by a "quantised alignment" in the relative angular momentum (or incremental) alignments without involving any special dynamical symmetry as e.g. pseudo-spin symmetry. Conditions for a non-trivial manifestations of the quantised incremental alignement together with the criteria of "being identical" found in the literature are discussed.

Since the discovery of the so-called identical (or twinned) rotational bands in neighboring superdeformed (SD) ^{152}Dy and ^{151}Tb nuclei [1] the origin and the abundance of this effect have been investigated vigorously. The effect, as often presented in the literature, consists in nearly vanishing differences between the γ-ray energies, $E_\gamma^A(I_A)$ and $E_\gamma^B(I_B)$ corresponding to rotational bands in two neighboring nuclei, say A and B. More precisely, defining four types of differences denoted

$$\Delta E^{AB}_{direct} \equiv E^A_\gamma(I_A) - E^B_\gamma(I_B) \tag{1.a}$$

$$\Delta E^{AB}_{average} \equiv E^A_\gamma(I_A) - \frac{1}{2}\left[E^B_\gamma(I_B+2) + E^B_\gamma(I_B)\right], \tag{1.b}$$

$$\Delta E^{AB}_{1,3} \equiv E^A_\gamma(I_A) - \frac{1}{4}\left[E^B_\gamma(I_B+2) + 3E^B_\gamma(I_B)\right], \tag{1.c}$$

$$\Delta E^{AB}_{3,1} \equiv E^A_\gamma(I_A) - \frac{1}{4}\left[3E^B_\gamma(I_B+2) + E^B_\gamma(I_B)\right], \tag{1.d}$$

one finds in numerous sets of experimental data that $|\Delta E^{AB}| \lesssim 3\,\mathrm{keV}$ over many transitions, often up to 15 or so. This phenomenon was found later in SD nuclei of

the $A \sim 190$ mass range, cf. e.g. ref.[2] and references therein, those with $A \sim 130$, ref.[3], in several "normal deformed" nuclei, refs.[4,5], and in particular in nuclei with very different masses, ref.[6].

Moreover, it has been suggested[7,2] that the incremental alignment Δi^{AB} defined as $\Delta i^{AB} = i_{AB} - [I_B(\omega_B) - I_A(\omega_B)]$, i_{AB} denoting the relative alignment between the two bands considered, may become approximately quantised i.e. $\approx k\frac{1}{2}$ where k is an integer. Such a suggestion seemed attractive, under some additional conditions discussed below, yet not unambigously justified by the present data since the spins of the SD bands are not known (precisely enough). The suggestion has been criticised on this ground in refs.[8,9] where the indications have been given that if the alignment $i_{AB} = 0$ (which could be the case in the strong coupling limit) then the incremental alignment of refs.[2,7] is trivially quantised and the whole problem looses an important part of its possibly new physical interest.

The purpose of the present article is to analyse the same problem theoretically. We use a simplified hamiltonian believed to imitate the most important mechanisms governing the rotational motion of deformed nuclei: the pairing, the angular momentum alignment and the nuclear deformation. We will solve the problem of N particles moving in an ensemble of the single-j orbitals with the hamiltonian

$$\hat{\mathcal{H}} = \sum_m e_m c_m^+ c_m - G \sum_m \sum_{m'} c_m^+ c_{\bar{m}}^+ c_{\bar{m}'} c_{m'} + \hat{\mathcal{H}}_{rot} \tag{2}$$

where

$$\hat{\mathcal{H}}_{rot} = -\omega \hat{j}_x \qquad \text{and} \qquad \hat{\mathcal{H}}_{rot} = \frac{(I - \hat{j}_x)^2}{2J}. \tag{3}$$

In the above expressions $e_m = \chi\,[3m^2 - j(j+1)]/[j(j+1)]$ is the spectrum of an axially-symmetric quadrupole-deformed single j-shell, χ represents the deformation, G is the monopole pairing constant and $\hat{\mathcal{H}}_{rot}$ stands for a standard, two-dimensional induced rotation hamiltonian (cranking and particles-rotor, respectively, cf. Eq.(2)). The corresponding hamiltonian will be diagonalised using the many-body basis $\{c_1^+ c_2^+ \cdots c_N^+ \mid 0\rangle, N \leq 2j+1\}$. Since no other approximation corresponding to neglect of some terms in the hamiltonian (such as e.g. Hartree-Fock-Bogolyubov approximation) is going to be used this algorithm is sometimes refered to as "exact".

