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Abstract

In this paper we first discussed the properties of the gauge magnetic field of anyons in Fermi representation, and found that there is no Lorentz force experienced by every particles. Based on this fact, we used the plain wave as the perturbative expansion basis and gave the the results of linear response theory of an ideal anyons gas in the random phase approximation. No Meissner effect and Goldstone mode have been found, which are opposite to the result of the mean field theory. At last, we make some comments on the mean field theory of anyons.

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§1.Introduction

Currently, the theory of fractional statistics becomes more and more noticeable. The properties of anyons (obeying the fractional statistics), such as whether its ground state of anyon gas is superconducting, are expected to be solved by a lots of theoretical physicists. In 1977, Leinaas and Myrheim[1] tried to discuss the possibility of existence of new quantum statistics in two-dimensional space, but, unfortunately it did not attract attention in academic circle at that time. Until 1982, some of the most intriguing ideas in physics, such as the magnetic monopole, the Aharonov-Bohm effect and the fractional quantization of charge and angular momentum *et al*, led Wilczek to propose again the concept of anyons, a new kind of identical particles, which obey neither Fermi-Dirac statistics nor Bose-Einstein statistics but obey a novel statistics, *i.e.*, the fractional statistics[2]. In 1984, Halperin[3] suggested that the excitations in the theory of fractional quantum Hall effect proposed by Laughlin behave like anyons, which was confirmed lately [4]. This was a turn point in the study on the fractional statistics. For the first time, anyons are observed in three-dimensioal realistic system inexplicitly. The discovery of high temperature oxides superconductors made a great progresses in theory of condensed matter, especially, in the theory of two-dimensional strongly correlated system. Soon after the discovery, P. W. Anderson[5] pointed out that the high temperature superconductors do not behave like conventional metals in their normal, nonsuperconducting state. His suggestion led Laughlin[6] to propose that many of those unusual

properties may be described and understood in terms of peculiar properties of a collection, or gas, of anyons. Furthermore, he proposed the mechanism of anyons superconductivity: the ground state of semions, interpolating between Fermi and Bose statistics, may be superfluiding. This suggestion was just confirmed in the mean field theory[7]. In the random phase approximation the full diamagnetism and a Goldstone mode was found, which was regarded as a hallmark of superconductivity in a semion gas. But unfortunately, in the same approximation, no phase transition had been found at any finite temperature[8], which indicates the anyon mechanism may not be related with the high-T_c oxides superconductor even if the mean field theory be correct. Some revised revisions have been proposed by sveral authers, such as Khveshchenko and Kogan[9], who used a BCS-type variational ground state to study the anyons system. Furthermore, a lot of experimental results approaches to deny this mechanism, because time-reversal T and parity P violation, which is a fundamental property of anyons, was failed to observed in high-T_c material[10]. Based on these experimental results authors proposed a two-component anyon model, in which Kosterlitz-Thouless phase transition may be responsible for superfluidity[11]. These facts make us have to consider again whether the ground state of anyon is superconducting.

Until now, Laughlin's ideal was just confirmed in the mean field theory. In this approximation particles move in a uniform magnetic field that is proportional to number density of particles and their energy spectrum is dis-

crete, unlike the single-body spectrum of ideal gas of bosons and fermions. As for the concept of the mean field, it is still worth studying further. An immediate, but unphysical conclusion of the mean field theory is that there does exist a nonzero Hall conductivity in an ideal anyon system, this is only because in classical sense no Lorentz force arises between anyons, even if they projected into Fermi representation and oppositely in the mean field theory every particle moves in a uniform gauge magnetic field. Thus we have to discussed this theory again. In this paper, we made a different approach to study the properties of ideal gas of anyons and found some different results opposite to the mean field theory. Using the plain wave as the perturbative expansion bases, we found no Meissner effect in the random phase approximation. Thus an ideal gas of anyon may not be superconducting.

In Section II, we discussed the properties of the gauge vector potential, which transmutes the statistics of anyons to fermions or bosons and gave a compact form of Hamiltonian as a starting-point to study the system in the perturbed method. In Section III. we introduced the random phase approximation in anyon system. In Section IV, we introduced the linear response theory of anyon system in random phase approximation briefly. In Section V, we listed the results of the random phase approximation in anyon system. In Section VI, we gave some discussions on the mean field theory. In Appendix, the detailed calculation of current-current function of ideal two-dimensional fermion gas were presented.

§2. Hamiltonian of the system

As usual, we start from the Hamiltonian for an anyons (obeying μ-statistics) in the Fermi representation[12]

$$H = \sum_i \frac{1}{m}(P_i + \frac{e}{c}A_i)^2 \tag{1}$$

where A_i is the gauge vector potential originating from statistical transmutation from anyons to fermions. Such a potential has special property: its corresponding gauge magnetic field is closely related to the density of particles

$$\begin{aligned} B_i &= \nabla_i \times A_i \\ &= (1-\nu)\frac{\hbar c}{e}\sum_{j\neq i}^{N}\delta(\vec{r}_i - \vec{r}_j) \\ &= (1-\nu)\frac{\hbar c}{e}\rho(\vec{r}_i) \end{aligned} \tag{2}$$

Obviously, this field has a lot of singularities in the configuration space. If we denote the ground state wave function of the system by $\Phi(\vec{r}_1, \cdots, \vec{r}_N)$, the principle of identical particles must forbid all the diagonal elements in the configuration space of wavefunction[13]:

$$|\Psi(\cdots, \vec{r}_i, \cdots, \vec{r}_j = \vec{r}_i), \cdots|^2 \equiv 0 \tag{3}$$

