Magnification Bias and Gravitational Lensing Statistics

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Abstract: We review the phenomenon of magnification bias in gravitational lensing and discuss its influence in (1) multiple-image quasars, (2) quasar-galaxy associations, (3) BL Lacs as lensed OVV quasars, and (4) the quasar luminosity function. We discuss the effects of macrolensing and microlensing separately. 11

1 Introduction

As the number of known examples of gravitational lensing (Schneider, Ehlers, & Falco 1992, Blandford & Narayan 1992), and the number of phenomena identified with lensing have increased, so too has there been increased interest in the statistics of lensing. An important aspect of statistical investigations is the identification of selection effects. Of these, one effect in particular, magnification bias, has received much attention. Ideas related to magnification bias are found occasionally even in the literature predating the discovery of the first lens (e.g. Barnothy 1966, Gott & Gunn 1974), but after the discovery of the double quasar Q0957+561 in 1979 there has been a great deal of interest in the subject, and new applications and variants of the bias have been introduced at regular intervals (e.g. Turner 1980, Canizares 1981, Turner, Ostriker & Gott 1984, Ostriker & Vietri 1985, Borgeest, Linde & Refsdal 1991).

In this article we review the physical basis of magnification bias and describe several contexts in which it has been applied to gravitational lensing. Throughout the paper we distinguish between the influence of macrolensing and microlensing and attempt to analyze the effect of each on the various applications of bias. We begin in §2 with a discussion of the origin of magnification bias, followed by reviews of the quasar luminosity function (§3) and the variation of lensing cross-section with magnification (§4). We then discuss the influence of magnification bias in multiply-imaged quasars (§5), quasar-galaxy associations (§6), BL Lacs (§7), and the quasar luminosity function (§8). In the concluding section (§9) we summarize the various arguments, paying particular attention to the relative importance of macrolensing and microlensing in the applications of magnification bias.

2 Origin of Magnification Bias

It is well-known that a gravitational lens can magnify a distant source and thus cause an increase in the flux received by an observer. Because of this, in any flux-limited sample, the magnified lensed sources will be drawn from a fainter source population than the unlensed sources. (Lensing can also demagnify sources, but for the applications discussed here the instances of magnification dominate.) As a consequence, the number of lensed sources detected is not simply the total number of sources in the sample multiplied by the probability that any random source is lensed, but is greater than this number. This phenomenon is referred to as *magnification bias.* Frequently in the literature the term amplification bias is also used. However, as emphasized by B.F. Burke (*private communication*), the effect is really one of magnification rather than amplification.

The degree of magnification bias that one sees depends on the properties of two functions. The first is the source luminosity function, which can be written in several forms. In differential form, $\Phi(L,z)d\log L$ indicates the number of sources at redshift z per unit comoving volume per logarithmic luminosity interval $d\log L$; $\Phi(M,z)dM$ is the equivalent function expressed per absolute magnitude interval dM. Alternatively, we can consider the cumulative forms, $\Phi(>L,z)$ and $\Phi(<M,z)$, which give the number density of sources brighter than luminosity L or magnitude M. The second relevant function is the dependence of lensing cross-section on magnification μ. This is described by the differential lens cross-section $\sigma(\mu)d\mu$ or equivalently, the differential "optical depth" $\tau(\mu)d\mu$, which differs from $\sigma(\mu)d\mu$ by a proportionality constant. Again it is sometimes useful to consider the cumulative lens cross-section $\sigma(>\mu)$ which is the cross-section corresponding to all magnifications greater than μ. The total cross-section for lensing is of course $\sigma_{tot}=\sigma(>0)$.

Consider a sample of sources at redshift z corresponding to a flux limit S_o and a luminosity limit $L_o=S_o d_L^2(z)$, where $d_L(z)$ is the luminosity distance to the sources. The magnification bias in the lensed sources of this sample can be quantified by the following bias factor, expressed in two equivalent ways:

$$B(>S_o,z)=\frac{\int_0^\infty \sigma(>L_o/L)\Phi(L)d\log L}{\sigma_{tot}\Phi(>L,z)}=\frac{\int_0^\infty \sigma(\mu)\Phi(>L_o/\mu,z)d\mu}{\sigma_{tot}\Phi(>L_o,z)}. \tag{1}$$

The bias is the factor by which the true number of lensed sources differs from the naive estimate obtained by multiplying the number of sources brighter than L by the probability that each source is lensed. The denominator represents the latter incorrect estimate. The numerator gives the correct way of calculating the number of lensed sources, that is, for each luminosity class of sources we compute the probability that these sources would be lensed with the necessary magnification to bring them within the flux limit of the sample.

Because of flux conservation the mean magnification must be unity (Weinberg 1976), provided we compute magnifications relative to a universe where the mass in the lenses is smoothly distributed. (Other normalizations are possible where the mean magnification is not unity, *e.g.* Ehlers & Schneider 1986). In many circumstances, therefore, the bias factor in (1) will not differ very much from unity and it would be difficult to identify the effects of magnification bias in the observations. However, the bias can be extremely large when a particular combination of conditions is met (see Fig. 1), viz.
(i) the luminosity function $\Phi(L)$ rises steeply for faint L,
(ii) the lens cross-section $\sigma(>\mu)$ has a tail extending to large μ, and

