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The Theoretical Analysis of Lens Surveys

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Abstract: We describe a maximum likelihood approach for analyzing lens surveys designed to use all the data provided by the surveys to constrain both lens and cosmological models. For flat cosmologies we place a 90% confidence upper limit on the cosmological constant of $\Omega_\Lambda < 0.8$ using the most pessimistic systematic assumptions and $\Omega_\Lambda < 0.45$ with the most optimistic. In the Einstein-DeSitter cosmology, the best fit velocity dispersion for an L_* E/S0 galaxy is $\sigma_* = 245$ km s^{-1} with a 90% confidence range of 210 km s^{-1} $< \sigma_* <$ 270 km s^{-1}. Thus the observations are inconsistent with the $(3/2)^{1/2}$ correction to the velocity dispersion, and this is supported by a more careful calculation of the dynamics of a luminous galaxy in an isothermal halo. The measured comoving density of galaxies or dark halos on this velocity dispersion scale is $n_* = 6.7 \times 10^{-3} h^3$ Mpc^{-3} with a 90% confidence interval from 2.0×10^{-3} to $1.7 \times 10^{-2} h^3$ Mpc^{-3} consistent with the observed number of galaxies. The observed statistics are consistent with the SIS model, although there are signs that the model has insufficient magnification bias.

1 Introduction

Besides finding lenses, the goal of lens surveys is to use the absolute incidence of lenses to determine the cosmological model. Unfortunately this is a difficult problem because of uncertainties introduced by mass estimates for galaxies, galaxy structure, and selection effects. Our best hope for disentangling all the problems is to use the least amount of *a priori* information in analyzing the lens statistics, and allow the lenses themselves to eliminate the uncertainties in the parameters. Where this is not possible, we can check the sensitivity of the statistical model to the input parameters. In our study we include the optical surveys for lensed quasars by the Snapshot survey (Bahcall *et al* 1992, Maoz *et al* 1992, 1993ab), Yee *et al* (1993), and Crampton *et al* (1993). The three surveys include 584 unlensed and five lensed quasars. We did not include the ESO/Liège Survey (Surdej *et al* 1993) for a variety of technical reasons discussed by Kochanek (1993a). Because the optical surveys overlap to a large extent, including the ESO/Liège Survey would only increase the sample by about 10%. We also include the separations of the eight radio lenses from the MIT/Green Bank (Burke *et al* 1992) and Jodrell (Patnaik *et al* 1992) lens surveys.