In another word, we have excluded the possibility that two identical particles occupy the same site. By using this properties, we obtain the expecta-

tion of the gauge magnetic field on the *ith* (for instance, $i = 1$) particle:

$$\begin{aligned} \overline{B_1} &\equiv \int dr_2 \cdots dr_N B(\vec{r}_1)|\Psi(\vec{r}_1, \cdots, \vec{r}_N)|^2 \\ &= 0 \end{aligned} \tag{4}$$

coinciding with the common sense of no classical Lorentz force experienced by any particle. Usually, an anyon can be regarded as a magnetic flux-tube carrying charge, which radius is zero. Thus, the magnetic field carried by the tube is restricted within the tube. Out of the tube, there is, obviously, no magnetic field, of course, though there does exist a vector potential. Therefore, in such a system, it is not strange that every particle is not affected the magnetic field, though the vector potential does exist. However, for an arbitrary position in space deviated from particle position we obtain a finite nonzero magnetic field

$$\begin{aligned} \overline{B(r)} &= (1-\nu)\frac{\hbar c}{e}\int dr_2 \cdots dr_N \sum_i \delta(\vec{r} - \vec{r}_i)|\Psi(\vec{r}_1, \cdots, \vec{r}_N)|^2 \\ &= (1-\nu)\frac{\hbar c}{e}\rho(\vec{r}) \end{aligned} \tag{5}$$

Here $\rho(\vec{r}) = \rho$ is the particle number density of the anyon system, *i.e.* the probability of a particle appearing at an arbitrary position $\vec{r}$ in the many particle distribution. Based on this result, several authors have calculated the collective mode and linear response in the random phase approximation, which is called the mean field (average field) theory[7]. In this case, every particle moves in a uniform gauge field, which is rigorously related with the density of the particle. For some special systems $\nu = 1 - \frac{1}{n}$, the n Landau level is fully filled, which leads to diamagnetism for the system.

Now we try to give another approach to study the properties of anyon system. We accord to the fact of Eq.(4). by choosing the phase wave-function as our perturbative expansion base and in the second quantum representation. The Hamiltonian can be written as

$$H = H_0 + H_I \tag{6}$$

where

$$H_0 = \int \frac{d\vec{r}}{2m}\Psi^\dagger(\vec{r})P^2(r^2)\Psi(\vec{r}) \tag{7}$$

$$\begin{aligned} H_I &= \frac{1}{2}\int d\vec{r}\frac{1}{m}\Psi^\dagger(\vec{r})(P(\vec{r})\cdot a(\vec{r}) + a(\vec{r})\cdot P(\vec{r}))\Psi(\vec{r}) \\ &+ \frac{1}{2}\int d\vec{r}\frac{1}{m}\Psi^\dagger(\vec{r})a(\vec{r}^2)\cdot a(\vec{r}^2)\Psi(\vec{r}) \end{aligned} \tag{8}$$

where Ψ is the spinless Fermi field operator and $\vec{a}$ is the gauge vector potential

$$\vec{a}(\vec{r}) = (1-\nu)\hbar\hat{z} \times \int d\vec{r'}\frac{\vec{r} - \vec{r'}}{|\vec{r} - \vec{r'}|^2}\Psi^\dagger(\vec{r'})\Psi(\vec{r'}) \tag{9}$$

The perturbative part of Hamiltonian contains a two-body interaction and triple-body interaction. The two-body interaction can be reduced into a compact form:

$$H_{I1} = \frac{1}{2}\int d\vec{r}d\vec{r'}\{j_i(\vec{r})v_{i0}(\vec{r} - \vec{r'})j_0(\vec{r'}) + j_0(\vec{r})v_{0i}(\vec{r} - \vec{r'})j_i(\vec{r'})\} \tag{10}$$

where

$$j_i(\vec{r}) = \frac{1}{2}\{\Psi^\dagger(\vec{r})\frac{P_i(\vec{r})}{m}\Psi(\vec{r}) - \Psi(\vec{r})\frac{P_i(\vec{r})}{m}\Psi^\dagger(\vec{r})\}, \qquad i = 1, 2; \tag{11}$$

$$j_0(\vec{r}) = \Psi^\dagger(\vec{r})\Psi(\vec{r}); \tag{12}$$

$$v_{i0}(\vec{r} - \vec{r'}) = -v_{0i}(\vec{r} - \vec{r'}) = (1-\nu)\hbar\frac{\varepsilon_{ij}(\vec{r} - \vec{r'})_j}{|\vec{r} - \vec{r'}|^2}. \tag{13}$$

The triple-body interaction is

$$H_{I2} = \frac{1}{2} \cdot \frac{(1-\nu)^2\hbar^2}{m} \int d\vec{r}d\vec{r'}d\vec{r''} j_0(\vec{r}) \frac{(\vec{r}-\vec{r'})\cdot(\vec{r}-\vec{r''})}{|\vec{r}-\vec{r'}|^2|\vec{r}-\vec{r''}|^2} j_0(\vec{r'}) j_0(\vec{r''}) \quad (14)$$

Let us reorganize H_{I2} into two pieces, using

$$j_0(\vec{r}) = \rho_0 + (j_0(\vec{r}) - \rho_0) \quad (15)$$

The first term of resulting expression is expected to dominate, when fluctuation in density are relatively small. Its meaning becomes transparent upon doing the integral

$$\int d\vec{r} \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^2} \cdot \frac{\vec{r}-\vec{r''}}{|\vec{r}-\vec{r''}|^2} = -2\pi \ln|\vec{r'}-\vec{r''}| + \text{constant} \quad (16)$$