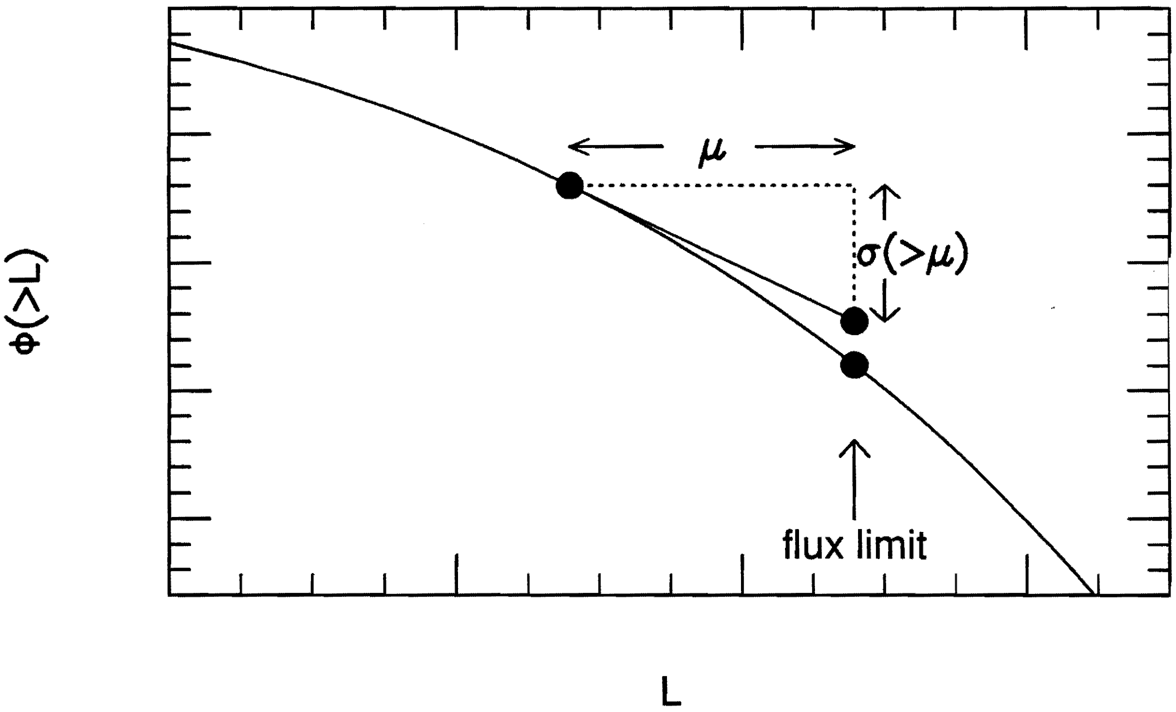

Figure 1: Strong magnification bias. The luminosity function $\Phi(L)$ is here steeper than the cumulative lens cross-section $\sigma(>\mu)$. Consequently, a flux-limited sample of sources will have a large number of intrinsically faint sources which are brought into the sample by being magnified through lensing.

(iii) the logarithmic slopes of the two functions satisfy the condition

$$\left|\frac{d\ln\Phi(L)}{d\ln L}\right| > \left|\frac{d\ln\sigma(>\mu)}{d\ln\mu}\right|. \tag{2}$$

As a simple example, suppose that the differential quasar luminosity function and the cumulative lens cross-section have the following forms:

$$\begin{aligned}\Phi(L)d\ln L &= \Phi_*(L/L_*)^{-\alpha}, && L \geq L_*,\\ &= 0, && L < L_*. \end{aligned} \tag{3}$$

$$\begin{aligned}\sigma(>\mu) &= \sigma_{tot}, && 0 \leq \mu \leq \mu_{min},\\ &= \sigma_{tot}(\mu/\mu_{min})^{-\gamma}, && \mu_{min} < \mu \leq \mu_{max},\\ &= 0, && \mu > \mu_{max}. \end{aligned} \tag{4}$$

Let us suppose that μ_{max} is much greater than unity and that L_* is much smaller than the luminosity limit $L_o = S_o d_L^2(z)$ corresponding to the flux limit S of the observations. Then, leaving out inessential terms, the bias factor in (1) becomes

$$B(>S_o) \sim \frac{1}{(\alpha-\gamma)}\left[\mu_*^{\alpha-\gamma} - 1\right], \tag{5}$$

where μ_* represents an effective magnification cut-off,

$$\mu_* = \min(S_o d_L^2/L_*, \mu_{max}). \tag{6}$$

In other words, the integral in the numerator of eq.(1) is cut off either by the turnover in the luminosity function at L_* or by the cut-off of the lens cross-section at μ_{max}, whichever occurs sooner. The main point of (5) is that if μ_* is large and if $\alpha > \gamma$, then the bias factor can be very large. We will use the term *strong magnification bias* to refer to such situations.

Magnification bias naturally operates at any wavelength, but it need not be considered for only one wavelength at a time. Borgeest *et al.* (1991) discuss the properties of double magnification bias, in which having two independent magnification biases in two different wavelengths produces a higher total bias factor than either individual bias. This assumes the two luminosities are not correlated. They suggest applying this concept to surveys of quasars in the radio and optical, but because of small samples and incompleteness in the radio, they were unable to demonstrate the presence of double magnification bias using current surveys. Nevertheless, double magnification is a strong candidate for explaining some of the more dramatic observations related to quasar-galaxy associations (§6), and future more complete surveys may well reveal evidence for an enhanced bias.

Before proceeding to a discussion of $\Phi(L)$ and $\sigma(>\mu)$, a few points are in order. First, it is straightforward to integrate (1) over source redshift to obtain the bias factor for a flux-limited sample that is not selected with respect to z. Second, the cross-sections $\sigma(\mu)$ and $\sigma(>\mu)$ are themselves usually integrals over lens redshift extending from the observer to the source. Third, the cross-sections can refer to any specific selection bias one may impose. For instance, we may be interested in a very detailed question such as knowing how many lensed quasars will have five images where no two images are closer together than 0.5 arc second, and where the fifth image is fainter than a hundredth of the brightest. Equation (1) will of course provide the answer to this question if we define the cross-section appropriately. However, this bias factor cannot then be applied to any other question. The point is that the bias factor is specific to each selection criterion, so that there is no universal bias factor which can be used for all applications.

3 Quasar Luminosity Function

In this article, we concentrate primarily on quasars. The optical luminosity function of quasars has been studied for many years (see reviews by Warren & Hewett 1990, Hartwick & Schade 1990, and Boyle 1993), and recently some degree of consensus has emerged. Boyle (1992) shows that the observations are consistent with a two-power-law luminosity function with a sharp break:

$$\Phi(L,z)d\log L = \frac{\Phi_*}{(L/L_*)^\alpha + (L/L_*)^\beta}. \tag{7}$$

Both Φ_* and L_* are functions of z, but apparently the power-law indices α and β are relatively independent of z. The function (7) has a logarithmic slope of $-\alpha$ for $L \gg L_*$ and a slope $-\beta$ for $L \ll L_*$. The observations give

$$\alpha \ = \ 2.6 \pm \ 0.1, \qquad \beta \ = \ 0.5 \pm 0.2. \tag{8}$$

Thus, by the discussion of §2, we may expect strong magnification bias for bright quasars if $|d\ln\sigma(>\mu)/d\ln\mu| \lesssim 2.6$, whereas for faint quasars we will need $|d\ln\sigma(>\mu)/d\ln\mu| \lesssim 0.5$.