The problem posed is: can the most standard hamiltonian used in the literature of the hight spin physics produce <u>at all</u> the identical band mechanism and the quantised angular momentum alignment ? Since the quantities of primary interest in such an analysis are some small differences of some big numbers we find it more adequate to attack a simplified hamiltonian "exactly" rather than the more sophisticated

hamiltonian approximately. Moreover, we will use the freedom of the parameters in (2) to simulate the conditions felt by the nucleons in a superdeformed nucleus.

In published discussions of identical bands various authors used distinct (at least two) criteria of "being identical": either the one specified above according to which two bands are considered "twinned" if $|\Delta E^{AB}| < \varepsilon \sim 3$ keV, or the "dynamical moment criterion": $J_A^{(2)}(I_A) = J_B^{(2)}(I_B)$ (more precisely: $|\Delta J_{AB}^{(2)}| \equiv |J_A^{(2)}(I_A) - J_B^{(2)}(I_B)| < \varepsilon \sim$ a few $\hbar^2 \mathrm{MeV}^{-1}$). To distinguish between the consequences of applying these distinct criteria let us remind the following ($I_A \equiv I, I_B \equiv I + \delta I$):

$$\Delta J_{AB}^{(2)} \equiv J_B^{(2)}(I_B) - J_A^{(2)}(I_A) \equiv J_B^{(2)}(I+\delta I) - J_A^{(2)}(I)$$

$$\equiv \frac{dI_B}{d\omega_B} - \frac{dI_A}{d\omega_A} \equiv \frac{d(I+\delta I)}{d\omega_B} - \frac{dI}{d\omega_A} = \frac{dI}{d\omega_B} - \frac{dI}{d\omega_A}$$

$$= \left[\frac{d\omega_B(I+\delta I)}{dI}\right]^{-1} - \left[\frac{d\omega_A(I)}{dI}\right]^{-1}$$

$$= \left[J_A^{(2)}(I) J_B^{(2)}(I+\delta I)\right] \frac{d}{dI}\left[\omega_A(I) - \omega_B(I+\delta I)\right] . \tag{4}$$

For twinned-band relation defined according to the $|\Delta J_{AB}^{(2)}| < \varepsilon$ criterion we have from Eq.(4) and in the limit of an exat equality

$$\Delta J_{AB}^{(2)} = 0 \quad \Rightarrow \quad \omega_A(I) - \omega_B(I+\delta I) = const, \tag{5}$$

or, using the definition $\omega \equiv \dfrac{dE}{dI}$ and integrating

$$E_A(I) = E_B(I+\delta I) + const.\, I + const' . \tag{6}$$

(A particular case of the above expression is the decoupled-rotor formula used e.g. in ref.[10]).

Conversely, applying e.g. the direct degeneracy criterion $\Delta E_{direct}^{AB} = 0$, we have, after noting that $dI \to \Delta I = 2$,

$$0 = E_\gamma^A(I_A) - E_\gamma^B(I_B) = \left[E^A(I_A+2) - E^A(I_A)\right] - \left[E^B(I_B+2) - E^B(I_B)\right]$$

$$\simeq 2\left[\frac{dE^A}{dI_A} - \frac{dE^B}{dI_B}\right] = 2\left[\omega_A(I) - \omega_B(I+\delta I)\right] . \tag{7}$$

Thus $\omega_A(I) \simeq \omega_B(I+\delta I)$, what implies that in such a case we must have $const. = 0$ in Eqs.(5,6). By differentiating (7) over I we get $J_A^{(2)}(I) = J_B^{(2)}(I+\delta I)$. Consequently:

the criterion of the band "identity" $\Delta E^{AB} = 0$ implies $\Delta J^{(2)}_{AB} = 0$ but the inverse is generally not true, as it indicate Eqs.(5-6), since in general $const. \neq 0$.