The triple-body interaction is approximately reduced to an effective repulsive Coulomb interaction:

$$H_{I2} \approx \frac{1}{2} \int d\vec{r}d\vec{r'} j_0(\vec{r}) v_{00}(\vec{r}-\vec{r'}) j_0(\vec{r'}) \quad (17)$$

where

$$v_{00}(\vec{r}-\vec{r'}) = -2\pi \frac{\rho_0}{m}(1-\nu)^2\hbar^2 \ln|\vec{r}-\vec{r'}| \quad (18)$$

Thus, we are left with a compact form of the interaction only

$$H_I = \frac{1}{2} \int d\vec{r}d\vec{r'} j_\mu(\vec{r}) v_{\mu\nu}(\vec{r}-\vec{r'}) j_\nu(\vec{r'}) \quad (19)$$

In the Fourior space, $\hat{v}$ takes a simple form

$$\begin{aligned} \hat{v}(\vec{q}) &= \int d\vec{r'} \hat{v}(\vec{r}-\vec{r'}) e^{i\vec{q}\cdot(\vec{r}-\vec{r'})} \\ &= \begin{pmatrix} \frac{\rho}{m}(1-\nu)^2\hbar^2\frac{(2\pi)^2}{q^2} & 0 & (1-\nu)\hbar\frac{i2\pi}{q} \\ 0 & 0 & 0 \\ -(1-\nu)\hbar\frac{i2\pi}{q} & 0 & 0 \end{pmatrix}. \end{aligned} \quad (20)$$

Here we take the momentum vector $\vec{q}$ with component only in x-direction. Obviously, we have a relation

$$v_{02} \cdot v_{20} = \frac{m}{\rho} v_{00} \quad (21)$$

§3. Random phase approximation in anyon system

In this section, we discuss the mechanics of calculation in the random phase approximation (RPA). As we have shown in Section II, the interactions of anyons system contains an effective repulsive Coulomb interaction, which suggests the importance of summing bubble graph. In general electron gas, the density-density correlation plays an important role in the calculation of the effective interaction, from which we can get the collective mode. Before we discuss the correlation function of j in anyon gas, we just give the form of effective interaction in random phase approximation. Using the rules for evaluating Feynman graphs[14], we have

$$v(\vec{k},\vec{k'},\vec{q},\omega) = v(\vec{k},\vec{k'},\vec{q}) + \sum_{\vec{p}} v(\vec{k},\vec{p}+\vec{q},\vec{q})\Pi(\vec{p}+\vec{q},\vec{p},\vec{q},\omega)v(\vec{p},\vec{k'},\vec{q},\omega) \quad (22)$$

The equation above is an integral equation. As v has the form

$$v(\vec{k},\vec{k'},\vec{q}) = (1, \frac{k_\parallel - \frac{q}{2}}{m}, \frac{k_\perp}{m})\hat{v}(\vec{q})(1, \frac{k'_\parallel + \frac{q}{2}}{m}, \frac{k'_\perp}{m})^T \quad (23)$$

Thus, the integral equation is reduced into an algebraic equation:

$$\hat{v}(\vec{q},\omega) = \hat{v}(\vec{q}) + \hat{v}(\vec{q}) \cdot \hat{\Pi}^0(\vec{q},\omega) \cdot \hat{v}(\vec{q},\omega) \quad (24)$$

where

$$v(\vec{k},\vec{k'},\vec{q},\omega) = (1, \frac{k_\parallel - \frac{q}{2}}{m}, \frac{k_\perp}{m})\hat{v}(\vec{q},\omega)(1, \frac{k'_\parallel + \frac{q}{2}}{m}, \frac{k'_\perp}{m})^T \tag{25}$$

and

$$\hat{\Pi}(\vec{q},\omega) = \int \frac{d\vec{p}}{(2\pi)^2}(1, \frac{k_\parallel + \frac{q}{2}}{m}, \frac{k_\perp}{m})^T(1, \frac{k_\parallel + \frac{q}{2}}{m}, \frac{k_\perp}{m}) \times \Pi(\vec{p}+\vec{q},\vec{p},\vec{q},\omega) \tag{26}$$

The simplest form of $\Pi(\vec{q}+\vec{p},\vec{p},\vec{q},\omega)$ is

$$\begin{aligned}\Pi(\vec{q}+\vec{p},\vec{p},\vec{q},\omega) &= \int \frac{d\omega'}{i2\pi} G_0(\vec{p}+\vec{q},\omega+\omega')G_0(\vec{p},\omega') \\ &= \frac{\theta(|\vec{k}+\vec{q}|-k_F)\theta(k_F-k)}{\omega-\varepsilon(\vec{k}+\vec{q})+\varepsilon(k)+i0^+} - \frac{\theta(k_F-|\vec{k}+\vec{q}|)\theta(k-k_F)}{\omega-\varepsilon(\vec{k}+\vec{q})+\varepsilon(k)-i0^+}\end{aligned} \tag{27}$$

After tedious algebraic, the matrix of $\hat{\Pi}(\vec{q},\omega)$ has the form

$$\hat{\Pi}(\vec{q},\omega) = \begin{pmatrix} \Pi^0_{00}(\vec{q},\omega) & \Pi^0_{01}(\vec{q},\omega) & 0 \\ \Pi^0_{10}(\vec{q},\omega) & \Pi^0_{11}(\vec{q},\omega) & 0 \\ 0 & 0 & \Pi^0_{22}(\vec{q},\omega) \end{pmatrix} \tag{28}$$

The detailed calculation of $\hat{\Pi}(\vec{q},\omega)$ is presented in the appendix.

