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Abstract.

Accreting white dwarfs have long been considered as candidates for Type Ia supernova progenitors. With the discovery of luminous supersoft X-ray sources (LSXSs), a new class of white dwarf accretor has been proposed as a candidate progenitor system. This class, close-binary supersoft sources (CBSSs) is studied here. We combine a population synthesis analysis with detailed evolutionary calculations for individual systems. We find that there are uncertainties related to the possibility that some of these systems may experience a common envelope phase. Systems that do experience a common envelope that is fatal to further hydrogen accretion, can be expected to then undergo either C-O/He core mergers, or else a subsequent epoch of helium accretion onto a C-O white dwarf.

The evolutionary calculations are essential to a meaningful computation of the rate of possible Type Ia explosions, and to reliable estimates of the uncertainties. In addition, the evolutionary calculations allow us to compute the amount of matter ejected prior to the explosion, and the history of local ionization due to the source. Some secondary characteristics of the supernova can therefore be predicted. Further, the possible existence of a pre-supernova nebula may allow surveys for nebulae in distant galaxies to successfully identify some progenitors.

1. Luminous Supersoft X-Ray Sources as Type Ia Progenitors

Accreting white dwarfs have long been recognized as possible Type Ia supernova progenitors. (See, e.g., Whelan & Iben 1973, Iben *et al.* 1987, Tutukov, Yungelson, & Iben 1992, Rappaport, Di Stefano & Smith 1994, Branch *et al.* 1995, Wheeler 1996, and references therein.) Yet, in spite of birthrates more than high enough to accommodate the observed supernova rates, some white dwarf accretor scenarios have been discovered to have problems which may prevent the white dwarf from accreting sufficient matter. In cataclysmic variables (CVs), for example, a low mass donor contributes matter at low to moderate rates. Generally these accretion rates are low enough that the matter doesn't burn as it accumulates. Instead, after the build-up of ΔM (a function of the white dwarf mass, M, and the accretion rate, $\dot{M}$), a nova explosion occurs. During the explosion, some of the accreted matter is ejected from the system. Indeed, it has been conjectured that in some cases, even more matter than was accreted can be ejected. (See Prialnik & Livio 1995 and references therein.) It would therefore seem that CVs have problems of two sorts if they are to become Type Ia supernovae. First, their companions are not rich in mass to donate; second, they have an unfortunate tendency to lose whatever matter is proffered. Symbiotics are a second class of accreting-white-dwarf binary that has been well-studied. In at least one subset of these systems, a giant donor of moderate mass donates matter to a white dwarf companion via a stellar wind. Although often the donor could, in principle, contribute enough mass to bring the white dwarf over the Chandrasekhar limit, the capture fractions tend to be small and the lifetime of the mass transfer phase is short (Kenyon *et al.* 1993). Yungelson et al. (1995) have shown that symbiotics are likely to make a negligible contribution to the rate of Type Ia supernovae if the white dwarf needs to achieve the Chandrasekhar mass in order for an explosion to occur. Even if the accretion of as little as $0.1 M_{\odot}$ could lead to a supernova, symbiotics can account for at most 1/3 of the rate inferred from observations.

ROSAT's identification of a new class of X-ray source, luminous supersoft X-ray sources (LSXSs), has led to the study of a new class of white-dwarf-accretor. LSXSs are thought to be white dwarf accretors because the apparently thermal spectra of the sources, characterized by effective temperatures on the order of tens of eV, and the bolometric luminosities ($\sim 10^{37} - 10^{38}$ ergs s^{-1}), suggest effective radii comparable to that typical of white dwarfs. (See, e.g., Hasinger 1994 for references.) Yet, although some LSXSs are now known to be either symbiotic novae or novae in short orbital period binaries (and one is apparently a planetary nebula), several of the first LSXSs to be studied were clearly typical neither of CVs nor

symbiotics. Instead, consistent models could be constructed in which the observed luminosity was provided by the steady nuclear burning of matter as it accreted through Roche-lobe overflow from a more massive ($\sim$ a few $M_\odot$), slightly evolved companion in a close ($\sim$ days) orbit. van den Heuvel *et al.* (1992; vdHBNR) worked out such a model, and considered the systems as possible progenitors for accretion-induced collapse. Rappaport, Di Stefano, & Smith (1994; RDS) worked out the details necessary to perform a population synthesis study, and considered the systems as possible Type Ia progenitors. Six of the LSXSs with optical identifications have properties consistent with those predicted by the CBSS model.

CBSSs are promising Type Ia progenitor candidates because the mass of the donor is typically large enough that the Chandrasekhar-mass explosion of its white dwarf companion would be possible, even if some mass escapes the system. Also, because the transferred mass is funneled through the L1 point, the capture fraction is large, compared to wind-mediated mass transfer systems. Most important of all is the fact that the rate of mass transfer can be compatible with steady nuclear burning of the accreted matter, allowing the white dwarf to grow in mass. In the course of their population synthesis calculations, RDS computed the rate of Type Ia supernovae that might be associated with CBSSs. They found that, depending on model assumptions, a broad range of rates could be accommodated, including numbers roughly consistent with the rate inferred from observations. Their uncertainty limits were large because of uncertainties about fundamental physical processes that determine the evolution of these systems. Here we describe ongoing efforts to understand the evolutionary paths open to CBSSs and to narrow the uncertainty limits.

In addition to being perhaps the most promising class of binary to become supernovae through a white dwarf's accretion of hydrogen, some of the channels in CBSS evolution that do not lead to successful hydrogen-accretion supernovae do lead to candidates for supernovae or other types of explosion through helium accretion, or through C-O/He mergers, for example. We therefore keep track of the numbers of systems that follow each channel of CBSS evolution.

2. Key Elements of the Computation

Our approach involves two steps. First we perform a population synthesis analysis to identify systems that might pass through an epoch in which they exhibit the behavior that defines close-binary supersoft sources. Second, we take each CBSS candidate and carry out a detailed binary evolution calculation. As RDS pointed out, this second step is crucial to a reliable estimate of the rate of possible Type Ia explosions associated with the CBSS

model. We demonstrate in detail the difficulties that must be addressed in order to perform the evolutionary calculations, which for these systems are complicated by the fact that the donor is more massive than the white dwarf, and may also be slightly evolved. Thus, the work described here leads to significant improvements over estimates which have not addressed the key evolutionary issues. (See, e.g., the summary in Branch *et al.* 1995.)

