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QCD and Experiment: Status of α_s

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1. Introduction

This is a concise review of the present status of the experimental determination of the QCD coupling α_s [1]. In 1989 I summarised the situation at that time [2]. Since then many important developments have taken place, both in theory and experiment. In particular, LEP was commissioned and LEP experiments have much contributed to precise QCD tests [3]. In other experimental areas, the picture has become considerably clearer, for example in deep inelastic scattering. Many relevant theoretical calculations, often of unprecedented complexity, have been completed. Some erroneous results were corrected. As an outcome of 3 years of really remarkable progress, our confidence in QCD has been further consolidated and the determination of α_s is now more precise and a lot of additional checks from many different processes have become possible. In this presentation I will mostly concentrate on new developments and much of the general material of ref.2 is not repeated here.

Because of the asymptotic freedom property of QCD a perturbative approach to hard processes is possible and consequently the expansion parameter, i.e. the running coupling $\alpha_s(Q)$, can be measured, Q being of the order of the typical energy scale of the measured hard process. The experimental results on $\alpha_s(Q)$ are often expressed in terms of $\Lambda_{\overline{MS}}$ which is useful in order to compare $\alpha_s(Q)$ at different values of Q. However, it is important to keep in mind that what is directly measured is $\alpha_s(Q)$. For example, for a given precision in $\alpha_s(Q)$, the accuracy on $\Lambda_{\overline{MS}}$ decreases with Q. Thus, sometimes experiments

at small Q may erroneously appear to be more sensitive to $\Lambda_{\overline{MS}}$ or to $\alpha_s(m_Z)$, i.e. the value of α_s at a much larger value of Q. But, of course, at small values of Q, the relation between $\alpha_s(Q)$ and what is actually measured is in general much less clear. In most cases, this apparent sensitivity is abundantly overcompensated if theoretical ambiguities are correctly evaluated. For measuring α_s, the real good experiments are those that directly lead to precise values of $\alpha_s(Q)$ at large Q.

At the present level of accuracy, when a value of $\Lambda_{\overline{MS}}$ is quoted it is extremely important to completely specify its relation with $\alpha_s(Q)$. In the following we shall always refer to the expression [1]:

$$\alpha_s(Q) = \frac{1}{b_f \log Q^2/\Lambda^2} \left[1 - \frac{b'_f \log\log Q^2/\Lambda^2}{b_f \log Q^2/\Lambda^2} \right] \qquad (1)$$

with $\Lambda = \Lambda_{\overline{MS}}$ and $b_f = \frac{33-2f}{12\pi}$, $b'_f = \frac{153-19f}{2\pi(33-2f)}$ where f is the number of light (approximately massless) quarks with m<<Q. Note that in order to relate α_s at different values of the scale one can also use the direct solution of the renormalisation group equation. To the same accuracy as in (1) one has (see ref.1, eq.3-27):

$$\frac{\alpha_s(\mu)}{\alpha_s(Q)} = 1 + b_f\, \alpha_s(\mu)\, t + b'_f\, \alpha_s(\mu) \log[1 + b_f\, \alpha_s(\mu)\, t] \qquad (2)$$

where $t = \log Q^2/\mu^2$. Small numerical differences between (1) and (2) are due to truncation of the perturbative series and are part of the theoretical uncertainties. A more substancial source of theoretical error is encountered when comparing $\alpha_s(Q)$ at values of Q where different numbers for f are to be taken. For example, on the one hand, the results of experiments on deep inelastic scattering, on Υ-decays etc. are usually expressed giving the value of $\Lambda^{(4)}$, defined by eq.1 written in terms of b_4 and b'_4. On the other hand, experiments at PEP, Petra, Tristan and LEP naturally express their results by using $\Lambda^{(5)}$, defined by eq.1 written in terms of b_5 and b'_5. In the threshold region the running of $\alpha_s(Q)$ is deformed by non perturbative effects. As the physical running coupling is continuous, the matching between the perturbative domains below and above threshold is obtained by assuming a smooth extrapolation of the beta function. In practice, one relates $\Lambda^{(4)}$ and $\Lambda^{(5)}$ by imposing continuity of $\alpha_s(Q)$ at some threshold value of Q, $Q_{th} \cong o(m_b)$: $\alpha_s(Q_{th})_{f=4} = \alpha_s(Q_{th})_{f=5}$. Slightly different procedures can also be envisaged (for an example, see ref.2).The resulting difference between $\Lambda^{(4)}$ and $\Lambda^{(5)}$ is quite important. Indeed, for realistic values of Λ, it amounts to approximately

$$\Lambda^{(5)} \cong 0.65\, \Lambda^{(4)} \qquad (3)$$

Clearly, some theoretical error is involved when the results at low energy, for, say, Q=3-5 GeV, are compared with LEP results at $Q=m_Z$. By varying Q_{th} in the range (0.75-2.5) m_b, with m_b=4.8 GeV and using eq.1 or eq.2, we estimate an error

$$\Delta\alpha_s(m_Z) \simeq \pm 0.0015 \qquad (4)$$

associated with translating a value of $\alpha_s(Q)$ measured below the beauty threshold (but above charm) into a corresponding figure for $\alpha_s(m_Z)$. A larger error on $\Delta\alpha_s(m_Z)$ would be implied if one would try to go from $Q \simeq 1$ GeV, i.e. below charm threshold, up to $Q=m_Z$.

We now consider the various methods to determine $\alpha_s(Q)$, starting from the totally inclusive processes.

2. Deep Inelastic Scattering

Leptoproduction at large momentum transfer is in principle a very solid and powerful method. It is completely inclusive and the theory of scaling violations via the renormalisation group and the light cone operator expansion is particularly rigorous and clear [1]. Moreover several structure functions can be measured, with different beams and targets. The Q^2 dependence can be tested at each value of x. Thus deep inelastic scattering provides a whole laboratory for quantitative tests of QCD, for determining α_s and for measuring the quark and gluon densities in nucleons.

For each structure function $F(x,Q^2)$, perturbative QCD predicts the Q^2 evolution but not the x shape. However, the structure of the evolution equations is such that Q^2 evolution and x shape are interrelated. The maximum of emphasis on specifically testing the pattern of scaling violations predicted by QCD is obtained by focusing on the logarithmic slopes. For each x-bin, one derives the best fit for the slope of a straigth line through the data on $\log F(x,Q^2)$ plotted versus $\log Q^2$. The Q^2 span and precision of the data are not sensitive to the curvature, so that for each x-bin one has one number for $d\log F(x,Q^2)/d\log Q^2$. The QCD evolution equations relate, for each given x, the slopes to $\alpha_s(Q)$. In the simplest non singlet case one has:

$$\frac{d\log F(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q)}{2\pi}\int_x^1 \frac{dy}{y}\frac{F(y,Q^2)}{F(x,Q^2)}\left[P_{qq}\left(\frac{x}{y}\right) + \text{two-loops}\right] \qquad (5)$$

A single value of Λ should fit the slopes at all values of x. The x shape of the structure function is obtained in terms of a suitable parametrisation by fitting the whole of the data.