Although the above model of the quasar luminosity function seems to be well supported by much of the observations, some disturbing trends have recently emerged. Goldschmidt *et al.* (1993) showed that there are many more quasars at very bright magnitudes ($M_B \lesssim 16$) than predicted by the Boyle model. This has the effect of reducing α below 2.6. Also, Hawkins & Véron (1993) found that a variability-selected sample of quasars showed very little evidence for a break at L_*, and moreover appeared to indicate a slope α much less than 2.6. These modifications, if confirmed, will have severe consequences for magnification bias. The subsequent discussion in this article is based on the Boyle model (eqs. 7 and 8), but we warn the reader that some of the results we quote are very sensitive to minor changes in the bright end slope of $\Phi(L, z) d \log L$.

There is also some information available on the X-ray and radio luminosity functions of quasars. In both bands, the basic form (7) appears to be valid and the power-law indices too are similar (Boyle 1993). In X-rays, $\alpha = 2.4 \pm 0.1, \beta = 0.7 \pm 0.2$, while in radio (flat spectrum sources), $\alpha = 2.0 \pm 0.1, \beta = 0.8 \pm 0.2$.

4 Variation of Cross-Section with Magnification

In gravitational lensing, it is common to distinguish between *macrolensing* and *microlensing*. Macrolensing refers to lensing by masses $\gtrsim 10^{10}$ $M_\odot$, i.e. mass distributions on the scale of galaxies, clusters of galaxies, or even larger scales. Multiple images created by such lenses have angular separations on the order of a fraction of an arc second or more and are therefore resolvable with normal telescopes. Microlensing on the other hand refers to lensing by masses $< 10^{10}$ $M_\odot$. If the lenses have masses in the range $M \sim 10^6\text{-}10^{10} M_\odot$, then the multiple images can be distinguished with milliarcsecond resolution, whereas for $M < 10^6$ $M_\odot$, the multiple images cannot be resolved at all. (Sometimes the lenses in the upper mass range are referred to as millilenses and the term microlens is reserved for the latter; we do not make this distinction.) It is usually assumed that microlenses are point masses (stars or black holes), but at the upper end of the mass range microlenses may also be globular clusters or dwarf galaxies.

4.1 Macrolensing Cross-Sections

As discussed in §2, magnification bias is sensitive to the form of the lensing cross-section, $\sigma(\mu)$ or $\sigma(>\mu)$, at large μ. In this limit it is known that the cross-section is dominated almost entirely by caustics with certain generic scaling laws (Benson & Cooke 1979, Blandford & Narayan 1986).

There are two primary caustics that are relevant for gravitational lens statistics: the *fold caustic* and *cusp caustic*. The fold has a cross-section that scales as

$$\sigma(\mu) d\mu \sim \mu^{-3} d\mu, \quad \sigma(>\mu) \sim \mu^{-2}, \tag{9}$$

(Blandford & Narayan 1986, Kovner 1987, Blandford & Kovner 1988, Kayser & Witt 1989), while the cusp has a component that scales as

$$\sigma(\mu) d\mu \sim \mu^{-7/2} d\mu, \quad \sigma(>\mu) \sim \mu^{-5/2} \tag{10}$$

(Mao 1992, Schneider & Weiss 1992).

A generic elliptical lens displays scalings appropriate to both types of caustics (e.g. Blandford & Kochanek 1987, Wallington & Narayan 1993, Kassiola & Kovner 1993). Such a lens can produce multiple images with either 5 or 3 images. Five image configurations always follow the fold-like scaling in (9), while three-image configurations can have contributions showing both the fold and cusp scalings, with a transition from one to the other at some finite value of μ. In addition, an elliptical lens can also produce a brightened single image due to the effect of a "naked cusp." These cases will exhibit the scaling in (10). Thus, we have the following:

$$\begin{aligned} 5\ \text{image}: \quad & \sigma(>\mu) && \sim \mu^{-2}, \\ 3\ \text{image}: \quad & \sigma(>\mu) && \sim \mu^{-5/2},\ \mu^{-2}, \\ 1\ \text{image}: \quad & \sigma(>\mu) && \sim \mu^{-5/2}. \end{aligned} \tag{11}$$

Comparing these scalings with the values of α and β in eq. (8) and noting the inequality in eq. (2), we expect the following:

(i) Five image configurations of lensed quasars should display strong magnification bias in bright samples of quasars.

(ii) Three image configurations should also most often show strong bias.

(iii) Cases where the lens produces a brightened single image should have marginally strong magnification bias, but not as strong an effect as in the multiply-imaged quasars.

(iv) Magnification bias should be relatively weak in faint quasars with $L \lesssim L_*$.

Since the X-ray and radio luminosity functions of quasars are similar to the optical function, the above results must be valid in these bands as well. However, the finite size of radio quasars, particularly steep spectrum sources, reduces the strength of the bias for these sources. This is because very high magnifications are obtained only when the source is extremely close to a caustic. If the source has a finite size, it is not possible for the whole source to participate in large magnification, and as a result there is a maximum magnification μ_{max} possible (see eq. 13 below). As the source size increases, μ_{max} decreases. Since the degree of strong magnification bias depends on μ_{max} (eq 5) this means that broad sources such as steep spectrum radio sources will have much weaker magnification bias compared to truly point-like sources. Optical and X-ray quasars, and possibly also flat spectrum radio sources, are essentially point-like as far as macrolensing is concerned. Therefore, μ_{max} is effectively infinite and the cut-off of magnification bias is due solely to the break in the luminosity function. Other factors being equal these sources should have significantly stronger magnification bias than steep spectrum radio sources.

4.2 Microlensing Cross-Sections

In the case of microlensing, large magnifications are again dominated by caustics and hence the scalings of $\sigma(\mu)$ and $\sigma(>\mu)$ with μ follow the laws given in eqs. (9), (10). One important point, however, is that collective effects connected with the correlated action of many microlenses become important even at modest optical depth. There are a few analytical results and many numerical investigations of these effects (*e.g.* Young 1981, Nityananda & Ostriker 1984, Paczyński 1986, Kayser *et al.* 1989, Wambsganss 1990, Witt 1990, Rauch *et al.* 1992). Wambsganss (this volume) reviews the subject.