2.1. POPULATION SYNTHESIS STUDY

To compute the rate of Type Ia supernovae associated with CBSSs, we must first compute the rate at which CBSSs are formed. Our assumption is that these sources evolve from primordial binaries. The evolutionary pathways via which a primordial binary can become a CBSS were mapped out in RDS. Here they will be briefly sketched.

In order for a primordial binary to become a CBSS, the primordial primary must become a white dwarf, and the primordial secondary, of mass m, must be close enough to its evolved white dwarf companion that it can overfill its Roche-lobe while itself becoming only slightly evolved. This suggests the following evolutionary sequence. (i) The primary overfills its Roche lobe when it is evolved enough that its envelope is convective; (ii) Roche lobe overflow of the primary leads to a common envelope; (iii) during the common envelope phase the secondary spirals closer to the core of the primary; (iv) at the end of the common envelope phase, m is larger than the mass of the primary's white dwarf remnant, M, and the orbital period is on the order of a day; (v) the evolution of the secondary, in combination with magnetic braking, causes it to overfill its Roche lobe; (vi) there is an epoch of mass transfer that takes place on a time scale roughly governed by the thermal time scale of the donor; (vii) the rate of mass transfer is such that, at least until the masses equalize, hydrogen accreted by the white dwarf can be steadily burned.

As one might imagine, there are uncertainties associated with every step. For those stages of the evolution that precede the final epoch of mass transfer (from the donor to the white dwarf), these include: our uncertainties about the distributions of properties among primordial binaries; the efficiency with which the common envelope can be ejected; the magnitude of the influence of magnetic braking. RDS explored the parameter space associated with all these effects and found that only the assumption that the secondary mass should be chosen from the IMF independently of the primary mass led to a relatively small number of systems. The reason for this is clear: m must be greater than the white dwarf mass and, if it is to evolve in a Hubble time, must be greater than $0.8M_\odot$; yet, when m is chosen from the IMF, masses smaller than $0.8M_\odot$ are favored. Such an in-

dependent choice, however, does not seem to be favored by observations of primordial binaries; we therefore do not explicitly consider it here. We do however, compute the number and properties (m, M, P_{orb}) of the candidate CBSSs that are generated by each of a set of 6 Monte Carlo simulations. Each simulation is defined by a specific set of assumptions about the distribution of properties among primordial binaries, together with assumptions about dynamical processes, such as magnetic braking and the termination of the common envelope phase. Every primordial binary passing through the complete sequence outlined above is a possible supersoft source (CBSS candidate). If we assume that stellar systems with mass between 1.8 and $8.0 M_\odot$ form in the Galaxy at a rate of 0.3 per year, and that half of the stars in this mass range are the more massive components of binaries, we find an average birthrate for CBSS candidates of $\sim$ 0.5 per century. The rate is doubled to 1 per century if we make contact with other population synthesis studies in which all primordial systems are assumed to be binaries (e.g., Yungelson *et al.* 1995). Whether or not the candidate systems pass through an epoch during which the mass transfer rate is consistent with steady nuclear burning, how long any such epoch will last, and whether or not the white dwarf will eventually explode, depend on the details of their further evolution. RDS were unable to follow this further evolution in detail. In this paper we outline the problems and begin to address them.

2.2. ORBITAL EVOLUTION

We assume that the donor continuously fills its Roche lobe. Its radius, r, can therefore be expressed as $f(q)a$, where $q \equiv m/M$, a is the orbital separation, and

$$f(q) = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} \,. \tag{1a}$$

Orbital angular momentum is drained from the system through the certain action of gravitational radiation, and the less certain action of magnetic braking. [Note that the variables in equation (1*b*) are given in solar units.]

$$\frac{\dot{J}_{GR}}{J} = -8.3 \times 10^{-10} \left[\frac{mMM_T}{a^4}\right] \quad \mathrm{yr}^{-1} \,;$$

$$\frac{\dot{J}_{MB}}{J} = -2.3 \times 10^{-6} f_{MB} \left[\frac{M_T^2\, r^\gamma}{M\, a^5}\right] \quad \mathrm{yr}^{-1} \,. \tag{1b}$$

where f_{MB} is a function that accounts for the change in the moment of inertia of the donor. Our uncertainty about the mechanism of magnetic braking is expressed through inclusion of the adjustable parameter γ; in the simulations described in §4, we have taken $\gamma = 4$. If, during the mass

transfer process, some mass is also ejected from the system, the ejected mass can be expected to carry angular momentum with it. Let β denote the fraction of the mass lost by the donor which is retained by the white dwarf. Then the rate of loss of angular momentum due to mass ejection can be written as

$$\frac{\dot{J}_{ej}}{J} = \frac{\dot{m}}{m}\frac{(1-\beta)q^{\eta}}{(1+q)} . \tag{1c}$$

$\dot{J}_{ej}$ is the rate at which angular momentum is carried from the system by the ejected matter. If the ejected mass carries the specific angular momentum of the white dwarf [donor], $\eta = 2$ [$\eta = 0$]. We have performed simulations for different values of η.

Invoking conservation of total angular momentum, we may derive an expression for $\dot{m}/m$.

$$\frac{\dot{m}}{m}\mathcal{D} = \mathcal{N} . \tag{2a}$$

$\mathcal{N}$ is a function of $\dot{J}_{GR}/J, \dot{J}_{MB}/J$, and the fractional change in r due to both nuclear and thermal processes. $\mathcal{D}$ is a function of q, β, η (a parameter related to the amount of angular momentum carried away by ejected matter), and the adiabatic index, ξ_{ad}. The value of ξ_{ad} determines whether the donor expands or shrinks in response to adiabatic mass loss: $\xi_{ad} = (d[\log(r)]/d[\log(m)])_{ad}$.

$$\mathcal{D} = \xi_{ad} + 2(1-\beta q) - q(1+\beta q)\frac{d\ln f}{d\ln q} + \frac{(1-\beta)}{(1+q)}(q - 2q^{\eta}) . \tag{2b}$$

Typically, one computes $\dot{m}$ by choosing a value of β consistent with the accretor's ability to retain mass, and then choosing the corresponding appropriate value of η. If, for example, the accretor is a white dwarf, and $\dot{m}$ is small, we would expect that over the duration of typical time steps, all of the matter is ejected via nova explosions; thus, $\beta = 0$, $\eta = 2$. If q and ξ_{ad} are known, $\mathcal{D}$ can be evaluated. If $\mathcal{D} > 0$, then dividing both sides of equation (2a) by $\mathcal{D}$ determines $\dot{m}$.