As already mentioned, now after about 25 years of experimental work on scaling violations in deep inelastic scattering a clear pattern is finally emerging. In short, the important progress over the situation of a couple of years ago is listed in the following. On leptoproduction from muon (or electron) beams, the famous discrepancy between the results of BCDMS and EMC on H and D targets at low x has been to a great extent clarified

by a reanalysis of the EMC data, by comparison with the SLAC data and by new data from the NMC collaboration. The results obtained on H, D, C and Fe targets by the various experiments are quite consistent among them. On neutrino experiments, the new data on the structure function F_3 on Fe by CCFR contradict previous results by CDHSW, which showed poor agreement with the QCD predictions at low x. The analysis of scaling violations in the CCFR data lead to values of $\alpha_s(Q)$ in perfect agreement with those obtained from charged lepton beams. Also, the discussion of theoretical errors in the determination of $\alpha_s(Q)$ from deep inelastic scattering has been considerably improved.

The BCDMS collaboration [4] has measured F_2 with muon beams on C, H and D targets. This experiment has the largest statistics at large Q^2. The C data are for Q^2>25 GeV^2 in the range 0.275<x<0.75, while for H and D, Q^2>8 GeV^2 for 0.06<x<0.16, Q^2>14 GeV^2 for 0.16<x<0.25 and Q^2>20 GeV^2 for 0.25<x<0.80. The QCD analysis of the BCDMS data is very solid and consistent results are obtained from singlet and non singlet structure functions on C, H and D targets.

There are no news on the BCDMS analysis of C data [4]. These data, at x>0.275 where the gluon density is negligible, were analysed in the non singlet approximation. The results obtained for $\alpha_s(Q)$ are reported in Table 1 (only experimental errors are included in Table 1).

More recently a combined analysis of the BCDMS [4] and the SLAC [5] data on H and D targets has been performed [6]. The SLAC data were collected in the period 1970-85. They were recently reanalysed with a better set of radiative corrections and a more refined measurement of R (the ratio of longitudinal to transverse structure functions).The Q^2 and x range of the SLAC data is Q^2=0.5-30 GeV^2 for x=0.07-0.75. The SLAC data are important to check the normalisation and to model higher twist effects that cause the main systematic error at large x. The minimum value of Q^2 is too low but, for the determination of $\alpha_s(Q)$, only the data at Q^2>5-20 GeV^2 were used. The average value of Q^2 is 75 GeV^2 for the BCDMS sample and 50 GeV^2 for the combined SLAC-BCDMS set of data used for $\alpha_s(Q)$. The higher twist contribution is sizeable at low values of Q^2. The higher twist effect on F_2 is parametrised according to $F_2(x,Q^2)=F_2^{LT}(x,Q^2)\,[1 + C(x)/Q^2]$, where F_2^{LT} is the lower twist structure function (with target mass corrections included) and C(x) is a function fitted to the data. The measured pattern of higher twist corrections is reasonable [6]. C(x) is of order 1 in GeV^2 at x>0.6 and compatible with zero within errors at small x. The result of the QCD analysis of the measured logarithmic slopes, combined for H and D data (which lead to quite consistent results), is shown in fig.1 [6]. The data extend down to very low values of x, so that a singlet analysis was performed, with a gluon density also fitted to the data. The uncertainties associated with varying the gluon density within errors and the effect of the higher twist correction are also shown in fig.1.The corresponding results on $\alpha_s(Q)$ are displayed in Table 1. As seen, the determinations of $\alpha_s(Q)$ from C and H,D targets agree very well.

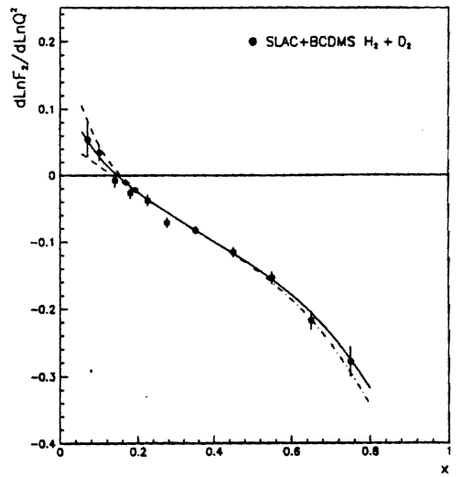

Fig. 1: Logarithmic slopes from the combined SLAC-BCDMS data on H and D [6]. The QCD fit, corresponding to the value of $\Lambda^{(4)}$ shown in Table 1, is also displayed. The uncertainties from varying the fitted gluon density within the errors (at small x) and the effect of higher twist terms (at large x) are also presented.

Table 1

Experiment	$\Lambda^{(4)}$ (MeV)	$\alpha_s(m_Z)$
	μ-beams	
BCDMS C	230 ± 63	0.110 ± 0.005
SLAC-BCDMS H,D	263 ± 42	0.113 ± 0.003
EMC H	211^{+117}_{-108}	0.108 ± 0.009
NMC D	307 ± 72	0.117 ± 0.005
	ν-beams	
CCFR Fe	210 ± 50	0.111 ± 0.004
CHARM $CaCO_3$	310 ± 157	0.114 ± 0.009

Table 1. Results on $\Lambda_{\overline{MS}}$ for f=4 and the corresponding values of $\alpha_s(m_Z)$ obtained by the various experiments. The indicated errors are purely experimental, with statistical and systematic errors added in quadrature.

The famous discrepancy between the BCDMS and the EMC [7] data on H has now been clarified to a large extent. First, the EMC data were recently reanalysed[8]. One important difference is in the treatment of $R = \sigma_L/\sigma_T$. In the original analysis the position R=0 was made, while in the new analysis $R=R_{QCD}$ was taken (as was done by BCDMS as well) and the normalisation was consequently recomputed. The logarithmic slopes obtained by the new EMC analysis and their QCD fit are shown in fig.2 [8]. The corresponding results for $\alpha_s(Q)$ are shown in Table 1. Second, the new NMC (New Muon Collaboration) [9] data on H and D targets are in good agreement with the BCDMS results but still show some disagreement with the reanalysed EMC data (see fig.3). A similar indication in favour of BCDMS had already been obtained by comparison with the SLAC data. However in that case the comparison was only based on an extrapolation, because there is no overlap in x and Q^2 between the SLAC and the BCDMS data.

The new NMC data cover a wide kinematic range, which extends down to very small values of x: $0.008<x<0.7$ and $0.8<Q^2<50$ GeV^2. The preliminary analysis of the logarithmic slopes is presented in fig.4 [9]. We see that the gluon density effect is quite substancial, because the data are at particularly low values of x. The results on $\alpha_s(Q)$ are reported in Table 1.

We now consider deep inelastic scattering from neutrino or antineutrino beams. Here the news are represented by the measurement of xF_3 on Fe by the CCFR collaboration [10] at $\langle Q^2\rangle=25$ GeV^2. At low x, the CCFR data are in disagreement with the previous measurements of the same structure function on the same target by CDHSW [11]. The logarithmic slopes measured by CCFR are in good agreement with the QCD predictions (which was not the case for the CDHSW results), as can be seen from fig.5, with results on $\alpha_s(Q)$ in perfect agreement with those obtained from muon beams and also,but with larger errors, with the old CHARM data [12] (see Table 1).