To illustrate now the properties of the "identical band" mechanism in our simplified quantum-mechanical many-body hamiltonian we set, as an example, $j = 11/2$. We represent a model nucleus "A" by a core with the moment of inertia $J = 80\,\hbar^2\mathrm{MeV}^{-1}$, a value typical for a superdeformed nucleus from the mass $A \sim 150$ range, surrounded by $N = 6$ particles. We take similarly $N = 7$ particles with the same core for a nucleus "B". We set $\chi = 6.0$ to simulate the splitting of the $j = 11/2$ orbital at the quadrupole deformation $\beta \simeq 0.6$ and fix $G = 0.6$, what insures that the calculated moment of inertia $J^{(1)}_x(\mathrm{yrast}, G = 0.6) \simeq 0.5\, J^{(1)}_x(\mathrm{yrast}, G = 0)$ at the limit of vanishing rotation. The latter condition follows from an analogy with the empirical observation that the measured ground-state moments of inertia are significantly smaller than the rigid body values, due to the pairing correlations. Within our single-j shell basis the many-body hamiltonian matrices have the dimensions $\mathcal{N}_6 = 924$ and $\mathcal{N}_7 = 792$, for the systems with 6 and 7 particles, respectively.

Before presenting the calculation results some comments may seem appropriate. Firstly, the hamiltonian used describes a two- (rather than three-) dimensional rotation. However, since we are interested here in the high-spin limit, $I \sim (20 - 70)$, the Coriolis and centrifugal forces are expected to play a leading role. In such a case one should expect that the maximum alignment is taking place so that $I \simeq < \hat{I}_x >$, $< I_y > \simeq 0$ and $I \gg < \hat{I}_z >$. In other words – the best possible conditions to apply the nuclear two-dimensional rotation approximation are met in this case. Secondly, in the calculations, the same value of the rotor's moment of inertia J is taken for "A" and "B" nuclei. Since the name "moment of inertia" is used in this context it will be important to distinguish between J playing a role of a parameter of the hamiltonian and the kinematical and dynamical moments of inertia which will be defined as usual by

$$J^{(1)} \equiv I_x \Big/ \left(\frac{dE}{dI_x}\right) \qquad \text{and} \qquad J^{(2)} \equiv \left[\frac{d^2E}{dI_x^2}\right]^{-1} . \tag{8}$$

Both E and I_x are calculated for A and B systems separate ly, (in particular $I^A_x \equiv < \psi_A|\hat{I}_x|\psi_A >$ and $I^B_x \equiv < \psi_B|\hat{I}_x|\psi_B >$) and the corresponding energies and moments, cf. also Eqs.(3-6), depend obviously on the distinct nucleonic configurations in the "A" and "B" systems. Consequently in general $J^{(1)}_A \neq J^{(1)}_B$, $J^{(2)}_A \neq J^{(2)}_B$, none of them being equal to J, or, there is no trivial equality relation between the theoretically calculated moments in (3-6) caused by the same J-value in the hamiltonians of "A" and "B".