One conceptual problem with this scheme is that β is not always known *a priori*; it can be a function of $\dot{m}$, the quantity one is attempting to compute. A further problem is that, if we view the set of quantities $\{q, \xi_{ad}, \beta, \eta\}$ as defining a parameter space, there are regions in this space where $\mathcal{D} \leq 0$. In such regions, the formalism that assumes that the donor can continuously fill its Roche lobe breaks down. Physically, the Roche lobe is shrinking relative to the surface of the donor. This leads to a dynamical instability which may itself lead to a common envelope. Regions in which $\mathcal{D} \leq 0$ tend to be characterized by large values of q and small values of ξ_{ad}. For CBSSs, q is greater than unity, typically ranging between 1.5 and 10. ξ_{ad} can be

estimated through comparisons with Henyey-like calculations. We find that a formula:

$$\xi_{ad} = \tilde{\xi}_{ad}\left[1 - \left(\frac{m_c}{\tilde{m}_c}\right)^2\right] , \tag{3}$$

seems to fit the numerical data well, with the preferred values for $\tilde{\xi}_{ad}$ and $\tilde{m}_c$ equal to 4.0 and 0.20, respectively. Technically, $\tilde{m}_c$ depends on the value of m, but its dependence is relatively weak and was not implemented in this calculation. Since the most massive cores found among donors in CBSSs are close to $0.2M_\odot$, ξ_{ad} can be small. Thus, CBSSs are at high risk for having the Roche-lobe-filling formalism break down. Note however, that even though the Roche lobe shrinks with mass transfer, for a significant subset of CBSSs, ξ_{ad} is large enough that the donor can shrink enough on comparable time scales to allow us to use the Roche-lobe-filling formalism.

In general, unless $\eta > 2$, the value of $\mathcal{D}$ decreases as β increases. Although it is possible that in some novae, more matter is ejected in an explosion than has been accreted since the last explosion, we take our lower limit to be $\beta = 0$. Thus, any systems for which $\mathcal{D} \leq 0$ when $\beta = 0$ cannot be evolved using the Roche-lobe-filling formalism.

We define β_{crit} to be the maximum value of β for which $\mathcal{D} > 0$. Each CBSS candidate falls into one of three categories: Class I ($\beta_{crit} < 0$); Class II ($0 \leq \beta_{crit} \leq 1$), Class III ($\beta_{crit} > 1$). With $\tilde{\xi}_{ad} = 4$ and $\tilde{m}_c = 0.2$, we find that, typically, $\sim \frac{1}{3} - \frac{1}{2}$ of our CBSS candidates are in Class I; these systems cannot be evolved using Eqs. (1) and (2). Another $\sim 20\%$ fall into Class II. Systems in class II can be evolved only as long as $\beta < \beta_{crit}$. The remaining systems fall into Class III, and can be successfully evolved throughout by using equations (1) and (2). The systems in class III tend to be characterized by smaller values of q and less evolved donors; hence, the mass transfer rates tend to be too low to lead to significant accretion. Figure 1 illustrates the evolution of the relevant time scales for a system in class III.

For the computation of the supernova rate, understanding how to deal with the first two formalism-defying categories is crucial. The reason for this is that such systems tend to be characterized by larger values of q; thus the donor can, in principle, donate more matter to the white dwarf.

3. Computing the Retention Factor

Because β plays an important role in determining the outcome of the calculations, we devote special attention to the physical considerations that determine its value. There are three that are of particular interest. The first two are directly related to the response of the white dwarf to accreted matter. (1) When hydrogen is incident on the white dwarf surface, will it

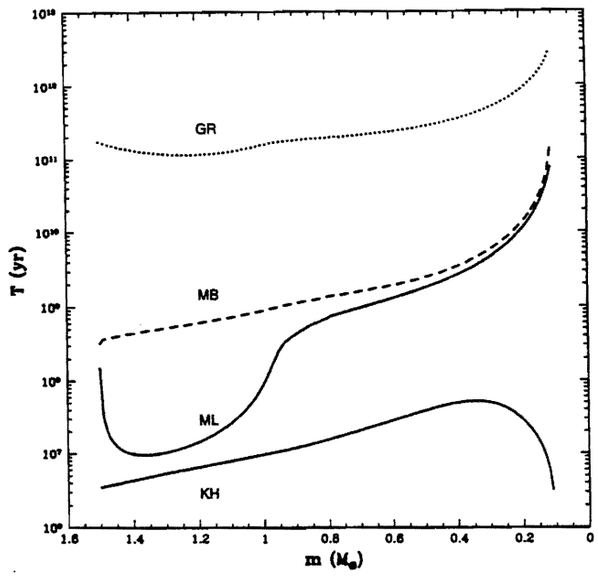

Figure 1. The evolution of the relevant time scales of a binary system initially consisting of a $1.5M_\odot$ donor star that has exhausted its central hydrogen and a $1.0M_\odot$ accretor. The labels corresponding to each curve denote the following time scales: GR – gravitational radiation ($J/\dot{J}_{GR}$); MB – magnetic braking ($J/\dot{J}_{MB}$); ML – mass loss ($m/\dot{m}$); and KH – Kelvin-Helmholtz (thermal). Mass transfer from the secondary initially proceeds on its Kelvin time until the mass ratio (q) of the components is approximately unity. After this point, the rate of mass-loss from the secondary is largely governed by the magnitude of magnetic braking. Nuclear evolution causes the orbital period to increase during the final stages of the binary evolution.

burn sporadically (and then possibly be lost through hydrodynamic events associated with nova explosions), or will it burn more-or-less steadily, and therefore likely be retained? (2) What fraction of the donated matter actually makes it down to the surface of the white dwarf? For example, the steady burning of matter will generate radiation-driven winds which eject some of the donated hydrogen before it reaches the white dwarf surface. (3) The third physical consideration comes from the orbital evolution of the binary during mass transfer, since smaller values of β are more likely to be associated with positive values of $\mathcal{D}$.

