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PERSISTENT INTERACTIONS OF QUANTUM FIELD THEORIES

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Abstract

A large class of quantum field theories, including spinor electrodynamics and self interacting gauge theories, have only persistent interactions. The coupling constants of the theories enter the fields only through the boundary conditions. The proof is independent of any method of solution of the field equations.

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Quantum field theory and its accompanying diagrammatic language has become an almost universal medium for the expression of ideas of microscopic physics (1). The quanta of these fields and the interactions which they experience are characterized by coupling constants, numerical parameters which describe the strength or relative probability with which a system undergoes a transition in which new degrees of freedom are energized or absorbed. These coupling constants appear in the field equations or interaction Hamiltonian or Langrange densities. In this note the detailed dependence of the fields on the coupling constants for several important field theories is shown to originate in the boundary conditions placed on the fields, independent of any particular method of solution of the field equations. Thus, the interactions persist into the future and

originate in the distant past if, as is typically the case, the boundary conditions are introduced at asymptotically large times.

As a first example consider the electromagnetic theory of a spin 1/2 object with minimal electromagnetic interaction. The field equations are

$$(i\gamma^{\mu}\partial_{\mu}-m)\psi \quad -eA^{\mu}\gamma_{\mu}\psi = 0 \tag{1}$$

$$\partial_{\mu}\partial^{\mu}A_{\nu} \quad = e\tilde{\psi}\,\gamma_{\nu}\psi \tag{2}$$

where e is the electric charge of the fermion. As is well known (1), e, the mass m and the fields A_{ν} and ψ must be renormalized in order to make the covariant scattering theory finite and sensible. That doesn't affect the

argument given here in any substantial way. These quantities can be regarded as unrenormalized or renormalized.

In addition to the field equations the theory also has equal time (anti) commutators as is appropriate:

$$\partial_t A_\nu(\vec{x},t)A_\mu(\vec{y},t) - A_\mu(\vec{y},t)\partial_t A_\nu(\vec{x},t) = P_{\mu\nu}(\vec{x},t)\delta(\vec{x}-\vec{y}) \tag{3}$$

$$\psi_\alpha(\vec{x},t)\psi^\dagger_\beta(\vec{y},t) + \psi^\dagger_\beta(\vec{y},t)\psi_\alpha(\vec{x},t) = \delta(\vec{x}-\vec{y})\delta_{\alpha\beta} \tag{4}$$

where $P_{\mu\nu}(\vec{x},t)$ is chosen to be consistent with the gauge conditions on A_μ .

In order to study the dependence of these fields on e introduce new fields

$$B_\mu = e^r A_\mu \tag{5}$$

$$\chi = e^s \psi \tag{6}$$

so that the field equations become

$$(i\partial_\mu\gamma^\mu - m)\chi - e^{1-r} B_\mu\gamma^\mu\chi = 0. \tag{7}$$

$$\partial_\mu\partial^\mu B_\nu = e^{1-2s+r} \tilde{\chi}\gamma_\nu\chi \tag{8}$$

Clearly, if $r=s=1$, the charge disappears from the field equations. The solutions χ and B_μ are independent of e unless it appears in the boundary conditions.

Consequently, the fields ψ and A_μ contain negative powers of e or contain e in their boundary conditions. In either case the interactions are persistent.

The dependence on negative powers of e is inconsistent with the (anti) commutation relations.

Reexpressing equations (3) and (4) in terms of B_μ and ψ gives

$$\partial_t B_\mu(\vec{x},t)B_\nu(\vec{y},t) - B_\nu(\vec{y},t)\partial_t B_\mu(\vec{x},t) = P_{\mu\nu}(\vec{x},t)e^2\delta(\vec{x}-\vec{y}) \tag{9}$$

$$\chi_\alpha(\vec{x},t)\chi_\beta^\dagger(\vec{y},t) + \chi_\beta^\dagger(\vec{y},t)\chi_\alpha(\vec{x},t) = \delta_{\alpha\beta}\delta(\vec{x}-\vec{y})e^2 \tag{10}$$

In both equation (9) and (10) the right side contains e while the left side is independent of e. Thus, this alternative is unsatisfactory. The conclusion is that the charge e enters the fields through the boundary conditions. The argument is entirely independent of any

method of construction of the solutions of the field equations.

Similar results obtain for a self interacting field such as Yang-Mills (2), for which the field equations are

$$\partial^{\mu}\partial_{\mu}W^{a}_{\nu} - \partial_{\mu}\partial_{\nu}W^{\mu a} + gf^{a}{}_{bc}\partial^{\mu}(W^{b}_{\mu}W^{c}_{\nu}) =$$

$$gf^{a}{}_{bc}\partial_{\mu}W^{b}_{\nu}W^{\mu c} - gf^{a}{}_{bc}\partial_{\nu}W^{b}_{\mu}W^{\mu c} + g^{2}f^{a}{}_{bc}f^{b}{}_{dv}W^{d}_{\mu}\,W^{v}_{\nu}W^{\mu c} \qquad (11)$$

where the latin indices range over the number of infinitesimal generators of the symmetry group and $f^{a}{}_{bc}$ are the structure constants of the group. The equal time commutators are similar in appearance to equation (3) but contain additional information on the right side referring to the gauge and group indices (3). [illegible] the coupling constant g disappears with the choice

$$\bar{W}^{a}_{\mu} = g^{-1}w^{a}_{\mu} \qquad (11)$$

so that we are presented with the same alternatives as before. Either W^a_μ is inversely proportional to g or g enters the fields in the boundary conditions. The former choice is equally unsatisfactory as the case of fermion electrodynamics since, constructing equal time commutators for the W^a_μ and their conjugate momentum fields gives g behavior on the right side of the equation and no g dependence on the left side. Thus, we conclude that the g dependence enters only through the boundary conditions. This conclusion is independent of any method of solution of the field equations.

The two examples given have dimensionless coupling

constants $e=(e^2/\hbar c)^{\frac{1}{2}}$ and $g=(g^2/\hbar c)^{\frac{1}{2}}$ for $\hbar=c=1$. However, it is easy to see that an interaction such as the four fermion interaction of the Fermi theory of weak interactions leads to similar results. Thus, for several important classes of quantum field theories (including spin zero mesons interacting with fermions, which except for the γ matrices has the same appearance as fermion electrodynamics) we see that the coupling constants must enter the theories through the boundary conditons. This conclusion is independent of the method of solution construction.

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3. Discussion of various gauges and quantization can be found in R. Gaigg, W. Kummer and M. Schweda, Physical and Nonstandard Gauges (Springer, Heidelberg 1990). In particular, for a discussion of the inequivalence of path integral quantization and canonical quantization see the paper of H.Cheng in this volume.