In this approximation, $\Pi^0(\vec{q},\omega)$ is not but the unperturbed current-current correlation function and can be expressed in this way

$$\Pi^0_{\mu\nu}(\vec{q},\omega) = \int \frac{d\omega'}{i2\pi} < 0|j_\mu(\vec{q},\omega+\omega')j_\nu(-\vec{q},\omega')|0> . \tag{29}$$

Define the current-current correlation function as

$$\Pi_{\mu\nu}(\vec{q},\omega) = \int \frac{d\omega'}{i2\pi} < \Phi|j_\mu(\vec{q},\omega+\omega')j_\nu(-\vec{q},\omega')|\Phi> . \tag{30}$$

we can obtain a Dyson equation about $\Pi_{\mu\nu}(\vec{q},\omega)$ in the random phase approximation

$$\hat{\Pi}(\vec{q},\omega) = \hat{\Pi}^0(\vec{q},\omega) + \hat{\Pi}^0(\vec{q},\omega)\hat{v}(\vec{q})\hat{\Pi}(\vec{q},\omega) \tag{31}$$

Its solution is

$$\begin{aligned}\hat{\Pi}(\vec{q},\omega) &= [1-\hat{\Pi}^0(\vec{q},\omega)\hat{v}(\vec{q})]^{-1}\hat{\Pi}^0(\vec{q},\omega) \\ &= \frac{1}{\varepsilon(\vec{q},\omega)} \times \begin{pmatrix} \Pi^0_{00} & \Pi^0_{10} & \Pi^0_{00}v_{02}\Pi^0_{22} \\ \Pi^0_{10} & \Pi^s_{11} & \Pi^0_{10}v_{02}\Pi^0_{22} \\ \Pi^0_{22}v_{20}\Pi^0_{00} & \Pi^0_{22}v_{20}\Pi^0_{01} & \Pi^0_{22}-\Pi^0_{00}v_{02}\Pi^0_{22} \end{pmatrix}\end{aligned} \tag{32}$$

where

$$\varepsilon(\vec{q},\omega) = 1 - \Pi^0_{00}(\vec{q},\omega)v_{00}(\vec{q}) - \Pi^0_{22}(\vec{q},\omega)v_{20}(\vec{q})\Pi^0_{00}(\vec{q},\omega)v_{02}(\vec{q}) \tag{33}$$

and

$$\Pi^s_{11}(\vec{q},\omega) = \varepsilon(\vec{q},\omega)\Pi^0_{11}(\vec{q},\omega) + \Pi^0_{10}(\vec{q},\omega)v_{00}(\vec{q})\Pi^0_{01}(\vec{q},\omega) \tag{34}$$

$$+ \Pi^0_{10}(\vec{q},\omega)v_{02}(\vec{q})\Pi^0_{22}(\vec{q},\omega)v_{20}(\vec{q})\Pi^0_{01}(\vec{q},\omega) \tag{35}$$

Hitherto we have calculated the current-current correlation function $\hat{\Pi}(\vec{q},\omega)$ in random phase approximation, but it is not quite object we want in the linear response theory. The physical current density in anyon system is $J = j + a$, not only j. Fortunately, they are closely related.

§4. The electromagnetic response kernel $K_{\mu\nu}(\vec{q},\omega)$

In the linear response theory[15], consider a system in the presence of a weakly external applied electromagnetic field described by the vector and

scalar potential $A(\vec{r},t)$ and $\varphi(\vec{r},t)$, respectively. For convenience, we write

$$A_\mu(x) = \begin{cases} c\varphi(x), & \mu = 0 \\ A_i(x), & \mu = i = 1,2 \end{cases} \tag{36}$$

where $x = (\vec{r},t)$. In the anyon system, the paramagnetic current is defined as

$$j^p_\mu(x) = \begin{cases} j^p_i(x) j_i(x) + \frac{1}{mc} a_i(x) j_0(x); \\ j^p_0 = j_0(x) \end{cases} \tag{37}$$

and the diamagnetic current density j^d is given by

$$j^d_\mu(x) = \begin{cases} \frac{1}{mc} j_0(x) A_i(x); \\ 0 \end{cases} \tag{38}$$

Thus, the physical current density in the presence of A is the sum

$$j_\mu(x) = j^p_\mu(x) + j^d_\mu(x) \tag{39}$$

In the linear response theory, the expectation value of the current density J_μ can be expressed by

$$J_\mu(x) = -\frac{c}{4\pi} \sum_\mu \int K_{\mu\nu}(x,x') A_\nu(x') dx' \tag{40}$$

where the electromagnetic response kernel $K_{\mu\nu}$ is given by

$$\begin{aligned} K_{\mu\nu}(x,x') &= -\frac{i4\pi}{c^2} < \Phi | j^p_\mu(x) j^p_\nu(x') | \Phi > \theta(t-t') \\ &- \frac{4\pi}{mc^2} < \Phi | j_0(x) | \Phi > \delta^3(x-x') \delta_{\mu\nu}(1-\delta_{\nu 0}) \end{aligned} \tag{41}$$

In Fourier space,

$$K_{\mu\nu}(\vec{q},\tau) = \frac{4\pi}{c^2} R_{\mu\nu}(\vec{q},\tau) + \frac{4\pi}{mc^2} \rho_0 \delta_{\mu\nu}(1-\delta_{\nu 0}) \tag{42}$$

where

$$R_{\mu\nu}(\vec{q},\tau) = -i < \Phi | j^p_\mu(\vec{q},\tau) j^p_\nu(-\vec{q},0) | \Phi > \theta(\tau) \tag{43}$$