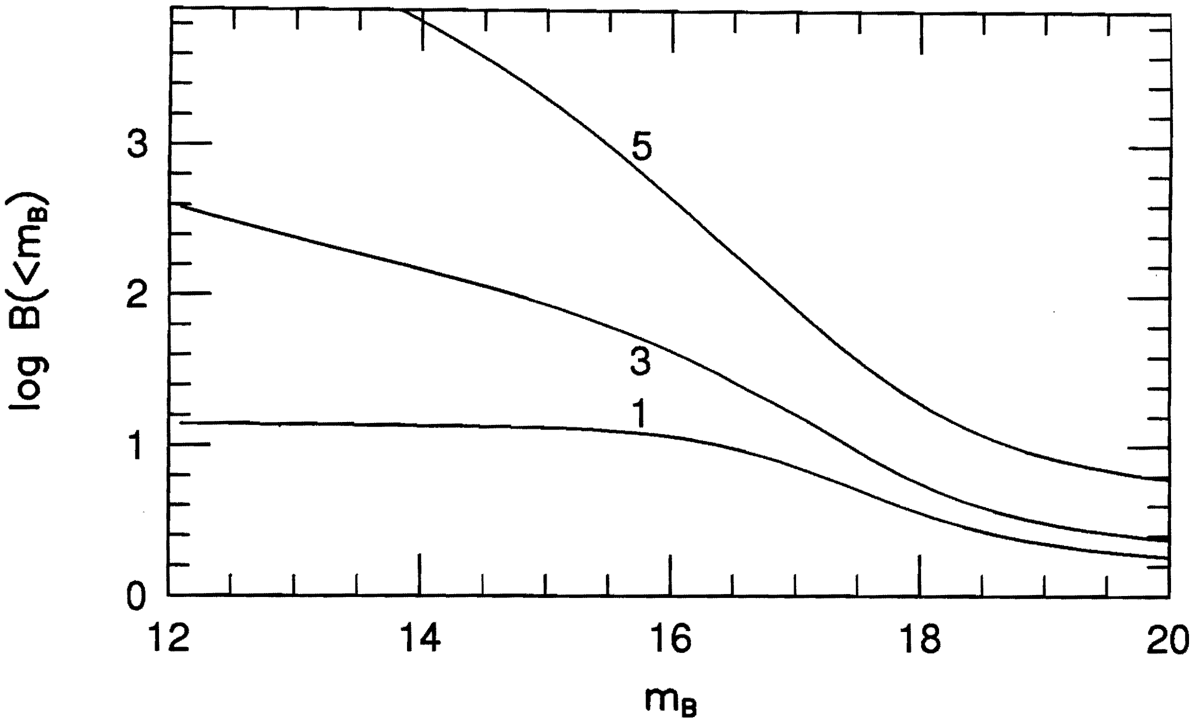

Figure 2: Bias factor for gravitationally lensed quasars as a function of apparent magnitude. The three curves show the bias factor for five image, three image, and brightened single image configurations. (From Wallington & Narayan 1993)

In the case of microlensing, the finite size of the source plays a much more important role than in macrolensing. This is because the linear scale of the caustics in the source plane is generally comparable to the Einstein radius ξ_0 of the lens, which tends to be quite small in the case of a microlens:

$$\xi_0 \sim 0.02 \left(\frac{M}{\mathrm{M}_\odot}\right)^{1/2} \mathrm{pc}, \tag{12}$$

where M is the mass of an individual microlens. If the source has a linear size ξ_s, then the maximum magnification μ_{max} that can be obtained scales as

$$\mu_{max} \sim \mathrm{few} \left(\frac{\xi_0}{\xi_s}\right)^{1/2} \sim \mathrm{few} \left(\frac{M}{\mathrm{M}_\odot}\right)^{1/2} \left(\frac{\xi_s}{10^{-2}\mathrm{pc}}\right)^{-1/2}. \tag{13}$$

When M is small, μ_{max} cannot become large unless the source size is also extremely small. Since strong magnification bias requires a large μ_{max} (cf. eq. 5), this means that the bias will be severely restricted under the action of microlensing, unless M/ξ_s is sufficiently large. In particular, for $M \lesssim 1M_\odot$, we need source sizes $\ll 10^{-2}$ pc for strong magnification bias.

5 Magnification Bias in Multiply-Imaged Quasars

5.1 Bias versus Apparent Magnitude

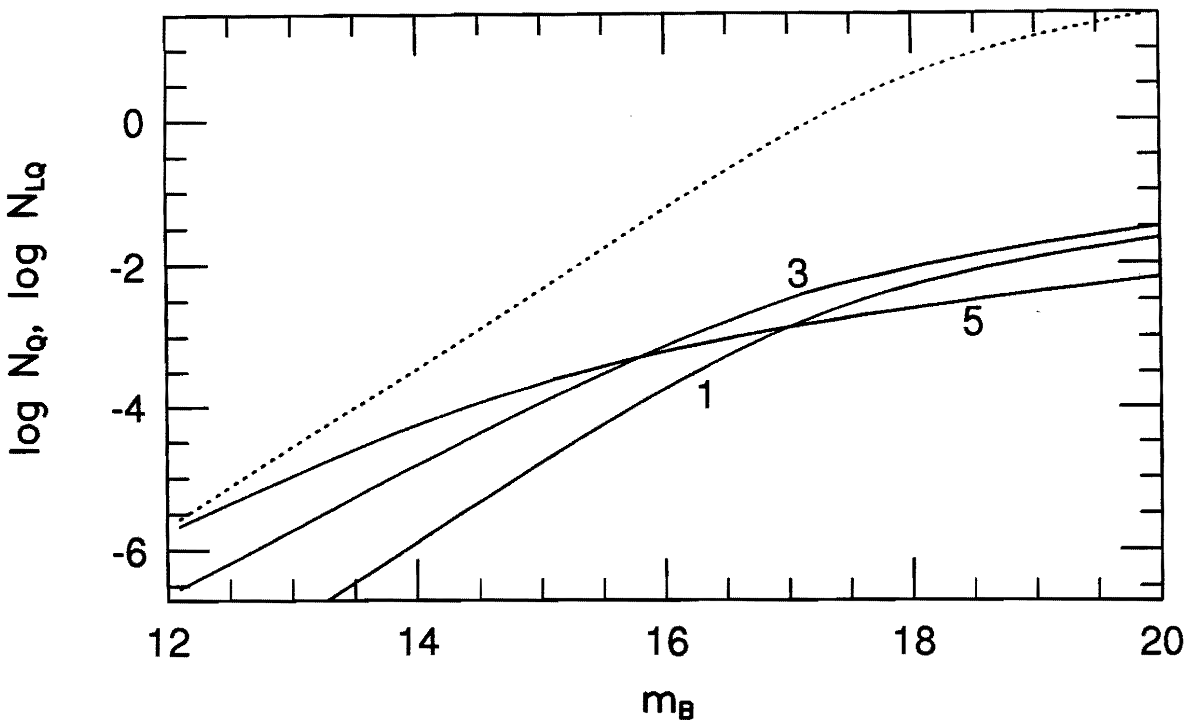

Figure 3: The dotted line shows the quasar luminosity function, $N_{\rm Q}$, and the three solid lines show the estimated numbers of lensed quasars, $N_{\rm LQ}$ per square degree with five, three, and one (brightened) image as a function of apparent magnitude. Note that the fraction of lensed quasars at magnitudes fainter than 19^m is about 10^{-4}, whereas the fraction is about 10^{-2} at around 16^m. This is consistent with the observations. Note also that at bright magnitudes, the number of multiply-imaged quasars significantly exceeds the number of quasars with brightened single images, a verification of the Magnification Multiplicity Conjecture. (From Wallington & Narayan 1993)

