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Maximum Entropy Lens Inversion

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Abstract: We present a one dimensional algorithm for inverting extended gravitational lens images using the maximum entropy method (MEM). We find the method accurately inverts simulated one-dimensional "ring" images, and is effective at solving for lens parameters. With sources which are larger than the lens critical radius there are systematic "glitches" in the source reconstructions. We find these errors to be intrinsic to MEM, but show that they can be removed by filtering. 11

1 Introduction

In inverting gravitational lens images, we are seeking to solve for the unknown source and the lens potential simultaneously. With multiply imaged objects, the knowledge that a single source produces the images provides the constraints necessary to solve for both unknown quantities. Extended lenses, such as rings and arcs, provide the best opportunity for detailed inversion, as they sample the lensing potential at many points and provide many more constraints than quasar lenses do (see reviews in Blandford & Narayan 1992, Schneider, Ehlers, & Falco 1993). There are currently two algorithms that produce detailed inversions with no *a priori* assumptions about the structure of the source: the "Ring Cycle" algorithm, (Kochanek *et al.* 1989), which assumes the lensed image is a true surface brightness map, and LensClean (Kochanek & Narayan 1992), which uses the principles of Clean (Hogbom 1974) to include the effects of finite resolution. The success of the maximum entropy method in regular image processing (see Burch, Gull, & Skilling 1983, Skilling & Bryan 1984, Cornwell & Evans 1985, and review by Narayan & Nityananda 1986), and its complementary properties compared with Clean motivated us to apply MEM techniques to the inversion of gravitational lenses. A more detailed description of the algorithm appears in Wallington, Narayan & Kochanek (1993).

2 Equations and Solution Method

We begin with a quick review of plain MEM. Let a *source*, S, be observed by means of a telescope with a point spread function or *beam*, B. The measured image or *data*, D, will then be of the form

$$D = S * B + \sigma, \tag{1}$$

where '$*$' represents a convolution and σ is random noise. The problem of image restoration is that given D and B, many sources fit the data to within the noise, so that inversions are not unique.

Maximum entropy resolves this problem by maximizing a linear combination of the "entropy" of the source and a measure of fit to the data. If the entropy is suitably defined, then the reconstruction

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overlapping data points for any plausible time delay test value in an autocorrelation calculation, and yet all such calculations show only a broad minimum with a width of 100 days or more (Thomson and Schild 1993). Thus with so much data presently available, although it should be possible to specify the time delay to a fraction of a day, the errors are still a week or two and we attribute this to the complication that both image components are affected by this cuspy residual, which has an amplitude of several percent.

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is a map that is in some sense the most probable. We use one of the most common expressions of entropy, $H = -\sum_k S_k \ln S_k$, where the array S_k represents source intensities at positions $k = 1$ through n. For goodness of fit we use a standard χ^2 difference between the data D and the image I, $\chi^2 = \sum_i (I_i - D_i)^2/\sigma_i^2$. We thus maximize

$$J = -\sum_k S_k \ln S_k - \lambda \sum_i \left(\frac{I_i - D_i}{\sigma_i}\right)^2, \tag{2}$$

where λ is a Lagrangian multiplier, and the array I_i represents the noise-free *beam convolved image* of the source,

$$I = S * B. \tag{3}$$

The parameter λ allows the user to adjust the relative importance of the two terms in eq. (2).

The image in a gravitational lens is a more complicated function of the unlensed source intensity distribution than the relation given in equation (3). The lens can be thought of as another screen distorting the true image of the source. The observed data, D_i, are thus given by the convolution of the beam, B, and the *lensed image*, L, with additive noise, σ,

$$D = L * B + \sigma, \tag{4}$$

where L itself is a linear transformation of the source intensity S through the lens mapping

$$L_k = \sum_j F_{kj} S_j. \tag{5}$$

We refer to $L * B$ as the *beam-convolved image* I:

$$I = L * B. \tag{6}$$

The coefficient F_{kj} in eq (5) is the contribution of source pixel, j, to the intensity of the lensed image pixel, k.