Thirdly, there is no room for any selfconsistency mechanism allowing the system to "self-adjust" with progressing alignment within the present formalism (we keep all the three coupling constants (χ, G, J) independent of spin). Although the last condition might seem restrictive, such an impression is to some extent only apparent: *a.* given the fact that the superdeformed nuclei do behave in many cases as excellent rotors, very often with practically linear I-vs.-ω dependence; and, *b.* the primary intention of this study: to find out whether the *simplest standard effective hamiltonians* used for a long time in nuclear structure physics may or may not lead to *the band identity* and to the *quantised incremental alignment mechanisms.*

Figure 1 illustrates the global features of the twinned-band mechanisms. We consider all possible combinations between all the 924 bands of the $\mathcal{N} = 6$ and 792 bands of the $\mathcal{N} = 7$ systems and qualify the bands of "A" and "B" systems as twinned if $|\Delta E^{AB}| \lesssim 3$ keV is satisfied for at least 5 transitions. The corresponding "statistics" for direct, average, (1/4, 3/4) and (3/4, 1/4) twinned bands is given in the Figure. We conclude that even within the relatively small space of a single j-shell aligning orbitals the percentage of identical bands is quite high.

While the comparison in Fig.1 is related to the criterion of Eq.(7), Fig.2 illustrates the "$J^{(2)}$-criterion" (cf. Eqs.(4-6)). There are four cases taken for this illustration: defining $\delta J^{(2)}_{AB} \equiv |J^{(2)}_A - J^{(2)}_B|$ we compare the total numbers of identical bands satisfying $|\delta J^{(2)}_{AB}| < \varepsilon = (1,\ 2,\ 3$ and $4)\ \hbar^2\mathrm{MeV}^{-1}$. The figure gives some impression about "how restrictive the $\delta J^{(2)}_{AB}$-criterion may be" when decreasing the acceptance parameter ε.

Let us recall that applying directly any of the four forms of the "ΔE^{AB}_{γ}-criterion" leads automatically to the quantised incremental alignment and thus brings no new information. Indeed, as discussed in [7], the incremental alignment satisfies

$$\Delta i^{AB} = 2\ \frac{E^B_\gamma(I+2) - E^A_\gamma(I+\delta I)}{E^B_\gamma(I+2) - E^B_\gamma(I)} \tag{9}$$

Applying the condition of "twinned bands" as $\Delta E^{AB}_{direct} \simeq 0$, $\Delta E^{AB}_{average} \simeq 0$, $\Delta E^{AB}_{1,3} \simeq 0$ or $\Delta E^{AB}_{3,1} \simeq 0$, implies, after substituting into Eq.(9): $\Delta i = 0$, $\Delta i = +1$, $\Delta i = 3/2$ and $\Delta i = 1/2$, respectively, and so the quantisation of the incremental alignment is implied trivially. Therfore in what follows we will discuss the consequences $\delta J^{(2)}_{AB}$-criterion, ($\delta J^{(2)}_{AB} \simeq 0$) only.

It will be instructive to illustrate an individual behavior of selected couples of bands. Figure 3 compares the four physical quantities relevant in this case for the

yrast bands of the systems composed of $N_A = 6$ and $N_B = 7$ valence particles. These are: the behavior of $\Delta J^{(2)}_{AB}$ and ΔE^{AB}_{γ} introduced above, as well as the incremental alignment Δi^{AB}_x and the relative alignment $\Delta s^{AB}_x \equiv j^{(B)}_x(\omega) - j^{(A)}_x(\omega)$. While $|\Delta J^{(2)}_{AB}|$ remains smaller than 2 units in this case, the difference ΔE^{AB}_{γ} corresponds to a constant (relatively large) of $\sim$ 189 keV; at the same time the incremental alignment and the relative alignment remain to a very good approximation "quantised". The corresponding differences are approximately equal to the *integer* numbers although the systems considered have an even and an odd particle numbers, respectively.

A similar illustration but for a couple of bands selected in two even systems with $N_A = 4$ and $N_B = 6$ is given in Fig. 4. This time the "quantised alignment" corresponds to a half-integer number although both numbers of valence particles are even. Of course in numerous other examples the quantised alignments in even-A and even-B systems are often integer while for the odd vs.even systems they are often half-integer.

The calculations with the cranking model hamiltonian (instead of the particles-rotor hamiltonian) give very similar results; the presentation of these results will not be repeated here.