From the previous section it is clear that magnification bias due to macrolensing should be quite dramatic for bright quasars, and that the evidence should be particularly strong in the multiply-imaged quasars (Kochanek 1991, Fukugita & Turner 1991, Wallington & Narayan 1993). Figure 2 shows the estimated bias factor as a function of apparent magnitude m_B. Notice how the bias becomes quite substantial at $m_B \sim 16$. The observations clearly confirm the presence of a large effect. From the measured counts and velocity dispersions of galaxies, it is estimated that the probability that a high redshift quasar will be multiply-imaged is $\sim$ few $\times 10^{-4}$ (Kochanek 1993, see Fig. 3). On the other hand, the observed lensing frequency is $\sim 10^{-2}$ at bright magnitudes. The enhanced lensing can be explained by magnification bias, as shown in Fig. 3.

Theoretical calculations (e.g. Fig. 2) clearly indicate that magnification bias in multiply-imaged quasars should be a strong function of apparent magnitude. This is very nicely confirmed by observations. Figure 4 shows a plot of m_B and redshift z of a complete sample of quasars which have been imaged with sufficient resolution to detect multiple imaging. The particular sources in this sample which are known to be lensed are separately identified. Notice how the lensed quasars are strongly segregated towards the bright m_B end. This shows that magnification bias has a gradient such that it strongly favors $m_B \lesssim 17$ compared to $m_B \sim 18-19$. This is exactly the pattern expected for a luminosity function of the form (7). As shown by

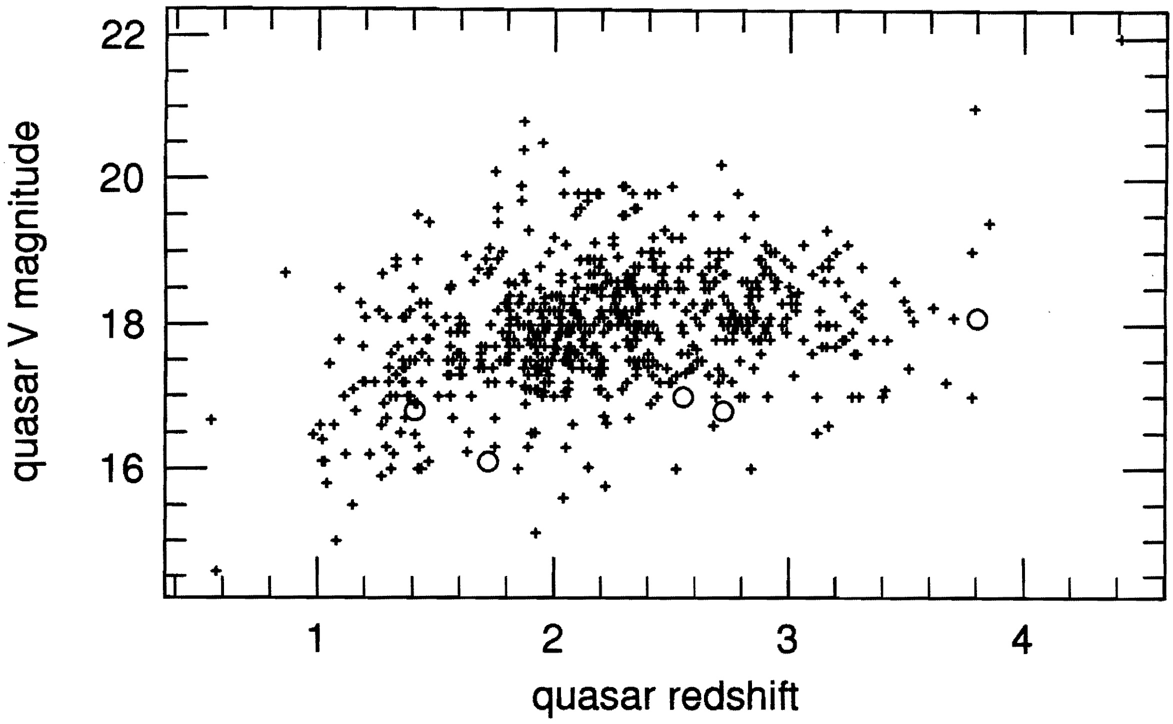

Figure 4: Shows all quasars from the samples studied by Crampton *et al.* (1992), Surdej *et al.* (1993), Yee *et al.* (1993), and Maoz *et al.* (1993). (Plot courtesy C. Kochanek) The multiply-imaged quasars are indicated by circles and the rest of the quasars by pluses. Note how the circles are not uniformly distributed among the pluses but are concentrated towards bright magnitudes. This is consistent with the predictions of magnification bias.

eq. (5), the magnitude of the bias factor depends on μ_*. Since μ_{max} is effectively infinite for macrolensing, μ_* is determined primarily by how much fainter than the flux limit one has to go to reach the break in the luminosity function. At $m_B \sim 16-17$, there is a factor of several tens in the luminosity before the break is reached and therefore the integral in the numerator of (1) receives a large contribution from a wide range of μ. The bias factor is thus quite large. But at $m_B \sim 18-19$, the integral cuts off for $\mu_* \sim$ few because the break is close to the flux limit, and there is only a weak magnification bias. Note that this explanation of the observations requires two features in the quasar luminosity function. First, the bright quasars counts must be a very steep function of apparent magnitude in order to produce a large bias. Equally importantly, the luminosity function must have a break at around $19-20^m$ in order for there to be a gradient in the bias as a function of magnitude.

Interestingly, there is a slight indication (Kochanek 1993) that the evolution of magnification bias with m_B seen in the observations is larger than that predicted. This might mean that either the slope α in eq. (1) is greater than 2.6, or that the break occurs at brighter magnitudes than in the Boyle model. Note that this is exactly in the opposite sense to the modifications proposed by Hawkins & Vèron (1993). They propose a shallower slope, which will reduce the quantitative magnitude of bias. More importantly, their data seem to suggest that there is no break at apparent magnitudes up to $m_B \sim 21$. This is in serious conflict with our understanding of the magnification bias effect, according to which a luminosity model must have a break in order to produce the variation of lensing frequency with m_B seen in Fig. 4. If Fig. 4 is a true representation of lensing probability, *i.e.* if there are no unknown selection effect by which multiple imaging may have been missed in the fainter quasars in this sample, then there has to be a significant break in the quasar optical luminosity function at $m_B \sim 19 - 20$.