A logical extension of MEM to include the effects of lensing is to maximize exactly the same quantity as in (2),

$$J = -\sum_k S_k \ln S_k - \lambda \sum_i \left(\frac{I_i - D_i}{\sigma_i}\right)^2, \tag{7}$$

where now the first summation is over intensities in the source plane while the second summation considers the difference between the observed and model intensities in the image plane.

We maximize the function J in (7) with respect to the source intensities S_k using the conjugate gradient method. This method performs a series of one dimensional line maximizations along directions which are determined from the gradient vector $\partial J/\partial S_\ell$ and chosen to be non-interfering, or conjugate, with respect to one another. The Lagrangian multiplier λ is optimized as the iterations proceed.

We use a parametrized model for the gravitational lens potential of the form

$$\phi = \frac{\alpha^{2-\gamma}}{\gamma}(r_c^2 + (x - x_\ell)^2)^{\gamma/2}, \tag{8}$$

which corresponds to an isothermal sphere model when $\gamma = 1$ and a Plummer model when $\gamma = 0$. In addition to γ there are three other parameters, *viz.* the critical radius α, the core radius r_c, and the lens position x_ℓ. Elliptical variants of this potential in two dimensions were originally devised by Blandford and Kochanek (1987) to model elliptical lenses.

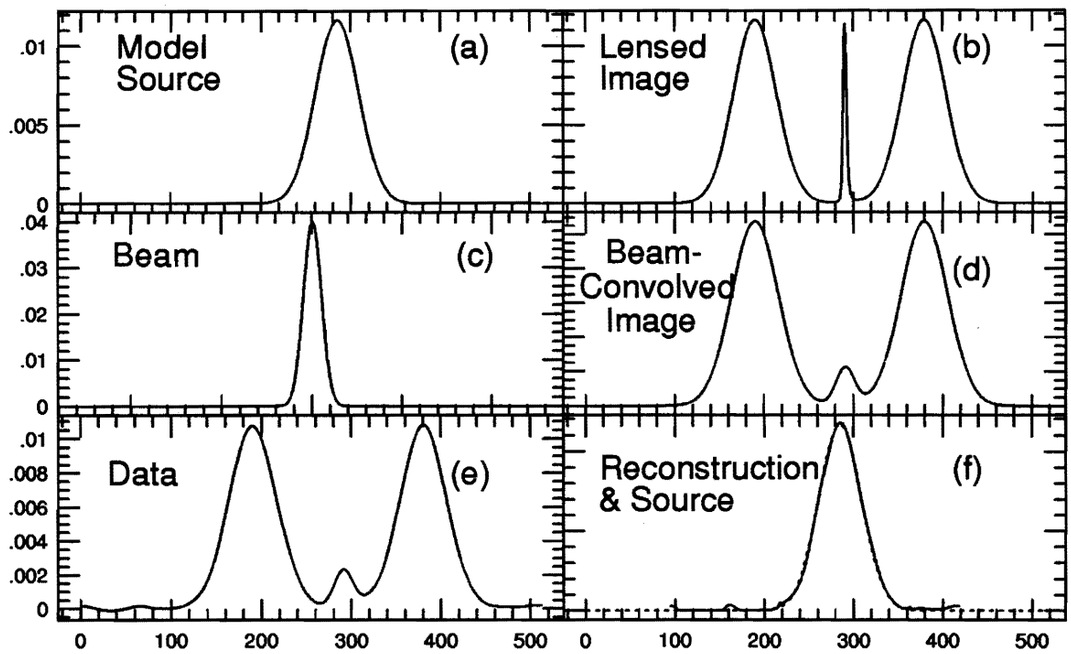


Figure 1: Progression of source reconstruction. The source (a) is acted upon by the lens, producing the lensed image (b). The beam (c) acts on the lensed image to produce the beam-convolved image (d). The addition of noise gives the data (e). When the data are processed through the MEM routine, the reconstructed source, (f) is obtained (solid line). The dotted line in (f) shows the original source.

