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Xunjun LI and Jerzy DUDEK

CENTRE DE RECHERCHES NUCLEAIRES
STRASBOURG

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LOUIS PASTEUR

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Xunjun LI and Jerzy DUDEK

Centre de Recherches Nucléaires, IN2P3–CNRS/Université Louis Pasteur
BP 20, F–67037 Strasbourg Cedex 2, France
CRN

Abstract

As it is well known, the single-nucleonic levels in a nucleus manifest either the Kramers degeneracy $d = 2$, or, if a nucleus is spherical, a trivial "magnetic" degeneracy $d = 2j + 1$. It will be shown using the results of the realistic total nuclear energy calculations that a possibility of a four-fold degenerate levels exists in a number of $N \sim 136$ isotones due to their high intrinsic symmetry. Those exotic states are predicted to be isomeric; they lie only a few hundreds of keV above the ground state. Other possible nuclear regions where the same mechanism may take place are indicated.

The *classical concept* of a nuclear deformation and that of a geometrical form of a nucleus have become important theoretical tools in testing various *quantum mechanisms* in nuclear structure. For example if there exists a symmetry of the nuclear field, consisting in its invariance with respect to $\hat{R}_x \equiv \exp(i\pi\hat{j}_x)$ operation (rotation of a coordinate frame through an angle of π around an axis, say $\mathcal{O}_x$ axis), it results in a conservation of the signature[1] quantum number, r. Such a symmetry implies the existence of two families of rotational bands characterised by $I = 0, 2, 4, \ldots$ and $I = 1, 3, 5, \ldots$ for an even-even nucleus (similarly $I = 1/2, 5/2, 9/2 \ldots$ and $I = 3/2, 7/2, 11/2, \ldots$ for an odd-A case). Conversely, by observing manifestations of those two types of bands in experiment the existence of the R_x-symmetry can be deemed.

In this article we suggest the possible existence of highly-symmetric shape-isomeric states in several isotones with $N \sim 136$. The underlying symmetry group is the spinor-group T_d. It has an attractive feature of implying the existence of three families of nucleonic multiplets: two of them double and one *quadruple* degenerate. It is precisely the latter degeneracy which is "unusual" or "exotic" in the nuclear context, the "usual" one, characteristic of deformed nuclear orbitals being the double (Kramers) degeneracy.

To introduce the mathematical arguments let us first present the results of the realistic nuclear total energy calculations performed by using Strutinsky method[2] with the Woods-Saxon Hamiltonian. The latter has been generalised (with respect to ref.[3] from which we take the "universal" set of parameters of the potential) to include spherical harmonics $Y_{\lambda\mu}$ with $\lambda = 3, 4, 5, \ldots$ and $-\lambda \leq \mu \leq +\lambda$ in the nuclear shape definition. The spherical harmonics define the nuclear surface Σ and therefore the underlying geometrical symmetry of the average field hamiltonian *via* the standard expansion (for details see e.g. ref.[3])

$$\Sigma : \; R(\vartheta, \varphi) = R_0 c(\{\alpha\}) \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\vartheta, \varphi) \right] . \tag{1}$$

In most of the calculations known in the literature the $\mu \neq 0$ components in the equation of the nuclear surface Σ, for $\lambda \geq 3$, have usually been neglected. Some selected combinations of the ($\lambda = 3$, $\mu \neq 0$) components in the nuclear shapes have been employed [4] in a search for the octupole correlations in the superdeformed nuclei of Hg-Pb region. Another study of $\mu \neq 0$ effects using a parametrisation that involves a nonaxiality parameter and limits the corresponding highly-dimensional space to a 2-dimensional nonlinear projection has been also presented in [5]. The expansion (1) using (α_{20}, $\alpha_{3\mu}$) dependence of the total energy surfaces to predict the existence of the $\alpha_{3\mu}$-unstable, superdeformed (SD) isomers was used in [6], while the discussion of similar symmetries in the metallic cluster context can be found in [7]. Here we are going to present for the first time the calculation results for $\lambda \leq 5$ with all the μ-values included in the total energy cross-sections.