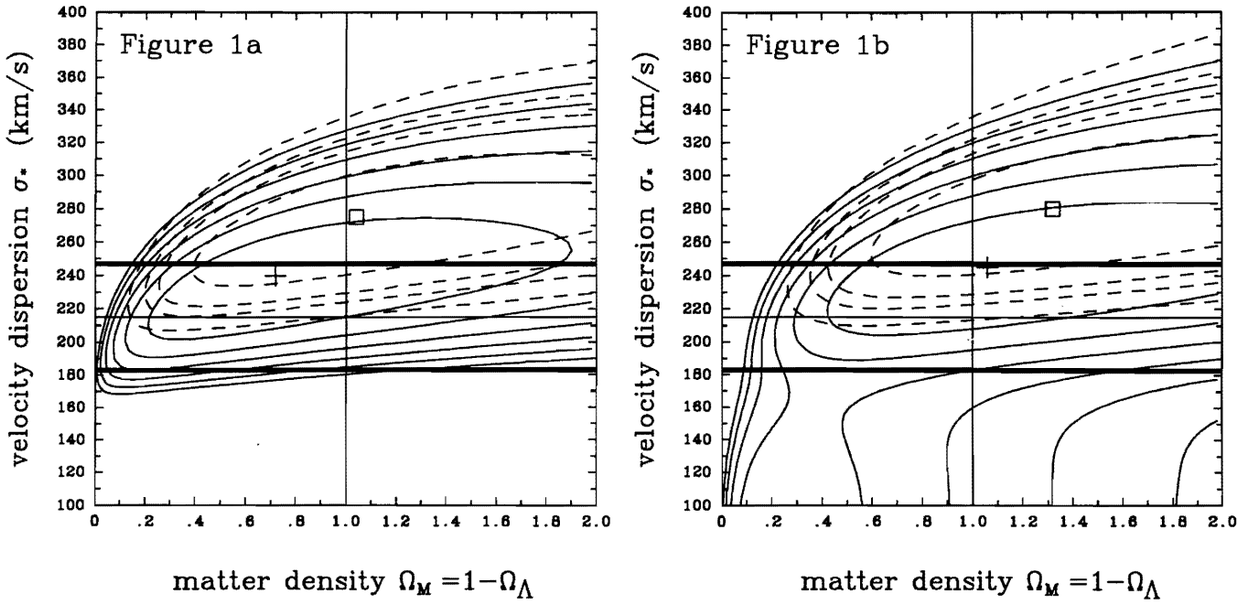


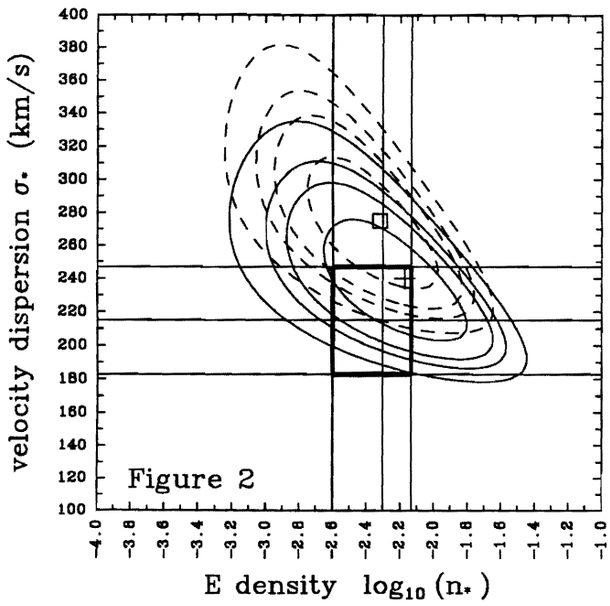
Figure 1: Likelihood contours for flat cosmologies in the space of the velocity dispersion scale σ_* and the matter density $\Omega_M = 1 - \Omega_\Lambda$. The solid lines show the likelihood contours excluding 0957+561, and the dashed lines show the likelihood contours including 0957+561. The peak likelihood with 0957+561 is marked by the square, and the peak without 0957+561 is marked by the cross. Figure 1a includes only E/S0 galaxies, while figure 1b also includes spiral galaxies with an effective velocity dispersion of $\sigma_* = 150$ km s^{-1}. Contours lie at the 68% (1σ), 90%, 95% (2σ), and 99% confidence levels of the likelihood ratio for two parameters. The vertical line marks the default Einstein-DeSitter cosmology. The light horizontal line is the dynamical estimate of σ_*, and the heavy horizontal lines mark the dynamically estimated range for σ_*.

2 Maximum Likelihood Methods for Lens Statistics

All analyses of lens surveys to date used only Poisson statistics to evaluate whether the number of lenses observed agrees with a particular theory. Poisson statistics cannot distinguish between different ways of producing a fixed number of lenses because it is sensitive only to the mean probability that the quasars in the sample were lenses $\langle p_i \rangle$ – in particular you cannot distinguish between changes in the numbers of lenses produced by cosmology, lens structure, and magnification bias. Yet these effects can be disentangled if the statistical approach is sensitive to not only the number of lenses, but also to their distribution and properties. Cosmology also effects the redshift distribution, magnification bias also effects the magnitude distribution, and lens structure also effects the morphologies, magnitudes, and separations.

Maximum likelihood techniques are intrinsically sensitive to the shapes of probability distributions, so they will be much more powerful discriminants between statistical models than Poisson statistics. Suppose that we have N_U unlensed quasars, and N_L lensed quasars and that the probability that quasar i is lensed in the current statistical models including all selection effects is p_i. The likelihood function for finding this set of lensed and unlensed quasars is