In summary: Our calculations indicate that the deformed standard average-field plus pairing hamiltonian with an induced rotation term generates numerous twinned bands according to criteria employed in the literature. The appearance of the twinned bands is accompanied by the quantised incremental alignment under the condition $\Delta E^{AB} \sim 0$ trivially and under the condition $\Delta J^{(2)}_{AB} \sim 0$, to our understanding, non-trivially. The identical bands lie often quite low in energy. Our calculations suggest, as possible origin of the quantisation of the incremental alignment, the appearance of the maximally aligned angular momenta of one or more nucleons. They bring naturally the multiples of some half-integer numbers into the corresponding relative quantities (differences) such as relative or incremental alignments.

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FIGURE CAPTIONS

Fig. 1

Dependence of the number of "identical" bands in function of the number of transitions obtained using the criterion $|\Delta E_{\gamma}^{AB}| < \varepsilon = 3$ keV (for details see text). The lines are marked as follows: full line – $|\Delta E_{\rm direct}^{AB}| < \varepsilon$; dotted line – $|\Delta E_{\rm average}^{AB}| < \varepsilon$; dashed line – $|\Delta E_{1,3}^{AB}| < \varepsilon$ *and* $|\Delta E_{3,1}^{AB}| < \varepsilon$, together.

Fig. 2

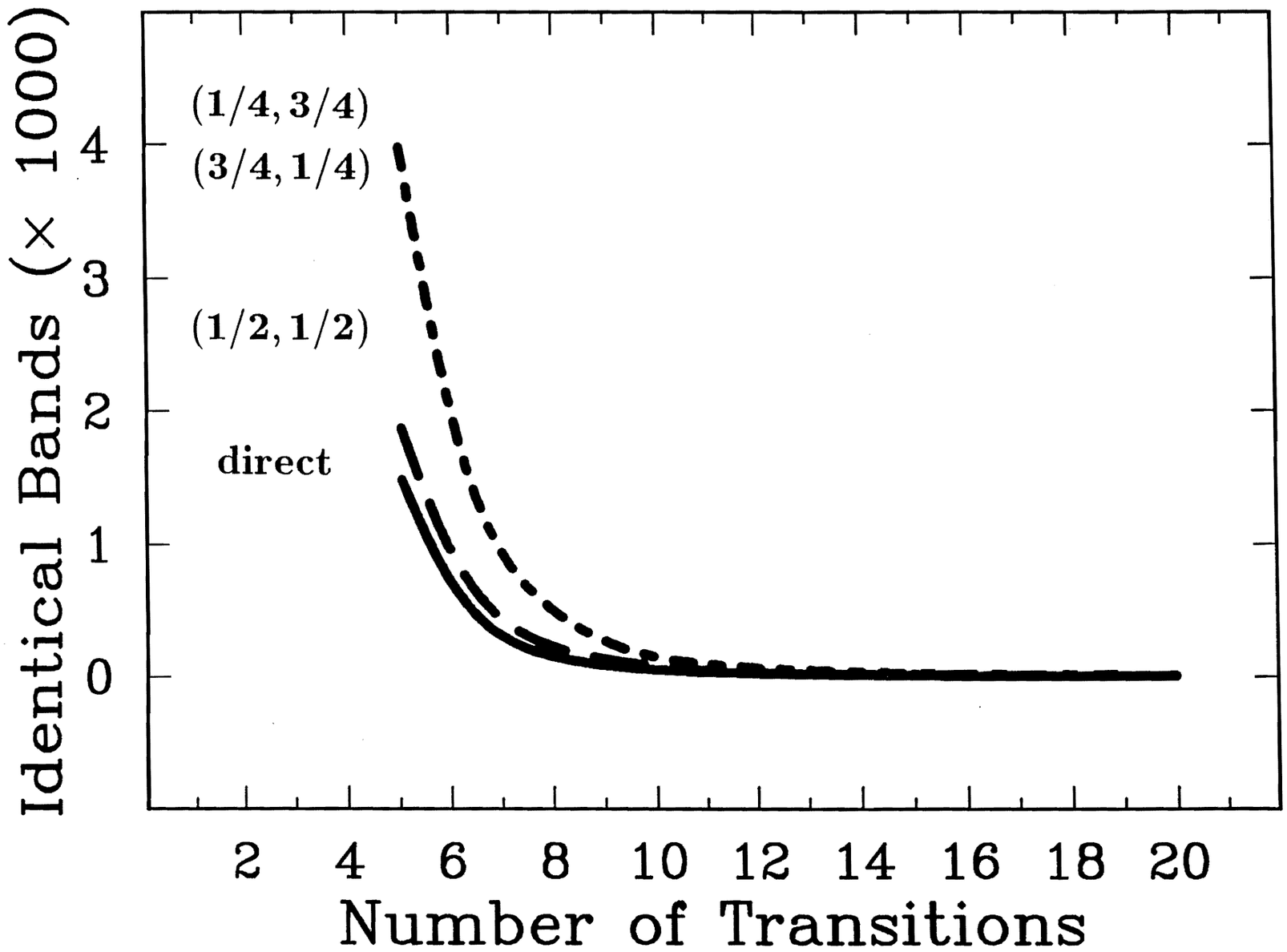
Dependence of the number of "identical bands" on the number of transitions using the criterion $|\Delta J_{AB}^{(2)}| < \varepsilon$ for $\varepsilon = (1,\ 2,\ 3$ and $4)\,\hbar^2\,\mathrm{MeV}^{-1}$. The total number of pairs of bands for $N = 6$ and $N = 7$ particles is given by the product of dimensionalities of the corresponding hamiltonians $\mathcal{N}_6 \cdot \mathcal{N}_7 = 924 \cdot 792 = 660\,708$. It should be noted in particular that the low-lying "identical bands" are numerous.