Figure 1 illustrates the total energy surface for ${}^{222}_{86}\mathrm{Rn}_{136}$ nucleus in function of α_{20} and α_{32} deformation parameters indicating the coexistence between two minima: the ground-state one at $\alpha_{20} \approx 0.16$ and the excited one at $\alpha_{20} \sim 0$ and $\alpha_{32} \simeq 0.15$. The results for $N \sim 134 - 138$ and the neighboring isotones with $Z \sim 84 - 90$ are similar. This particular $\alpha_{3\mu\neq 0}$ projection has been selected since the corresponding isomeric

minimum lies lowest in energy. To present more precisely the total energy behaviour around this interesting minimum several energy cross-sections have been done.

Figure 2 presents the cross-sections of the total energy surface corresponding to the $\alpha_{3\mu}$, $\alpha_{4\mu}$ and $\alpha_{5\mu}$ dependence suggesting that the exotic octupole deformation $\alpha_{32} \simeq 0.15$ remains a dominating deformation component in the ^{222}Rn. A similar result remains true also for several neighboring nuclei.

Let us consider first an ideal case of a nucleus with $\alpha_{32} \neq 0$ and all other deformations vanishing (small deviations of α_{32} from zero will imply possible, small deviations from the ideal case prediction). Such a nucleus posesses, as one can easily show, a very high geometrical symmetry composed of 24 symmetry elements. They are, besides the trivial identity operation: three "signature-type" two-fold rotations about each of the principal axes, $\hat{R}_x(\pi)$, $\hat{R}_y(\pi)$ and $\hat{R}_z(\pi)$; six four-fold rotary-reflexion operations along the three principal axes, denoted sometimes $\hat{S}_x(\pi/4)$ and $\hat{S}_x(3\pi/4)$, $\hat{S}_y(\pi/4)$ and $\hat{S}_y(3\pi/4)$ and, $\hat{S}_z(\pi/4)$ and $\hat{S}_z(3\pi/4)$; eight three-fold symmetry axes passing through the center of the nucleus and finally six plane-reflexions.

These symmetry elements form a classical group traditionally denoted T_d. To insure that the fermion transformation properties are satisfied in the *quantum case*, one usually introduces[8] a special "symmetry" element $\hat{Q}$ such that $\hat{Q}^2 = 1$. With the help of this special element one requires that e.g. n-fold symmetry operation $\hat{R}^n(2\pi/n) = \hat{Q}$ and $\hat{R}^{2n}(2\pi/n) = \mathbb{1}$. The corresponding extended group composed of 25 elements is called "fermion T_d" or T_d^D (cf. ref.[8]). The T_d^D group posesses 3 irreducible representations (irreps), one 4-dimensional and two non-equivalent two dimensional ones.

To illustrate the differences in the intrinsic structure of the nucleonic wave functions corresponding to those three irreducible representations and therefore also three distinct symmetries, let us schematize a construction of the related irreps using the $\{|n\,\ell\,j\,m>\}$ basis, one of the most standard ones in nuclear structure physics. For that purpose we define a coordinate system in such a way that the z-axis coincides with one of the three-fold axes and remains at the same time the m-projection axis for the $\{|n\,\ell\,j\,m>\}$ basis. We see immediately that the corresponding $R'_z(2\pi/3) = \exp[i(2\pi/3)\hat{j}'_z]$ operation reduces to a multiplication by the phase-factor $\exp[i(2\pi/3)m]$. After some transformations, and using the above phases, we arrive at distinguishing three subsets of the whole $\{|n\,\ell\,j\,m>\}$ ensemble: (set 1) $\{|n\,\ell\,j = \ell + 1/2,\, m = 1/2,\, 5/2,\, 7/2,\, 11/2,\, 13/2, \ldots\}$; (set 2): $\{|n\,\ell\,j = \ell - 1/2,\, m = 1/2,\, 5/2,\, 7/2,\, 11/2,\, 13/2, \ldots\}$ and (set 3): $\{|n\,\ell\,j,\, m = 3/2,\, 9/2,\, 15/2, \ldots>\}$. The ensembles (1) and (2) span the bases of two non-equivalent two-dimensional representations denoted E and E^* while some special combinations of all of them contribute to the G (four-dimensional) representation.