We now consider the important issue of estimating the theoretical error associated with the measurements of $\alpha_s(m_Z)$ in Table 1[F1]. There are many possible sources of errors in the relation between what is actually measured and $\alpha_s(Q)$.

First, there are terms down by powers of Q^2 (e.g. ambiguities on target mass corrections and from higher twist terms), and uncertainties on the gluon density and on the x-parametrisation of structure functions. In ref.[6] these sources of errors are included in the quoted experimental error. We have already discussed the treatment of $1/Q^2$ corrections in ref. 6, which was precisely aimed at obtaining an estimate of the corresponding error directly from the combined SLAC-BCDMS data. The resulting effects are presented in fig.1. The error from the gluon parametrisation is also shown in fig.1. For the other experiments mentioned in our previous summary, the discussion of such errors is less transparent.

F1 For theoretical errors we must decide what we mean: a maximum error, similar to 3σ for experimental errors, or a 68% c.l. error similar to 1σ. In the following we talk of 1σ errors, although there is no real way of quantifying a confidence level for theoretical errors.

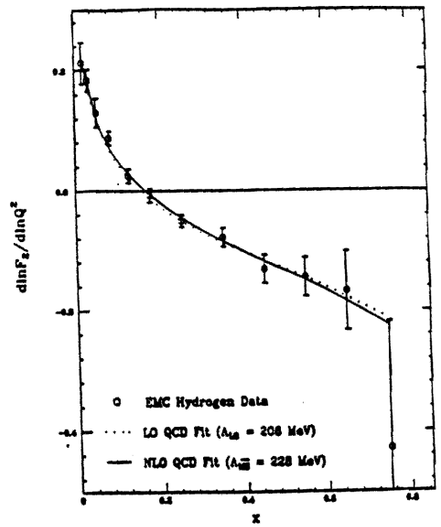

Fig. 2: Logarithmic slopes from the new analysis of the EMC data on H [8] and their QCD fit (see Table 1).

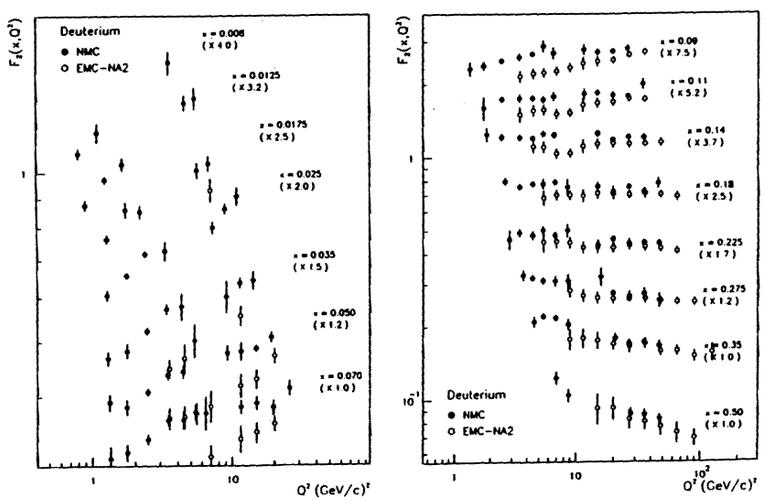

Fig. 3: Comparison between the NMC [9] and the reanalised EMC data [8] on D.

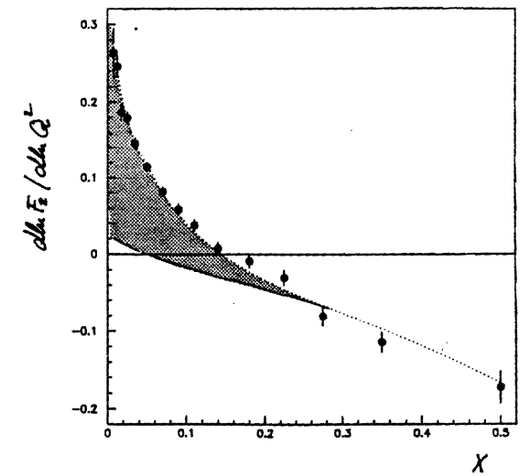

Fig. 4: Logarithmic slopes from the NMC data on D[9] and their QCD fit (see Table 1). The shaded area is the difference due to the gluon density.

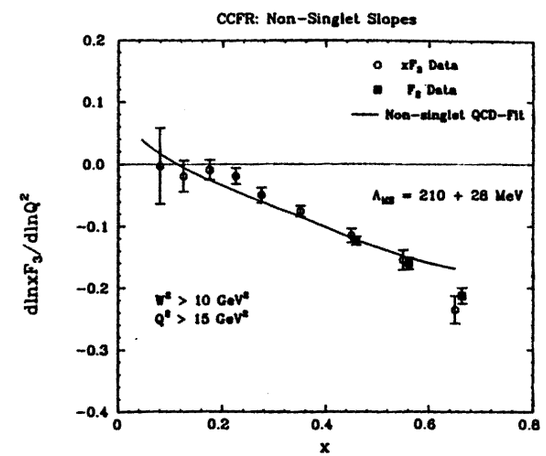

Fig. 5: Logarithmic slopes from the CCFR data on F_3 [10] (which is a purely non-singlet structure function) and their QCD fit (see Table 1).

Second, there are theoretical errors associated with neglected higher order terms in $\alpha_s(Q)$ in the QCD evolution equations. The most usual way to obtain an estimate of these higher order terms is by varying the renormalisation scale μ (i.e. express the perturbative expansion in terms of $\alpha_s(\mu)$) in some fiducial interval around the natural scale of the hard process (Q in deep inelastic scattering) and observe the change in the value of Λ or of $\alpha_s(m_Z)$. More in general some other reparametrisation of the perturbative series can be tried, for example a change of the factorisation scale (the scale of parton densities). When the scale is changed in the leading term in $\alpha_s(Q)$ the subleading terms are correspondingly modified in order to lead to the same result, whithin the accuracy of the known terms in the expansion. The compensation is not exact because the series is truncated. This is a reasonable method, provided that it is correctly interpreted. Of course, the result depends on the range of μ which is chosen, so that some subjective element is immediately evident. In general, μ can be varied in a range such that $\alpha_s(Q)\log Q^2/\mu^2$ remains a small quantity (so that no new large logs are generated). In principle, μ could depend on the parameters which are fixed when Q^2 varies, e.g. on x in deep inelastic scattering, but it is usually taken as a constant. The method only provides an indication of a lower bound on the error from higher order terms: such error cannot be smaller than what is obtained in this way because otherwise a contradiction would be manifest. One obvious remark on this method is that it extrapolates the behaviour of the unknown terms in the series from those that are known: the scale dependence is considered small if the first corrective term is small. But, clearly there is no theorem that guarantees that higher order terms are small when the first correction is small. The scale dependence in the determination of $\alpha_s(m_Z)$ from the actual experiments of deep inelastic scattering was studied in refs.[6, 13]. The result is something like $\Delta\alpha_s(m_Z)=\pm 0.004$. The total theoretical error which is usually quoted is close to this estimate [3,6,13]: $\Delta\alpha_s(m_Z)=\pm 0.005$, when the error from translating $\alpha_s(Q)$ measured at Q=5-10 GeV into $\alpha_s(m_Z)$ is also included (see eq.4).