3.1. WHAT FRACTION OF $\dot{M}$ CAN THE WHITE DWARF BURN?

The physics of the accretion and burning of hydrogen on a white dwarf has been studied by a number of authors. (See, for example, Paczyński 1970; Sion, Acierno, & Tomcszyk 1979; Taam 1980; Nomoto 1982; Iben

1982; Fujimoto 1982; Fujimoto and Sugimoto 1982; Fujimoto & Taam 1982; Fujimoto and Truran 1982; Sion & Starrfield 1986; Livio, Prialnik, & Regev 1989; Prialnik & Kovetz 1995)

Their results generally indicate that there is a critical range of values of accretion rates, $\dot{m}_{min}(M) < \dot{m} < \dot{m}_{max}(M)$, within which accreted mass can be burned steadily, thereby steadily increasing the mass of the white dwarf. Although the values of $\dot{m}_{min}$ and $\dot{m}_{max}$ are mass-dependent, they each have values on the order of $10^{-7} M_{\odot}$ yr^{-1}. For $\dot{m} < \dot{m}_{min}$, matter will burn sporadically in nova explosions. There are indications that at least some of the accreted matter (and in some cases even more matter than was accreted since the last explosion) is blown away during the nova explosion. (See, e.g., Prialnik & Livio 1995 and references therein.) For $\dot{m} > \dot{m}_{max}$, not all of the accreted matter can burn. It is an open question whether the excess matter will begin to form an envelope around the white dwarf which could possibly continue to grow until the white dwarf itself fills its Roche lobe, or whether the excess matter can be ejected. vdHBNR suggested that an envelope might form, but that once the white dwarf filled its Roche lobe, mass donation might stop for some time, leading to a sort of limit cycle behavior.

3.2. WHAT FRACTION OF $\dot{M}$ CAN THE WHITE DWARF EJECT?

Even for $\dot{m}_{min} < \dot{m} < \dot{m}_{max}$, i.e., in the 'steady burning region', it seems unlikely that all of the donated matter can be burned. (See, e.g., Livio, M. 1995, Fujimoto & Taam 1982, Kato, Saio, & Hachisu 1989, Shara & Prialnik 1994.) Because the luminosity is close to the Eddington limit, radiation pressure will be large and some of the incident matter is likely to be ejected in the form of a wind. Mass may also be lost due to helium shell flashes. Thus, the upper limit for β is unity only under the most conservative of assumptions. A more realistic upper limit is almost certainly lower.

The physical considerations that place a lower limit on β are the amount of energy available to eject matter, and the fraction of the available energy that can be harnessed for that purpose. Energy for ejection can come from nuclear burning of the matter retained, for example. Let Φ be the gravitational potential energy of ejected mass before it leaves the system. If we consider only the energy associated with nuclear burning, and let α represent the nuclear burning efficiency factor, and f the fraction of the energy released through nuclear burning that can be devoted to mass ejection, then

$$f\beta\alpha\dot{m}c^2 \geq (1-\beta)\dot{m}|\Phi| \ . \qquad (4a)$$

If we write $|\Phi| \cong M_{\rm eff}G/R_{\rm ej}$, where $R_{\rm ej}$ is the radius from which mass is ejected and $M_{\rm eff}$ roughly represents the mass enclosed by $R_{\rm ej}$, then this

equation can be written in the suggestive form:

$$\frac{\beta}{(1-\beta)} > \frac{1}{2f\alpha}\frac{R_s}{R_{\rm ej}}\,, \tag{4b}$$

where R_s is the Schwarzschild radius associated with $M_{\rm eff}$. Larger values of f, allow smaller values of β. For $f = 0.01$ [0.1], β is (depending on assumptions about R_{ej}, for example) ~ 0.01 [~ 0.001], with the exact value depending on the distribution of matter within the system, and on the point of "exit" for matter which is ejected.

3.3. THE VALUES OF β AND THE ORBITAL EVOLUTION

On the face of it, it might seem as if the best way to "grow" a white dwarf is to encourage large values of β—the larger the fraction of incident mass retained, the better. However, since large values of β can lead to mass-transfer instabilities, they can lead to a common envelope phase and the effective termination of mass accretion. Thus, the ability to gain matter during a long epoch of mass transfer favors $\beta < \beta_{crit}$.

3.4. THE VALUES OF β IN OUR SIMULATIONS

As §3.1 and §3.2 indicate, there are significant uncertainties about what fraction of the donated matter can actually be accreted. The early calculations of nuclear burning considered matter that was actually incident on the white dwarf surface at a constant rate. The influence of nuclear burning on matter infalling to the white dwarf was not explicitly included. For example, although early considerations of the fate of matter in excess of what the white dwarf can burn ($\dot{m} > \dot{m}_{max}$) led to the suggestion that it would form an envelope around the white dwarf (Nomoto 1982), solutions in which it is ejected in a steady wind have recently been found (Nomoto 1996). Since the amount of mass ejected for $\dot{m} > \dot{m}_{min}$ has a pivotal influence on the rate of Type Ia explosions predicted by the CBSS model, we must approach the calculations for the evolution of the CBSS candidates with care.

We have therefore concentrated on attempting to define the uncertainty limits associated with the range of possible physical assumptions. In one approach we attempt to compute the minimum rate of Type Ia explosions predicted by the CBSS model. We do this by assuming that the considerations associated with nuclear burning, as delineated in §3.1, determine the value of β. To this end, we have subjected each of the 6 data sets that emerge from the population synthesis analyses to 8 different "treatments" characterized by conservative assumptions about the value of β. The common features in these 8 conservative treatments, which are described in the

caption to Table 1, are that β is set equal to unity when $\dot{m}_{min} < \dot{m} < \dot{m}_{max}$, and is assumed to have a functional form

$$\beta = \left(\frac{\dot{m}}{\dot{m}_{min}}\right)^{b_1} \tag{5a}$$

for $\dot{m} < \dot{m}_{min}$, while for $\dot{m} > \dot{m}_{max}$, we use

$$\beta = \left(\frac{\dot{m}_{max}}{\dot{m}}\right)^{b_2}. \tag{5b}$$

b_1 and b_2 are chosen to have different values in different "treatments". We refer to this set of treatments as "conservative". Although they are indeed associated with phases of conservative mass transfer, the motivation for the name is that they lead to a conservative estimate of the rate of Type Ia supernovae.