The results of the diagonalisation of the deformed Woods-Saxon hamiltonian are presented in Figure 3 for the neutrons where the single-particle levels are plotted in function of the α_{32} deformation. A similar illustration for the protons is presented in Figure 4. Both figures suggest that the isomerism of the α_{32}-type illustrated in the present article may repeat itself for the proton-neutron (Z-vs.-N) combinations corresponding to $Z \sim 56$, 70, 90 and similarly $N \sim 70$, 90, 112, 136. In addition, the shell closures at $Z = 50$, 64, 82 and 100 and those at $N = 64$, 82, 100 and 126 are likely to be unstable or susceptible to the α_{32} deformation thus increasing the chance

of observing some manifestations of the T_d^D symmetry in nuclei. The corresponding detailed calculations are in progress.

The figures 3 and 4 illustrate at the same time the content of the irreps of the T_d^D group in the nucleonic orbitals which, at the zero deformation, are labelled using the traditional spectroscopic notation. One can read for example that the $s_{1/2}$ and $p_{1/2}$ levels are related to the E and E^* representations respectively; the $h_{9/2}$ and $g_{9/2}$ orbitals both contain 2 members of the G-representation (8 states) while the remaining two states belong again to E (in $h_{9/2}$) and E^* (in $g_{9/2}$), etc.

In summary: An existence of an island of low lying nuclear shape isomers manifesting (at least approximately) the high-symmetry of the spinor-T_d group is predicted at $84 \lesssim Z \lesssim 90$, $N \sim 136$ nuclei. Similarly, a possible instability of the ${}^{146}_{64}\mathrm{Gd}_{82}$ with respect to α_{32}-exotic octupole deformation (which expresses the spinor-T_d symmetry) is suggested. The isomers of this symmetry should produce (in an ideal case) the single particle or quasiparticle spectra with an exact four-fold degeneracy.

Figure captions

Fig. 1
The total-energy surface calculated for the $^{222}Rn_{136}$ nucleus in function of the α_{20} and α_{32} deformations. (A comparison of the results in Figs. 3 and 4 indicates that *a priori* the best chances for the $\alpha_{32} \neq 0$ isomeric minimum occur at $Z = 90$ and $N = 136$ and not at $Z = 88$. Indeed, calculations show that the corresponding minima are deeper and the corresponding potential barriers are higher in Thorium as compared to Radium nuclei. However, those corresponding to the $Z = 90$ case are unstable with respect to the hexadecapole deformation and thus the conditions for the four-fold degeneracy of the nucleonic levels will not be optimal there).

Fig. 2
The total energy cross-sections in function of the $\alpha_{3\mu}$, $\alpha_{4\mu}$ and $\alpha_{5\mu}$ deformations at the $\alpha_{32} = 0.15$ local (isomeric) equilibrium characteristic for several isotopes and isotones of the Rn_{136} nucleus. Observe that the isomeric minimum in Fig. 2 turns out to be stable against any single-multipole distortion. (The couplings between various components have been studied and will be published elsewhere. The preliminary results confirm the tendency to stabilise the isomeric minimum in question.) Calculation results for α_{32} (not displayed) confirm also the stability against this degree of freedom as well.

Fig. 3
Neutron single-particle levels in function of the octupole α_{32} deformation. This diagram is illustrative for the isomeric minima like the one in Fig. 1 at $\alpha_{20} \sim 0$ and $\alpha_{32} \neq 0$. The solutions corresponding to the three irreducible representations are marked by G (four-fold degeneracy) – full lines: E (two-fold degeneracy) – dashed lines and E^* (a two-fold degeneracy related to an irrep. non equivalent to E) – dotted lines. Observe the gaps in the spectra corresponding to the strong octupole α_{32} effects at $N = 90$, 100, 112, 126 and 136.

Fig. 4
Similar to Fig. 3 but for the protons. The strong α_{32}-octupole effects are expected at $Z = 56$, 64, 70 and 90.