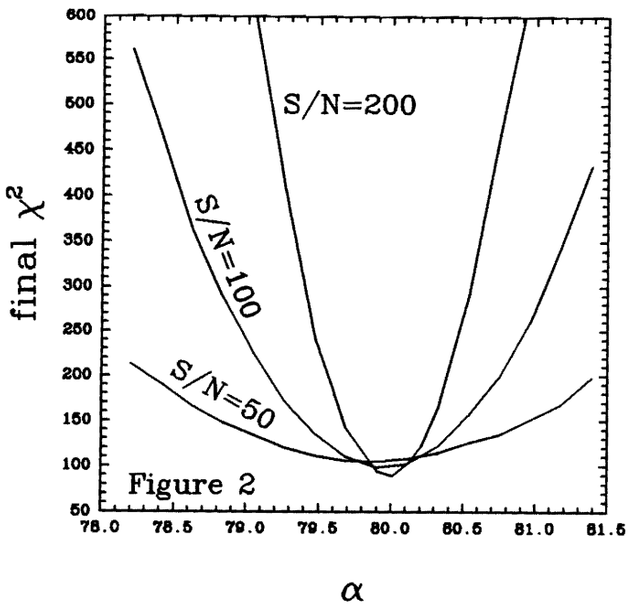
3 Tests of the MEM Algorithm

The examples of lensing of extended sources found so far all involve rings or arcs in which the intrinsic size of the source is smaller than the lens critical radius. To test our algorithm, we created one-dimensional "rings" using an isothermal lens potential ($\gamma = 1$) with a finite core radius, r_c, and a critical radius, α, larger than the width of the model source.

Figure 1 shows the result of applying our MEM reconstruction algorithm to this simulated image. The algorithm clearly recovers the source well, and most of the residuals come from noise in low lying regions. We have been able to produce good reconstructions for ring-like cases with a wide variety of parameters.

Our inversion algorithm calculates a final χ^2 once the stopping criteria have been satisfied for a given lens model. The lens parameters can then be varied and the maximization performed for each choice of parameter. By mapping χ^2 as a function of the parameters and finding the lowest χ^2, we can identify the optimum lens model. Figure 2, for instance, shows the residuals as a function of α near the minimum for several noise levels. The minimum χ^2 is very close to the true value of $\alpha = 80$ in all cases. As the noise level decreases, the parabola becomes tighter, meaning that the accuracy with which α can be estimated increases. Figure 3 shows another example of the behavior of χ^2, in this case a contour plot as a function of α and γ. Again there is a smooth dependence with nearly elliptical contours surrounding a minimum very close to the true parameter values.

We also tried a number of non-ring-like source and lens configurations. In these source models, extensive portions of the source are unlensed, and only a small region of the source in the middle is



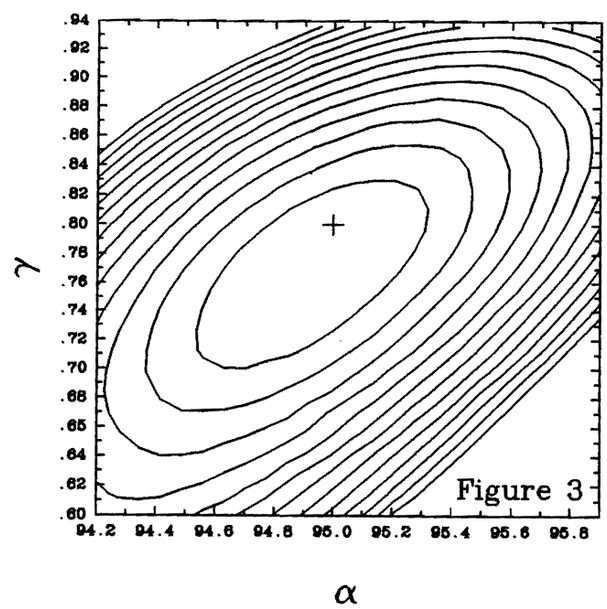


Figure 2: Final χ^2 as a function of the lens parameter α. The minimum χ^2 is obtained close to the true value of $\alpha = 80$. The curvature of the parabolic minimum determines the accuracy with which α can be estimated. High SN data constrain lens parameters more tightly than low SN data, as expected.