5.2 Doubles versus Quadruples

Calculations of magnification bias indicate that five-image configurations have a significantly larger bias factor than three-image configurations (Fig. 2). The former are observed as quadruples and the latter as doubles. In fact, even though the intrinsic total cross-section σ_{tot} for quadruples is significantly less than σ_{tot} for doubles, the theory still predicts that the probabilities of observing the two configurations are roughly equal near $m_B \sim 16 - 17$ (Kochanek 1991, Wallington & Narayan 1993, Kassiola & Kovner 1993, see Fig. 3). This prediction is confirmed by the observations. Among the multiply-imaged quasars shown in Fig. 4, two are quadruples and three are doubles. As we go to fainter samples of lensed quasars, we predict that doubles will predominate over quadruples.

5.3 Magnification Bias and Image Separation

For pure galaxy lensing there is no significant magnification bias effect favoring large angular separations of images in multiply-imaged quasars. Large separations are produced by large lensing galaxies and small separations by smaller galaxies, but within each class of lens the distribution of magnifications is expected to be the same. Therefore there is no significant differential bias as a function of separation. However, if lensing by a galaxy is assisted by a surrounding cluster, then in fact there is a strong positive correlation between image separation and magnification (Turner, Ostriker & Gott 1984), and there will be a magnification bias favoring larger image separations. This effect presumably plays a role in the case of Q0957+501, but it is not clear if it is important for general lens statistics.

6 Quasar-Galaxy Associations

A number of authors have claimed that quasars preferentially occur in the vicinity of galaxies (Arp 1981, Hammer & Notale 1986, Stocke *et al.* 1987, Webster *et al.* 1988, Fugmann 1988, 1989, Drinkwater *et al.* 1991). The overdensity is quite significant in some of these cases, and has been explained as the result of magnification bias (Canizares 1981, Vietri & Ostriker 1983, Schneider 1986). The lensing action of a galaxy brightens background quasars in its vicinity and thus brings some dim quasars into the sample which would not be observed without the presence of the lens. These extra sources are thus preferentially found near galaxies and account for the observed associations.

Given a region of the sky with a certain magnification, μ, the factor by which quasars are overdense is given by (Narayan 1989)

$$q(\mu, L) = \frac{1}{\mu}\frac{\Phi(> L/\mu)}{\Phi(> L)}. \qquad (14)$$

This result reflects two opposing effects of magnification on the observed number density of a population. For a set of sources with constant luminosity, there will actually be an observed underdensity by a factor μ due to the fact that the part of the source plane being observed has been "stretched out". The counteracting effect is that by magnifying the source population, sources which were otherwise too dim can be brought into a flux limited sample. The latter effect will predominate and an overdensity will be observed when the logarithmic slope of the luminosity function is steeper than unity. The quasar luminosity function meets this criterion at the bright end, and significant overdensities can occur. However, as with most applications of magnification bias, the effect diminishes considerably as one gets close to the break in the luminosity function (Narayan 1989, Kovner 1989, Schneider 1989). Indeed, for quasars fainter than the break, there can actually be an underdensity of quasars near galaxies.

At the Gravitational Lenses meeting in Hamburg (Kayser *et al.* 1991), there was a joint discussion of observational and theoretical issues related to quasar-galaxy associations (Narayan 1992). It appears that the observational evidence for associations is generally (i) stronger for bright samples than for faint samples, and (ii) stronger when the search is restricted to within a few arcseconds of the foreground galaxy than when expanded to a larger area. Both of these indications are consistent with what is expected for magnification bias. However, the quantitative strengths of the effect claimed by various groups are not yet understood in detail and may not necessarily be compatible. For instance, it is a matter for concern that the sensitive search carried out by Yee *et al.* (1993) revealed no evidence for an overdensity even though the sample had a number of quite bright quasars.

It is unclear whether macrolensing or microlensing is more important in producing quasar-galaxy associations. In the case of macrolensing the bias is straightforward to calculate since the cross-sections are known. A detailed calculation has not yet been done and is worthwhile. In general, we expect macrolensing to produce an effect only out to a few Einstein radii from the center of a lens. How then does one explain the discovery by Fugmann (1990) that quasars are overdense in the vicinity of Lick galaxies out to several arc minutes? Only with the inclusion of lensing by large scale structure of the universe and with a liberal amount of double magnification bias might macrolensing be able to produce associations at such a large distance (Bartelmann & Schneider 1992).

Microlensing by compact halo objects can produce an effect out to a fairly large distance from a galaxy, almost as far as the halo extends. Also, if most of the dark matter in galaxies is due to microlenses then the cross-section for microlensing will be greater than for macrolensing alone. Theoretical estimates of the effect of microlensing on quasar-galaxy associations have been carried out (e.g. Vietri & Ostriker 1983, Schneider 1986, Linder & Schneider 1988). Compared to the case of pure macrolensing, these calculations have greater uncertainties because the cross-sections are less well understood, particularly when the finite size of the source is considered. One obvious point is that microlensing cannot account for the associations reported for 3C radio sources (Hammer & LeFevre 1990), because these sources have a very extended size. Therefore, if all the published claims for associations are taken at face value, then microlensing alone cannot be the whole story and we probably need to consider the combined effects of macrolensing and microlensing.

7 BL Lacs as Lensed OVVs

Ostriker & Vietri (1985, 1990) proposed the interesting idea that BL Lacs are actually distant OVV quasars which have been brightened by a foreground lensing galaxy. They proposed that the redshifts that have been measured in several BL Lacs do not correspond to the sources at all, but really represent the redshifts of foreground galaxies acting as lenses. The discovery of

Stickel *et al.* (1988, 1989) that some BL Lacs have higher redshifts than the galaxies with which they are apparently associated, adds considerable weight to this theory. Using a very simple lens model consisting of a disk of matter of constant surface density, Ostriker & Vietri found that the magnification of a background quasar has two peaks corresponding to lens positions near the observer and near the source. This implies that the lensing galaxies would preferentially have low or high redshifts. There is indeed evidence for a statistical overdensity of BL Lac host galaxies at low redshift, which appears to strongly validate this picture.