$$\ln L = -\sum_{i=1}^{N_U} p_i + \sum_{j=1}^{N_L} \ln p_j$$



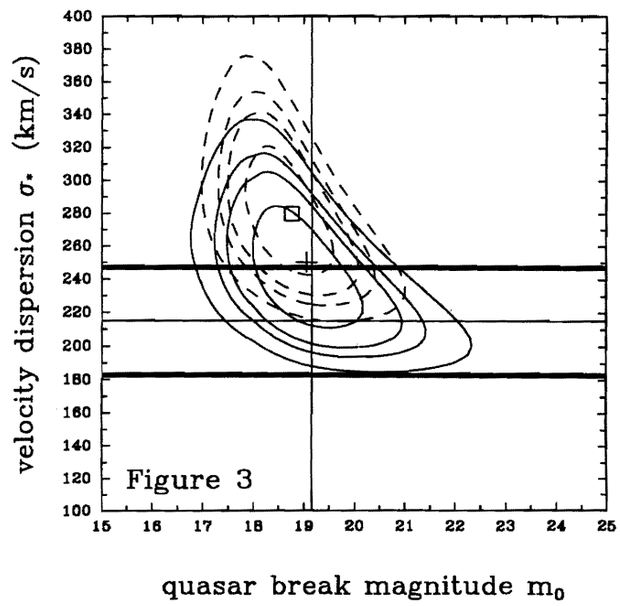


Figure 2: Likelihood contours in the space of velocity dispersion scale σ_* and the number density of ellipticals n_* (in units of h^3 Mpc^{-3}) in an Einstein-DeSitter cosmology. The vertical line marks the observed density of E/S0 galaxies, with a 90% confidence error bar on the measured value. The heavy solid box is the allowed range estimated from galaxy number counts and dynamics. Contour spacing is the same as in figure 1.

Figure 3: Likelihood contours in the space of break magnitude of the quasar luminosity function and the velocity dispersion scale σ_*. The vertical line marks the observational value of $m_0 \simeq$ 19.15 B mag. Contour spacing is the same as in figure 1.

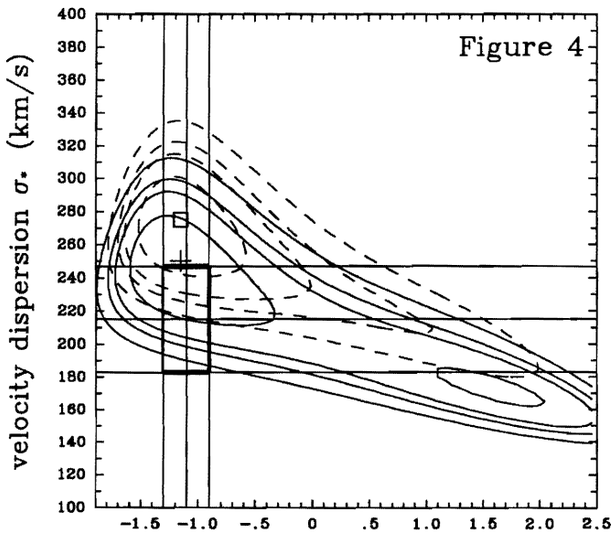
for $p_i \ll 1$. This expression is almost identical to the Poisson likelihood if the probability of being lensed is the same for all quasars. The real samples, however, are widely distributed in redshift and magnitude, so the probabilities vary by an order of magnitude or more over the sample.

This likelihood function still has trouble discriminating between numbers of galaxies, galaxy masses, and cosmological models because it has no sensitivity to the morphologies of the lenses. We therefore add a configuration probability p_{ci} for each of the lensed quasars. The configuration probability can include the image separation, the morphology, flux ratios, and lens redshift. For our purposes we include only the probability for a lens to have a given separation, because this is what we need to be sensitive to galaxy masses.

3 The Statistical Model

The statistical model popularized by Fukugita & Turner (1991) is a population of singular isothermal spheres (SIS) distributed based on Schechter (1976) function fits to local galaxy counts,

$$\frac{dn}{dL} = \frac{n_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha} \exp\left(-L/L_*\right) \qquad \alpha \simeq -1.1$$