In my opinion, the real theoretical error is probably somewhat larger, not only because of the previous words of caution about the scale dependence, or because of similar reservations on the estimate of higher twist terms, but also because other sources of errors can be imagined. For example, at realistic energies, when moving Q^2 at fixed x, one often goes through the threshold for production of heavy quarks in the final state, either charm or beauty, depending on the experiment and the Q^2 dependence of the structure function is altered in a non perturbative way. Thus I would be a little bit more conservative and assign a theoretical error $\Delta\alpha_s(m_Z)=\pm 0.007$ to $\alpha_s(m_Z)$ measured in deep inelastic scattering.

In conclusion, the situation of deep inelastic scattering is much clearer now. The very fundamental test of QCD based on measuring the small logarithmic deviations from scaling and comparing them with the quantitative theoretical predictions has been finally accomplished by several experiments to a reasonable degree of accuracy. The measured values of $\alpha_s(Q)$ can be expressed in the form:

$$\alpha_s(m_Z) = 0.112 \pm 0.002\ (\text{exp}) \pm 0.007(\text{th}) =$$
$$= 0.112 \pm 0.007 \qquad (6)$$

where the central value and the experimental error are obtained by combining the data in Table1. Note that the theoretical error is dominant.

3. Total Hadronic Production in e^+e^- Annihilation and Z Decay

The total hadronic cross section in e^+e^- annihilation, and its modern variant, the Z total hadronic width, are processes with the same gold-plated standing as deep inelastic scattering, as far as QCD tests and the measurement of $\alpha_s(Q)$ are concerned. Both are totally inclusive, both can be treated by the light-cone operator expansion plus renormalisation group approach. As already observed, deep inelastic scattering is wider in scope (many structure functions with Q^2 and x depedence), and the QCD scaling violations are relatively prominent (enhanced by logs). The total hadronic cross section in e^+e^- annihilation is just one function of Q^2 and the QCD effect is a small correction. But there are some advantages. This process is experimentally simpler and the theoretical calculations were possible up to the order $\alpha_s(Q)^3$. Actually an error in the first computation of the next-next to leading correction has now been corrected[14]. The new result for the third order coefficient is numerically moderate so that the expansion looks quite reassuring. For f=5 and pure photon exchange one has, at $Q=\sqrt{s}$:

$$R_{e^+e^-} = \frac{11}{3}\{1 + (1+\frac{3}{11}\mu)\frac{\alpha_s(Q)}{\pi} + 1.411\,(\frac{\alpha_s(Q)}{\pi})^2 - 12.76\,(\frac{\alpha_s(Q)}{\pi})^3 + ...\} \qquad (7)$$

with $\mu=4m_b^2/Q^2$ describing, in linear approximation, the effect of the finite b-quark mass [15].

The corresponding formula for the ratio Γ_h/Γ_l measured at LEP is given by [16]:

$$\frac{\Gamma_h}{\Gamma_l} = (\frac{\Gamma_h}{\Gamma_l})_{EW}\{1 + 1.05\,\frac{\alpha_s(m_Z)}{\pi} + (0.9\pm 0.1)\,(\frac{\alpha_s(m_Z)}{\pi})^2 - 13\,(\frac{\alpha_s(m_Z)}{\pi})^3 + ...\} \qquad (8)$$

where the b-quark mass correction (which is particularly large on the axial contribution) was taken into account in the first order coefficient. The second order coefficient is different for the vector and axial contributions to the width due to the effect on the axial part of the top-bottom mass splitting from diagrams as the one in fig.6 [17]. The corresponding error is induced by our ignorance of m_t.

By now the LEP measurement of Γ_h/Γ_l is the best experiment of this kind for the determination of $\alpha_s(m_Z)$: the energy is largest, the LEP detectors, being aimed at the very

precise electroweak physics, have a better control of the systematics (acceptance, efficiency...) in comparison to previous e^+e^- experiments. Also a better treatment of radiative corrections has been implemented in the LEP esperiments. The most recent experimental value of Γ_h/Γ_l, combined over the four LEP experiments, is given by [18]:

$$R_{exp} = \left(\frac{\Gamma_h}{\Gamma_l}\right)_{exp} = 20.86 \pm 0.08 \qquad (9)$$

where a common systematic error is also taken into account. Combining this experimental result with the theoretical prediction for $\alpha_s(m_Z)=0$ [16]:

$$R_{EW} = 19.97 \pm 0.03 \qquad (10)$$

where the error is due to our ignorance of m_t and m_H, one obtains:

$$\alpha_s(m_Z) = 0.132 \pm 0.012 \qquad (11)$$

The separate results of the four LEP experiments are:

$$\begin{aligned} \alpha_s(m_Z) &= 0.118 \pm 0.020 && \text{ALEPH} \\ &= 0.160 \pm 0.027 && \text{DELPHI} \\ &= 0.135 \pm 0.026 && \text{L3} \\ &= 0.127 \pm 0.026 && \text{OPAL} \\ &= 0.132 \pm 0.012 && \text{Average} \end{aligned} \qquad (12)$$

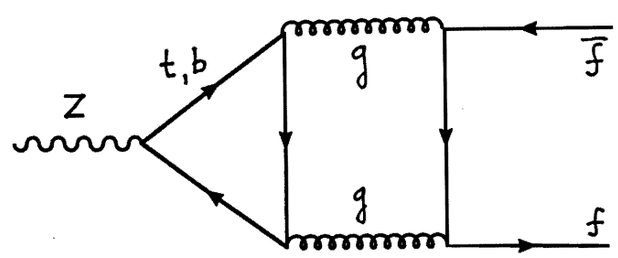

Fig. 6: A typical diagram that makes the difference between the QCD corrections for vector and axial couplings.

In this case the theoretical error is negligible with respect to the experimental error. In fact, first, the experimental error is large. Second, the contribution of terms down by powers of $1/Q^2$ is quite small and the scale dependence is also very limited because $\alpha_s(m_Z)$ is small and the perturbative series extends up to the third order term. The statistical error is still a sizeable component of the error so that further improvement of this measurement is possible.

Starting with the works of refs.[19] all available e^+e^- data at high energies have been analysed in order to extract values of $\alpha_s(Q)$. We consider a recent study by Branchina et al [20]. These authors use an updated set of data from Spear up to Tristan energies, fit $\alpha_s(Q)$ using the LEP values for m_Z, $\sin^2\theta_W$ and so on and make use of the new corrected formula eq.7 for the QCD correction. The results are given in Table 2 for two different fits, one for $\sqrt{s}$ >5 GeV and one for $\sqrt{s}$ >14 GeV.