A number of arguments have been advanced against the Ostriker & Vietri scenario (Gear 1991, Kayser 1992, Wallington & Narayan 1993, Merrifield 1993, Abraham *et al.* 1993). Wallington & Narayan (1993) showed that macrolensing alone cannot explain BL Lacs as lensed quasars. There are several arguments. First, at magnification levels which would produce a significant magnification bias, multiple imaging is more common than single image magnification. Thus for every BL Lac which is seen, one would expect to see many bright multiply imaged quasars (see Fig. 3). This is an application of the so-called *Magnification Multiplicity Conjecture* (Kovner 1990, Kochanek 1991), which states that the cross-section for multiple imaging at any given large magnification is generally larger than the cross-section to produce a single brightened image of the same magnification. Another argument is that, when the redshift distribution of the lenses is computed using a more realistic lens model than that used by Ostriker & Vietri, highly brightened single images arise over a wide range of lens redshifts, rather than just in two concentrated peaks near the observer and the source. Finally, Narayan & Schneider (1990) showed that the foreground galaxies of the BL Lacs observed by Stickel *et al.* (1988,1989) must have very large core radii if they are not to produce multiple images of the quasars. These large cores are at odds with other estimates of lens core radii (Wallington & Narayan 1993, Kassiola & Kovner 1993), derived from the absence of central images in multiply imaged quasars.

The above arguments seem to rule out macrolensing. However, Ostriker & Vietri (1985, 1990) did not claim that macrolensing alone would produce their effect. In fact, strong magnification bias associated with microlensing was an essential ingredient of their model. But Gear (1991) has argued that since BL Lacs are strong in the radio, and since microlenses are not expected to magnify the radio continuum significantly, this rules out the Ostriker & Vietri scenario. Similarly, Kayser (1992) argued that in microlensing, high magnification states would last only a few years for solar-mass microlenses, whereas several BL Lacs have remained bright for more than a decade. Both of these arguments rule out solar-mass microlenses. The arguments can be circumvented only if microlenses have masses $\gtrsim 10^6 M_\odot$.

An even more extreme proposal than that made by Ostriker & Vietri is the idea that perhaps all quasars are gravitationally lensed Seyfert nuclei (Barnothy 1966, Setti & Zamorani 1986). This is clearly much too extreme, but it leads to the issue we discuss in the next section.

8 Magnification Bias and the Quasar Luminosity Function

Turner (1980) suggested that strong magnification bias may be responsible for the apparent redshift evolution of quasars and that, particularly at the bright end, the sample of observed quasars could be dominated by lensed images. Later workers showed that the effect is not so powerful when they included the fact that flux is conserved in gravitational lensing (Avni 1981, Peacock 1982, Vietri 1985, Ostriker & Vietri 1986, Schneider 1987). Nevertheless, numerous

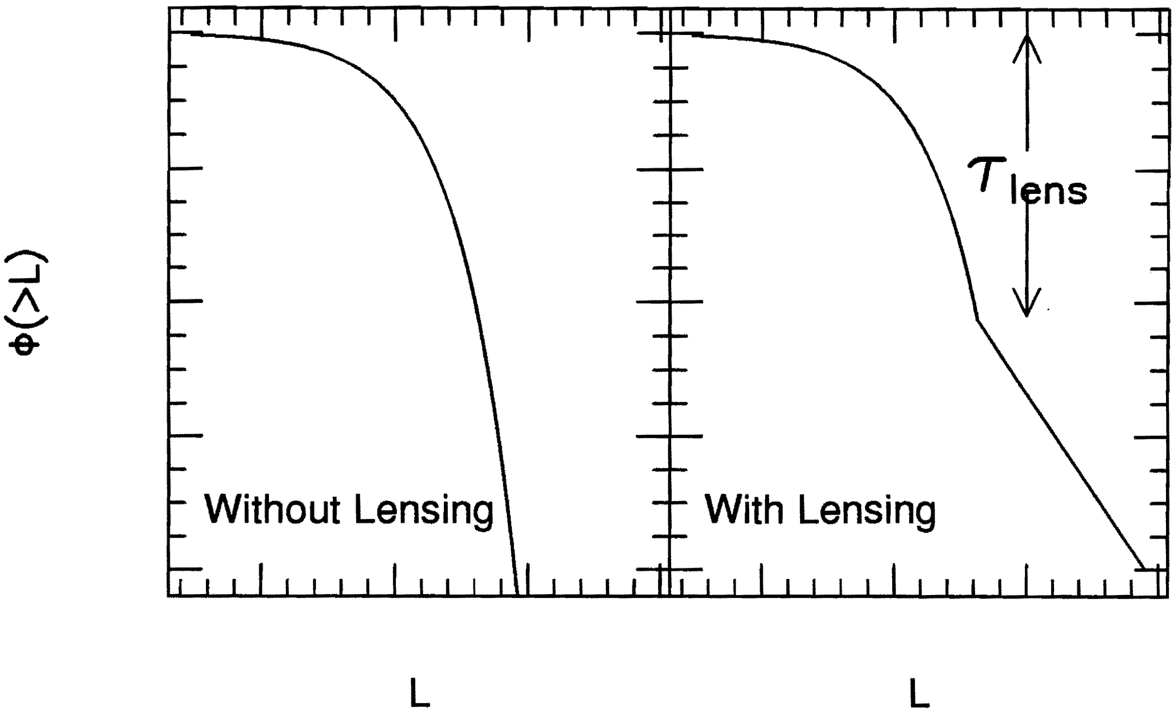

Figure 5: Shows the effect of lensing on the quasar luminosity function. The panel on the left is the intrinsic luminosity distribution of the sources. The panel on the right indicates the observed luminosity function if the sources are viewed through a universe with an optical depth $\tau_{\rm lens}$ in gravitational lenses. The excess sources at bright magnitudes will be preferentially at high redshift and their counts will have a specific slope corresponding to the asymptotic behavior of the lensing cross-section at high magnifications.

investigators have looked into the question of exactly how much the quasar luminosity function is modified by lensing.

If the entire bright end of the observed quasar luminosity function is produced by magnification bias, then the intrinsic slope is presumably much steeper than the observed slope. Lensing with magnification would then produce a bright tail to the luminosity function with an index, α, equal to either 2 or 5/2 depending on which type of caustic dominates (see eqs. 9,10). As shown schematically in Fig. 5, the normalization of the bright tail depends on the optical depth to lensing, $\tau_{\rm lens}$. If $\tau_{\rm lens} < 1$, there will be a segment of the luminosity function showing the intrinsic slope, and the lensed tail will take over only at a lower amplitude. However, the Boyle luminosity function (eq. 7) has a single break and a constant slope brightward of L_*. If the bright end slope is indeed due to lensing, then we require $\tau_{\rm lens} \sim 1$, which as we argue below is unlikely. Another point is that the lensing cross-section increases rapidly with increasing source redshift. This implies that low redshift quasars should show the intrinsic slope in their luminosity function while higher z quasars should be more affected by lensing and should show the lensing slope of 2 or 5/2. The observations, however, indicate essentially the same value of α at all redshifts.