The paramagnetic kernel $R_{\mu\nu}$ can be given in term of the time-ordered quantity

$$P_{\mu\nu}(\vec{q},\omega) = -i \int_{-\infty}^{+\infty} < \Phi | T\{ j^p_\mu(\vec{q},\tau) j^p_\nu(-\vec{q},0) \} | \Phi > e^{i\omega\tau} d\tau \tag{44}$$

where T is the time-order operator. Their relations are

$$ReR_{\mu\nu}(\vec{q},\omega) = ReP_{\mu\nu}(\vec{q},\omega); \tag{45}$$

$$ImR_{\mu\nu}(\vec{q},\omega) = sgn(\omega) ImP_{\mu\nu}(\vec{q},\omega). \tag{46}$$

Now, we come to calculate $P_{ij}(\vec{q},\omega)$. According to the definition, it can be written as

$$P_{ij}(1,2) = -i < \Phi | T\{ [j_i(1) + j_0(1) a_i(1)][j_j(2) + j_0(2) a_j(2)] \} | \Phi > \tag{47}$$

Approximately, using ρ_0 instead of j_0, one obtains

$$\begin{aligned} P_{ij}(\vec{q},\omega) &= \Pi_{ij}(\vec{q},\omega) + \frac{\rho}{m} v_{i0}(\vec{q}) \Pi_{0j}(\vec{q},\omega) \\ &+ \frac{\rho}{m} \Pi_{i0}(\vec{q},\omega) v_{0j}(\vec{q}) + \frac{\rho^2}{m^2} v_{i0}(\vec{q}) \Pi_{00}(\vec{q},\omega) v_{0j}(\vec{q}) \end{aligned} \tag{48}$$

Thus, we have

$$P_{11}(\vec{q},\omega) = \Pi^0_{11}(\vec{q},\omega) + \frac{\Pi^0_{10}(\vec{q},\omega)\Pi^0_{01}(\vec{q},\omega)}{\Pi^0_{00}(\vec{q},\omega)} \cdot \frac{\varepsilon(\vec{q},\omega) - 1}{\varepsilon(\vec{q},\omega)}; \tag{49}$$

$$P_{12}(\vec{q},\omega) = \frac{\rho}{m} \frac{\Pi^0_{10}(\vec{q},\omega) v_{02}(\vec{q})}{\varepsilon(\vec{q},\omega)}; \tag{50}$$

$$P_{21}(\vec{q},\omega) = \frac{\rho}{m} \frac{v_{20}(\vec{q}) \Pi^0_{01}(\vec{q},\omega)}{\varepsilon(\vec{q},\omega)}; \tag{51}$$

$$P_{22}(\vec{q},\omega) = -\frac{\rho}{m} + \frac{\Pi^0_{22}(\vec{q},\omega) + \frac{\rho}{m}}{\varepsilon(\vec{q},\omega)} \tag{52}$$

§5. Results of the random phase approximation calculation

In anyon system, one of the problem we are interested in is whether Meissner effect does arise, which can judge the ground state of anyon gas is superconducting. In the mean field theory, Fetter *et al* and latter Chen *et al*[7] have discussed this problem and found some evidences to sipport the existence of this effect. Their physical pictures is that the gauge field is considered as a uniform magnetic field, approximately. Every particle moves in a uniform magnetic field and interacts with a residual potential. In the case of $\nu = 1 - \frac{1}{n}$, the n lowest Landau levels are filled fully. Their unperturbative base is Landau level, which is every different from the plane wave as we used.

As Schafroth[16] has shown, the Meissner effect requires that the transverse part of the kernel $K_{\mu\nu}(\vec{q},\omega)$ remains finite in the long wavelength limit $q \to 0$ for frequency $\omega = 0$. In this limit, we find

$$P_{11}(\vec{q},\omega=0) = -\frac{\rho}{m}; \tag{53}$$

$$P_{12}(\vec{q},\omega=0) = P_{21}(\vec{q},\omega=0) = 0; \tag{54}$$

$$P_{22}(\vec{q},\omega=0) = -\frac{\rho}{m} + \frac{1}{6}\cdot\frac{\rho}{m}\frac{q^2}{1+\frac{(1-\nu)^2}{12}}. \tag{55}$$

Putting them into K_{ij}, we obtain

$$K_{ij}(q,\omega=0) = (\delta_{ij} - \frac{q_i q_j}{q^2})K(q) \tag{56}$$

where, unfortunately, $K(q)$ has the form in the long wavelength limit

$$K(q,\omega=0) = \frac{\rho}{m}\frac{q^2}{6+\frac{(1-\nu)^2}{2}} \to 0, \quad \text{as } q^2 \to 0 \tag{57}$$

The factor $(\delta_{ij} - \frac{q_i q_j}{q^2})$ in K_{ij} guarantees that K_{ij} is a purely transverse in this case. Therefore, we find no Meissner effect in anyon gas in our approximation. Another interested quantity is the collective mode, which is related with the pole in the response function. From appendix we derive in the limit $q \to 0$,

$$\Pi^0_{00}(\vec{q},\omega) \approx \frac{m}{\pi}\cdot\frac{q^2}{\omega} + O(\frac{q^2}{\omega^2}); \tag{58}$$

$$\Pi^0_{22}(\vec{q},\omega) \approx \frac{1}{4m\pi}\cdot\frac{q^2}{\omega} + O(\frac{q^2}{\omega^2}). \tag{59}$$

Then,

$$\varepsilon(q,\omega) = 1 - \frac{(1-\nu)^2}{\omega^2} - \frac{1}{\omega^2}O(\frac{q^2}{\omega^2}) \tag{60}$$

There is evidently a pole at the point,

$$\omega^2 \approx (1-\nu)^2 + O(\frac{q^2}{\omega^2}), \quad \text{as } q^2 \to 0. \tag{61}$$

which means that the collective excitation spectrum has an energy gap in the limit $q^2 \to 0$. This result coincides with two-dimensional electron gas with a logarithmic interaction.