A separate "treatment" is designed to cater to the needs of successful orbital evolution. We effectively place a lower limit, $\mathcal{D}_{min}$ on $\mathcal{D}$. For $\mathcal{D}_{min}$ equal to a particular value, say ~ 0.01, we track the evolution using the conservative assumptions given above. If $\mathcal{D}$ decreases below $\mathcal{D}_{min}$, however, we adjust β appropriately. This allows us to follow every system. This treatment doesn't affect the evolution of systems in class III ($\beta_{crit} > 1$) at all, since $\mathcal{D}$ never decreases below $\mathcal{D}_{min}$ anyway. It has an important influence on systems in class II ($0 < \beta_{crit} < 1$). These are systems whose evolution can be begun for any "treatment", but which tend to fail during the evolution as β approaches β_{crit}. In fact there is reason to believe that some of these systems do indeed survive (Di Stefano 1996; Di Stefano & Nelson 1996), so this "optimistic" treatment may be more realistic for systems in class II than the conservative treatments. For systems in class I ($\beta_{crit} < 0$), it is less clear that the "optimistic" treatment applies. Formally, we need to set β to negative values early in the evolution of these systems in order to respect the lower limit on $\mathcal{D}$. Physically, it is likely that these systems would have to survive as viable mass-transfer binaries through an initial phase in which mass and angular momentum are lost at rates greater than those calculated in the Roche-lobe-filling formalism (equations [1] and [2]). Hence, overall, we expect the "optimistic" treatment to yield an upper limit on the rate of Type Ia supernovae associated with close-binary supersoft sources.

4. Evolutionary Computations

4.1. LESSONS FROM INDIVIDUAL SYSTEMS

It is instructive to study the evolution of individual systems that can be continuously followed through use of the Roche-lobe-filling formalism. In

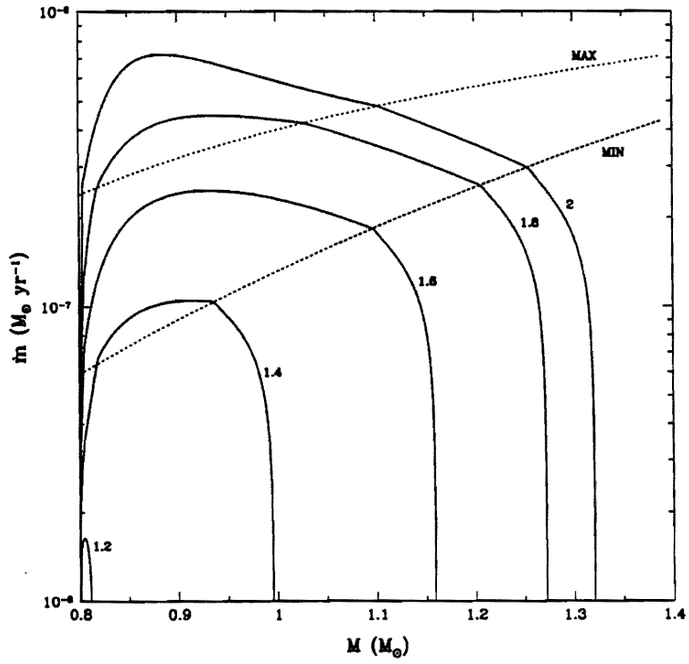

Figure 2. Evolutionary tracks showing the mass donation rate as a function of the mass of the accretor (white dwarf). The dotted lines, denoted by MAX and MIN, indicate the maximum and minimum accretion rates for which stable nuclear burning can occur on the surface of the white dwarf. Below this region mass is lost as a result of nova explosions, while above this region we assume that the 'excess' mass is expelled from the system (Eq. [5]). For each track, the initial mass of the white dwarf was taken to be $0.8M_\odot$ and the helium core mass of the donor was taken to be zero (TAMS). The initial mass of the secondary for each track is labeled in units of the solar mass. Note that the upper curve corresponds to an evolution for which the mass of the white dwarf nearly reaches the Chandrasekhar limit.

Figures 2, 3, and 4 we track the evolution of several sets of systems for which $\beta_{crit} > 1$ (class III). Figure 2 illustrates the effect on total mass retention of increasing the value of q. For a white dwarf with $m = 0.8M_\odot$, and a main-sequence donor, we find that q must be greater than ~ 1.7 in order for significant accretion to occur; yet had we increased q much beyond 2.5 ($m > 2M_\odot$), the evolution would have become unstable. In Figure 3, the time history of the a set of similar systems is examined. The epoch of supersoft X-ray emission will cease near the time of mass equalization, but orbital evolution continues until the donor has exhausted its envelope.

Figure 4 demonstrates the effect of the donor's state of evolution on the mass accretion rates and on the total mass accreted by the white dwarf.