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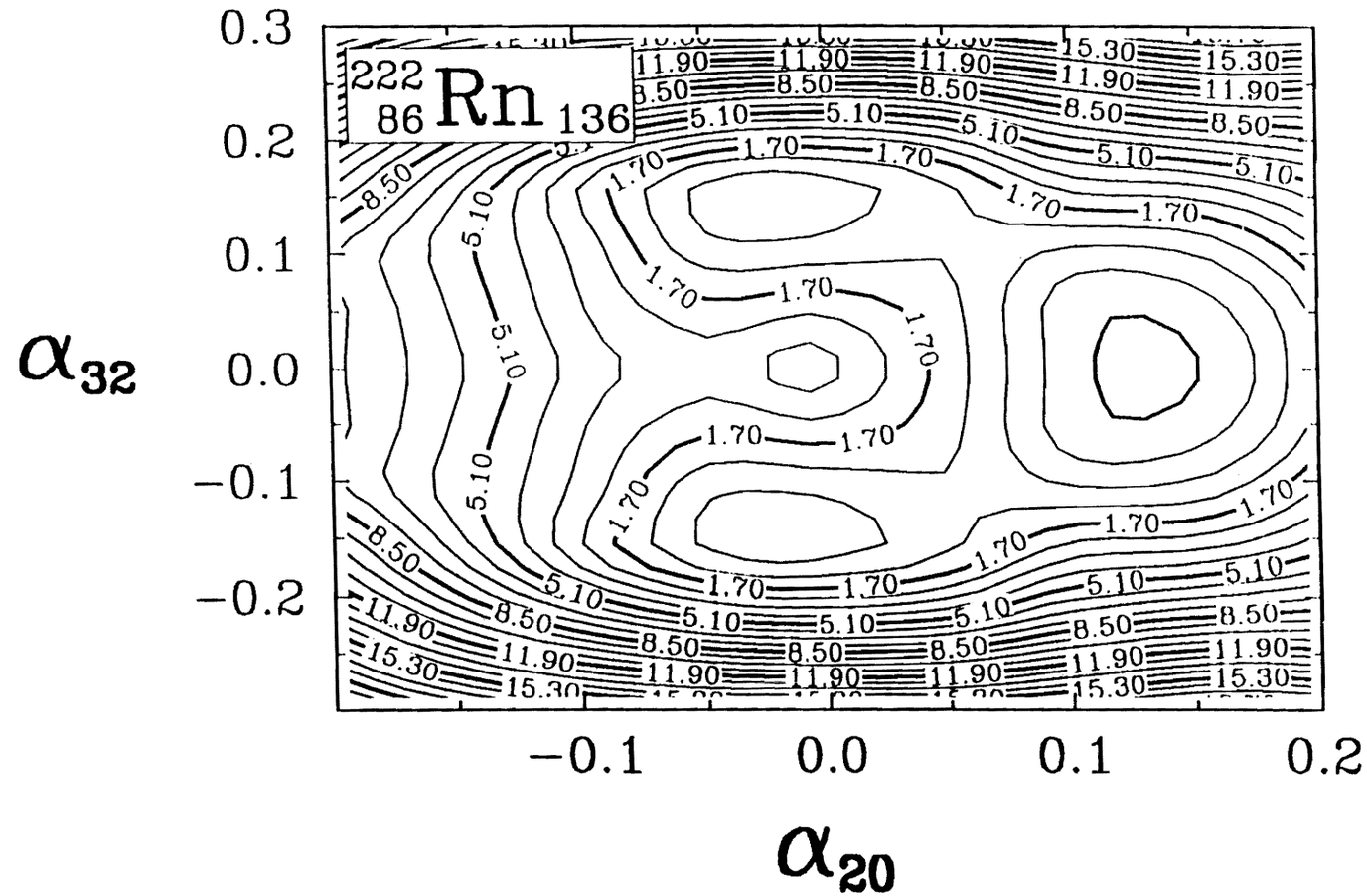
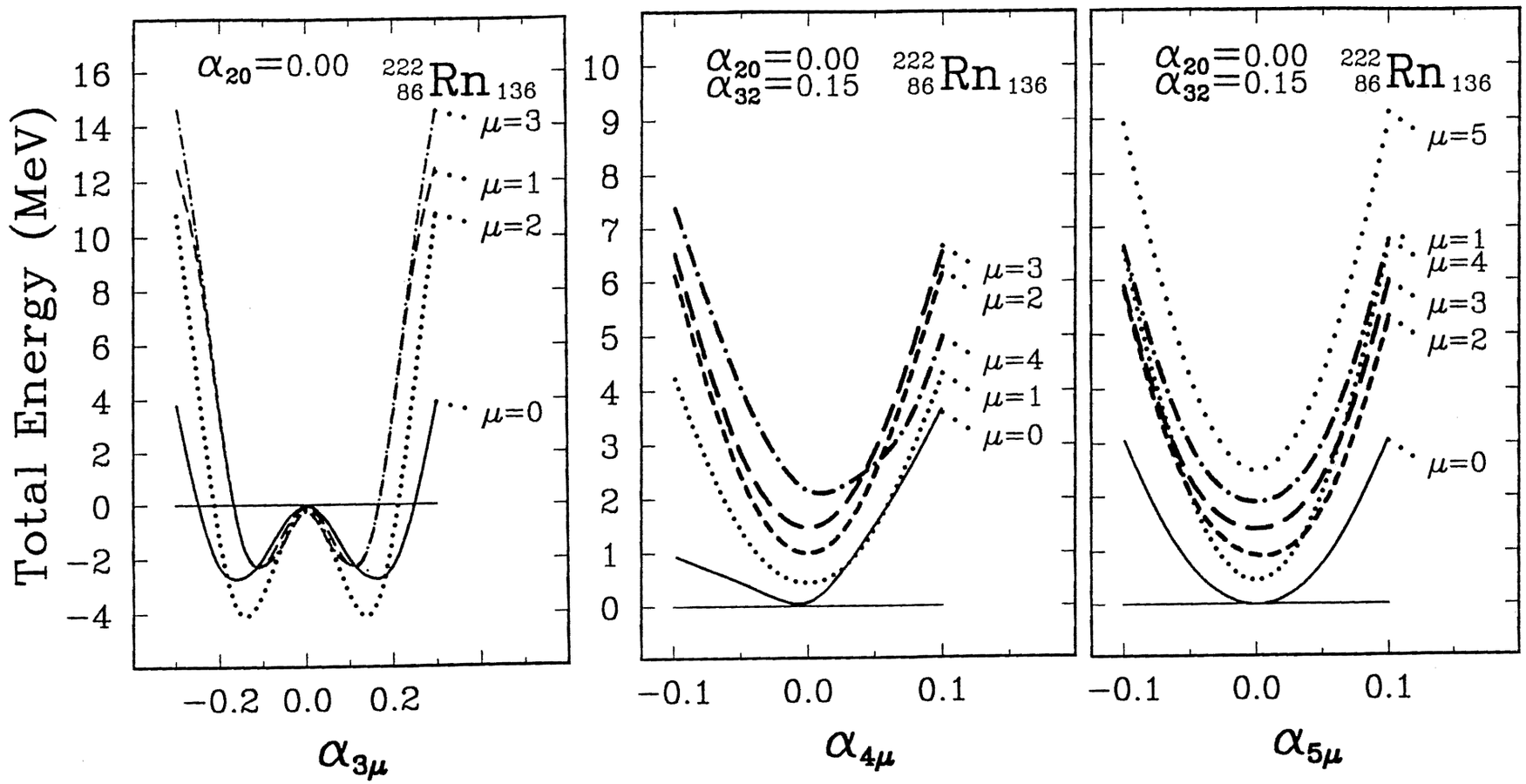
Total Energy Surface