§6. Conclusion and discussion

In this paper, our approach is very different from that of mean field theory as we have chosen the plain wave instead of Landau level as the

perturbed expansion basis. We have not obtained some results of the mean field theory. No meissner effect and Goldstone mode were found in such a perturbative approach, and the anyon gas does not manifest the superfluidity in the ground state.

In the mean field theory, some conclusion s are obviously unphysical, one of which in the nonzero Hall conductivity[7]. It is impossible that there is a nonzero Hall conductivity because no magnetic field in an ideal anyon gas. One of important evidences to support anyon superconductivity in the meanfield theory is the exsitence of full diamagnetism. Unfortunately, in the same approximation, no any phase transition was found at any finite temperature. Therefore, in the sense of the mean field theory, such kind of superconductivity (even if possible) might be only regarded as an anormaly, just as the two-dimensional system of ideal bosons, in which there is no Bose-Einstenin condensatoin at any finite temperature[17]. It is impossible that this mechanism should occurs in the high-T_c materials. Another of important evidences is the existenc of a linear Goldstone mode. According to the Landau theory of superfluidity, it should be no doubted correct. But some results of interacted anyon system indicates that there is no relation between full diamagnetism and linear Goldstone mode. In the anyon system with Coulomb interaction ($\frac{1}{r}$), Yang *et al*[18] found that the collective mode is proportional to $\sqrt{q}$ in the long wavelength limit, and the ground state is still of full diamagnetism. Thus in the mean field theory of ideal anyon gas, the coexistence of full diamagnetism and Goldstone mode

belongs to coincidence. Recently, we found that the collective excitation in a short wavelength limit approaches to a constant quantity, and unexpectly is restrictly stable in the mean field theory. The minimum of $\frac{\omega}{q}$ is zero[19]. Thus according to the Landau's theory of superfluidity, any small flow of such fluid, suppose its velocity v, will lead to some excitation with $\omega < vq$, which will cost some energy to make the flow slower[20]

Since we just make a random phase approximation in our approach, we can not draw a final conclusion that the ground state of anyon gas is not superconducting. The validity of the mean field is still worth studying further. Now we are devoting ourselves to do a more complicate calculation in a high-order approximation in order to understand this problem.

Appendix: The Detailed Calculation of the Current-Current Correlation Function

Here we present the detailed calculation of the unperturbed correlation function $\Pi^0_{\mu\nu}(\vec{q},\omega)$. From the definition of $\Pi^0_{\mu\nu}(\vec{q},\omega)$ we can write it

$$\begin{aligned}\Pi^0_{\mu\nu}(\vec{q},\omega) &= \int \frac{d\vec{k}}{(2\pi)^2}(\vec{k}+\frac{\vec{q}}{2})_\mu(\vec{k}+\frac{\vec{q}}{2})_\nu \\ &\times \left\{\frac{\theta(|\vec{k}+\vec{q}|-k_F)\theta(k_F-k)}{\omega+\varepsilon(k)-\varepsilon(\vec{k}+\vec{q})+i0^+} - \frac{\theta(k_F-|\vec{k}+\vec{q}|)\theta(k-k_F)}{\omega+\varepsilon(k)-\varepsilon(\vec{k}+\vec{q})-i0^+}\right\}\end{aligned} \tag{62}$$

where

$$k_\mu = \begin{cases} 1, & \mu = 0 \\ \frac{k_i}{m}, & \mu = i = 1,2 \end{cases} \tag{63}$$

Using the formula,

$$\frac{1}{x \pm i0^+} = P\frac{1}{x} \mp i\pi\delta(x) \tag{64}$$

Then, the real and imaginary part of $\Pi^0_{\mu\nu}(\vec{q},\omega)$ can be written as

$$\begin{aligned} Re\Pi^0_{\mu\nu}(\vec{q},\omega) &= P\int \frac{d\vec{k}}{(2\pi)^2}\left\{(\vec{k}+\frac{\vec{q}}{2})_\mu(\vec{k}+\frac{\vec{q}}{2})_\nu \frac{\theta(k_F-k)}{\omega+\varepsilon(k)-\varepsilon(\vec{k}+\vec{q})}\right. \\ &+ \left.(\vec{k}-\frac{\vec{q}}{2})_\mu(\vec{k}-\frac{\vec{q}}{2})_\nu \frac{\theta(k_F-|\vec{k}-\vec{q}|)}{\omega+\varepsilon(\vec{k}-\vec{q})-\varepsilon(k)}\right\} \end{aligned} \tag{65}$$

and

$$\begin{aligned} Im\Pi^0_{\mu\nu}(\vec{q},\omega) &= -\pi\int \frac{d\vec{k}}{(2\pi)^2}k_\mu k_\nu \delta(\omega+\varepsilon(\vec{k}-\frac{\vec{q}}{2})-\varepsilon(\vec{k}+\frac{\vec{q}}{2})) \\ &\times\left\{\theta(|\vec{k}+\frac{\vec{q}}{2}|-k_F)\theta(k_F-|\vec{k}-\frac{\vec{q}}{2}|)+\theta(k_F-|\vec{k}-\frac{\vec{q}}{2}|)\theta(|\vec{k}-\frac{\vec{q}}{2}|-k_F)\right\} \end{aligned} \tag{66}$$