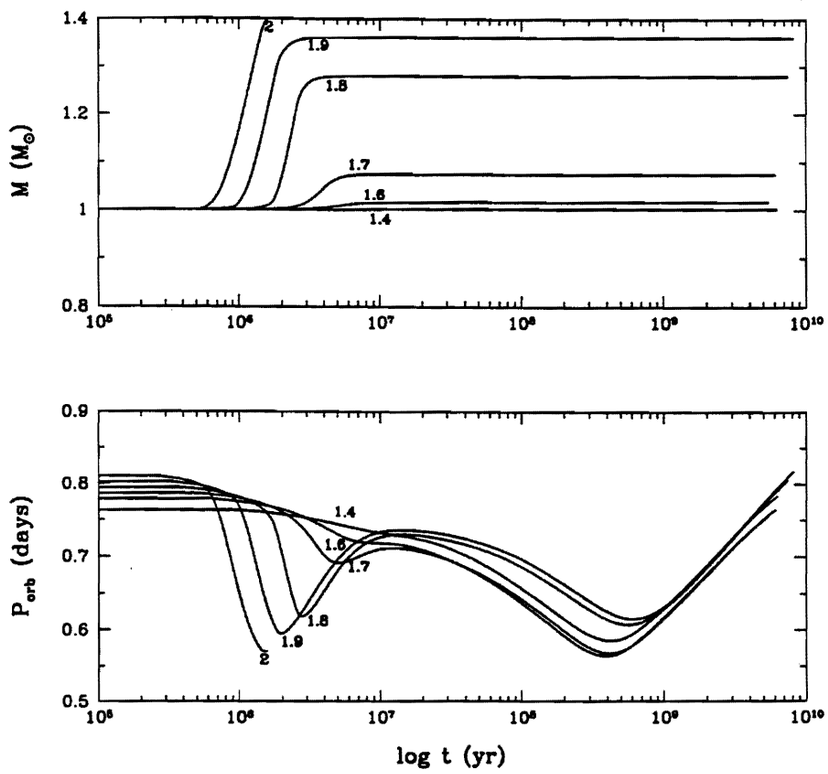

Figure 3. Temporal evolution of the mass of the accretor (white dwarf) and the orbital period of the system. In the upper panel, the mass of the white dwarf is followed as a function of the logarithm of time for several different systems. In each case, the donor initially had zero core mass (at TAMS) and was transferring matter to a $1.0M_\odot$ accretor. The initial mass of the donor is labeled in solar units (from $1.4M_\odot$ to $2.0M_\odot$). Note that the white dwarf accretes most of its mass over a time period of less than ten million years. The lower panel shows the evolution of the orbital period as a function of the logarithm of time. The same family of curves is illustrated as in the upper panel. Note that the $2.0M_\odot$ curve terminates at the point where the white dwarf has accreted enough mass to exceed the Chandrasekhar limit.

Increasing donor core mass is associated with higher mass accretion rates, and larger values of the total mass accreted by the white dwarf. Candidate systems with core mass larger than $0.2M_\odot$, however, are at greater risk of dynamical instabilities that will either place them in class II or class I. Note that some of the white dwarfs that start with $M = 1M_\odot$, can reach the Chandrasekhar mass.

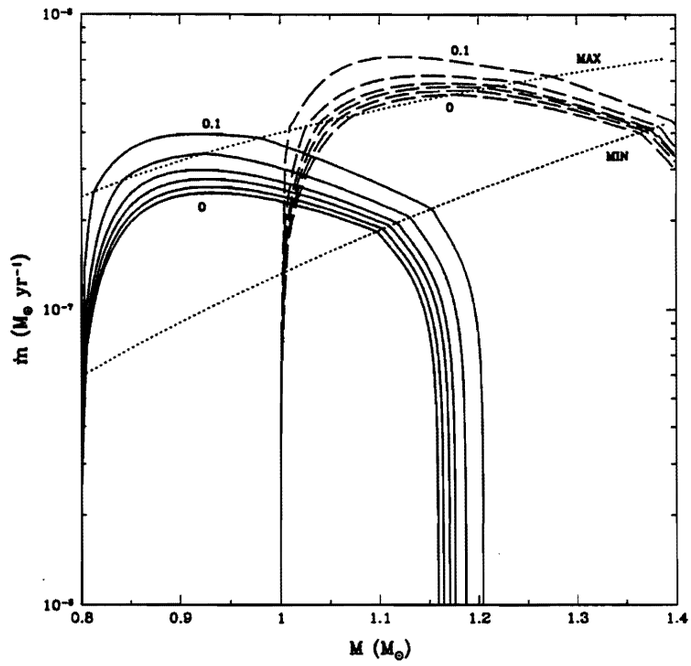

Figure 4. Evolutionary tracks showing the mass donation rate as a function of the mass of the accretor (white dwarf). The dotted lines correspond to the maximum and minimum accretion rates for which stable nuclear burning can occur on the surface of the white dwarf. The solid curves correspond to evolutionary tracks for which a $1.6M_\odot$ donor initially transfers mass to a $0.8M_\odot$ white dwarf. Each curve in the set corresponds to a different initial value of the helium core mass (in solar units). The initial core mass has been incremented from zero to $0.1M_\odot$ in steps of $0.02M_\odot$. The dashed curves correspond to a $2.0M_\odot$ donor that initially transfers mass to a $1.0M_\odot$ white dwarf. The initial core mass has been incremented from zero to $0.1M_\odot$ in steps of $0.02M_\odot$. Note that relatively little mass is accreted by the white dwarf when the mass transfer rate is less than the minimum required for stable burning (i.e., in the regime where nova explosions are expected).

4.2. THE RATE OF TYPE IA SUPERNOVAE

To compute the rate of Type Ia supernovae associated with the CBSS model, we evolve each system that emerges from the population synthesis analysis. Applying our full range of treatments to each of the 6 population synthesis results (i.e., 6 "data sets") allows us to compute the rate associated with a large number of different input assumptions. The range of results provides an estimate of the uncertainty limits. We find that, for the conservative assumptions about β, the fraction of systems that emerge from the population synthesis study and go on to become potential su-

pernovae is not very sensitive either to the population synthesis model or to the treatment. The results are summarized in Table 1. To use this table to compute the birthrates, the numbers in the relevant columns should be multiplied by $\sim 0.5 - 1$ per century. As explained earlier, the upper number of 1 per century incorporates the assumption that all primordial stellar systems are binaries. Two features are apparent. First, even in the conservative treatments, some white dwarfs do achieve $1.4M_{\odot}$. Second, the rate at which this occurs is small, ranging from $\sim$ 2 to 5% of the rate inferred from observations. This rate is a rock bottom estimate, however. Even lowering the value of β in the steady burning region from unity to 0.7 might change these numbers significantly. The "optimum" treatment is likely to derive the upper limits to the rate of Type Ia supernovae that may be associated with the CBSS model. Independent of the population synthesis model, these rates are comparable to the inferred rates of supernovae. Together, the conservative and optimum treatments provide a large belt of uncertainty in the rate of Chandrasekhar mass supernovae predicted by the CBSS model. The issues that need to be more clearly delineated in order to narrow this range have been identified through the evolutionary analysis. Most critical, is the value of β. For example systems in which $0 < \beta_{crit} < 1$ are a promising source of Type Ia progenitors (Di Stefano 1996, Di Stefano & Nelson 1996), which is not tapped in our conservative set of treatments. We note that the value of ξ_{ad} also plays an important role (§2), although it is potentially more straightforward to compute.