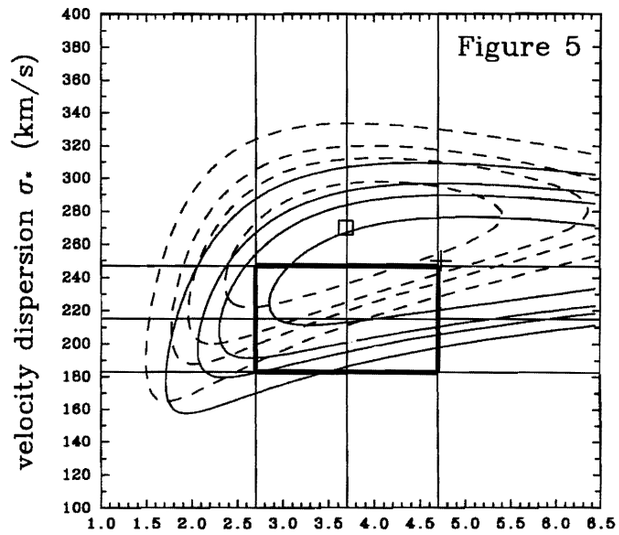


Figure 4: Likelihood contours in the space of Schechter exponent α and velocity dispersion scale σ_* for a fixed galaxy number density n_* equal to the density of E/S0 galaxies. The vertical line marks the observational value for $\alpha \simeq -1.1$, with a 90% confidence error bar on the measured value. The heavy solid box is the allowed range estimated from galaxy number counts and dynamics. Contour spacing is the same as in figure 1. There is a secondary minimum in the results without 0957+561 (solid lines) near $\alpha = 1.5$.

Figure 5: Likelihood contours in the space of Tully-Fisher exponent γ and velocity dispersion scale σ_* for a fixed galaxy number density n_* equal to the density of E/S0 galaxies. The vertical line marks the assumed value of $\gamma \simeq 4$ and a 90% confidence error level in the estimated value. The heavy solid box is the allowed range estimated from galaxy number counts and dynamics. Contour spacing is the same as in figure 1.

and the Tully-Fisher/Faber-Jackson relations to relate the velocity dispersion scale of the SIS mass model to the characteristic luminosity of the galaxies

$$\frac{L}{L_*} = \left(\frac{\sigma}{\sigma_*}\right)^{\gamma} \qquad \gamma \simeq 4.$$

Turner, Ostriker, and Gott (1984) first advocated increasing the dynamical dispersion in the SIS lens model for E/S0 galaxies to $(3/2)^{1/2}$ of the measured value based on the effects of dark halos (Gott 1977). A more careful analysis of this problem by Kochanek (1993b) shows that this correction is too large, and that under most circumstances the central velocity dispersion is a reasonable estimate for the SIS dispersion with errors of 10-20%. While it is true that the average dispersion of the luminous matter is smaller than that of the SIS halo by the $(3/2)^{1/2}$ factor, the central velocity dispersion in a typical aperture is comparable to the halo dispersion.

Unfortunately these 10-20% errors cripple attempts to limit cosmological models because the expected number of lenses scales as σ_*^4 leading to 50-100% errors in the expected number of lenses. By using the measured lens separations, we can use the lenses themselves to independently estimate the appropriate value of σ_*. All probabilities are calculated including the selection function for the individual surveys. Where the same quasar was observed by several

surveys, the superposition of all the selection functions is used. The details of the calculation are contained in Kochanek (1993b).

4 Conclusions

The most important statistical result is that the lens surveys are inconsistent with a large cosmological constant, with $\Omega_\Lambda < 0.8$ at a 90% confidence level. This is independent of the treatment of 0957+561 and spiral galaxies. Figure 1 shows likelihood contours in the two dimensional space of matter density $\Omega_M = 1 - \Omega_\Lambda$ and the estimated velocity dispersion of L_* E/S0 galaxies σ_* both without (figure 1a) and with (figure 1b) spiral galaxies. This constraint is weaker than suggested by Fukugita & Turner (1991) because the value of σ_* is lower than the $(3/2)^{1/2}$ corrected value they used. The high value of $\sigma_* \simeq 270$ km s^{-1} used by Fukugita & Turner (1991) is only marginally consistent with the lens data from which we infer a best fit value of $\sigma_* = 245$ km s^{-1} with a 90% confidence range from 210 km s^{-1} $< \sigma_* <$ 270 km s^{-1} for the Einstein-DeSitter cosmology. Figure 2 shows the likelihood contours fitting the velocity dispersion scale σ_* and the local comoving density of E/S0 galaxies n_*. The estimate for n_* from lens statistics is consistent with the locally measured value, although the error bars from the lens measurement are larger, and it strongly limits the existence of compact, dark matter concentrations resembling galaxies.