Table 2

$\sqrt{s}$ range (GeV)	5-61.4	14-61.4
$\Lambda^{(5)}$ (MeV)	$397 \pm^{256}_{193}$	$990 \pm^{608}_{485}$
$\alpha_s(m_Z)$	0.129 ± 0.013	0.151 ± 0.017

Table 2. Values of $\Lambda^{(5)}$ or, equivalently $\alpha_s(m_Z)$, obtained from the fits of ref.[20] in the two indicated ranges of energy.

I think that the errors that appear in Table 2, and in other similar analyses, is not the whole error. To me is in fact a bit surprising that an error almost as small as the one from the combined LEP measurement of $\alpha_s(m_Z)$, given in eq.11, could be obtained from a compilation of low energy experiments. In fact the errors in Table 2 are obtained by assuming uncorrelated systematic errors among the various experiments. But it is not difficult to imagine important correlated systematics. For example, most old experiments used first order electromagnetic radiative corrections for initial state photon emission. Since the effect of initial state radiation is large, the contribution of multiphoton radiation is still sizeable. By neglecting multiphoton initial state radiation the value of R_{e+e-} is overestimated. In fact when a hard photon is emitted, the invariant mass of the hadronic system can fall below the corresponding cut. The second order correction reduces the probability for hard photon emission. Thus correcting for the cut on the basis of the lowest order estimate leads to a larger value of R_{e+e-}. The magnitude of this effect was studied in ref.[21]. It was found that it can lead to a 1% change in R_{e+e-}. Similarly many corrections are implemented using the same methods and, often, the same computer codes. We also see that the central value of $\alpha_s(m_Z)$ depends sensitively on the range of energies which are included in the fit.

In conclusion, I think that the low energy experiments are less reliable and precise than the LEP experiments. With the further progression of LEP this statement will become more and more true. The errors on $\alpha_s(Q)$ obtained from the assumption of uncorrelated

systematics are probably optimistic. From $R_{e^+e^-}$ at energies below LEP I would quote as a result something like:

$$\alpha_s(m_Z) = 0.14 \pm 0.02 \tag{13}$$

4. τ Decay

The possibility of measuring α_s from τ-decay has been extensively studied in a series of interesting papers [22], in particular by Braaten, Narison and Pich. The relevant quantity is $R_\tau = \Gamma(\tau \to \nu_\tau + \text{hadrons})/\Gamma(\tau \to \nu_\tau + l\nu)$, with $l=e,\mu$. It is claimed [23] that the present experimental value

$$(R_\tau)_{exp} = 3.53 \pm 0.04 \tag{14}$$

implies $\alpha_s(m_\tau) = 0.32 \pm 0.04$ and, based on this result it is argued that $\alpha_s(m_Z) = 0.118 \pm$
0.004
0.006 ·

Given that m_τ is so small this determination of $\alpha_s(m_Z)$ appears a bit too precise! In defence of this method one can certainly point out that R_τ has several combined advantages. First, R_τ is even more inclusive than $R_{e^+e^-}(s)$. Dropping some inessential complications R_τ is an integral over a spectral function, analogue to $R_{e^+e^-}(s)$:

$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \operatorname{Im} \Pi_\tau(s) \tag{15}$$

One expects that the asymptotic regime is more precocious for more inclusive quantities. Second, one can use analyticity in order to transform the above integral into an integral over the circle $|s| = m_\tau^2$:

$$R_\tau \sim \frac{1}{2\pi i} \int_{|s| = m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \Pi_\tau(s) \tag{16}$$

This not only gives some confidence that the relevant scale of energy for the evaluation of $\Pi_\tau(s)$ is of order m_τ, but also shows that the integration over the low energy domain helps very much in smearing out the complicated behaviour in the resonance region. Also important is the phase space factor in front of $\Pi_\tau(s)$ in eqs.15, 16 that kills the sensitivity near Re $s = m_\tau^2$, where there is a gap. Note that one could naively think of gaining even more (in terms of increasing the effective energy scale) by going from the circle to some

other curve in the complex plane with $|s|>m_\tau^2$ except at the tip where Re $s = m_\tau^2$. But, as observed in ref.[24], one would not obtain an advantage because the integrand at large $|s|$ becomes much larger than the result, so that there must be important cancellations and the precision of any asymptotic approximation on the leftover would not improve. This argument is used in ref.[24] to show that the choice of the circle is actually not conclusive on what the effective energy scale is, and that it can well be smaller than m_τ. Third, $\Pi_\tau(s)$ can be evaluated at $|s| \sim m_\tau^2$ by controlling non perturbative corrections by using the operator product expansion and some estimate (either experimental or by some model) of the dominant condensates, in the spirit of the QCD sum rules[25]. Finally the perturbative component of $\Pi_\tau(s)$ is known up to terms of order $\alpha_s(m_\tau)^3$.

This series of virtues of R_τ is indeed real but would not be sufficient in itself to justify the precision on $\alpha_s(m_\tau)$ which is claimed. The real point is another one which I now discuss. By applying the previous strategy eventually one arrives at an expression for R_τ of the following form (schematically) [22]:

$$R_\tau = R_\tau^0 \left\{1 + \frac{\alpha_s}{\pi} + 5.202 \left(\frac{\alpha_s}{\pi}\right)^2 + 26.37 \left(\frac{\alpha_s}{\pi}\right)^3 + \ldots + c\frac{m^2}{m_\tau^2} + d\frac{\langle m\,\bar{q}\,q\rangle}{m_\tau^4} + \ldots + g\frac{\langle O_6\rangle}{m_\tau^6} + \ldots\right\} \tag{17}$$

where the perturbative coefficients are appropriate for f=3 and $\alpha_s = \alpha_s(m_\tau)$, c,d and g are known coefficients, m is indicative of light quark masses and <O> stands for the condensate (i.e. the vacuum expectation value) of the operator O. The crucial ingredient that allows a precise determination of $\alpha_s(m_\tau)$ is that one replaces m by the current masses of u and d quarks (s quarks are essentially irrelevant, because the corresponding final state is Cabibbo suppressed in τ decay). With m ~ 5-10 MeV it is clear that the dangerous $1/m_\tau^2$ corrections are completely negligible. This replacement of m by the current masses is based on the assumption, usually made in the QCD sum rule approach [25], that all non perturbative effects are in operator matrix elements. According to this assumption the Wilson coefficients are determined by perturbation theory also at the non leading level in $1/Q^2$, where they are not protected by scale invariance against effects of order Λ^2/Q^2, with Λ some non perturbative scale, typically the confinement scale $\Lambda_{\overline{MS}}$. What is special about the τ–decay channel is that operators of dimension 2 are absent in the expansion of two electroweak currents. If not for this gap the above assumption would not play such a dramatic role, because in presence of an operator of dimension 2 what is non leading c-number coefficient and what is operator matrix element is largely conventional. If one accepts this assumption there are no $1/Q^2$ corrections in the massless quark limit neither in τ decay nor in e^+e^- annihilation (in the totally inclusive rates). To appreciate the impact of this assumtion in the case of τ decay, note that $\alpha_s(m_\tau)/\pi \sim 0.10 \sim (500\ \text{MeV}/m_\tau)^2$ and that $\alpha_s(m_\tau)$=0.32 corresponds to $\Lambda^{(3)}$ = 378 MeV.

with A_i and B_i computed. Recently, for some of the observables, an improvement has been implemented. Near the boundary of phase space logarithmic terms like $\alpha_s \log(1-T)$ and $\alpha_s^2 \log(1-T)$ at leading or next to leading level become large (here T is thrust and T=1 corresponds to the 2-jet limit). Leading and next to leading logs of this type have been computed and resummed for energy-energy correlations [37], thrust [38] , heavy-jet mass [39], and the multiplicity of jets [40]. The advantage is to increase the fiducial domain where the perturbative calculations can be trusted. In some cases the apparent preference for surprisingly low fitted values of μ, the scale parameter, is eliminated when the resummation is included.