Figures 5 and 6 illustrate which systems go on to achieve $1.4M_{\odot}$, for both the conservative and "optimum" assumptions. The results are consistent with the insights gained from the study of individual systems, as depicted in Figures 2 and 4.

5. Prediction From the CBSS Model

An important feature of the evolutionary calculations of CBSSs is that they lead to predictions that go beyond the computation of birthrates. These predictions are potentially testable in observations that occur after, and possibly even before, the explosion. For example, the amount of matter ejected into the surrounding interstellar medium by each system, and the time history of the ejection, is computed. Some general patterns emerge from the work described here, and are being studied in more detail in ongoing work. For example, as Figure 4 suggests, a significant number of systems in which a white dwarf gains $0.2M_{\odot}$, but does not achieve $1.4M_{\odot}$, do not experience significant mass loss. For systems which do "make it" to $1.4M_{\odot}$, more mass is typically ejected from the system. For the uppermost curve in Figure 4, for example, roughly $0.5M_{\odot}$ is ejected during

TABLE 1. Summary of the results of the evolutionary calculations. "Set" refers to the data sets of CBSS candidates that emerge from each of the 6 population synthesis studies we have carried out. Note that there is relatively little variation among the results derived for different data sets. "Case" refers to the class of evolutionary "treatment" used to evolve the CBSS candidates. There are two classes of treatment, conservative (CON) and optimum (OPT). The numbers in each column represent the average fraction of systems that fall into the category indicated by the column headings. A treatment is characterized by the values of the parameters used in the evolution of the CBSS candidates. These include b_1 and b_2 (Eq. 5), and the values of $\tilde{\xi}_{ad}$, $\tilde{m}_c$, and η (which determines the value of J_{ej}). In rows $1-6$, the average of the results for 9 separate conservative treatments is shown. In our standard conservative treatment, $\tilde{\xi}_{ad} = 4$, $\tilde{m}_c = 0.2$, $\eta = 2$, $b_1 = 2$, and $b_2 = 1$. Although the results for individual treatments are not shown, we note that the results among the conservative treatments are not generally dramatically different for different treatments. The exception is for $\tilde{\xi}_{ad} = 10$. This case tends to maximize the value of $\mathcal{D}$, so that all systems can be evolved; we find however, that the mass transfer rates tend to be so low that no system reaches $1.4M_{\odot}$. In row 7, the results for the optimum treatment, which has been applied here only to data set 6, are shown. The evolutionary parameters are the same as those for the standard conservative treatment; when $\mathcal{D} < 0$, however, β is chosen so as to set $\mathcal{D}$ equal to $\mathcal{D}_{min}$.

Set	Case	$\beta_{crit} < 0$ Class I	$\beta_{crit} \in [0,1]$ Class II	$\beta_{crit} > 1$ Class III	$\mathbf{M \rightarrow 1.4M_{\odot}}$	$\mathbf{\Delta M \geq 0.2}$
1	CON	0.72	0.10	0.18	**0.012**	**0.10**
2	CON	0.67	0.12	0.21	**0.009**	**0.02**
3	CON	0.73	0.10	0.17	**0.012**	**0.10**
4	CON	0.53	0.15	0.32	**0.015**	**0.16**
5	CON	0.51	0.18	0.31	**0.016**	**0.14**
6	CON	0.45	0.20	0.36	**0.018**	**0.18**
6	OPT	0.45	0.20	0.36	**0.55**	**0.81**

a $\sim 10^6$ year interval which ends roughly 10^5 years before the explosion. The amount and distribution of circumstellar matter can be checked via radio observations, for example. (See, e.g., Boffi & Branch 1995, Eck *et al.* 1995.) Our calculations also allow us to compute the ionization state of the local ISM (as well as that of ejected material) prior to the explo-

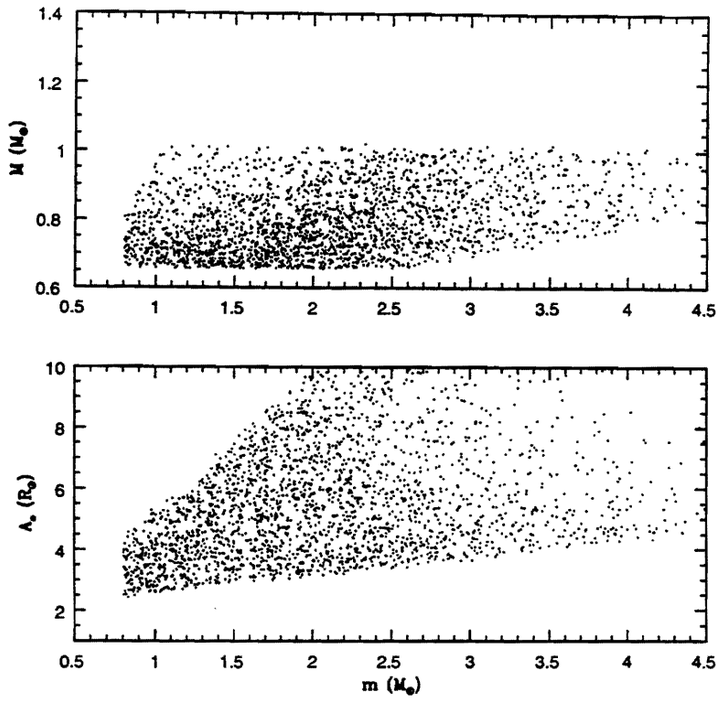

Figure 5. In the upper panel, the initial mass of the white dwarf (accretor) is plotted against the initial mass of the donor for all CBSS candidates surviving the common envelope phase. In the lower panel, the separation after the common envelope phase is plotted against the initial mass of the donor. Each dot represents a single supersoft system. The systems generated by the Monte Carlo simulation were drawn from a primordial population in which the probability distribution of q was proportional to $q^{1/4}$ and the orbital periods were uniformly distributed in logarithmic intervals. The efficiency factor associated with ejection of the envelope was taken to be equal to 0.1.