Figure 3: Contours of final χ^2 for models with a range of values of α and γ. The χ^2 has quadratic behavior around a minimum close to the true value, indicated by a +.

lensed. Such sources do not correspond to any currently observed example of lensing. We find our method does not work as well for sources larger than the lens critical radius as it does for the narrower ring sources. We again used an isothermal potential with a finite core for these simulations, except that the critical radius is now much smaller than the source size. The result of a typical inversion of this type is shown in Figure 4a.

The problem with this reconstruction is clear—significant "glitches" appear at the boundaries of the lensed region. We tried a number of methods to eliminate them, but the problem appears to be intrinsic to the MEM method. Glitches were clearly present on most wide sources we tried, but were worst in cases of low resolution and high noise. Figure 5 shows a montage of reconstructions in which noise and resolution have been varied. The glitches scale with the noise in the limit of high resolution, but for poorly resolved cases they are present even in noise-free models.

We have been unable to prevent glitches from occurring in our reconstructions of wide sources, but we have had some success in treating the glitches after they have formed by smoothing the reconstructed source by a suitably selected beam. In gravitational lens inversions there is an extra complication which is not present in normal image restoration. Given a certain beam in the image plane, the corresponding beam in the source plane is generally different due to lens magnification. To allow for this magnification effect, we calculated the largest average magnification over a beam in the image plane and smoothed the source with a correspondingly narrower beam in the source plane. Any structure in the reconstruction on scales smaller than this beam cannot be real, and thus smoothing the source on this scale does not destroy any real information. The results are shown in Figure 4b. The glitches are not eliminated completely, but they are reduced to noise-level bumps.

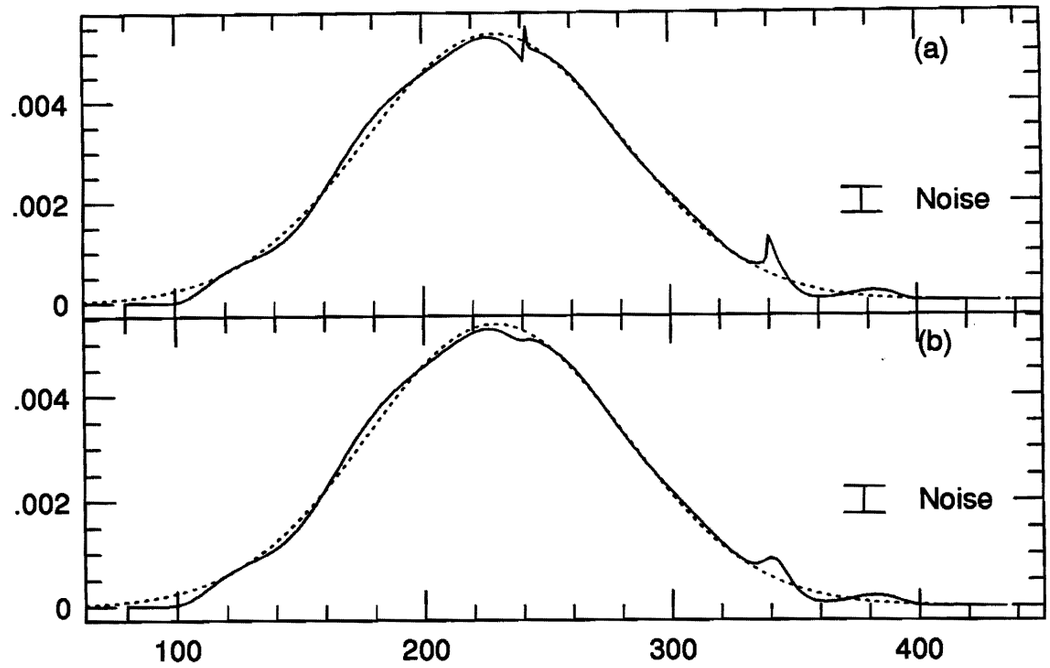


Figure 4: Reconstruction of a wide source. In panel (a) the solid line shows the reconstruction and the dotted line the real source. The two glitches are at the positions of the caustics, where the image multiplicity changes, and occur for most reconstructions of such wide sources. Panel (b) shows the same reconstruction after smoothing. The amplitude of the glitches has decreased to approximately the noise level.