Take Fermi momentum k_F and Fermi energy $\varepsilon_F = \frac{k_F^2}{2m}$ as units:

$$k_i \to \frac{k_i}{k_F}; \omega \to \frac{\omega}{\varepsilon_F}$$

We now precede to evaluate each component of $\Pi^0_{\mu\nu}(\vec{q},\omega)$ in turn. First

$$\begin{aligned} lcc\ \ Re\ \ \Pi^0_{00}(\vec{q},\omega) &= 2m\int \frac{d\vec{k}}{(2\pi)^2}\left(\frac{\theta(1-k)}{\omega-2kq\cos\varphi-q^2}-\frac{\theta(1-k)}{\omega-2kq\cos\varphi+q^2}\right) \\ &= m\int_0^1 \frac{kdk}{2\pi q}\int_0^{1\pi}\frac{d\varphi}{2\pi}\left(\frac{1}{k\cos\varphi-a_+}-\frac{1}{k\cos\varphi+a_-}\right) \\ &= -\frac{m}{2\pi q}\int_0^1 kdk\left\{\frac{1}{\sqrt{a_+^2-k^2}}\frac{a_+}{|a_+|}\theta(a_+^2-k^2)+\frac{1}{\sqrt{a_-^2-k^2}}\frac{a_-}{|a_-|}\theta(a_-^2-k^2)\right\} \end{aligned}$$

$$\begin{aligned} &= \frac{m}{2\pi q}\left\{\sqrt{a_+^2-1}\frac{a_+}{|a_+|}\theta(a_+^2-1)-a_+ + \sqrt{a_-^2-1}\frac{a_-}{|a_-|}\theta(a_-^2-1)-a_-\right\} \\ &= -\frac{m}{2\pi}+\frac{m}{2\pi q}\left\{\sqrt{a_+^2-1}\frac{a_+}{|a_+|}\theta(a_+^2-1)+\sqrt{a_-^2-1}\frac{a_-}{|a_-|}\theta(a_-^2-1)\right\} \end{aligned} \tag{67}$$

where

$$a_\pm = \frac{q^2 \pm \omega}{2q} \tag{68}$$

The imaginary part of $\Pi^0_{00}(\vec{q},\omega)$ is

$$\begin{aligned} Im\ \ \Pi^0_{00}(\vec{q},\omega) &= -2\pi m\int\frac{d\vec{k}}{2\pi}\delta(\omega-kq\cos\varphi) \\ \times &\left\{\theta(|\vec{k}+\frac{\vec{q}}{2}|-1)\theta(1-|\vec{k}-\frac{\vec{q}}{2}|)+\theta(|\vec{k}-\frac{\vec{q}}{2}|-1)\theta(1-|\vec{k}+\frac{\vec{q}}{2}|)\right\} \\ &= -\frac{m}{2\pi}\int kdk\frac{2}{\sqrt{4k^2q^2-\omega^2}}\theta(4k^2q^2-\omega^2) \\ \times &\left\{\theta(k^2+\frac{\omega}{2}+\frac{q^2}{4}-1)\theta(1-k^2+\frac{\omega}{2}-\frac{q^2}{4})+\theta(k^2-\frac{\omega}{2}+\frac{q^2}{4}-1)\theta(1-k^2-\frac{\omega}{2}-\frac{q^2}{4})\right. \end{aligned} \tag{6}$$

As $\Pi^0_{00}(\vec{q},-\omega) = \Pi^0_{00}(\vec{q},\omega)$, we just take ω as positive

$$Im\Pi^0_{00}(\vec{q},-\omega) = \frac{1}{2\pi q}\{\sqrt{1-a_+^2}\theta(1-a_+^2)-\sqrt{1-a_-^2}\theta(1-a_-^2)\} \tag{70}$$

Now we pass to other component of $\Pi^0_{\mu\nu}$. As $k_{\mu=0}=1$ and $k_{\mu=1}=\frac{k_1}{m}$, then

$$\begin{aligned} &Re\ \ \Pi^0_{10}(\vec{q},\omega) \\ &= 2\int\frac{d\vec{k}}{(2\pi)^2}\left\{\frac{k\cos\varphi+\frac{q}{2}}{\omega-2kq\cos\varphi-q^2}-\frac{k\cos\varphi-\frac{q}{2}}{\omega-2kq\cos\varphi+q^2}\right\}\theta(1-k) \\ &= \frac{\omega}{2q^2}\int\frac{d\vec{k}}{(2\pi)^2}(\frac{1}{k\cos\varphi-a_+}-\frac{1}{k\cos\varphi+a_-})\theta(1-k) \\ &= \frac{\omega}{2mq}Re\Pi^0_{00}(\vec{q},\omega) \end{aligned} \tag{71}$$

and

$$
\begin{aligned}
Im\ \ \Pi^0_{10}(\vec{q},\omega) &= -2\pi\int\frac{d\vec{k}}{(2\pi)^2}k\cos\varphi\delta(\omega-2kq\cos\varphi)\\
\times\ &\left\{\theta(|\vec{k}+\frac{\vec{q}}{2}|-1)\theta(1-|\vec{k}-\frac{\vec{q}}{2}|)+\theta(|\vec{k}-\frac{\vec{q}}{2}|-1)\theta(1-|\vec{k}+\frac{\vec{q}}{2}|)\right\}\\
&= \frac{\omega}{2mq}Im\Pi^0_{00}(\vec{q},\omega)
\end{aligned}
\tag{72}
$$