An interesting recent development is the discovery by Goldschmidt *et al.* (1993) that there is an excess of bright quasars at $\sim 16^m$ relative to the Boyle luminosity function (which used the Palomar-Green Survey to normalize the bright end). It would be very interesting to see whether this turnup of the counts is due to lensing. It is the sort of effect that is expected with strong magnification bias for a small $\tau_{\rm lens} \sim$ few $\times 10^{-3}$ (*e.g.* compare with Fig. 3, which corresponds to $\tau_{\rm lens} \sim$ few $\times$ 10^{-4}). If the turnup in the counts seen by Goldschmidt *et al.* is due to lensing, then there are some clear predictions. The luminosity function brightward of the turnup should show a slope of $\alpha = 2$ or $5/2$, most likely the former. Moreover, the excess quasars should be dominated by high redshift sources.

How much effect can macrolensing have on the quasar luminosity function? Certainly, macrolensing is not strong enough to produce the entire bright end of the luminosity function. The reason is that the cross-section for galaxy lenses is so small (Kochanek 1991, Wallington & Narayan 1993, Surdej *et al.* 1993), that only a small fraction of the quasar population would feel the effect of lensing. By the Magnification Multiplicity Conjecture mentioned earlier, if there were a population of lenses such that a significant proportion of observed quasars were brightened by lensing, then there should be a correspondingly large number of multiple-image quasars (Kovner 1990, Kochanek 1991). Since relatively few multiply imaged quasars have been observed, it is not likely that a large number of highly brightened singly imaged quasars exist.

Microlensing is a more likely candidate for causing a significant alteration of the quasar luminosity function, since it can produce strong magnifications without detectable splitting. Several authors have therefore considered this possibility (*e.g.* Schneider 1987, Bartelmann & Schneider 1992). However, as we have seen with several of the applications of magnification bias, the main stumbling block lies in the radio observations. The luminosity function of radio quasars is similar to that of optical quasars which means that very likely both functions are produced by the same effect. However, microlensing should have very little effect on radio sources, because of their large sizes. In addition, there are already interesting limits on the optical depth $\tau_{\rm lens}$ in the universe due to microlensing objects of various masses (Canizares 1982, Nemiroff 1991, Kassiola, Kovner & Blandford 1982, Surdej *et al.* 1993, Schneider 1993, see Nemiroff, this volume). As these constraints improve, the possibility that microlensing causes a large effect on the observed quasar luminosity function will become less attractive.

9 Summary

In §§ 5–8 we discussed four different phenomena in gravitational lensing where a significant effect due to magnification bias has been claimed. Table 1 summarizes our assessment of the weight of the current evidence. We feel that the case for a strong magnification bias in the multiply-imaged quasars is compelling (§5), while there is practically no evidence for a significant effect on the overall quasar luminosity function (§8). We consider the case to be intermediate in the other two phenomena — there are intriguing indications in the observations for a measurable excess of galaxies in the vicinity of quasars (§6) and for BL Lacs to be lensed (§7), but the theoretical situation is somewhat confused.

Discussions of magnification bias in the literature tend to consider both macrolensing and microlensing. We summarize below our views on the relative merits of the two hypotheses.

9.1 Macrolensing

There is no question that magnification bias due to macrolensing is important and that it has been observed. In fact, as we showed in §5, the effect is quite dramatic in the case of the multiply-imaged quasars. On the other hand, it is unlikely that macrolensing can make

Table 1: Applications of Magnification Bias

Type of Phenomenon	Observational Evidence	Theoretical Situation
Multiple-image quasars	Strong evidence for MB	Consistent with macrolensing
Quasar-galaxy associations	Moderately convincing	Possible with macrolensing Microlensing cannot explain radio
BL Lacs by lensing	Intriguing but not convincing	Unlikely with macrolensing Perhaps $> 10^6 M_\odot$ microlenses?
Quasar luminosity function	No evidence	Impossible with macrolensing Perhaps $> 10^6 M_\odot$ microlenses?

all BL Lacs by lensing distant OVVs (§7), or that it can produce a wholesale modification of the quasar luminosity function (§8). The key argument is the Magnification-Multiplicity Conjecture, which argues against preferentially producing brightened single images instead of multiple images. Since the total optical depth in the universe to multiple-imaging is limited, there is a tight limit to how much effect we can expect from macrolenses. In the case of quasar-galaxy associations it is quite likely that macrolensing can explain some of the observations. This is because the claimed effect is relatively weak and could plausibly be produced by fairly modest macrolensing magnifications. This is yet to be demonstrated with detailed calculations.

9.2 Microlensing

Theoretically, microlensing offers a much greater potential to produce a significant magnification bias compared to macrolensing. There are two reasons for this:

(i) Until recently, there was in principle no limit to the optical depth to microlensing that one could assume in the models. Indeed, many optimistic studies postulated nearly closure density in microlenses of some favorable mass range.

(ii) There is no need to restrict attention to singly-imaged configurations, since even the multiply-imaged configurations will be unresolved and will appear as brightened single images. This increases the cross-section and eliminates the Magnification-Multiplicity Conjecture.

Despite these advantages, recent developments have weakened the case for large scale effects due to microlensing.

(i) The density of the universe in microlenses of various masses is beginning to be constrained by a variety of observations and arguments. Indeed, there is currently no mass range where $\Omega \sim 1$ is allowed, and the limits may become quite stringent in the near future.

(ii) Several of the claimed phenomena (*e.g.* quasar-galaxy associations, BL Lacs, quasar luminosity function, §§ 6–8) seem to be seen both in optical and radio quasars. The only direct observational evidence we have for microlensing is from the variability of optical quasars (e.g. Q2237+0305: Corrigan *et al.* 1991, Q0957+561: Schild, this volume). It has been established that this variability must be produced by sub-solar mass microlenses, but such lenses cannot possibly magnify radio sources. Radio sources can have a significant magnification bias only if microlenses have masses $M \gtrsim 10^6 M_\odot$, but there is no evidence at all for a significant mass density in the universe in such objects. In fact it is precisely this mass range where limits on the number density of lenses has become quite tight.

For the above reasons, we are inclined to believe that the role of microlensing in magnification bias is fairly limited.

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