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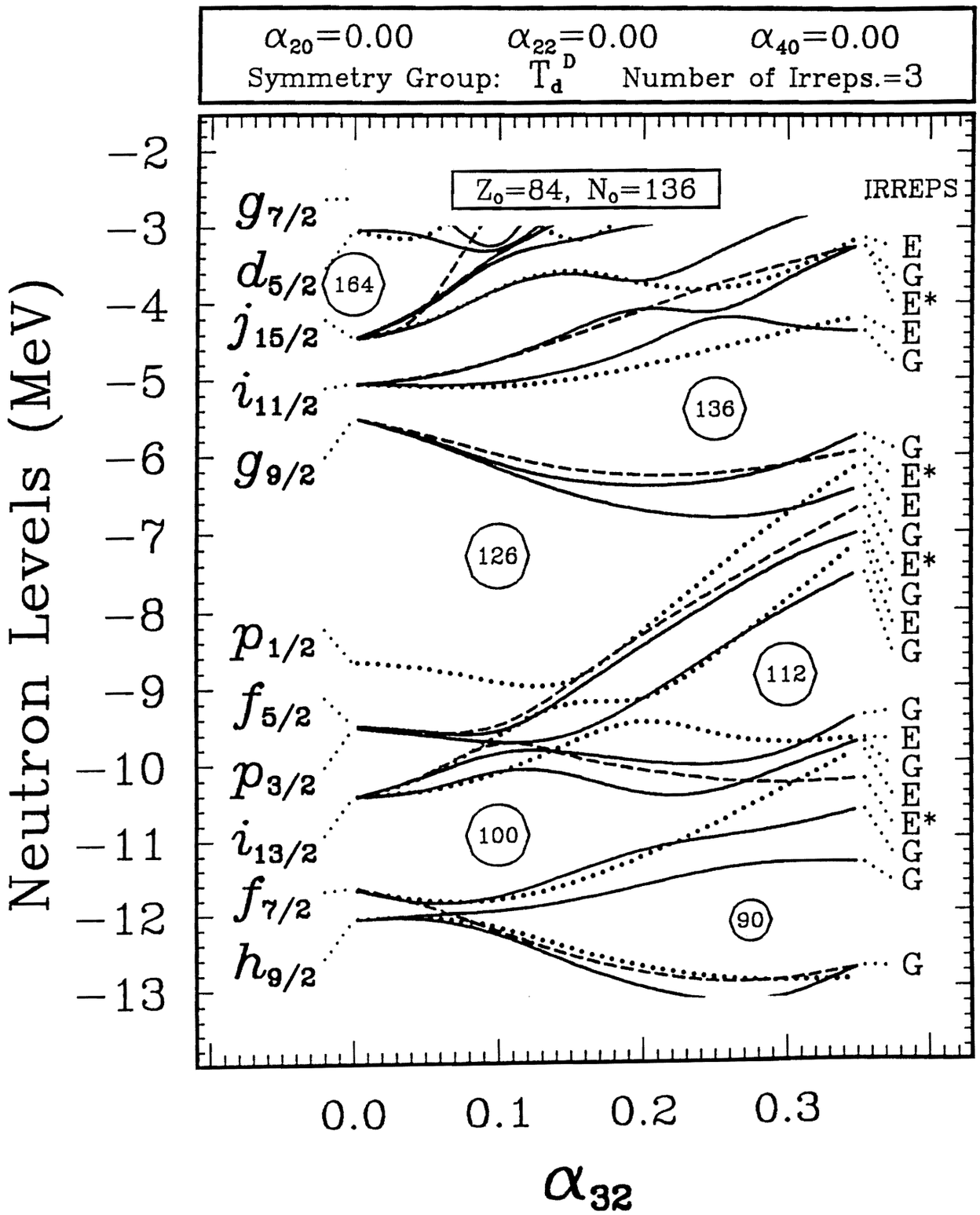
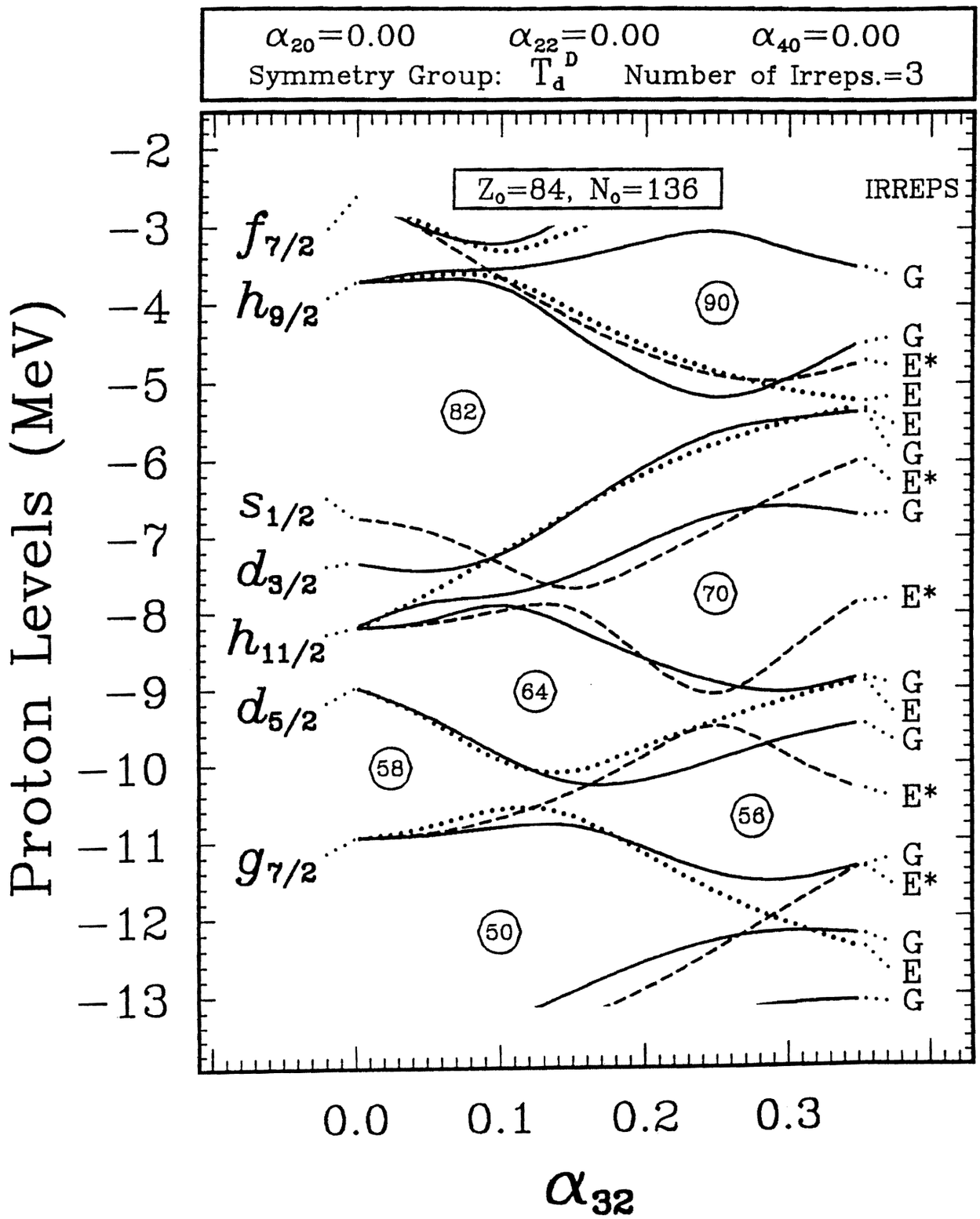
μ=3
μ=1
μ=2
μ=0
$\alpha_{3\mu}$
α_{20}=0.00
α_{32}=0.15
$^{222}_{86}$Rn$_{136}$
μ=3
μ=2
μ=4
μ=1
μ=0
$\alpha_{4\mu}$
α_{20}=0.00
α_{32}=0.15
$^{222}_{86}$Rn$_{136}$
μ=5
μ=1
μ=4
μ=3
μ=2
μ=0
$\alpha_{5\mu}$