The problem of hadronisation corrections appears when the theoretical calculations in terms of partons have to be compared with the experimental observations in terms of hadrons.In general the effect of hadronisation is simulated by Monte Carlo generators. For each observable the fit range is restricted to an interval of values where the impact of hadronisation is relatively small (fig.7). The magnitude at LEP of the hadronisation corrections is of order 5-15% . The error on the correction is estimated by comparing the 3 or 4 sufficiently sophisticated and updated generators which are available. In general the performance of these generators in reproducing the observed distributions is really good. In spite of this, since many of the ingredients of these codes are similar, I think that the errors on hadronisation corrections estimated by this comparison could be optimistic.

The theoretical error from our ignorance of higher order terms in the perturbative expansion is estimated, as usual, from the scale dependence of the fitted value of $\alpha_s(m_Z)$ (figs.8,9) or in other similar ways.The observed scale dependence is in most cases less pronounced when resummed formulae are used. Without resummation the best fit is usually obtained for very small values of μ, often so small that $\alpha_s \log\mu^2/m_Z^2$ becomes rather large. In fact the points that drive the fit toward small values of μ sit in a region where perturbation theory is of marginal validity. Since small values of μ in general lead to relatively small values of $\alpha_s(m_Z)$, there has been some tendency to argue that values of $\alpha_s(m_Z)$ on the low side are preferred by the data. With the advent of resummed formulae it has been convincingly shown that the preference for small values of μ disappears. In the region near the boundary of phase space, where the preference for small values of μ was originated, the corrective terms are non negligible and their effect removes the need for small μ. As a result the fits with resummed formulae indicate a larger central value for $\alpha_s(m_Z)$.

The results obtained by the four LEP collaborations and by SLD on $\alpha_s(m_Z)$ from second order perturbative formulae are summarised in Table 3, while the results from expressions including the resummation of logs, which are only available for a subset of observables (see, for example, fig.10), are displayed in Table 4. The split up of the total error into experimental and theoretical, when possible, is also shown.

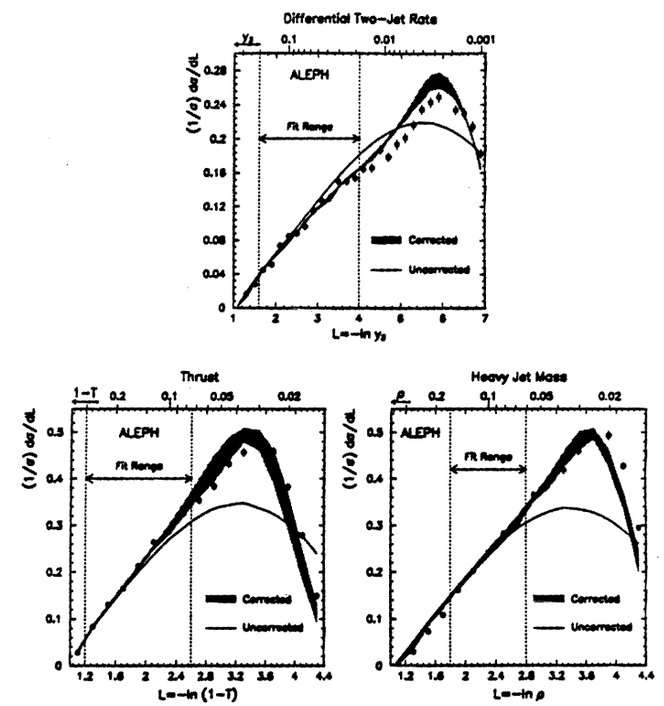

Fig. 7: Experimental distributions (statistical errors only) together with bands covering the predictions using three hadronisation models. The curves refer to the same values of α_s but without hadronisation corrections (from ref.41).

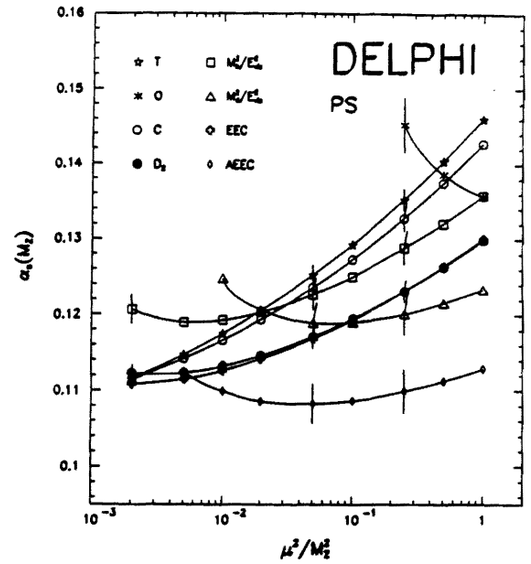

Fig. 8: Observed scale dependence on the results for $\alpha_s(m_Z)$ from different observables measured by DELPHI and fitted with second order QCD predictions, i.e. with no resummation (from ref.42).

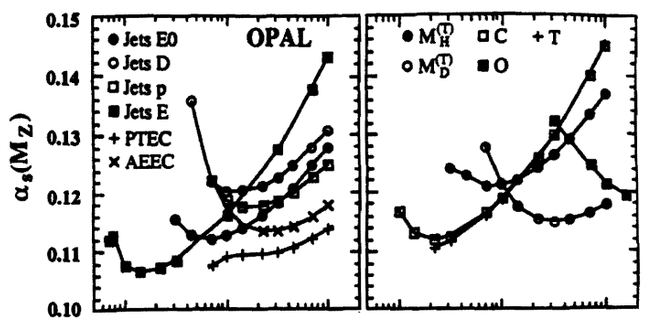

Fig. 9: Observed scale dependence on the results for $\alpha_s(m_Z)$ from different observables measured by OPAL and fitted with second order QCD predictions, i.e. with no resummation (from ref.44).