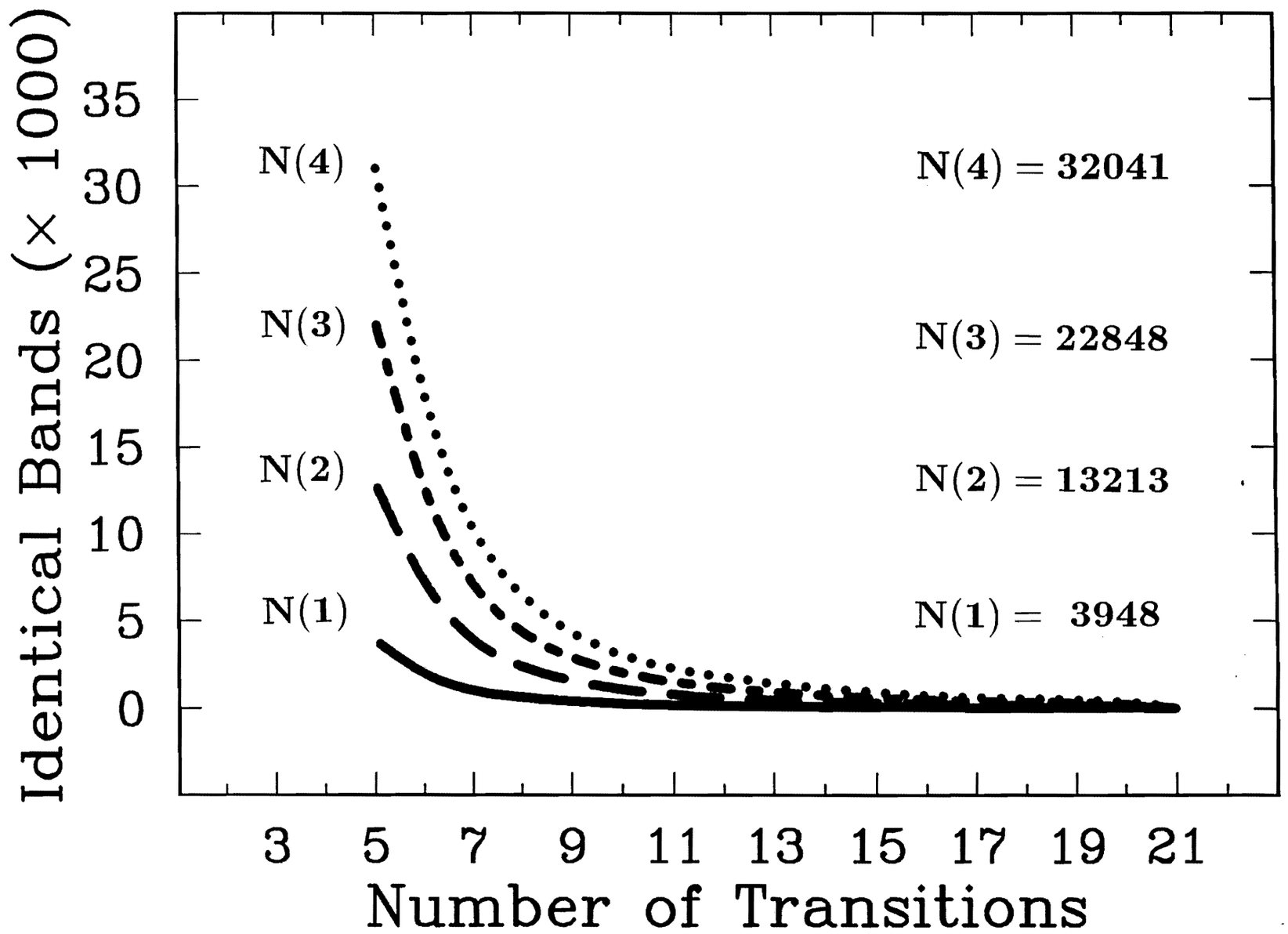