Similarly,

$$
Re\Pi^0_{11}(\vec{q},\omega) = -\frac{1}{4\pi m}+\frac{\omega^2}{4q^2m^2}Re\Pi^0_{00}(\vec{q},\omega); \tag{73}
$$

$$
Im\Pi^0_{11}(\vec{q},\omega) = -\frac{\omega}{4q^2m^2}Im\Pi^0_{00}(\vec{q},\omega). \tag{74}
$$

As for Π^0_{22},

$$
\begin{aligned}
&Re\ \ \Pi^0_{22}(\vec{q},\omega)\\
&= \frac{2}{m}\int\frac{d\vec{k}}{(2\pi)^2}k^2\sin^2\varphi(\frac{1}{\omega-2k\cos\varphi-q^2}-\frac{1}{\omega-2kq\cos\varphi+q^2})\theta(1-k)\\
&= \frac{2}{m}\int\frac{d\vec{k}}{(2\pi)^2}\frac{1}{2q}(-a_+-a_-+\frac{a_-^2-k^2}{k\cos\varphi+a_-}-\frac{a_+^2-k^2}{k\cos\varphi-a_+})\theta(1-k)\\
&= -\frac{1}{4\pi m}-\frac{1}{6\pi mq}\left\{(a_+^2-1)^{\frac{3}{2}}\frac{a_+}{|a_+|}\theta(a_+^2-1)-a_+^3\right.\\
&+\ \left.(a_-^2-1)^{\frac{3}{2}}\frac{a_-}{|a_-|}\theta(a_-^2-1)-a_-^3\right\}
\end{aligned}
\tag{75}
$$

and

$$
Im\Pi^0_{22}(\vec{q},\omega) = \frac{1}{6\pi mq}\left\{(1-a_+^2)^{\frac{3}{2}}\theta(1-a_+^2)-(1-a_-^2)^{\frac{3}{2}}\theta(1-a_-^2)\right\} \tag{76}
$$

Summarily, the results of calculation are listed as following

$$
\begin{cases}
Re\Pi^0_{00}(\vec{q},\omega) = \\
-\frac{m}{2\pi}+\frac{m}{2\pi q}\{\sqrt{a_+^2-1}\frac{a_+}{|a_+|}\theta(a_+^2-1)+\sqrt{a_-^2-1}\frac{a_-}{|a_-|}\theta(a_-^2-1)\}\\
Im\Pi^0_{00}(\vec{q},\omega) = \\
\frac{m}{2\pi q}\{\sqrt{a_+^2-1}\theta(a_+^2-1)+\sqrt{a_-^2-1}\theta(a_-^2-1)\}
\end{cases}
\tag{77}
$$

$$
\begin{cases}
Re\Pi^0_{10}(\vec{q},\omega) = \\
-\frac{\omega}{4\pi q}+\frac{\omega}{4\pi q^2}\{\sqrt{a_+^2-1}\frac{a_+}{|a_+|}\theta(a_+^2-1)+\sqrt{a_-^2-1}\frac{a_-}{|a_-|}\theta(a_-^2-1)\}\\
Im\Pi^0_{10}(\vec{q},\omega) = \\
\frac{\omega}{4\pi q^2}\{\sqrt{a_+^2-1}\theta(a_+^2-1)+\sqrt{a_-^2-1}\theta(a_-^2-1)\}
\end{cases}
\tag{78}
$$

$$
\begin{cases}
Re\Pi^0_{11}(\vec{q},\omega) = \\
-\frac{1}{4\pi m}+\frac{\omega^2}{8\pi q^3m}\{\sqrt{a_+^2-1}\frac{a_+}{|a_+|}\theta(a_+^2-1)+\sqrt{a_-^2-1}\frac{a_-}{|a_-|}\theta(a_-^2-1)-a_+-a_-\}\\
Im\Pi^0_{11}(\vec{q},\omega) = \\
\frac{\omega^2}{8\pi q^3m}\{\sqrt{a_+^2-1}\theta(a_+^2-1)+\sqrt{a_-^2-1}\theta(a_-^2-1)\}
\end{cases}
\tag{79}
$$

$$
\begin{cases}
Re\Pi^0_{22}(\vec{q},\omega) = \\
-\frac{1}{4\pi m}+\frac{1}{6\pi qm}\{a_+^3+a_-^3-(a_+^2-1)^{frac32}\frac{a_+}{|a_+|}\theta(a_+^2-1)-(a_-^2-1)^{\frac{3}{2}}\frac{a_-}{|a_-|}\theta(a_-^2-1)\}\\
Im\Pi^0_{11}(\vec{q},\omega) = \\
\frac{1}{6\pi qm}\{(a_+^2-1)^{\frac{3}{2}}\theta(a_+^2-1)+(a_-^2-1)^{\frac{3}{2}}\theta(a_-^2-1)\}
\end{cases}
\tag{8}
$$

There exist some relations,

$$\begin{cases} Re\Pi^0_{10}(\vec{q},\omega) = \dfrac{\omega}{2qm} Re\Pi^0_{00}(\vec{q},\omega), & Im\Pi^0_{10}(\vec{q},\omega) = \dfrac{\omega}{2qm} Im\Pi^0_{00}(\vec{q},\omega) \\ Re\Pi^0_{11}(\vec{q},\omega) = -\dfrac{1}{4\pi m} + \dfrac{\omega^2}{4q^2m^2} Re\Pi^0_{00}(\vec{q},\omega), & Im\Pi^0_{11}(\vec{q},\omega) = \dfrac{\omega^2}{4q^2m^2} Im\Pi^0_{00}(\vec{q},\omega) \end{cases} \tag{81}$$

where $a_\pm = \dfrac{q^2 \pm \omega}{2q}$.

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