sion; post-explosion limits on these quantities are also possible to obtain. (See, e.g., Kirshner, Winkler, & Chevalier 1987, and Smith *et al.* 1991.) The possible existence of an ionization nebula pre-explosion suggests that surveys for nebulae in distant galaxies may be identifying some CBSS Type Ia progenitors (Rappaport *et al.* 1994; Remillard *et al.* 1994; Di Stefano *et al.* 1995). It is therefore important to check the catalogs compiled by such surveys to determine whether a pre-explosion nebula had been observed at the coordinates of the supernova. While none of these observations alone is likely to definitively identify a progenitor as a close-binary supersoft source, a combination of such observations for a number of sources, should help to constrain the model.

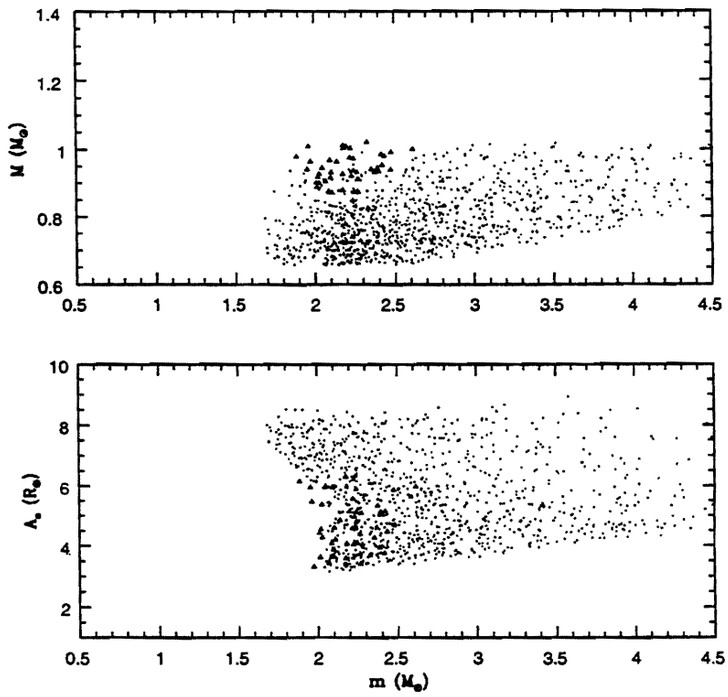

Figure 6. In the upper panel, the initial mass of the white dwarf (accretor) is plotted against the initial mass of the donor for CBSS candidates in Figure 5 that undergo a Type Ia supernova explosion as a result of having attained a mass greater than $1.4M_{\odot}$. Each dot represents a supersoft system whose evolution has been followed assuming that a dynamical instability can be avoided ("optimum" case; see text for details). The small triangles denote those systems for which the white dwarf attained a mass greater than $1.4M_{\odot}$ based on the standard ("conservative") evolution described in the text. In the lower panel, the separation after the common envelope phase is plotted against the initial mass of the donor for the same respective sets of systems. It is clear from the top panel that systems for which the original mass of the white dwarf is approximately $1.0M_{\odot}$ and for which the original mass of the donor is approximately $2.0M_{\odot}$ are favored.

6. Summary and Conclusions

We have begun the process of assessing the likelihood that close-binary supersoft sources are progenitors of Type Ia supernovae. The birthrate of CBSS "candidates", as computed from a Monte-Carlo population synthesis analysis like that done by RDS, is roughly 2-3 times larger than the rates of SN Ia inferred from observations. The new feature we have added is an analysis of the evolution of each of the "candidates". Such an analysis is crucial if we are to compute the rate of supernova explosions. In principle

it should also enable us to draw a profile of those systems which do become supernovae, and to compute properties (such as the amount, placement, and ionization state of circumstellar matter associated with the pre-explosion binary evolution) which can be checked through pre- and post-explosion observations.

In practice, the evolutionary calculations are complicated by the fact that large values of q and the donor's state of evolution, which may place it near the base of the giant branch, each cause a large fraction of our CBSS candidates to flirt with dynamical instability. They thereby risk a common envelope phase of evolution with an uncertain outcome. To derive reliable results we have attempted to address three central issues: How does the donor respond to mass loss? What fraction of the donated mass can the white dwarf accrete? What is the fate of donated mass that is not accreted?

We have addressed the first question through parameterizing the results of a series of Henyey-like calculations. Our approach to the second question is described in §3. Briefly it has consisted of considering two extreme sets of assumptions about the value of β. Using our most conservative assumptions about β we find that we cannot even begin to evolve $\sim \frac{1}{2}$ to $\frac{2}{3}$ of our CBSS candidates (class I), and that $\sim \frac{1}{5}$ of the candidates "fail" because the value of β exceeds $\beta_{\rm crit}$ at some point during the evolution (class II). The remaining $\sim 15 - 30\%$ of the candidates can be successfully evolved (class III). In total, the rate at which white dwarfs reach the Chandrasekhar mass under these conservative assumptions is a few percent of the SN Ia inferred from observations. The rate at which white dwarfs complete the process of accreting and burning 0.2 $M_\odot$ of hydrogen is a few tens of percent of the rate of SN Ia explosions. C-O white dwarf mergers with He cores occur at a rate that could be roughly twice as large as the observed supernova rate.

If, on the other hand, we assume that physical processes such as radiation-driven winds will push β toward small values, enhancing the possibility of stable binary evolution, we find that the rate at which white dwarfs achieve the Chandrasekhar mass is comparable to the rate of Type Ia supernova explosions.

In summary, the work described here indicates that CBSSs are promising candidates for progenitors of Type Ia supernovae, even if an accreting white dwarf needs to achieve the Chandrasekhar mass in order for an explosion to occur. The progress lies in the fact that the detailed evolutionary considerations have allowed us to derive well-defined upper and lower limits, and to identify the physical issues that need to be addressed in order to narrow the range of uncertainties. Work along the lines of investigation suggested here is underway.

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