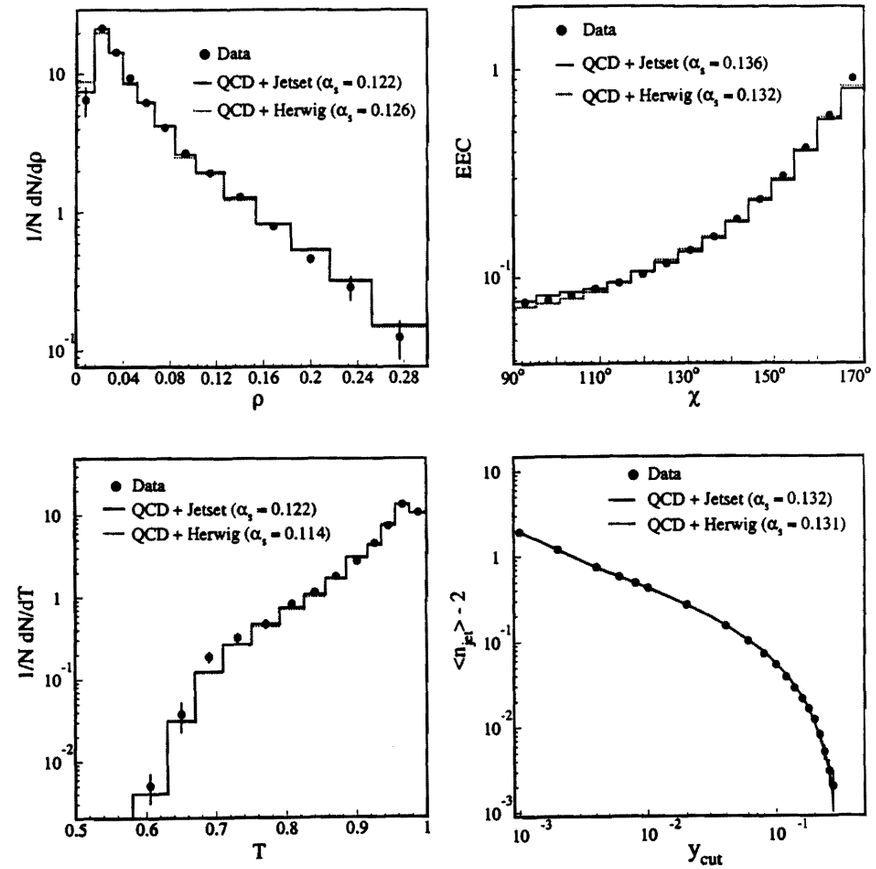

Fig. 10: Heavy-jet mass (ρ), thrust (T), energy-energy correlation (EEC) and average jet multiplicity ($\langle n_{jet} \rangle$) distributions measured by L3 and compared with QCD predictions with resummation (from ref.43).

Table 3

Experiment	$\alpha_s(m_Z)$	Exp. Error	Theor.Error
ALEPH [41]	0.118 ± 0.009		
DELPHI [42]	0.113 ± 0.007	± 0.002	± 0.007
L3 [43]	0.118 ± 0.010		
OPAL [44]	$0.122 \pm^{0.006}_{0.005}$	± 0.001	$\pm^{0.006}_{0.005}$
SLD [45]	0.121 ± 0.014	± 0.009	$\pm^{0.012}_{0.009}$

Table 3. Values of $\alpha_s(m_Z)$ from second order perturbative formulae.

Table 4

Experiment	$\alpha_s(m_Z)$	Exp. Error	Theor.Error
ALEPH [41]	0.125 ± 0.005	± 0.002	± 0.004
DELPHI [42]	0.125 ± 0.006	± 0.002	± 0.006
L3 [43]	0.125 ± 0.009	± 0.003	± 0.008
OPAL [44]	$0.122 \pm^{0.003}_{0.006}$	± 0.001	$\pm^{0.003}_{0.006}$

Table 3. Values of $\alpha_s(m_Z)$ from resummed formulae.

The "official" averages presented [46] at the winter conferences are:

$$o(\alpha_s^2): \quad \alpha_s(m_Z) = 0.119 \pm 0.006$$

$$o(\alpha_s^2) + \text{resumm.}: \quad \alpha_s(m_Z) = 0.123 \pm 0.005 \qquad (26)$$

As seen from Tables 3 and 4 the theoretical error is by far the dominant uncertainty. I think that a realistic error is somewhat larger. First, the uncertainty on the hadronisation corrections, which was estimated from the differences among a few Monte Carlo generators could well be larger, due to common biases, in spite of the good overall fitting. Second, the error from higher order perturbative terms, as estimated from scale dependence and other similar methods, should be taken as a lower bound on the error (as already discussed, the error cannot be smaller than that, otherwise one would be confronted with a contradiction among apriori equally reliable determinations of $\alpha_s(m_Z)$). In conclusion I would replace eqs.26 with something like:

$$\alpha_s(m_Z) = 0.122 \pm 0.009 \qquad (27)$$

7. Other Determinations of α_s

We now list a number of additional determinations of α_s which, for reasons either of precision or of reliability or simply because they are still in a rather exploratory phase, are not as important as the ones discussed in the previous sections.

$p\bar{p}$ ->W+jets[47].The UA2 Collaboration has measured $r = \sigma(W+1jet)/\sigma(W+0jets) = (3.91 \pm 0.40)\%$, with a specified jet-counting criterium. The value of α_s can be obtained from this quantity by taking advantage of some recent theoretical calculations, i.e. the 2-loop Drell-Yan QCD correction [48], the p_T distribution of W's at order α_s^2 [49] (see also [50]), which enters because there is a minimum p_T for the jet to be identified, plus a good deal of Monte Carlo simulation in order to properly correct for the underlying event, fragmentation effects and so on. The result, expressed in terms of $\alpha_s(m_Z)$, is given by:

$$\alpha_s(m_Z) = 0.121 \pm 0.024 \qquad (28)$$

$p\bar{p}$ ->$b\bar{b}$X [51]. The value of α_s is extracted from the $b\bar{b}$ production cross section with $p_T > p_{T\min}$, measured by UA1. The sample of $b\bar{b}$ events is subdivided into 2-body or 3-body, according to their azimuthal distribution. The 2-body are nearly back to back: $\Delta\phi > 150^0$, while for 3-body $\Delta\phi < 150^0$. The 2-body cross section σ_2 starts at order α_s^2 and the theoretical calculations are available [52] up to order α_s^3. The 3-body cross section σ_3 starts at order α_s^3 and the next to leading corrections are not known. Two main methods are described. One uses the ratio σ_3/σ_2, in order to reduce the dependence on the structure functions, the other is directly from σ_2. Clearly only the second method is acceptable, because σ_3 is not known beyond leading accuracy. The second method leads to:

$$\alpha_s(m_Z) = 0.109 \pm^{0.016}_{0.012} \qquad (29)$$

σ_2 is sensitive to α_s because of its quadratic dependence. The error allegedly includes the ambiguities from the poor knowledge of the gluon structure function, the scale dependence etc. There are some words of caution to be spelled because it is well known that conventional gluon structure functions fit the UA1 data but fail to fit the CDF data [53] on b production at large p_T. To reconcile the theory with the CDF data, that probe the gluon density at smaller values of x, the gluon density has to be modified [54]. It would be interesting to check that the resulting change on the present determination of α_s is still within the quoted error.