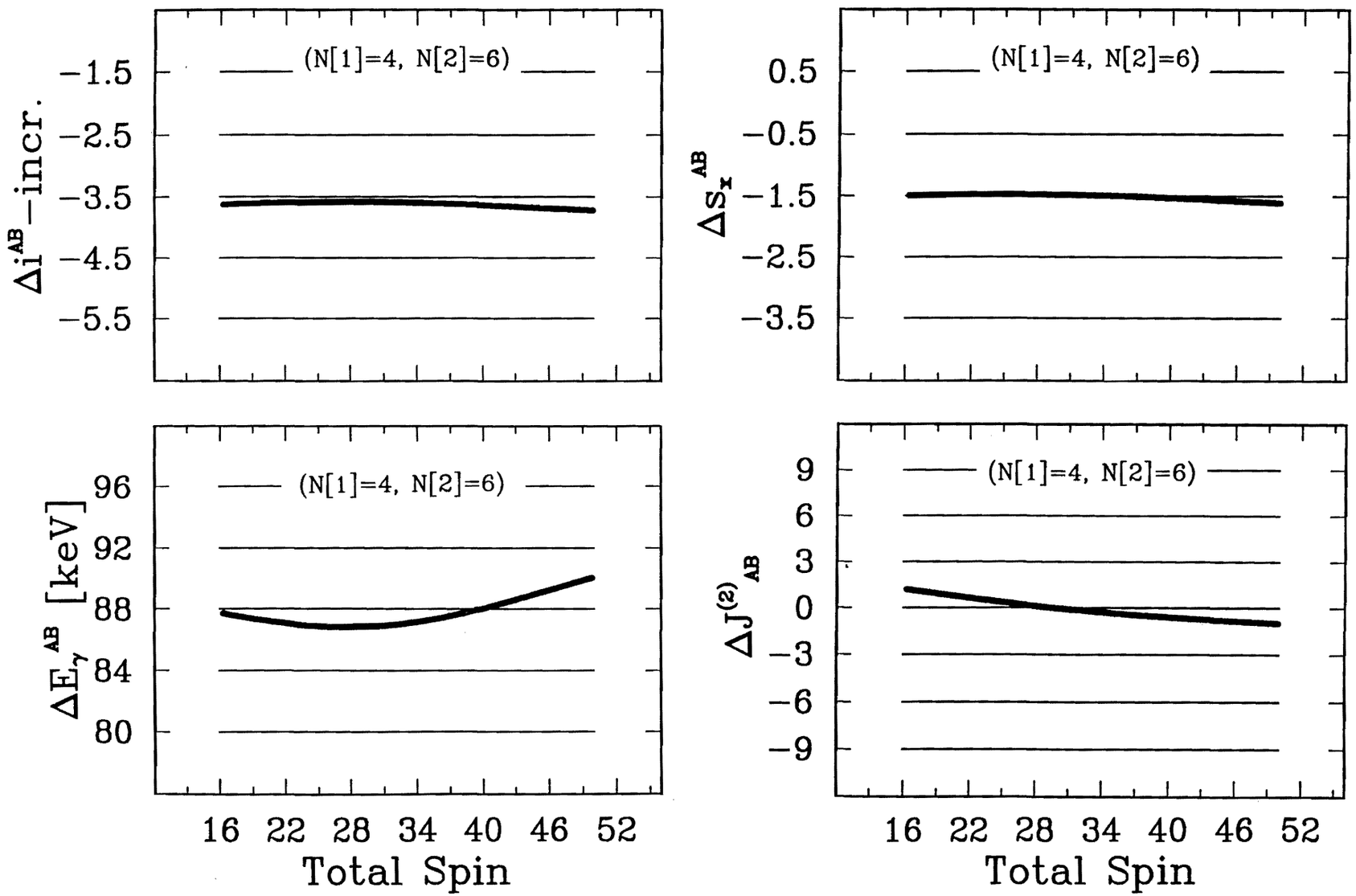
Fig. 3