The lens statistics must have a strong break in the quasar number count distribution (see figure 3) near $m_B = 19.15$ to produce the correct amount of magnification bias and magnitude distribution of the lenses. There are some signs that the degree of magnification bias at the bright end of the quasar distribution is being underestimated, and this is consistent with the effects of adding ellipticity or a core radius to the lens model. The inconsistency is not, however, statistically significant at the estimated 90% confidence limit. The statistics are completely inconsistent with quasar number counts that lack a break, because all the lenses are concentrated at the bright end of the quasar sample. If there was no break, the lenses would be more uniformly distributed in magnitude.

We tried estimating other parameters of the model such as the Schechter function slope α and the "Tully/Fisher" exponent γ. When everything except σ_* and one of these two variables is held fixed, we find that lens statististics are consistent with the measured values. There are not enough lenses to determine the shape of the distribution of image separations, so the statistics constrain these parameters weakly (see figures 4 and 5). The small number of lenses means that only the mean image separation is measured with any accuracy and even this depends on the treatment of 0957+561. Only with a larger lens sample will lenses be able to strongly constrain the distribution of galaxies.

Unfortunately the existing samples are not large enough to proceed further – Monte Carlo models resampling the data sets using the best fit model and then redoing the maximum likelihood analysis show that the likelihoods for the SIS model fitting the observed data are comparable to the median likelihoods found in the Monte Carlo studies. The greatest improvements in the models will come from finding more gravitational lenses because constraints on the velocity dispersion scale will become much more stringent if the number of lens separations is doubled. Estimating the amount of ellipticity directly from the data will be more practical if there are a larger number of four versus two image lenses – at the moment, attempts to estimate the ellipticity were swamped by the Poisson errors from the small numbers of lenses. Doubling the sample size will begin to constrain the value of Ω_M in normal ($\Omega_\Lambda = 0$) cosmologies.

Acknowledgements

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References

Bahcall, J.N., Maoz, D., Doxsey, R., Schneider, D.P., Bahcall, N.A., Lahav, O., & Yanny, B., 1992, *Ap J* 387, 56

Burke, B.F., Lehàr, J., & Connor, S.R., 1992, in *Gravitational Lenses*, R. Kayser, T. Schramm, & L. Nieser, eds., (Springer: Berlin) 237

Crampton, D., McClure, R.D., & Fletcher, J.M., 1992, *Ap J* 392, 23

Fukugita, M., & Turner, E.L., 1991, *MNRAS* 253, 99

Gott, J.R., 1977, *Ann Rev* 15, 235

Hartwick, F.D.A., & Schade, D., 1990, *Ann Rev* 28, 437

Kochanek, C.S., 1993a, *Ap J in press*

Kochanek, C.S., 1993b, *Ap J in press*

Maoz, D., Bahcall, J.N., Schneider, D.P., Doxsey, R., Bahcall, N.A., Lahav, O., & Yanny, B., 1992, *Ap J* 394, 51

Maoz, D., Bahcall, J.N., Doxsey, R., Schneider, D.P., Bahcall, N.A., Lahav, O., & Yanny, B., 1993a, *Ap J* 402, 69

Maoz, D., Bahcall, J.N., Schneider, D.P., Bahcall, N.A., Lahav, O., Djorgovski, S., Doxsey, R., Gould, A., Kirhakos, S., & Yanny, B., 1993b, *Ap J* 409, 28

Patnaik, A.R., Browne, I.W.A., King, L.J., Muxlow, T.W.B., Walsh, D., & Wilkinson, P.N., 1992, in *Gravitational Lenses*, R. Kayser, T. Schramm, & L. Nieser, eds., (Springer: Berlin) 140

Schechter, P., 1976, *Ap J* 203, 297

Surdej, J., Claeskens, J.F., Crampton, D., Filippenko, A.V., Hutsemékers, D., Magain, P., Pirenne, B., Vanderriest, C., & Yee, H.K.C., 1993, *AJ* 105, 2064

Tully, R.B., & Fisher, J.R., 1977, *A&A* 54, 661

Turner, E.L, Ostriker, J.P., & Gott, J.R., 1984, *Ap J* 284, 1

Yee, H.K.C., Filippenko, A.V., & Tang, D., 1993, *AJ* 105, 7