Fermilab Library
0 1160 0019173 8

CCAST-92-31
August, 1992

THE LOWEST COLLECTIVE EXCITATION IN AN IDEAL ANYONS GAS ($\nu = \frac{1}{2}$)

by
SHUN-QING SHEN

CO-AUTHOR: Xiao-Hui Wu (Fudan, Shanghai)

Chinese Centre of Advanced Science and Technology (World Laboratory),
P. O. Box 8730, Beijing 100 080, People's Republic of China

THE LOWEST COLLECTIVE EXCITATION IN AN IDEAL ANYONS GAS ($\nu = \frac{1}{2}$)

Shun-Qing Shen
Chinese Center of Advanced Science and Technology (World Laboratory)
P. O. Box 8730, Beijing 100 080, People's Republic of China

Xiao-Hui Wu
Department of Physics, Fudan University
Shanghai 200 433, People's Republic of China

ABSTRACT

It is presented the lowest collective excitation spectrum for an ideal anyon gas ($\nu = \frac{1}{2}$): the minimum of $\frac{\omega(q)}{q}$ approaches to zero, which, according to Landau's theory, indicates that ideal anyon gas ($\nu = \frac{1}{2}$) may not be superfluid in the mean field theory.

PACS number: 71.45-d, 74.65+n, 05.30-d

It is still an interesting theoretical problem whether the ground state of an ideal anyon gas ($\nu = \frac{1}{2}$) is superfluid, though some experimental data have shown that the high-T_c superconducting materials do not break the symmetry of time-reversal T and space-reflection P.[1] In his pioneer paper[2], Laughlin proposed a conjecture that for an anyon system of $\nu = \frac{1}{2}$, its ground state might be superfluid. His proposal was strengthened in a remarkable calculation by Fetter, Hanna and Laughlin[3], who established a mean field theory for anyons and found two evidences to support anyon superconductivity: one is a linear excitation spectrum in long wavelength limit, and another is the Meissner kernel $K_{RPA}(q) \to 1$ as $q \to 0$. These results were also recovered by Chen *et al.*[4] In another different way, Lee and Fisher[5] presented a bosonic formulation of anyon superconductivity. However, these calculations are all restricted in the limit of long wavelenth in the mean field approximation, and the behavior in the short-wavelength region have not been described until now. Usually, discussion on the behavior in the short wave-length region is physically meaningless. Generally speaking, excitations in many-body system arise in a low energy region, which corresponds to a long wave length region. In this paper, extended to a short wavelength region, an *unexpected* result is obtained in the calculation of the lowest collective excitation spectrum: $\omega(q) \to \omega_c$ as $q \gg 1$ and *this spectrum is also stable in the mean field theory.* The minimun of $\frac{\omega(q)}{q}$, obvoiusly, approaches to zero, which does not support anyon superfluid, according to Landau's theory of superfluidity[6].

In the mean field theory[3], the current-current correlation function of an ideal anyon gas ($\nu = \frac{1}{2}$) is

$$D_{RPA} = \frac{1}{\pi D}\begin{pmatrix} q^2\Sigma_0 & q\omega\Sigma_0 & -iq\Sigma_s \\ q\omega\Sigma_0 & \omega^2\Sigma_o - D & -i\omega\Sigma_s \\ iq\Sigma_s & i\omega\Sigma_s & \Sigma_2 + \Sigma_1^2 - \Sigma_0\Sigma_2 \end{pmatrix} \tag{1}$$

where

$$\begin{aligned}\Sigma_i(q,\omega) &= \frac{1}{2}\sum_{n=1}^{\infty}\frac{e^{-x}x^{n=1}}{(n-1)!(\omega^2-x^2)} \\ &\quad \Big((1-\delta_{n1})(n-x)^i + (n+1)^{-1}(n+1-x)^{2-i}[n(n+1)-(2n+3x+x^2)^i]\Big)\end{aligned} \tag{2}$$

$$x = \frac{q^2}{2} \tag{3}$$

$$\Sigma_s = D - 1 - \Sigma_1 + \Sigma_0 \tag{4}$$

and

$$D(q,\omega) = 1 + 2\Sigma_1 - \Sigma_0 + \Sigma_1^2 - \Sigma_0\Sigma_2 \tag{5}$$

The collective excitation spectrum is determined by the pole of the density-density correlation function $D^{00}_{RPA}(q,\omega)$, *i.e.*

$$ReD(q,\omega) = 0 \tag{6}$$

The result in the long wavelength limit is well known. Here we just discuss it in a short wavelength limit: $q \gg 1$. Usually, the collective spectrum with the short wavelength is unstable, such as in a normal metal[7]. Unexpectedly, such excitation in an ideal anyon gas ($\nu = \frac{1}{2}$) in the mean field theory

approaches to a constant and their corresponding dynamic structure factor has a strictly delta form, which means the spectrum is stable.