An example of the "twinned band" mechanism for the yrast solutions corresponding to the systems with $N = 6$ and $N = 7$ valence particles calculated with the particles-rotor hamiltonian. The $|\Delta J_{AB}^{(2)}| < \varepsilon = 2\,\hbar^2\,\mathrm{MeV}^{-1}$ criterion is very well satisfied while $\Delta E_{\gamma} \simeq 189$ keV, on the average, over ten consecutive transitions. Observe that $\Delta s_x^{AB} \equiv j_x^{(A)}(\omega) - j_x^{(B)}(\omega)$ is "quantized" with a very good approximation and satisfies $\Delta s_x^{AB} \simeq 1\,\hbar$, while the incremental alignment (as defined in Refs.[2,7]) is also quantised with a comparable accuracy. Note that although the two systems considered have *even* and *odd* particle numbers, the "quantisation" expresses itself in terms of *integer* numbers. In this example both compared configurations are the yrast ones.

Fig. 4

Similar to that in Fig. 3 but for "even systems" with $N = 4$ and $N = 6$ particles, respectively. Note that both the incremental alignment Δi^{AB} and the relative alignment Δs_x^{AB} are quantised (to a very good approximation) but in terms of *half-integer* numbers. (Of course in other examples, with different band combinations, one finds also the integer-number quantisation for the even systems).


(1/4, 3/4)
(3/4, 1/4)
(1/2, 1/2)
direct
Identical Bands (× 1000)
0
1
2
3
4
2
4
6
8
10
12
14
16
18
20
Number of Transitions


Identical Bands (× 1000)
35
30
25
20
15
10
5
0
N(4)
N(3)
N(2)
N(1)
N(4) = 32041
N(3) = 22848
N(2) = 13213
N(1) = 3948
3
5
7
9
11
13
15
17
19
21
Number of Transitions

Particles–Rotor: U(2j+1)