Scaling Violations in Fragmentation Functions . In principle the scaling violations for fragmentation functions are predicted by QCD in close analogy with the case of structure functions. An interesting first attempt at the measurement of α_s from the fragmentation function into the sum of all charged tracks has been presented very recently [55]. The necessary lever arm in Q^2 is obtained by combining LEP data with the results of PEP, Petra and Tristan. The error quoted in the determination of $\alpha_s(m_Z)$, which is $\pm$ 0.006, is in my opinion not plausible. One problem is that the value of α_s is not derived by measuring the logarithmic slopes and then comparing them with the QCD evolution equations, which relate the slopes (i.e. the derivatives of the fragmentation functions) with α_s. Rather the observed behaviour in Q^2 is directly compared with a Monte Carlo of the matrix element type, i.e. one that contain the exact QCD matrix element squared at the next to leading level. The Monte Carlo makes the analysis simpler because it also provides a built in model for the non perturbative component of the fragmentation functions, for the effect of the change of composition of the final state (for example, takes into account that one goes from photon exchange at low energies to Z dominance at LEP) and so on. But one has to trust the Monte Carlo. Among other things, the QCD resummation of logs is truncated at the second order, which is what is contained in the Monte Carlo. As for all cases where a Monte Carlo is crucial, it is difficult to understand what the real error can be. Thus I quote the result of ref.[55] with a big question mark added to the error:

$$\alpha_s(m_Z) = 0.119 \pm 0.006 \pm ? \qquad (30)$$

Photon Structure Function. There are no great news on this front. Apparently a small bug in the existing non leading corrections to the evolution equations was found and corrected [56]. The existing data sets and analyses [57] are not homogeneous and it is often difficult to understand what has been done and even what the precise result is (e.g. $\Lambda^{(3)}$ or $\Lambda^{(4)}$?). Also the uncertainties on the hadronic part remain [58]. It would be great if the data could be reanalysed with the present knowledge and experience. I will repeat here the tentative conclusion that I have proposed in ref.[2] , i.e. that $\Lambda^{(4)}$~50-300 GeV, or :

$$\alpha_s(m_Z) = 0.103 \pm 0.013 \qquad (31)$$

Charmonium Splittings and Lattice Calculations. In a recent paper [59] it is claimed that lattice calculations are by now sufficiently accurate that a precise determination of α_s is possible from the observed 1P-1S splitting in charmonium. The experimental input is the value $m_{h_c} - (3m_{J/\Psi}+m_{\eta_c})/4 = 458.6 \pm 0.4$ MeV, where h_c is the spin singlet 1P state. This splitting can be computed on the lattice in the quenched approximation. This is equivalent to determining a discrete version of Λ for zero flavours. Then one must apply corrections to go from the lattice to the continuum limit in the $\overline{MS}$ definition and from 0 to 3 or 4 flavours of quarks. The claim is that one obtains a result equivalent to $\alpha_s(m_Z) = 0.105 \pm 0.004$. I think that there are too many technical assumptions and it is difficult to assess the actual size of the error.Certainly I find the error by far too optimistic, but the result is interesting, as a promise for the future.

8. Summary and Conclusion

In the following I collect those which I consider the main methods of determining α_s with my subjective estimate of the theoretical errors (while, of course, the experimental errors were in all cases left intact!). For each process the label (Exp) or (Th) qualifies whether the dominant source of error is experimental or theoretical, respectively. We have:

R_τ	$\alpha_s(m_Z) = 0.117 \pm^{0.010}_{0.016}$	(Th)
Deep Inelastic Scattering	0.112 ± 0.007	(Th)
Υ Decays	0.11 ± 0.01	(Th)
Re^+e^- ($\sqrt{s}$<62 GeV)	0.14 ± 0.02	(Exp)
$p\bar{p}$ ->W+jets	0.121 ± 0.024	(Exp~Th)
Γ(Z->hadrons)/Γ(Z->l$\bar{l}$)	0.132 ± 0.012	(Exp)
Jets at LEP	0.122 ± 0.009	(Th)
Average	**0.118 ± 0.007**	(32)

The error on the average was set close to the typical size of the theoretical error involved in each of the best experiments, because I do not like to average theoretical errors (a naive average would give $\pm$ 0.004). The corresponding range of $\Lambda^{(5)}$ is:

$$\Lambda^{(5)} = (240 \pm 90) \text{ MeV} \qquad (33)$$

We see that in spite of the fact that I was extremely conservative at each step in this review, the resulting determination of α_s is very precise, with a 6% relative error on $\alpha_s(m_Z)$. While, on the one hand, I am supporting conservative estimates of theoretical errors, on the other hand I warn skeptical people that the value of $\alpha_s(m_Z)$ and the error given by eq.32 appear by now quite solid, and the error cannot be much larger than that (for example larger than 0.01). The consistency of the results on α_s from all the available experiments provides a remarkable quantitative test of the theory. The list of results on $\alpha_s(m_Z)$ in eq.(32) are reproduced in fig.11, while in fig.12 the corresponding value of α_s at the scale of each experiment was plotted.

In conclusion, the overall picture looks very favourably for QCD. The determination of α_s from many different experiments coincide within errors. The value of α_s found at LEP energies is small enough to correspond to a very mild interaction. This is the best evidence for asymptotic freedom and the running of α_s, because such a weak force could not explain the observed binding of hadrons. To me this is a better argument than the consistency with the running of α_s predicted by QCD which is visible in fig.12.

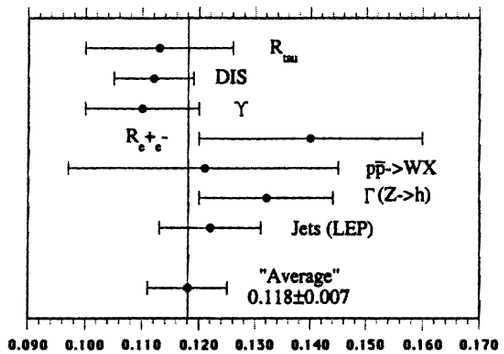

Fig. 11: Main measurements of $\alpha_s(m_Z)$ and their weighted average.

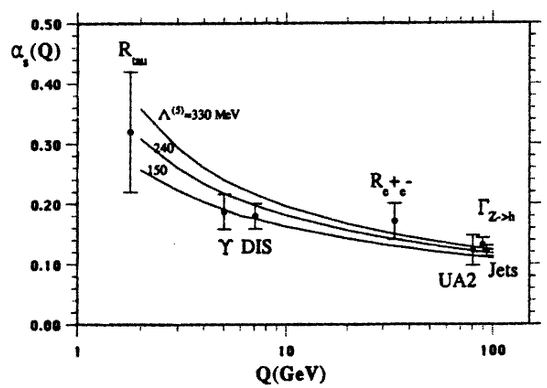

Fig. 12: Results of the main measurements of $\alpha_s(Q)$ plotted at the actual scale of the experiment.

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