Fortunately, the glitches do not appear to have any effect on optimization of the lens model. We have done many tests using wide sources and generally find quadratic behavior of χ^2 near the minimum similar to the results described for ring images (Figs. 2, 3). Thus, even though the reconstruction of a wide source is not ideal, the correct lens parameters can still be determined accurately.

4 Summary

We demonstrate a one dimensional algorithm for inverting gravitational lens images using the maximum entropy method. Our algorithm is very effective at solving for the parameters of the lens model. For each lens parameter we get a good estimate of its true value, and the χ^2 for deviations around the estimated value has a quadratic form (see Figs 2a, 2b). The algorithm also works very well at reconstructing sources. We have inverted a large number of typical ring-like images, and obtain source reconstructions with rms errors consistent with the noise.

The main surprise is that MEM produces glitches at the boundaries of the multiply imaged regions when reconstructing "wide" sources. We have found that the glitches are an intrinsic feature of the MEM technique, but despite many attempts, we have been unable to find a universal fix for the glitches. Fortunately, the glitches are prominent only in the limits of low beam resolution and low signal to noise. Furthermore, large amplitude glitches are invariably concentrated in unphysical sharp features that can be largely eliminated by smoothing the reconstruction with a Clean-like restoring beam. Since, in addition, images corresponding to the kinds of wide sources where glitches are most

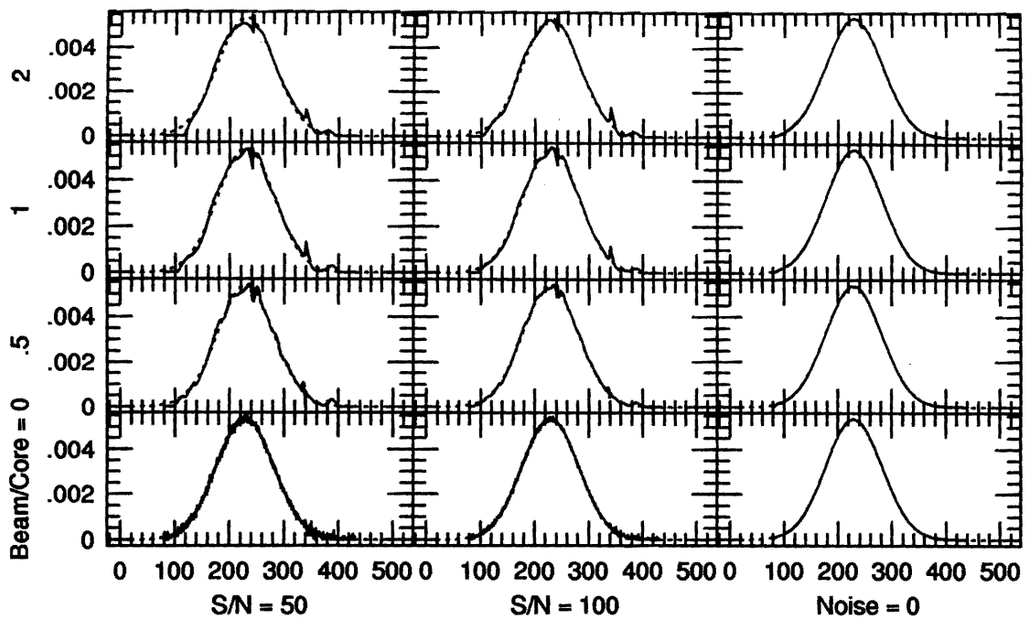


Figure 5: Montage of reconstructions of a wide source. The same lens and source are used in each case, but the noise level and beam width are varied as indicated. The noise level decreases to the right, and the resolution decreases towards the top. The glitches are clearly strongest when noise is high and resolution is low (upper left corner). In the limits of no noise and infinite resolution the glitches disappear.

prominent have never been observed, we feel that the occurrence of glitches is not a severe limitation of the method.

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