Now we discuss the behavior near the $\omega = 1$. In the unperturbed case, the lowest energy of the particle-hole excitation is just the interval of Landau level, i.e. $\omega = 1$. When $\omega \to 1$, $\frac{1}{\omega^2-1} \to \infty$. In this case, the only term ($n = 1$) remained approaximately in Eq.(2) is,

$$\Sigma_i(q, \omega \to 1) \simeq \frac{1}{4}\frac{e^{-\frac{q^2}{2}}}{\omega^2 - 1}(2 - \frac{q^2}{2})^{2-i}(2 - \frac{5}{2}q^2 + \frac{q^4}{4})^i \tag{7}$$

Substituting Eq.(7) into Eq.(6), one obtains a dispersion relation in a short wavelength limit,

$$\omega^2 \simeq 1 - \frac{q^6}{8}e^{-\frac{q^2}{2}}, \quad \text{as } q \gg 1 \tag{8}$$

At three points, $q = 0.68, 2$, and 2.73, the collective spectrum crosses with that of the particle-hole excitation. A numerical calculation is shown in Fig.1. Obviously, the minimum of $\frac{\omega(q)}{q} \to 0$ approaches to zero,

$$\frac{\omega}{q} \to 0, \text{as } q \gg 1 \tag{9}$$

Now whether the excitation is stable is a key problem expected to solve. The imaginary part of D^{00}_{RPA} for real positive ω is essentially the dynamic structure factor $S_{RPA}(q, \omega)$, that can be measured, for example, with inelastic neutron scattering. After a tedious calculation, one obtains

$$S_{RPA}(q, \omega) \simeq \begin{cases} (\frac{q}{2\sqrt{2}} + O(q^2))\delta(\omega - \sqrt{2}q - O(q^2)), (as\ \text{q}\to 0) \\ \frac{1}{32}q^6 e^{-\frac{q^2}{2}}\delta(\omega - 1 - \frac{q^6}{16}e^{-\frac{q^2}{2}} + O), (as\ \text{q}\to \infty) \end{cases} \tag{10}$$

The sharp delta function shows that this system has a restricted class of collective mode, which is opposite to the case of the ideal electron gas. Usually, the collective excitation only in a long wavelength region, where its momentum q is less than a certain value q_c, and out of the region ($q \geq q_c$). $Im\epsilon \neq 0$ and the spectrum is unstable and decays rapidly into particle-hole excitations.

The minimum of $\frac{\omega(q)}{q}$ is not favor of anyon superfluidity. In Landau's theory of superfluidity[6], suppose the velocity of a flowing superfluid liquid $\vec{v}$, then when a single excitation appears in the liquid, with a momentum $\vec{q}$, and energy $\omega(q)$, the change of the energy in liquid is

$$E = \omega(q) + \vec{q} \cdot \vec{v} + \frac{1}{2}Mv^2 \tag{11}$$

$\frac{1}{2}Mv^2$ is the original dynamic energy of the flowing liquid, and $\omega + \vec{q} \cdot \vec{v}$ is the change of energy due to the excitation appears. The change must be negative as the decrease of the energy: $\omega + \vec{q} \cdot \vec{v} < 0$. For a given $\vec{q}$, when $\vec{q}$ and $\vec{v}$ are antiparallel, the quantity on the left side of the inequality is minimum, $\omega - qv < 0$ or

$$v > \frac{\omega}{q} \tag{12}$$

If the minimum of $\frac{\omega}{q}$ is nonzero, then, for a velocity of the flow below a certain value, excitation cannot appear in the liquid. This means that the flow will not become slower, i.e. that the liquid exhibits the phenomenon of superfluidity, otherwise the flow will become slower, i.e. the liquid does not superfluid. Unfortunately, from the spectrum of an anyon gas, it is just

in the latter case.

In the mean field theory of anyon gas, we find that the lowest collective excitation is linear in a long wavelength limit and approaches to a plateau, which is almost independent of q, in a short wavelength limit. The short-wavelength part of excitation almost, but does not coincide with the lowest energy for particle-hole excitation. As this spectrum is stable and does not decay (at least in the mean field theory), the result of Eq.(9) indicates that any small flow of fluid, suppose its velocity v, will lead to some excitations with $\omega \leq vq$, which costs some energy to make the flow slower. The existence of a short wave length excitation will not be favor of superfluidity for an ideal anyon gas. Therefore, though the spectrum of the small excitation is linear, and combined with a finite compressibility in a long wavelength limit, we cannot draw a conclusion that the ground state of anyon gas is superfluid in the calculation of the mean field theory.

Acknowledgments One of authors (SQS) wish to thank Prof. R. B. Laughlin for useful discussion.

Referrences

[1] R. F. Kiefl *et al.*, Phys. Rev. Lett. **64**, 2082(1990); S. Spielman *et al.*, Phys. Rev. Lett. **65**, 123(1990);

[2] R. B. Laughlin, Science **242**, 525(1988); Phys. Rev. Lett. **60**, 1057(1988);

[3] A. L. Fetter, C. B. Hanna and R. B. Laughlin, Phys. Rev. **B39**, 9679(1989);

[4] Y. Chen, F. Wilczek, E. Witten and B. Halperin, Int. J. Mod. Phys. **B3**, 1001(1989);

[5] D. H. Lee and M. P. A. Fisher, Phys. Rev. Lett. **63**, 1903(1989);

[6] L. D. Landau and E. M. Lifshitz, Statistical Physics (Part 2), (Pergamon, 1980), P.88;

[7] G. Mahan, Many-Body Physics, (Plenum Press, New York, 1981).

Figure Caption
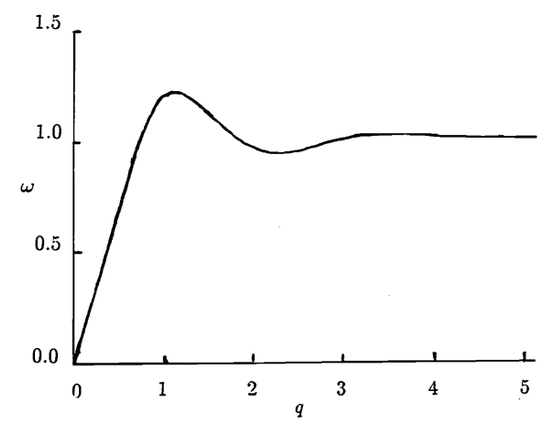
Figure 1: The lowest collective excitation spectrum for an ideal anyon gas ($\nu = \frac{1}{2}$).