α_{20}=0.00 α_{22}=0.00 α_{40}=0.00
Symmetry Group: T_d^D Number of Irreps.=3
Z_0=84, N_0=136
IRREPS
$g_{7/2}$
$d_{5/2}$
$j_{15/2}$
$i_{11/2}$
$g_{9/2}$
$p_{1/2}$
$f_{5/2}$
$p_{3/2}$
$i_{13/2}$
$f_{7/2}$
$h_{9/2}$
164
136
126
112
100
90
E
G
E*
E
G
G
E*
E
G
E*
G
E
G
G
E
G
E
E*
G
G
G
Neutron Levels (MeV)
−2
−3
−4
−5
−6
−7
−8
−9
−10
−11
−12
−13
0.0
0.1
0.2
0.3
α_{32}


α_{20}=0.00 α_{22}=0.00 α_{40}=0.00
Symmetry Group: T_d^D Number of Irreps.=3
Z_0=84, N_0=136
IRREPS
$f_{7/2}$
$h_{9/2}$
$s_{1/2}$
$d_{3/2}$
$h_{11/2}$
$d_{5/2}$
$g_{7/2}$
90
82
70
64
58
56
50
G
E*
E
Proton Levels (MeV)
−2
−3
−4
−5
−6
−7
−8
−9
−10
−11
−12
−13
0.0
0.1
0.2
0.3
α_{32}